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Journal of Banking & Finance 30 (2006) 669–678

Journal of BANKING & FINANCE

www.elsevier.com/locate/jbf

Portfolio selection using hierarchical Bayesian analysis and MCMC methods

Alex Greyserman [a,*], Douglas H. Jones [b], William E. Strawderman [c]

[a] Hite Capital Management LLC, 257 Park Ave South, 15th Floor, New York, NY 10010, USA
[b] Rutgers Business School, Rutgers, The State University of New Jersey, 111 Washington Avenue, Newark, NJ 07102, USA
[c] Department of Statistics, Rutgers, The State University of New Jersey, New Brunswick, NJ 08854, USA

Available online 8 August 2005

Abstract

This paper contributes to portfolio selection methodology using a Bayesian forecast of the distribution of returns by stochastic approximation. New hierarchical priors on the mean vector and covariance matrix of returns are derived and implemented. Comparison's between this approach and other Bayesian methods are studied with simulations on 25 years of historical data on global stock indices. It is demonstrated that a fully hierarchical Bayes procedure produces promising results warranting more study. We carried out a numerical optimization procedure to maximize expected utility using the MCMC (Monte Carlo Markov Chain) samples from the posterior predictive distribution. This model resulted in an extra 1.5 percentage points per year in additional portfolio performance (on top of the *Hierarchical Bayes* model to estimate $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ and use the Markowitz model), which is quite a significant empirical result. This approach applies to a large class of utility functions and models for market returns.

JEL classification: C11; C15

Keywords: Monte Carlo Markov Chain; Gibbs sampling; Hierarchical Bayes; Portfolio theory

* Corresponding author.
E-mail addresses: alex@mint.com (A. Greyserman), dhjones@rci.rutgers.edu (D.H. Jones), straw@stat.rutgers.edu (W.E. Strawderman).

0378-4266/$ - see front matter
doi:10.1016/j.jbankfin.2005.04.008

1. Introduction

Portfolio theory is concerned with the allocation of an individual's wealth among various available assets. The basic Markowitz version of the portfolio selection problem is (Markowitz, 1952):

$$\begin{aligned} &\text{Maximize} \quad \mathbf{w}'\boldsymbol{\mu} - \frac{\lambda}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}, \\ &\text{subject to} \quad \mathbf{w}'\mathbf{e} = 1, \end{aligned} \tag{1}$$

where $\mathbf{w} = (w_1, w_2, \ldots, w_m)' \in R^m$ is a column vector of proportions representing a portfolio of assets, $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ are the covariance matrix and mean column vector of asset returns $\mathbf{y} = (y_1, y_2, \ldots, y_m)'$, λ is the investor's risk-aversion parameter, and $\mathbf{e}$ is a column unit vector. For no short sales restrictions, an additional constraint, $w_i \geqslant 0$, can be added. This portfolio selection approach is termed the Mean–Variance (MV) method because it ranks portfolio weights by their mean–variance pairs. The set of optimal portfolios obtained as the level of risk-aversion, λ, varies is termed the *Markowitz efficient frontier*.

The Markowitz MV method can be viewed as maximizing expected utility. For example, if the investor's current wealth is W_0, his terminal wealth is

$$W = (1 + \mathbf{w}'\mathbf{y})W_0. \tag{2}$$

According to Von Neumann and Morgenstern axioms, the investor determines $\mathbf{w}$ by considering the expected value of a non-decreasing utility function of W. Using the exponential utility function,

$$U(W;\lambda) = -\exp[-\lambda W], \tag{3}$$

and assuming $\mathbf{y}$ is distributed multivariate normal $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the maximization of expected utility reduces to ranking MV portfolios using $(\boldsymbol{\mu}, \boldsymbol{\Sigma}; \lambda)$ in model (1).

