Estimation of Pedaling Torque for Electric Power Assisted Bicycles

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***Abstract*—Since electric power assisted bicycles have been released in Japan, their sales are increasing. However the prices are high because they are equipped with an expensive torque sensor to measure human pedaling torque to assist human by an electric motor. The purpose of this study is to develop an external power assist systems applicable for a normal bicycle that fits in the Japanese regulation. This paper proposes an instantaneous pedaling torque observer based on the disturbance observer and the fourier filtering in order to decouple the load torque and the pedaling torque. The validity of the proposed method is confirmed by numerical simulations and experiments.**

I. Introduction

Since electric power assisted bicycles have been released in 1993, they have become popular as safe vehicle comparing to motorcycles. The electric power assisted bicycles are categorized in an expensive class of bicycles. However, they have been extended widely for reasons that they are friendly to the environment. In Japan, the electric power assisted bicycles provide torque depending on the pedaling torque of the rider. The assist-ratio is limited in Japan as shown in Fig. 1. In recent years, there are several studies of estimating load torque without using torque sensors. These systems have been applied to various devices such as robotic manipulators, electric wheelchairs, and automotive electric power assisted steering systems [1] [2] [3]. In the electric power assisted bicycles, load torque of the motor is composed of pedaling torque and load torque due to wind, friction and gravity. The total load torque of the motor can be easily estimated by existing methods such as the disturbance observer. In this paper, we proposed a method to extract the pedaling torque from the total load torque. By the proposed method, we can remove the torque sensor from the bicycle, which lead to the reduction of the cost of the bicycles. In the proposed method, we focus on periodicity of the pedaling torque and its fundamental component in the frequency domain. By the proposed system, the normal bicycles can be easily converted to the electric power assisted bicycles by only replacing the front wheel with the motorized one without using the torque sensors.

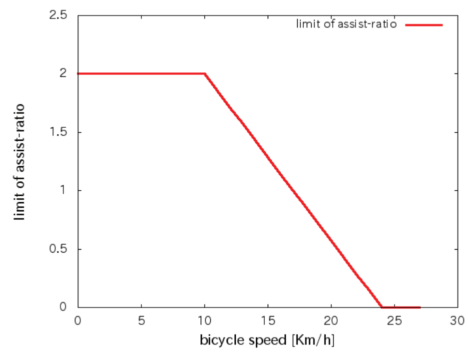

Fig. 1: Limit of assist ratio.

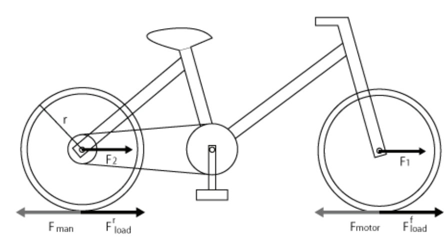

Fig. 2: Model of an electric power assisted bicycle.

II. Elctric Power Assisted Bicycle

In this study, we consider a front-wheel-assisted bicycle because the front wheel is easy to install a motor than the rear wheel. A model of the front-wheel-assisted-type bicycle is shown in Fig. 2. The assist torque is determined by the pedaling torque and the speed of the bicycle. The model of assist torque calculation is shown in Fig. 3.

A. Model of the electric power assisted bicycle

The equation of motion of the bicycle is given as follows. Parameters appeared in the equations are denoted in TABLE I.

$$M\dot{v} = F_{motor} + F_{man} - F_{load} \tag{1}$$

$$F_{load} = K_0 + K_1 v + K_2 v^2 \tag{2}$$

where K_0, $K_1 v$, and $K_2 v^2$ are inclined road force, viscous friction force, and aerodynamic drag, respectively. (1) can be equivalently transformed into the motion equation around the motor axis as follows.

$$J\dot{\omega} = \tau_{motor} + \tau_{man} - \tau_{load} \tag{3}$$

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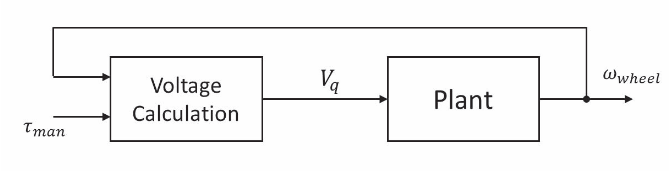


Fig. 3: Model of assisted torque calculation.

