STATISTICS FOR MANAGEMENT (IDS 570)

HOMEWORK 6
DUE DATE: THURSDAY, OCTOBER 23 AT 11:59 PM

Problem 1. (Expectation and Variance). Consider the following:

(a) [4 pts] For a binomial probability function with $p = 0.6$ and $n = 15$, find the probability that the number of successes is less than 8.

(b) [4 pts] For a binomial probability function with $p = 0.2$ and $n = 20$, find the probability that the number of successes is less than 12 but greater than 6.

(c) [4 pts] Find the mean and variance for a Bernoulli random variable X with probability of success $p = 0.3$?

(d) [4 pts] Determine the probability of more than 5 successes for a random variable X with a Poisson distribution with parameter $\lambda = 4.6$?

Problem 2. (Probability Distributions). A manager needs to decide whether to bid on a new project. The sunk cost of preparing the bid is \$14,000. The profit from successfully attaining this project is \$90,000. Typically 35% of bids are successful.

(a) [5 pts] Is it worth preparing the bid given the expected payoffs?

(b) [5 pts] Given the expected payoffs, what is the minimum probability necessary for a bid to be successful in order to convince the manager to prepare the bid?

Problem 3. (Probability Distributions). In the neighborhood coffee shop, people either order a hot beverage or a frappuccino. For both of these kinds of people, the distribution is a Poisson distribution with a mean of 30 per hour for a hot beverage and 18 per hour for the frappuccinos.

(a) [5 pts] What is the expected number of clients the coffee shop gets in a half-an-hour lunch break?

(b) [5 pts] Suppose the coffee shop received no client for frappuccinos for the first 10 minutes of the half- hour lunch break, what is the probability that it gets no client for frappuccinos for the rest of the break?

(c) [5 pts] What is the probability that the coffee shop gets less than 10 customers for hot beverages in the following 2 hours?

Problem 4. (pdfs vs. cdfs). Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. Suppose that $f(x) = kx^3$ for $0 \leq x \leq 1$ and zero everywhere else.

(a) [4 pts] What is the value of k?

(b) [4 pts] Find $F(x)$.

(c) [4 pts] Find $P(\frac{1}{2} \leq X \leq 2)$.

(d) [5 pts] Is it more likely that X is between 0 and $\frac{1}{2}$ or that X is between $\frac{1}{2}$ and 1?

(e) [4 pts] What is the median of X?

Problem 5. (pdfs and cdfs). Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{x}{4} & 1 \leq x \leq A \\ 0 & \text{otherwise} \end{cases}$$

Here A is a constant (that you will have to determine).

(a) [5 pts] Find A so that $f(x)$ is a pdf.
(b) [4 pts] Find the cdf $F(x)$.
(c) [4 pts] Find $E[X]$.
(d) [4 pts] Find $Var[X]$.
(e) [5 pts] Find the 37.5^{th} quantile of this distribution.

Problem 6. (The Uniform Distribution). Bob is a student at CMU and takes the bus to school on Mondays, Wednesdays and Fridays, and walks to school on Tuesdays and Thursdays. (He is lazy and never comes into school on the weekends.) Suppose that the amount of time it takes him to take the bus into school is uniformly distributed between 10 and 16 minutes and the amount of time it takes him to walk to school is uniformly distributed between 18 and 20 minutes. In addition, suppose that the amount of time that it takes Bob to get to school each day is independent of the amount of time that it takes Bob to get to school every other day.

(a) [5 pts] What is the expected value of the amount of time Bob spends getting to school each week.
(b) [5 pts] What is the variance of the amount of time Bob spends getting to school each week.
(c) [5 pts] Given that on Monday it takes the bus less than 15 minutes to get to school, what is the probability that it takes Bob more than 19 minutes to walk to school on Tuesday.
(d) [5 pts] Given that on a Wednesday Bob has already been on the bus for 13 minutes, what is the probability that his total time on the bus on this day is greater than or equal to 14.