Classical portfolio selection uses least-squares estimates of $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ in model (1). However, MV portfolio selection based on estimates of population moments leads to a problem of estimation risk that arises from the difference between the estimates and the true parameter values. It has been well documented that the problem of estimation risk is significant (Dickinson, 1974; Putnam and Quintana, 1991; Pari and Chen, 1985; Frankfurter et al., 1971; Jobson and Korkie, 1980). Empirical studies of estimation risk associated with least-squares estimates appear in Levy and Sarnat (1970), Solnik (1982), Board and Sutcliffe (1992), Chopra et al. (1993), Chopra and Ziemba (1993). All of these studies conclude that resulting portfolios involve either extreme volatility or lack of diversification.

The use of Bayes and empirical Bayes estimators to estimate $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ have been advocated by several researchers (Brown, 1976; Bawa et al., 1979; Jorion, 1986; Frost and Savarino, 1986). Jorion (1986, 1991) employs Bayes modifications of James–Stein shrinking formulas (James and Stein, 1960) to estimate $\boldsymbol{\mu}$, while Frost and Savarino (1986) employ empirical Bayes estimators of $\boldsymbol{\mu}$, assuming $\boldsymbol{\Sigma}$ has intraclass structure. They show through simulated and historical data that MV portfolios using their respective Bayes estimates in model (1) dominate MV portfolios using classical least squares estimates. See also Kadiyala and Karlsson (1997), Kandel et al. (1995), and Shaken (1987).

This paper examines a fully hierarchical Bayes model for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. These models are multivariate and thus can capture more complete information on the interdependence between assets than previous models. Although these models are dependence between assets than previous models. Although these models are cross-sectional, one-step forward forecasts based on the posterior predictive distribution of returns are available for ranking portfolios. The posterior predictive distribution has been proposed for forecasting univariate ARMA models since Zellner (1971); also see West and Harrison (1989). This paper will empirically demonstrate that Bayesian forecasts are superior to moment estimates in portfolio ranking. Moreover, this approach applies to any utility function.

Marriott et al. (1993) show how to obtain the predictive distribution for a vector of future values via the Gibbs sampler and Monte Carlo integration. Kim et al. (1998) exploit MCMC sampling methods to provide a practical likelihood based framework for the analysis of stochastic volatility models. These methods are used to compare the fit of stochastic volatility and GARCH models. Nakatsuma and Tsurumi (1996) compare small-sample properties of Bayes estimation and maximum likelihood estimation (MLE) of ARMA-GARCH models using MCMC sampling. McCulloch and Tsay (1994) use the Gibbs sampler for Bayesian analysis of AR models. This paper also exploits MCMC sampling methods to obtain a practical stochastic approximation to the posterior predictive distribution and its moments.

This paper is structured as follows. The definition of posterior predictive distributions is given in Section 2. Maximum expected utility is defined in Section 3. In Section 4, we describe data on eleven country-stock index funds provided by Morgan Stanley Capital International. In addition, designs for comparing the different Bayesian models are described. Bayesian data models including the fully hierarchical prior are explained in Section 5. Sections 6 and 7 describe the results and conclusion, respectively.

2. Bayes posterior predictive distributions

Denote observed returns on m assets by $\mathbf{y}$ and future, or unobserved, returns by $\tilde{\mathbf{y}}$. Let $\theta \in R^p$ and $\phi \in R^q$ denote p parameters and q hyperparameters, respectively. The parametric family of the joint likelihood of $\mathbf{y}$ and $\tilde{\mathbf{y}}$ will be denoted by $f(\mathbf{y}, \tilde{\mathbf{y}}|\theta, \phi) = f(\mathbf{y}, \tilde{\mathbf{y}}|\theta)$ and depends on the joint parameters only through the low-level parameter θ. Denote the prior distribution of (ϕ, θ) by $\pi(\phi, \theta) = \pi(\phi)\pi(\theta|\phi)$. Non-hierarchical models fix ϕ and compute posterior distributions using the prior $\pi(\theta|\phi)$, while hierarchical models compute posterior distributions using the joint prior $\pi(\phi, \theta)$.