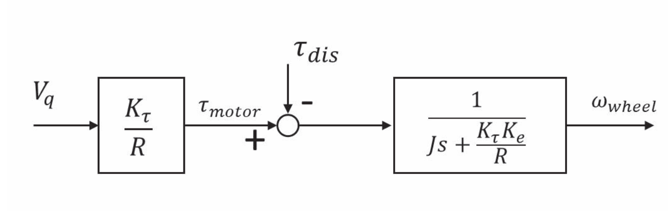


Fig. 4: Control model.

TABLE I: Parameter

V_q	q axis voltage of armature winding
I_q	q axis current of armature winding
R	Resistance of armature winding
K_e	Constant of back electromotive force
K_τ	Constant of torque
M	Mass of bicycle and rider
J	Inertia of rotor
J_n	Nominal value of inertia of rotor
r	Wheel radius
θ_c	Angle of crank
G	Gear ratio
v	Velocity of bicycle
ω	Angular velocity of wheel
F_{motor}	Motor force
F_{man}	Pedaling force
F_{load}	Load force
τ_{motor}	Motor torque
τ_{man}	Pedaling torque
τ_{load}	Load torque

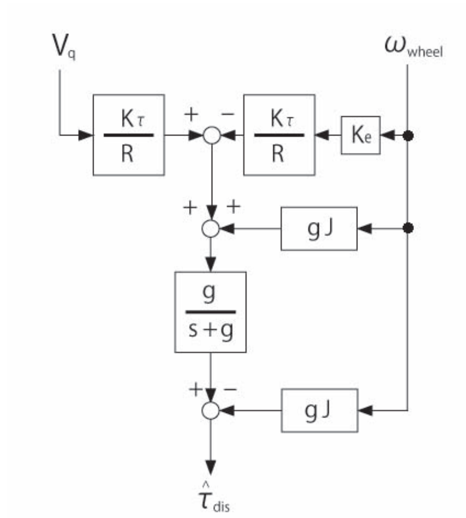


Fig. 5: Model of disturbance observer.

where $J = Mr^2$, $\tau_{motor} = rF_{motor}$, $\tau_{man} = rF_{man}$, $\tau_{load} = rF_{load}$, respectively. Assuming that the electrical time constant of the motor is short enough and the motor is not salient, q-axis current I_q and the motor torque τ_{motor} are described by

$$I_q = \frac{1}{R}(V_q - K_e\omega) \tag{4}$$

$$\tau_{motor} = K_\tau I_q \tag{5}$$

Eventually, the plant model(3) is given by

$$J\dot{\omega} = \frac{K_\tau}{R}V_q - \frac{K_\tau K_e}{R}\omega - \tau_{dis} \tag{6}$$

where τ_{dis} is the total disturbance torque defined as follows.

$$\tau_{dis} = \tau_{load} - \tau_{man} \tag{7}$$

III. Instantaneous pedaling torque estimator

A. Disturbance observer

The model of the controlled plant of the electric power assisted bicycle is shown in Fig. 4. The disturbance observer [4] can estimate the sum of the pedaling torque and the load torque applied to the front wheel axle.

$$\hat{\tau}_{dis}(t) = \frac{g}{s+g}(\frac{K_\tau}{R}V_q - \frac{K_\tau K_e}{R}\omega - J_n\dot{\omega}) \tag{8}$$

where g is a gain of the disturbance observer. The block diagram is shown in Fig. 5.

B. Instantaneous pedaling torque estimator

We need to extract the pedaling torque from the disturbance torque estimated by the disturbance observer. The pedaling torque has a double period per one revolution of the crank angle [5]. We pick it out by means of the Fourier series expansion to estimate the pedaling torque.

