STARISTICS FOR MANAGEMENT (IDS 570)

HOMEWORK 3 SOLUTION

Problem 1

(a) There are $2^3 = 8$ possible outcomes, which can be represented by sequences of the form DNN to mean the first type of defect occurred (D), but the second (N) and third (N) types of defects did not occur.

(b) The probability that the three defects occur is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

(c) The only outcomes, out of the possible 8, where exactly two defects occur are DDN, DND, NDD. Therefore the probability is $\frac{3}{8}$.

Problem 2

(a) An appropriate sample space is to consider whether or not it rains on each day. The sample space is then RR, RN, NR, RR where, for example, RN means it rains today and it doesn't rain tomorrow. Note that unlike Problem 1, these events are not equally likely. Additionally note that these events are mutually exclusive.

(b) The data from the problem can be summarized as follows: $P(RR) + P(RN) = 0.3$, $P(RR) + P(NR) = 0.2$, $P(NN) = 0.6$. Also, for clarity, let $R\#$ be the event that it rains on the first day and $\#R$ be the event that it rains on the second day, and similarly let $N\#$ ($\#N$) be the probability that it doesn't rain on the first (second) day. Hence, the probability that it doesn't rain on the first day is $P(N\#) = 1 - P(R\#) = 0.7$.

(c) The probability that it rains on at least one of the days is $1 - P(NN) = 0.4$.

(d) For this problem we make a table of outcomes:

	R(tomorrow)	N(tomorrow)	Prob.
R(today)	0.1^5	0.2^4	**0.3**
N(today)	0.1^2	**0.6**	0.7^1
Prob.	**0.2**	0.8^3	**1**

The boldface numbers are the probabilities given by the data in the problem statement. Following the superscripts, we can fill in the remaining probabilities as follows. From part (b) $P(N\#) = 0.7$. Then, since $P(N\#) = P(NR) + P(NN)$ we have $0.7 = P(NR) + 0.6$ and so $P(NR) = 0.1$. For ^{3}, $P(\#N) = 1 - P(\#R) = 1 - 0.2 = 0.8$. Then we can fill in ^{4} from $P(RN) + P(NN) = P(\#N)$, so that $P(RN) = 0.8 = 0.6 = 0.2$. Finally we can fill in ^{5} from $P(RR) + P(RN) = P(R\#)$, so that $P(RR) = 0.3 - 0.2 = 0.1$.

(e) By definition $R\#$ and $\#R$ are independent if $P(R\# \cap \#R) = P(R\#) \cdot P(\#R)$. From the data we know that $P(R\#) = 0.3$ and $P(\#R) = 0.2$. Furthermore, $P(R\# \cap \#R) = P(RR) = 0.1$, and since $0.3 \cdot 0.2 \neq 0.1$ the events are NOT independent.

Problem 3

(a) Since this student is selected first, the probability equals the number of female students over the total number or students, or $\frac{40}{100} = \frac{2}{5}$.

(b) In this problem order matters. We first find the probability that the first student selected is male, which is $\frac{60}{100}$ by the same reasoning as in the previous question. Then, of the remaining students (99), we need the number of male students that remain (59). Therefore, the probability that both selected students are male is $\frac{60}{100} \cdot \frac{59}{99} = \frac{59}{165}$.

Alternatively, we can use combinations for solving this problem. There are $C_2^{100} = 4,950$ ways to select two students from the class and $C_2^{60} = 1,770$ ways to select two male students. The probability of selecting two male students is thus $\frac{1,770}{4,950} = \frac{59}{165}$.

(c) For this problem we have to find $P(MF) + P(FM)$ where, for example, MF represents a male giving the presentation on Bayes' Theorem and a female giving the presentation on Tree Diagrams. Proceeding as in the previous answers, $P(MF) + P(FM) = \frac{60}{100} \cdot \frac{40}{99} + \frac{40}{100} \cdot \frac{60}{99} = \frac{8}{33} + \frac{8}{33} = \frac{16}{33}$.

Alternatively, we can use combinations to solve this problem. The total number of ways of picking the two students is $C_2^{100} = 4,950$. Of these, the number of events where the students giving the presentations are of different gender is the product of the number of ways of choosing one male student and one female students, or $C_1^{60} \cdot C_1^{40} = 60 \cdot 40 = 2,400$. The final probability is therefore $\frac{2,400}{4,950} = \frac{16}{33}$.