In the portfolio selection problem, $\theta = (\mu, \boldsymbol{\Sigma})$ and ϕ will represent a vector of hyperparameters in the prior for $(\mu, \boldsymbol{\Sigma})$. Portfolio selection using posterior predictive distributions addresses two unknown quantities, $\tilde{\mathbf{y}}$ and (ϕ, θ), with the primary goal being to gain information about $\tilde{\mathbf{y}}$ with (ϕ, θ) as nuisance parameters. The advantage of the hierarchical model, with priors instead of point estimates of hyperparameters, is that the posterior distributions will reflect the appropriate uncertainty in the hyperparameters. The disadvantage is that the posterior predictive distribution will not be analytically tractable usually; however, the method based on the MCMC sampler provides a stochastic approximation of the posterior predictive distribution.

According to the likelihood principal all evidence about $(\tilde{\mathbf{y}}, \phi, \theta)$ is contained in the joint likelihood function $f(\mathbf{y}, \tilde{\mathbf{y}}|\phi, \theta)$ (for an overview see Bjørnstad, 1990). Based on this likelihood, we wish to develop a posterior predictive distribution for $\tilde{\mathbf{y}}$, $\pi(\tilde{\mathbf{y}}|\mathbf{y})$, by eliminating (ϕ, θ) from the joint likelihood. The Bayes approach for this problem is to integrate out (ϕ, θ) using the joint prior. The resulting predictive distribution for $\tilde{\mathbf{y}}$ given the data, y, is the following:

$$\pi(\tilde{\mathbf{y}}|\mathbf{y}) = \int_{\phi \in R^q} \int_{\theta \in R^p} f(\tilde{\mathbf{y}}|\theta)\pi(\theta, \phi|\mathbf{y})\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{4}$$

A stochastic approximation of the posterior predictive distribution is generated by simulation, using the MCMC sampler if necessary, using $(\theta, \phi) \sim \pi(\theta, \phi|\mathbf{y})$ to generate $\tilde{\mathbf{y}}$ distributed with density $f(\mathbf{y}|\theta, \phi)$, and repeating these steps to obtain more simulated observations.

3. Maximum expected utility

For the reader's convenience, we repeat the basic notation. Let $w \in R^m$ denote portfolio weights satisfying $w_i \geqslant 0$, $i = 1, \ldots, m$; $\sum_{i=1,\ldots,m} w_i = 1$. The inner product $\mathbf{w}'\tilde{\mathbf{y}}$ is the portfolio-return on future investment performance. An investor will choose a utility for wealth. This utility is denoted by a monotonically increasing, concave function $U(W;\lambda)\colon W \to R$, where $\lambda \geqslant 0$ is a fixed parameter denoting risk-aversion. The posterior expected utility of $W = (1 + \mathbf{w}'\tilde{\mathbf{y}})W_0$, where W_0 is initial wealth, given the data $\mathbf{y}$ is:

$$E\{U((1 + \mathbf{w}'\tilde{\mathbf{y}})W_0;\lambda)|\mathbf{y}\} = \int_{\tilde{\mathbf{y}} \in R^m} U((1 + \mathbf{w}'\tilde{\mathbf{y}})W_0;\lambda)\pi(\tilde{\mathbf{y}}|\mathbf{y})\,\mathrm{d}\tilde{\mathbf{y}}. \tag{5}$$

This expectation exists under standard regularity conditions.

The *direct utility* (*DU*) optimal portfolio model is a solution to the following model:

$$\begin{aligned} &\text{Maximize} \quad E\{U((1 + \mathbf{w}'\tilde{\mathbf{y}})W_0;\lambda)|\mathbf{y}\},\\ &\text{subject to} \quad \mathbf{w}'\mathbf{e} = 1, \end{aligned} \tag{6}$$