1) Average pedaling torque estimator: The value of disturbance torque $\hat{\tau}_{dis}(t)$ estimated by the disturbance observer is a function with respect to time t. This function can be converted to a function with respect to the crank angle as follows.

$$\tilde{\tau}_{dis}(\theta_c(t)) = \hat{\tau}_{dis}(t) \tag{9}$$

θ_c is obtained from the wheel angle θ_{wheel} as follows.

$$\theta_c = G\theta_{wheel} \tag{10}$$

where G is gear ratio between the wheel and the crank. To estimate the average pedaling torque, we utilize the Fourier series expansion. We can pick out the periodic components of the pedaling torque by

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} \tilde{\tau}_{dis}(\theta_c)\cos 2\theta_c d\theta_c \tag{11}$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} \tilde{\tau}_{dis}(\theta_c)\sin 2\theta_c d\theta_c \tag{12}$$

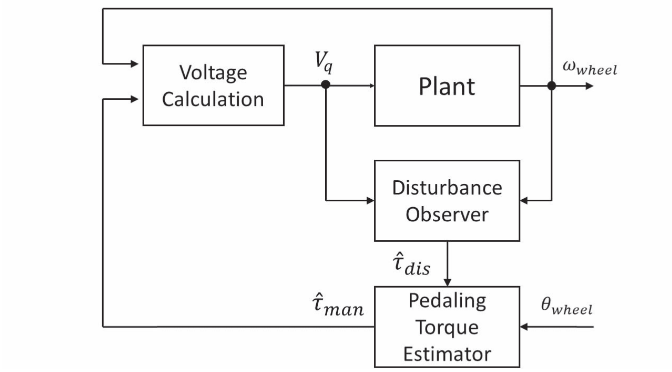


Fig. 6: Model of estimated pedaling torque.

then, the average pedaling torque $\tilde{\tau}_{man}(\theta_c)$ can be reconstructed by

$$\tilde{\tau}_{man}(\theta_c) = a_1 \cos 2\theta_c + b_1 \sin 2\theta_c + \sqrt{{a_1}^2 + {b_1}^2} \tag{13}$$

The term $\sqrt{{a_1}^2 + {b_1}^2}$ is a compensation term so that $\tilde{\tau}_{man}(\theta_c) \geq 0$ satisfies. $\tilde{\tau}_{man}(\theta_c)$ consists of the fundamental component of the pedaling torque in the preceding crank rotation.

2) Average load torque estimator: We assume that the load torque does not change rapidly. The average load torque of the preceding rotation is given by the following equation.

$$\hat{\tau}_{load}(t) = \frac{1}{2\pi}\int_0^{2\pi} (\tilde{\tau}_{dis}(\theta_c) + \tilde{\tau}_{man}(\theta_c))d\theta_c \tag{14}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \tilde{\tau}_{dis}(\theta_c)d\theta_c + \sqrt{a_1^2 + b_1^2} \tag{15}$$

3) Instantaneous pedaling torque estimator: We can estimate the instantaneous pedaling torque by taking the difference of the average load torque and the disturbance torque as follows.

$$\hat{\tau}_{man}(t) = \hat{\tau}_{load}(t) - \hat{\tau}_{dis}(t) \tag{16}$$

The model of the pedaling torque estimation is shown in Fig. 6. The pedaling torque is estimated by the disturbance observer and the pedaling torque estimator. From the estimated pedaling torque and the angular velocity, the voltage command V_q is determined. The angle and the angular velocity of the wheel are obtained by hall-effect sensors.

IV. SIMULATION

A. Estimation of pedaling torque without power assistance

In this section we consider a validity of the proposing system by numerical simulations. The pedaling torque and the load torque are given as follows [6].

$$\tau_{man} = 20(1 + \exp(-0.2t))(1 + \sin 2\theta_{crank}) \tag{17}$$

$$\tau_{load} = 0.089\omega^2 \tag{18}$$

Simulation parameters were selected as shown in TABLE II. Simulation results of the angle of the crank and the angular velocity of the wheel are shown in Fig. 7(a) and Fig. 7(b).

TABLE II: Simulation parameters.

J_n [kgm^2]	8.71 (M=80kg,r=0.33m)
D [Nms/rad]	5.81
K_τ [Nm/A]	2.794
R [Ω]	0.2
g [rad/s]	50

The estimated disturbance torque is shown in Fig. 7(c). The load torque and the estimated average load torque is shown in Fig. 7(d). The estimated pedaling torque is shown in Fig. 7(e). The pedaling torque is estimated approximately. The top part of the pedaling torque is underestimated because the average load torque has a steady-state error. However, this error is small enough and almost negligible.