(d) In this case the probability of picking a male student for the first presentation is exactly equal to the probability of picking a male student for the second presentation. Therefore, the probability that both presentation are given by men is $\frac{60}{100} \cdot \frac{60}{100} = \frac{9}{25}$.

Problem 4

(a) The sample space is the set of all possible ways of choosing 3 out of the 9 employees, or $C_3^9 = 84$. The number of groups of size 3 comprised only of employees who are stealing is $C_3^3 = 1$. Therefore, the probability that the owner selects all three employees who are stealing is $\frac{1}{84}$.

(b) The number of groups of 3 employees comprised only of employees who are not stealing is $C_3^6 = 20$. Therefore, the probability that the owner selects only employees who are not stealing is $\frac{20}{84} = \frac{5}{21}$.

Problem 5

(a) The sample space can be given by the 27 strings 000, 001, 002, 010, 011, 012, 020, 021, 022, 100, 101, 102, 110, 111, 112, 120, 121, 122, 200, 201, 202, 210, 211, 212, 220, 221, 222, where for example 102 means that the first show received 1 complaint, the second show received 0 complaints, and the third show received 2 complaints. Note that each of these events are equally likely. Since the only outcomes that correspond to exactly 3 complaints are 012, 021, 102, 111, 120, 201, 210 (there are 7 of these) the probability that the total number of complaints is 3 is $\frac{7}{27}$.

Alternatively, we can think of the sample space as strings of length 3 composed of the numbers 0, 1, 2. It follows that the total number of possible strings is $3^3 = 27$ because each digit of the string can be chosen from 3 possible values, 0, 1, or 2, and the string has length 3. Furthermore, in order to count the number of strings which have 3 complaints, we count two different possibilities. Either the string is 111, which occurs in C_3^3 possible ways, or the string is composed of one copy of each of the number from the set (0, 1, 2) in any order, or in other words the number of ways to order 3 elements from a set of size 3, or P_3^3 . Therefore, the number of strings that sum to 3 is $C_3^3 + P_3^3 = 1 + 6 = 7$ and we get the same probability by dividing by the total number of strings: $\frac{7}{27}$.

(b) Since the only outcomes which correspond to each show getting the same number of complains are 000, 111, 222 (there are 3 of these) the desired probability is $\frac{3}{27} = \frac{1}{9}$. Alternatively, we can count the number of strings having the same number in each digit. The only such strings are the strings with three copies of one number from (0,1,2). For each of the three symbols, there is $C_3^3 = 1$ such string, and thus there are 3 strings of interest. Dividing by the total number of strings yields $\frac{C_1^3}{27} = \frac{3}{27} = \frac{1}{9}$.

(c) Since the only strings which correspond to this event are 000, 001, 002, 010, 011, 020, 022, 100, 101, 110, 111, 112, 121, 122, 200, 202, 211, 212, 220, 221, 222 (there are 21 of these) the desired probability is $\frac{21}{27}$.

Alternatively, we can compute the probability of the complement event and then subtract this from one. The complement event is that all three shows get a different number of complaints. The only way that this can happens is if one of the shows gets 0 complaints, one of the shows gets 1 complaint and one of the shows get 2 complaints. In other words, it is the number of ways of ordering the elements of the set (0,1,2), which is $P_3^3 = 6$. Therefore, the probability of the complement event is $\frac{6}{27}$ and the desired probability is $1 - \frac{6}{27} = \frac{21}{27}$.

Alternatively, we can count the number of strings that have this property. There are 2 cases to consider. In the first case all of the digits are the same: $3 \cdot C_3^3$ since all 3 digits must be identical and there are 3 choices for which number we select from (0,1,2). In the second case, exactly 2 of the 3 digits are the same. To count this we first choose which 2 of the 3 digits in the sequence will have the same number, which can be done it C_2^3 ways (for example we can choose digit 1 and digit 3). Then we must select 2 out of the 3 possible values in (0,1,2) to occupy these slots, where order matters because we let the first number chosen occupy the two chosen slots and the second number chosen occupy the remaining slot (for example, if we choose 0 and then 1, the sequence is 010, and if we choose 1 and then 0, the sequence is 101). This is done in P_2^3 ways. Putting everything together yields $3 \cdot C_3^3 + C_2^3 \cdot P_2^3 = 21$, and the same probability follows by diving by the total number of possible sequences, or $\frac{3 \cdot C_3^3 + C_2^3 \cdot P_2^3}{27} = \frac{21}{27}$.