where $\mathbf{w} \in R^m$. Typically, model (6) must be solved with a non-linear optimization algorithm. Many standard algorithms exist, such as sequential quadratic programming (see Gill et al., 1981, p. 237; Schittowski, 1980, 1985) as implemented in *MATLAB*, enabling solutions for any utility function that is twice-continuously differentiable. In case the expected utility is not analytically tractable, it is necessary to contemplate samples from the posterior predictive distribution that can be used to approximate the expected utility. This is an advantage since portfolio selection can be carried out with a general utility function. Given a sample, $\tilde{\mathbf{y}}_1, \dots, \tilde{\mathbf{y}}_K$, from the predictive posterior distribution, the direct utility portfolio selection problem (6) is approximated by the model:

$$\begin{aligned} &\text{Maximize} \quad K^{-1}\sum_{k=1}^{K} U((1+\mathbf{w}'\tilde{\mathbf{y}}_k)W_0;\lambda), \\ &\text{subject to} \quad \mathbf{w}'\mathbf{e} = 1. \end{aligned} \tag{7}$$

For the empirical study we the use exponential utility, $U(W;\lambda) = -\exp[-\lambda W]$.

4. Empirical data analysis

The daily stock market indices for 11 different countries over the period 1975–2002 are used in this comparison of DU and MV using different data models. The countries include US, UK, Canada, Belgium, Australia, France, Japan, Austria, Spain, Germany, and Hong Kong. The indices are compiled and provided by Morgan Stanley Capital International. Monthly returns were computed as the percentage changes in the index between consecutive last days of the month. Morgan Stanley Capital International provided two indices per country, one in local currency and one in \$US. Our study is based on the returns in \$US.

The data are partitioned into five periods of five consecutive years. This allows a comparison of the means, standard deviations, within-country serial correlations and between-country correlations.

4.1. Comparison design

To examine the performance of different models, 276 out-of-sample periods of one months each, covering the period January 1980–December 2002, are used. That is, the first data set is from January 1975–December 1979 (60 monthly observations) and the first out of sample observation is for January 1980. Our last out-of-sample portfolio is for December 2002. For a given model, we run the MCMC sampler independently on 276 data sets, and use the individual posterior distributions to form the portfolio. Our procedure can be summarized as follows:

1. Use 60 observations (initially those for months 1–60) to generate the joint posterior distributions of the means and covariances (via the MCMC sampler) and, in accordance with the decision theory rules, compute the posterior means of these distributions.
2. For a given λ, the risk-aversion parameter, find the investment proportions w.
3. Apply these proportions to the actual returns observed in the next month to obtain the actual portfolio return for each model and value of λ.
4. Roll the sample forward by one month, e.g. months 2–61, and repeat steps (1)–(3).

This resulted in 276 sample periods being used. A common value, $\lambda = 0.02$, of the risk-aversion parameter was used. Computational time prevented expanding the procedure to a range of λ's in this study. This is planned for future research.

5. Bayesian data models

We will examine two hierarchical data models, and apply MCMC sampling to obtain estimates of the posterior distribution of $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\tilde{\mathbf{y}}$. These estimates will then be used to solve models (1) and (6) for MV and DU portfolios respectively.

All models, even non-Bayesian, can be specified within a hierarchical Bayesian structure. In addition, the MCMC sampler can be used to solve even the simplest model (while a closed-form analytical solution may exist, it may still be easier to run the MCMC sampler to generate the posterior distributions). The following three data models will be tested empirically:

5.1. Classical model

$$\begin{aligned} \mathbf{y}|\boldsymbol{\mu}, \sigma^2 &\sim N\left(\boldsymbol{\mu}, \sigma^2\mathbf{I}\right), \\ \mu_j &\sim iid \text{ Uniform}(-\infty, \infty), \quad j = 1, \ldots, 11, \\ \sigma^2 &\sim \text{Inv} - \text{Gamma } (\varepsilon_1, \varepsilon_2). \end{aligned} \tag{8}$$

The values of ε_1 and ε_2 are equal to 0.0001 allowing for a proper, diffuse hyperprior. The MCMC sampler with diffuse priors yields close approximations of the classical estimators.