B. Estimation of pedaling torque with power assistance

This section investigates the validity of the proposed system if the maximum assist torque is limited according to the regulation. Simulation results of the estimated pedaling torque and the motor torque are shown in Fig. 8(a). The velocity of the bicycle is shown in Fig. 8(b). It was confirmed that this system is valid regardless of existence of assist torque feedback.

V. FUNDAMENTAL MEASUREMENTS FOR EXPERIMENTS

In order to estimate the pedaling torque more accurately, we have measured the torque constant and the friction coefficient of our electric bicycle. These measurements were performed in the electric bicycle where the front wheel is equipped with the motor.

A. Torque constant

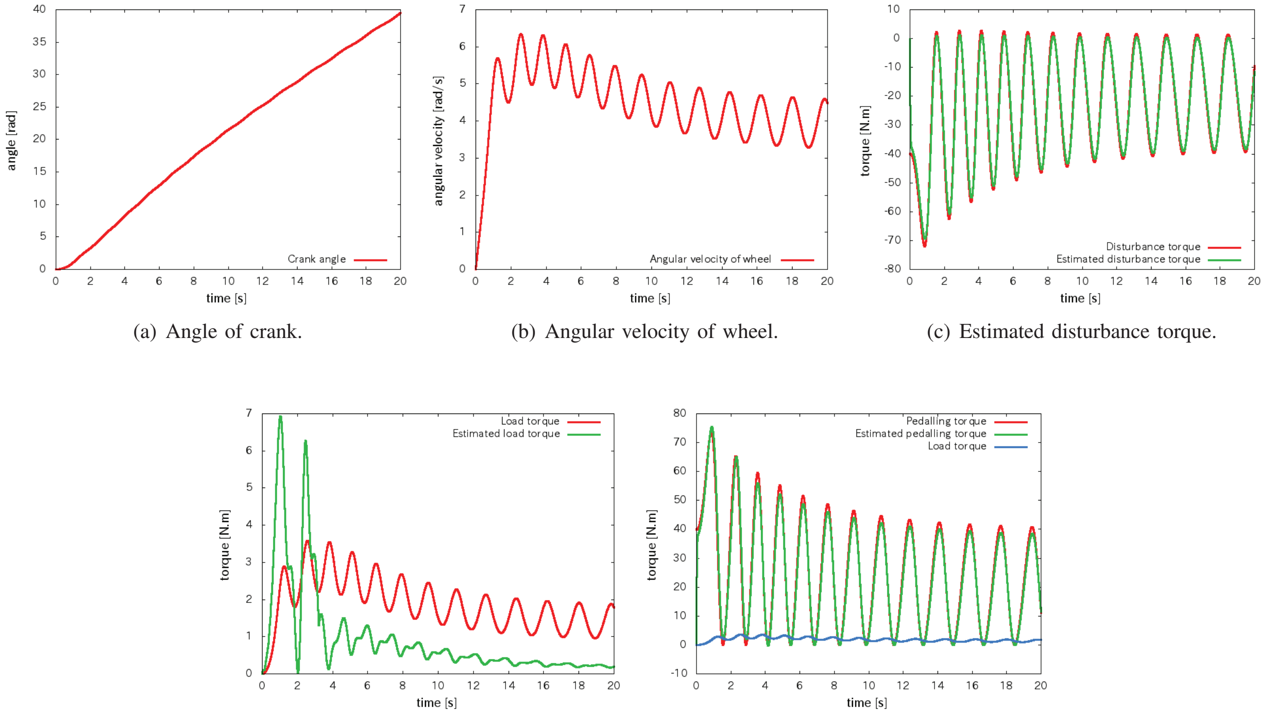
In general, the torque of motor is simply expressed in (5). Our experimental system of the electric power assisted bicycle utilizes a brushless dc motor whose control input is a terminal voltage given as a form of the duty ratio. If we assume that the electrical time constant is negligible compared to the mechanical time constant, (5) can be rewritten as follows.

$$\tau_{motor} = K'_\tau d - D\omega \tag{19}$$

d is duty ratio which we command. We measured points where the motor torque statically balances with the load torque arbitrary applied. Therefore $\omega = 0$. The result is shown in Fig. 9.