Problem 6

(a) What is the first number that Luis counts? Answer: 45. What is the second number that he counts? Answer: 45+3 = 48. What is the third number that he counts? Answer: 45 + (2× 3) = 51. If we continue this procedure, we can see the 15th number that Luis counts is 45 + (14× 3) = 45+42= 87.

(b) The word "TIGER" is 5 letters long. Therefore, we have $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

(c) $C_5^9 = \frac{9!}{4!5!} = 126$.

(d) The total different ways we can arrange the letters in the word "PRETTY" is 6!. However, some of the words are the same. For example, the words "PTREYT" and "PTREYT" are the same. So we need to remove these words that are counted twice. Therefore, the total number of ways that we can arrange the letters in "PRETTY" is $\frac{6!}{2!} = 360$.

Problem 7

(a) a) Row Percentages: There are six percent frequency distributions in this table with row per-

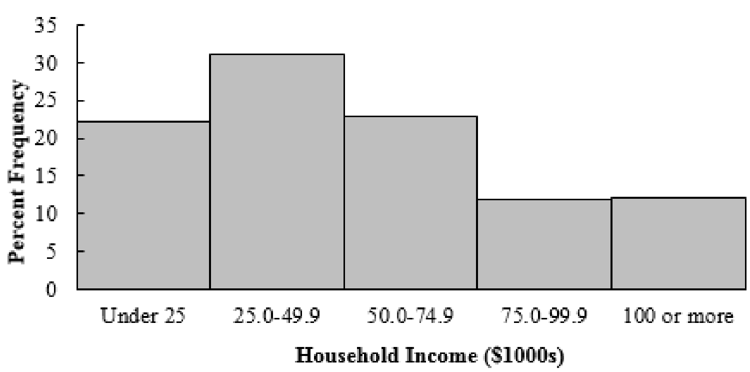
	Household Income ($1000s)					
Educational Level	**Under 25**	**25.0-49.9**	**50.0-74.9**	**75.0-99.9**	**100 or more**	**Total**
Not H.S. graduate	42.23	34.73	13.94	5.41	3.68	100.00
H.S. graduate	22.25	31.00	22.75	11.93	12.07	100.00
Some college	13.99	26.20	23.31	16.20	20.30	100.00
Bachelor's degree	6.42	15.19	20.66	18.72	39.02	100.00
Beyond bach. deg.	3.71	10.60	16.29	15.87	53.54	100.00
Total	17.77	25.08	20.64	13.90	22.62	100.00

centages. The first five give the percent frequency distribution of income for each educational level. The total row provides an overall percent frequency distribution for household income.

The second row, labeled H.S. Graduate, is the percent frequency distribution for households headed by high school graduates. The fourth row, labeled Bachelor's Degree, is the percent frequency distribution for households headed by bachelor's degree recipients.

(b) The percentage of households headed by high school graduates earning $75,000 or more is 11.93% + 12.07 = 24.00%. The percent of households headed by bachelor's degree recipients earning $75,000 or more is 18.72% + 39.02% = 57.74%.

(c) The percent frequency histogram for high school graduates. The percent frequency distribution



for college graduates with a bachelors degree.

The histograms show that households headed by a college graduate with a bachelors degree earn more than households headed by a high school graduate. Yes, there is a positive relationship between education level and income.

(d) Column Percentages:

	Household Income ($1000s)					
Educational Level	**Under 25**	**25.0-49.9**	**50.0-74.9**	**75.0-99.9**	**100 or more**	**Total**
Not H.S. graduate	32.10	18.71	9.13	5.26	2.20	13.51
H.S. graduate	37.52	37.05	33.04	25.73	16.00	29.97
Some college	21.42	28.44	30.74	31.71	24.43	27.21
Bachelor's degree	6.75	11.33	18.72	25.19	32.26	18.70
Beyond bach. deg.	2.21	4.48	8.37	12.11	25.11	10.61
Total	100.00	100.00	100.00	100.00	100.00	100.00

There are six percent frequency distributions in this table of column percentages. The first five columns give the percent frequency distributions for each income level. The percent frequency distribution in the "Total" column gives the overall percent frequency distributions for educational level. From that percent frequency distribution we see that 13.51% of the heads of households did not graduate from high school.

(e) The column percentages show that 25.11% of households earning $100,000 or more were headed by persons having schooling beyond a bachelor's degree. The row percentages show that 53.54% of the households headed by persons with schooling beyond a bachelor's degree earned $100,000 or more. These percentages are different because they came from different percent frequency distributions and provide different kinds of information.