5.2. James–Stein model

$$\begin{aligned} \mathbf{y}|\boldsymbol{\mu}, \sigma^2 &\sim N\left(\boldsymbol{\mu}, \sigma^2\mathbf{I}\right), \\ \boldsymbol{\mu}|\mu_0, \tau_0^2 &\sim N\left(\mu_0\mathbf{1}, \tau_0^2\mathbf{I}\right), \\ \mu_0 &\sim \text{Uniform}(-\infty, \infty), \\ \sigma^2 &\sim \text{Inv} - \text{Gamma}(\varepsilon_1, \varepsilon_2), \\ \tau_0^2 &\sim \text{Inv} - \text{Gamma}(\varepsilon_3, \varepsilon_4). \end{aligned} \tag{9}$$

The values of ε_1, ε_2, ε_3, and ε_4 are equal to 0.0001 allowing for a proper, diffuse hyperprior.

5.3. Hierarchical Bayes model

$$\begin{aligned} \mathbf{y}|\boldsymbol{\mu}, \boldsymbol{\Sigma} &\sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \\ \boldsymbol{\mu}|\mu_0, \boldsymbol{\Sigma} &\sim N(\mu_0\mathbf{1}, \boldsymbol{\Sigma}/\kappa_0), \\ \boldsymbol{\Sigma} &\sim \text{Inv} - \text{Wishart}_{v_0}\left(\tau_0^{-2}\mathbf{P}_0^{-1}\right), \\ \mu_0 &\sim \text{Uniform}(-\infty, \infty), \\ \tau_0^2 &\sim \text{Inv} - \text{Gamma}(\varepsilon_1, \varepsilon_2). \end{aligned} \tag{10}$$

The degrees of freedom parameter, v_0, is unrestricted other than $v_0 \geqslant m$, where m is the number of asset, equal to 11 in this application. The values of ε_1 and ε_2 are equal to 0.0001 allowing for a proper, diffuse hyperprior; and κ_0 is equal to 0.10 n where n represents the sample size. In addition, $\mathbf{P}_0$ is a known correlation matrix with structure:

$$\mathbf{P}_0 = \begin{bmatrix} 1 & \rho_0 & \cdots & \rho_0 \\ \rho_0 & 1 & \cdots & \rho_0 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_0 & \rho_0 & \cdots & 1 \end{bmatrix}. \tag{11}$$

In this application, the estimate of the correlation parameter ρ_0 is 0.5.

6. Results

We will employ a method of comparison based on portfolio performance. We use the posterior means of $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$ as inputs to the MV framework (1); or the posterior marginal predictive distribution as inputs to the direct utility framework (6). In either framework, we obtain a vector of weights $\mathbf{w}_t$. The actual performance of the portfolio, $\mathbf{w}_t'\mathbf{y}_t$, is then tracked over time $t = 1,\ldots,276$. Consequently, we obtain an actual simulation of how a model would have performed over the study period. This performance is compared for the various models: *Classical*, *James–Stein*, and *Hierarchical Bayes*. In addition to these three models, we include as a benchmark a heuristic portfolio-selection device that weighs each asset equally, denoted by the term *Weighted* in the ensuing figure.

Table 1 displays the portfolio performance comparisons summarized over the stream of 276 monthly returns. It is quite apparent that the *Classical models* under-perform all the other models, including a naïve equal weight portfolio. The *James–Stein model* portfolio produces superior results to the *Classical model* portfolio. The additional edge in performance due to the *Hierarchical Bayes model* is quite significant. In addition, the Direct Utility (DU) method increases performance significantly over methods that impute estimates into the MV model (1).

