STATISTICS FOR MANAGEMENT (IDS 570)

HOMEWORK 10 SOLUTION

Problem 1

(a) Since the results are given as statistically significant, the χ^2-statistic is likely to be large.
(b) Since the results are given as statistically significant, the p-value is likely to be small.
(c) In the week before the festival, the expected count is higher than the observed count. This tells us that some elderly people may be able to delay death.
(d) For the week before the festival, the contribution to the χ^2 statistic is

$$\frac{(\text{observed - expected})^2}{\text{expected}} = \frac{(33 - 50.82)^2}{50.82} = 6.249$$

(e) In the week after the festival, the observed count is higher than the expected count. This tells us that, although some elderly people are able to delay death, they don't delay it for very long.
(f) The control group allows us to attribute the difference specifically to the meaningful event (the Harbor Moon Festival) since the effect was only seen in the group who found this event meaningful.

Problem 2

(a) $MSG = \dfrac{SSG}{l-1} = \dfrac{11156}{2} = 5578$ and $MSE = \dfrac{SSE}{N-l} = \dfrac{1334}{12} = 111.$
(b) $F = \dfrac{MSG}{MSE} = \dfrac{5578}{111} = 50.16.$
(c) Look at the R code given below:

```
> examscores = c(70, 77, 83, 90, 97, 37, 43, 50, 57, 63, 3, 10, 17, 23, 30)
> drugs = c(rep("Memory Drug", 5), rep("Placebo", 5), rep("No Treatment", 5))
> dataset = data.frame(examscores, drugs)
> dataset
   examscores        drugs
1          70  Memory Drug
2          77  Memory Drug
3          83  Memory Drug
4          90  Memory Drug
5          97  Memory Drug
6          37      Placebo
7          43      Placebo
8          50      Placebo
9          57      Placebo
10         63      Placebo
11          3 No Treatment
12         10 No Treatment
13         17 No Treatment
14         23 No Treatment
15         30 No Treatment
> aov(examscores~drugs, data = dataset)
Call:
   aov(formula = examscores ~ drugs, data = dataset)

Terms:
                  drugs Residuals
Sum of Squares  11155.6    1334.4
Deg. of Freedom       2        12

Residual standard error: 10.54514
Estimated effects may be unbalanced
> summary(aov(examscores~drugs, data = dataset))
            Df Sum Sq Mean Sq F value   Pr(>F)
drugs        2  11156    5578   50.16 1.49e-06 ***
Residuals   12   1334     111
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(d) since p-value is very small (less than $\alpha = 5\%$), we reject the null-hypothesis. In other words, the treatments will have different effects.

Problem 3

(a) We find the required sum of squares using the shortcut formulas at the end of this section. We compare the group means to the overall mean:

$$SSG = \sum n_i \left(\bar{x}_i - \bar{x}\right)^2 = 6(36.00 - 38)^2 + 6(37.67 - 38)^2 + 6(35.83 - 38)^2 + 6(42.50 - 38)^2 = 174.4$$

We find the variability within the groups:

$$SSE = \sum (n_i - 1)s_i^2 = (6 - 1) \cdot 14.52^2 + (6 - 1) \cdot 12.40^2 + (6 - 1) \cdot 13.86^2 + (6 - 1) \cdot 17.41^2 = 4299.0$$

We find the total variability:

$$(n - 1)s^2 = (24 - 1) \cdot 13.95^2 = 4475.9$$

We see that $SSG + SSE = 174.4 + 4299.0 = 4473.4$. The difference from 4475.9 is due to rounding the means and standard deviations in the summary statistics.

(b) We are testing

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

vs

$$H_a : \text{Some } \ \mu_i \neq \mu_j$$

where the $_i$'s represent the mean numbers of ants for the four types of bread. We have 4 - 1 = 3 degrees of freedom for the groups, 24 - 4 = 20 degrees of freedom for the error, and 24 - 1 = 23 degrees of freedom for the total. We compute mean squares by dividing sums of squares by degrees of freedom, then take the ratio of the mean squares to compute the F-statistic.