From this result, we fit a straight line by the least squares method. The inclination of this line is K'_τ. We can see that there is a dead band between the duty ratio and the motor torque. No torque is appeared when $d \leq d_1 = 0.028$. The motor torque has property as follows in the state of $\omega = 0$.

$$\tau_{motor} = \begin{cases} K'_\tau(d - d_1) & (d > d_1) \\ 0 & (0 \leq d \leq d_1) \end{cases} \tag{20}$$



(a) Angle of crank.

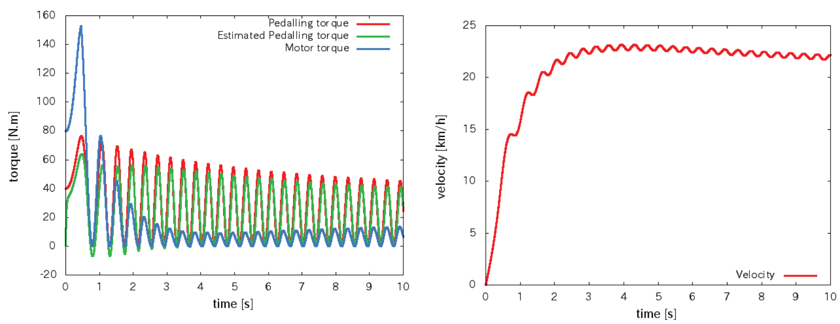
(b) Angular velocity of wheel.

(c) Estimated disturbance torque.

(d) Estimated load torque.

(e) Estimated pedaling torque.

Fig. 7: Simulation results of pedaling torque estimation without power assistance.



(a) Estimated pedaling torque with power assistance.

(b) Velocity of bicycle.

Fig. 8: Simulation results of pedaling torque estimation with power assistance.

B. Characteristic of angular velocity and torque

(6) can be simplified as follows.

$$J\dot{\omega} = -D\omega + \tau_{motor} - \tau_{dis} \tag{21}$$

When no load is applied to the motor and the system is in the steady state, (21) can be represented by

$$\tau_{motor} = D\omega + \tau_{f0} \tag{22}$$

where $D\omega + \tau_{f0}$ represents Coulomb and viscous friction. The measured result is shown in Fig. 10.

We fit a straight line by the least squares method. The static friction torque τ_{f0} is identified as 18.16[Nm]. The characteristic of the friction torque τ_f with respect to the angular velocity is represented as follows.

$$\tau_f = \begin{cases} D\omega + \tau_{f0} & (\omega > 0) \\ 0 & (\omega = 0) \end{cases} \tag{23}$$

C. Nonlinear disturbance observer

Taking these nonlinear characteristics into account, (8) can be converted as follows.

$$\hat{\tau}_{dis} = \begin{cases} \dfrac{g}{s+g}\{-(D\omega+\tau_f)+K_\tau'(d-d_1)-J_n s\omega\} \\ \qquad\qquad \text{if } (d > d_1, \omega > 0) \\ \dfrac{g}{s+g}\{-(D\omega+\tau_f)-J_n s\omega\}, \\ \qquad\qquad \text{if } (d \leq d_1, \omega > 0) \end{cases} \tag{24}$$

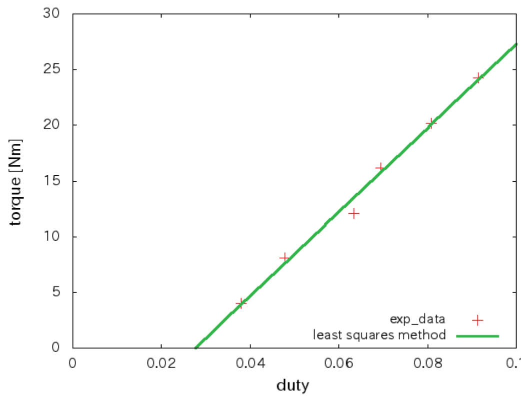


Fig. 9: Measured characteristics of torque v.s. duty ratio.

