STATISTICS FOR MANAGEMENT (IDS 570)

HOMEWORK 5
DUE DATE: SATURDAY, OCTOBER 18 AT 11:59 PM

Problem 1. (Expectation and Variance). You and your friend play the following game. You toss a six sided dice. Your score, denoted by X, is the number that appears on the dice and your friends score, denoted by Y is 6 minus the number that appears on the dice. The person who gets the higher score wins the game. In case both scores are equal, the game results in a tie.

(a) [5 pts] What is your expected score $E[X]$?
(b) [5 pts] What is the expected score of your friend $E[Y]$?
(c) [5 pts] What is $Var[X]$?
(d) [5 pts] What is $Var[Y]$?
(e) [5 pts] Let Z be a random variable such that $Z = X - Y$. What is $E[Z]$?
(f) [5 pts] What is the value of $Var[Z]$?
(g) [5 pts] What is the probability that Z is greater than zero? Note that this is the probability that you win the game.
(h) [5 pts] Now, suppose you play the following modified game. You and your friend both toss a six sided dice independently and let X denote the number that you get and Y denote the number that your friend gets. Again, let $Z = X - Y$. What is the value of $E[Z]$ and $Var[Z]$?

Problem 2. You and your friend play the following game. You toss a six sided dice and the number that appears on the dice is your score, denoted by X. Your friend tosses a fair coin six times and his score is the number of times the toss resulted in a heads, denoted by Y. The person getting a higher score wins. In case both scores are equal, the game results in a tie.

(a) [4 pts] What is the probability that the game will result in a tie, i.e. $P(X = Y)$?
(b) [4 pts] What is the probability that you win the game, i.e. $P(X > Y)$?
(c) [4 pts] Now suppose both you and your friend toss a six sided dice and whoever gets the higher score wins (if the two scores are equal, the game results in a tie). What is the probability that you win the game?

Hint: You can use the joint distribution $P(X = i, Y = j)$ to compute the above probabilities.

Problem 3. (Joint Distribution). Let X and Y be two random variables where $X \in \{1, 2\}$ and $Y \in \{1, 2, 3\}$. The following table shows the joint probability distribution of the two random variables with a missing entry.

	$X = 1$	$X = 2$
$Y = 1$	0.1	0.15
$Y = 2$	0.175	0.075
$Y = 3$	0.225	p

(a) [4 pts] What is the value of p?
(b) [4 pts] Compute the marginal probabilities: $P(X = 1)$ and $P(X = 2)$.
(c) [4 pts] Compute the marginal probabilities: $P(Y = 1)$, $P(Y = 2)$, $P(Y = 3)$.
(d) [4 pts] What is the value of $E[X]$?
(e) [4 pts] What is the value of $E[Y]$?
(f) [4 pts] Compute the covariance between X and Y.
(g) [4 pts] Are the random variables X and Y independent? Explain.

Hint: Notice that $\sum_{i=1}^{n}\sum_{j=1}^{m} P(X = x_i, Y = y_j) = 1$.