STATISTICS FOR MANAGEMENT(IDS 570)

HOMEWORK 4
DUE DATE: THURSDAY, OCTOBER 02 AT 11:59 PM

Problem 1. (Independence). According to a survey, the fraction of people happy with their boss is 0.5 and the fraction of people happy with their hours of work is 0.4. Also, the fraction of people unhappy with their boss AND unhappy with their hours of work is 0.3.

(a) [4 points] What fraction of people are BOTH happy with their boss and with their hours of work?
(b) [4 points] What is the conditional probability that a person is happy with their boss given that (s)he is happy with their hours of work?
(c) [4 points] Are the events being happy with your boss and being happy with your hours of work independent?

Problem 2. (Conditional Probability and Bayes' Theorem I). Suppose there are two urns, I and II. Urn I contains 2 black balls and 1 white balls. Urn II contains 1 black ball and 1 white ball. An urn is drawn at random and a ball is chosen at random from it.

(a) [4 points] What is the probability that a black ball is chosen?
(b) [4 points] If a white ball is chosen, what is the probability that it came from urn I?
(c) [4 points] Consider the event that urn I is chosen and the event that a black ball is chosen. Are these events independent?

Problem 3. (Conditional Probability and Bayes' Theorem II). In a certain company it is known from past years that if a new worker attends an introductory training program then the probability that the worker will meet the quota for sales in his/her first year of work is 0.8. If a new worker does not attend the training program, the probability that the worker will meet the quota for sales in his/her first year of work is 0.3. Suppose that $\frac{1}{2}$ of new workers attend the training program.

(a) [4 points] What is the probability that a randomly chosen new worker meets the quota for sales in his/her first year of work?
(b) [4 points] Suppose a new worker is chosen at random and (s)he meets his/her quota for sales in the
rst year. What is the probability that (s)he attended the training program?
(c) [4 points] Now suppose that $\frac{9}{10}$ new workers attend the meeting and all other numbers remain the same. What is the probability that a randomly chosen new worker meets the quota for sales in his/her first year of work?

Problem 4. (Conditional Probability and Bayes' Theorem III). During the winter season internal damage to pipes often occurs. Although we cannot check directly whether or not any internal damage has occurred, there are tests that can give us some indication as to whether or not there is any damage. The test however is not infallible. The test has probability 0.9 of detecting damage when it is present but it also has probability 0.3 of falsely indicating internal damage. Suppose that the probability that the winter will cause damage to a new pipe is 0.5.

(a) [4 points] What is the probability that a new pipe has damage after the winter season given that the test indicates that there is damage?

(b) [4 points] What is the probability that a pipe has damage given that the test indicates that there is no damage?

Problem 5. (Probability Rules). Suppose A and B are independent events.

(a) [7 points] Are A and $\bar{B}$ independent?
(b) [7 points] Are $\bar{A}$ and $\bar{B}$ independent?

Problem 6. [13 points] Consider independent trials consisting of rolling a pair of a fair dice over and over. What is the probabulity of sum of 5 appears before a sum of 7?

Problem 7. [13 points] A certain type of missile hits its target with probability $p = 0.3$. Find the minimum number of n of missiles that should be fired so that there is at least an 80 percent probability of hitting the target.

Problem 8. [16 points] Let E be an event for which $P(E) > 0$. Show that the conditional probability function $P(* \mid E)$ satisfies the axioms of a probability space, that is

1. For any event A, we have $P(A \mid E) \geq 0$.
2. For any certain event S, we have $P(S \mid E) = 1$.
3. For any two disjoint events A and B, we have

$$P\left(A \cup B \mid E\right) = P\left(A \mid E\right) + P\left(B \mid E\right)$$