Table 1
Portfolio yearly return performance comparisons

Data model	Method			
	Mean variance (MV)		Direct utility (DU)	
	Average	Std. dev.	Average	Std. dev.
Weighted	12.0	9.26	12.0	9.26
Classical	10.9	7.35	11.2	7.66
James–Stein	12.5	8.46	12.7	9.10
Hierarchical Bayes	14.5	10.38	16.0	11.18

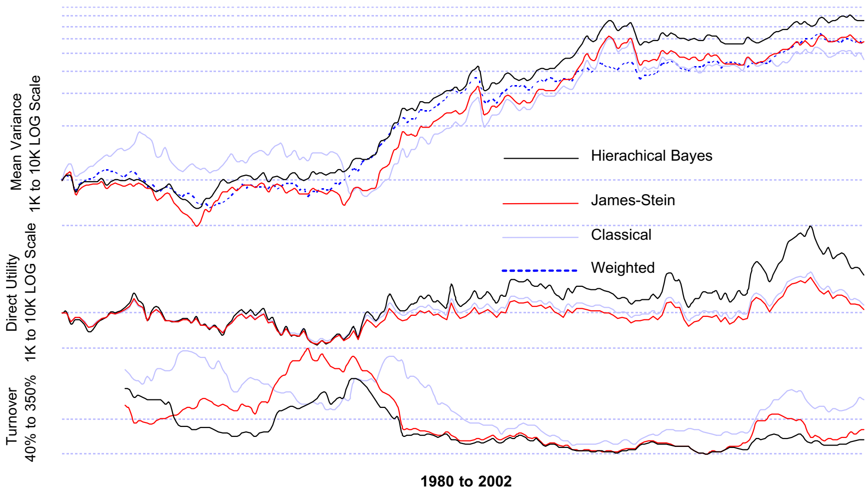

Fig. 1. Equity performance mean variance (top), direct utility factors (mid) and turnover rates (bottom).

Fig. 1 shows the equity performance of the various data models and their turnover rates (to be discussed shortly), where we show the growth of a hypothetical \$1000 portfolio. The top series display the MV portfolios, while the middle series display the DU factors. The actual performance of the DU portfolios are obtained as the DU factor times the performance of the MV performance. Note the *Hierarchical Bayes* factor remains above 1 after the mid-1980s.

An interesting point is to ask why the *James–Stein* and *Hierarchical Bayes models* under-perform the *Classical model* for the first 4 years and then significantly outperform the next 10 years (1985 and on). Closer inspection confirmed what we suspected from general knowledge of the history of these particular markets: some markets, such as the US, Japan, and Germany exhibited stable positive trends in the early 80s. The *Classical model* interprets the data of the late 70s to invest in these markets. The shrinking characteristics of the Bayes models, however, hurt their performance during this period.

Portfolio return and the resulting commission costs are obviously of great interest in practical applications. We define turnover as

$$PT_i = \sum_{i=1}^{11} (w_{i,t} - w_{i,t-1}), \tag{12}$$

that is, the portfolio turnover in a given month is the sum of the changes in portfolio weights from the previous month to that month. We will analyze the portfolio turnover for each of our models.

Fig. 1 (bottom series) compares the portfolio turnover rates of the three data models using the DU method. It is quite apparent that the *Classical model* under-performs all the other models most of the time. The *James–Stein* portfolio produces superior results to the *Classical* portfolio. The *Hierarchical Bayes* model results in significantly lower turnover rates. Note that the *Classical* portfolio showed significant gains in the early 1980s but at the cost of high turnover rates.

7. Summary and conclusion

A contribution of this paper was to employ practical hierarchical Bayesian models that incorporate a high degree of parameter uncertainty. A practical hierarchical Bayesian model accounting interclass covariance was applied to portfolio selection. The MCMC sampler was used to generate posterior prediction distributions and estimates of moments. The *James–Stein* model which has appeared previously in the finance literature is basically the Markowitz model using shrinking estimators in the mean, while the covariance matrix estimate is taken (independently of the mean) to be the sample covariance matrix. The *Hierarchical Bayes* model is a more general model, in both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.

We carried out a numerical optimization procedure to maximize expected utility using the MCMC samples from the posterior predictive distribution. This model resulted in an extra 1.5 percentage points per year in additional portfolio performance (on top of the *Hierarchical Bayes* model to estimate $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ and use the Markowitz model), which is quite a significant empirical result. This approach applies to a large class of utility functions and models for market returns.

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