HW5   U 19 10 007
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a)

		0	1	2
y	0	$\frac{25}{102}$	$\frac{13}{51}$	$\frac{1}{17}$
	1	$\frac{13}{51}$	$\frac{13}{102}$	0
	2	$\frac{1}{17}$	0	0

b) $P(x=0) = \frac{25}{102} + \frac{13}{51} + \frac{1}{17} = \frac{19}{34}$

$P(x=1) = \frac{13}{51} + \frac{13}{102} = \frac{13}{34}$

$P(x=2) = \frac{2}{34}$

$P(y=0) = \frac{25}{102} + \frac{13}{51} + \frac{1}{17} = \frac{19}{34}$

$P(y=1) = \frac{13}{51} + \frac{13}{102} = \frac{13}{34}$

$P(y=2) = \frac{1}{17} + 0 = \frac{2}{34}$

c) $P(x=0, y=0) \neq P(x=0) * P(y=0)$

$\frac{25}{102} \neq \frac{19}{34} * \frac{19}{34}$ Not Independent.

d) $E[x] = \frac{0 \cdot 19}{34} + 1 \cdot \frac{13}{34} + 2 \cdot \frac{2}{34} = \frac{1}{2}$

$E[y] = \frac{0 \cdot 19}{34} + 1 \cdot \frac{13}{34} + 2 \cdot \frac{2}{34} = \frac{1}{2}$

$Var[x] = Var[y] = 1^2 \frac{13}{34} + 2^2 \cdot \frac{2}{34} - \left(\frac{1}{2}\right)^2 = \frac{25}{68}$

e) $cov[x,y] = E[xy] - E[x] \cdot E[y]$

$cov[x,y] = \frac{13}{102} - \left(\frac{1}{2}\right)^2 = -\frac{25}{204}$

f) $cor\ r(x,y) = \frac{cov[x,y]}{\sqrt{var[x] \cdot var[y]}}$

$cor\ r(x,y) = -\frac{22}{204} : \left(\sqrt{\frac{25}{68} * \frac{25}{68}}\right) = -\frac{1}{3}$

g) I) $P(y=0 \mid x=0) = \dfrac{P(y=0, x=0)}{P(x=0)} = \dfrac{25}{57}$

$P(y=1 \mid x=0) = \dfrac{P(y=1, x=0)}{P(x=0)} = \dfrac{1}{19}$

II) $P(y=0 \mid x=1) = \dfrac{P(y=0, x=1)}{P(x=1)} = \dfrac{2}{3}$

$P(y=1 \mid x=1) = \dfrac{P(y=1, x=1)}{P(x=1)} = \dfrac{1}{3}$

$P(y=2 \mid x=1) = \dfrac{P(y=2, x=1)}{P(x=1)} = 0$

№2   a) $\int_0^L A\,x(L-x)\,dx = 1 \Rightarrow A = \dfrac{6}{L^3}$

b) $y = |2x - L|$

$F_y(y) = P(y \leq y) = \dfrac{3L^2 y - y^3}{2L^3}$

$f_y(y) = \begin{cases} \dfrac{3}{2L} - \dfrac{3}{2L^3 \cdot y^2} & , \ 0 \leq y \leq L \\ 0 & , \ elsewhere \end{cases}$

c) $E[y] = \int_0^L y\left[\dfrac{3}{2L} - \dfrac{3}{2L^2}y^2\right]dy = \dfrac{3}{8}L$

Q3 $f(x,y) = 4x(2-y)$, $0 \le x \le 1$ and $1 \le y \le 2$

a) $f_x(x) = \int_1^2 4x(2-y)\,dy = 2x$, $0 \le x \le 1$

b) $f_y(y) = \int_0^1 4x(2-y)\,dx = 2(2-y)$, $1 \le y \le 2$

c) $cov(x,y) = 0$

d) $f_{x|x=y}(x) = \dfrac{f(x,y)}{f_y(y)} = \dfrac{f_x(x)\,f_y(y)}{f_y(y)} = f_x(x)$

$y = 1,5 = P_{x|y=1,5}(x) = f_x(x)$