$$MSG = \frac{174.4}{3} = 58.13 \quad MSE = \frac{4299.0}{20} = 214.95 \quad F = \frac{MSG}{MSE} = \frac{58.13}{214.95} = 0.27$$

The ANOVA table summarizing these calculations is shown below.

Source	DF	SS	MS	F	P
Bread	3	174.4	58.13	0.27	0.846
Error	20	4299.0	214.95		
Total	23	4473.4			

Using an F-distribution with 3 and 20 degrees of freedom, we find the area beyond $F = 0.27$ gives a p-value of 0.846. This is a very large p-value, so we have no convincing evidence to conclude that the mean number of ants attracted to a sandwich depends on the type of bread.

Problem 4

(a) The hypothesis are
H_0: Type of tag is not related to survival
H_a: Type of tag is related to survival

(b) The table below shows the expected counts, obtained for each cell by multiplying the row total by the column total and dividing by $n = 356$.

	Metal	Electronic
Survived	47.4	53.6
Died	119.6	135.4

(c) We calculate the chi-square test statistic

$$\begin{aligned} \chi^2 &= \frac{(33-47.4)^2}{47.4} + \frac{(68-53.6)^2}{53.6} + \frac{(134-119.6)^2}{119.6} + \frac{(121-135.4)^2}{135.4} \\ &= 4.37 + 3.87 + 1.73 + 1.53 \\ &= 11.5 \end{aligned}$$

(d) We compare our test statistic of 11.5 from part (b) to a chi-square with 1 degree of freedom to get a p-value of 0.0007. There is strong evidence that the type of tag and survival rate of the penguins are related.

Problem 5

(a) The null hypothesis for an ANOVA always assumes the population means are equal. Hence, we may write the null hypothesis as:
$H_0 : \mu_1 = \mu_2 = \mu_3$ - The mean head pressure is statistically equal across the three types of cars. Since the null hypothesis assumes all the means are equal, we could reject the null hypothesis if only mean is not equal. Thus, the alternative hypothesis is:
H_a: At least one mean pressure is not statistically equal.

(b) The test statistic in ANOVA is the ratio of the between and within variation in the data. It follows an F distribution.
We have

$$SSG = 3(666.67 - 529.22)^2 + 3(473.67 - 529.22)^2 + 3(447.33 - 529.22)^2 = 86049.55$$

and

$$\begin{aligned} SSE &= (643 - 666.67)^2 + (655 - 666.67)^2 + (702 - 666.67)^2 \\ &+ (469 - 473.67)^2 + (427 - 473.67)^2 + (525 - 473.67)^2 \\ &+ (484 - 447.33)^2 + (456 - 447.33)^2 + (402 - 447.33)^2 \\ &= 10254. \end{aligned}$$

Therefore,

$$F = \frac{MSG}{MSE} = \frac{86049.55/(3-1)}{10254/(9-3)} = 25.17$$

(c) To find the critical value from an F distribution you must know the numerator (MSG) and denominator (MSE) degrees of freedom, along with the significance level.
F^* has df1 and df2 degrees of freedom, where df1 is the numerator degrees of freedom equal to $l-1$ and df2 is the denominator degrees of freedom equal to $N-l$. In our example, df1 = 3 − 1 = 2 and df2 = 9 − 3 = 6. Hence we need to find Critical Value $F_{2,6}$ corresponding to $\alpha = 5\%$. Using the F tables in your text we determine that $F^* = 5.14$.

(d) You reject the null hypothesis if: F (observed value) $> F^*$ (critical value). Since $25.17 > 5.14$, so we reject the null hypothesis. Since we rejected the null hypothesis, we are 95% confident $(1-\alpha)$ that the mean head pressure is not statistically equal for compact, midsize, and full size cars. However, since only one mean must be different to reject the null, we do not yet know which mean(s) is/are different. In short, an ANOVA test will test us that at least one mean is different, but an additional test must be conducted to determine which mean(s) is/are different.