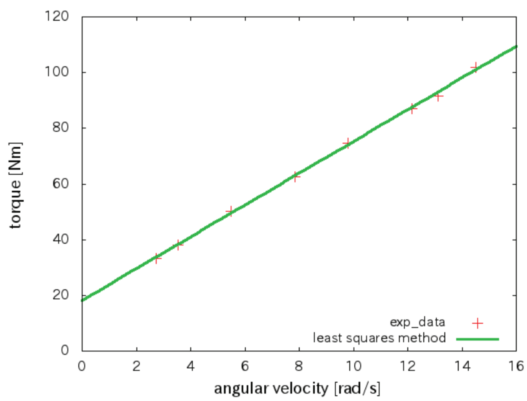


Fig. 10: Measured characteristics of torque v.s. angular velocity.

Using the estimated disturbance torque by the nonlinear disturbance observer, we can estimate the estimated instantaneous pedaling torque more accurately.

VI. Experiment

This section investigates the validity of the proposed system by experiments. An apparatus for the experiments is shown in Fig. 11. This bicycle is a front-assisted-type electric-power-assisted-bicycle sold in the market. The motor is a brushless DC motor whose rated output is 250W. The original controller was replaced with the proposed one. The conventional controller determines the motor torque from measured torque by a sensor. The proposed one estimates the pedaling torque and determines the motor torque based on the estimated value. The torque sensor is equipped with the crank box, but it is not used in our control. We obtain the angle of the wheel from the signal of hall sensors which are attached in the motor. Considering the gear ratio, the angle resolution of the wheel is 0.74[degree]. The experimental condition is straight road without inclination. Parameters are shown in TABLE III.

A. Experiment without power assistance

This section describes experiments of the pedaling torque estimation by the pedaling torque estimator. First, we estimate the pedaling torque without power assistance. We start the measurements when the speed becomes constant. We imposed the pedaling torque so that the bicycle runs at a constant speed. The result is shown in Fig. 12.

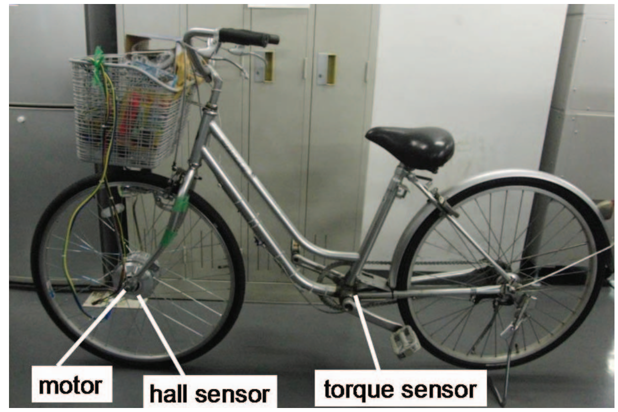


Fig. 11: Experimental system.

TABLE III: Experiment parameter.

J_n [kgm^2]	8.71 (M=80kg,r=0.33m)
D [Nms/rad]	5.81
K'_τ [Nm]	375
R [Ω]	0.2
g [rad/s]	50
d_1	0.028
τ_{f0} [Nm]	18.16

The pedaling torque is almost estimated by the pedaling torque estimator at $0 \leq t \leq 2$ and $t \geq 5$. However, there are some periods when the value of the estimated pedaling torque is higher than the actual value obtained by the torque sensor. There are several reasons of the error. One is a modeling error of friction of the bicycle. There is friction at the chain and the rear wheel and the front one. The other is that the resolution of the detection angle is quite low. The angular velocity is calculated by the information of the angle of wheel from the hall sensor. The angular velocity is very important in the proposed method to estimate the pedaling torque. In addition, the delay of estimation was observed slightly. There might be the delay to transmit the torque between the pedal and the front wheel. It is necessary to improve these problems in order to obtain a better accuracy.

B. Experiment with power assistance

We set the assist torque whose value is the half of the torque limited by the law. The actual pedaling torque and the estimated pedaling torque are shown in Fig. 13.

We obtained the similar results to those in the previous part. We can observe the periodically and intensely change of torque. From these results, the motor torque does not affect the pedaling torque estimator.

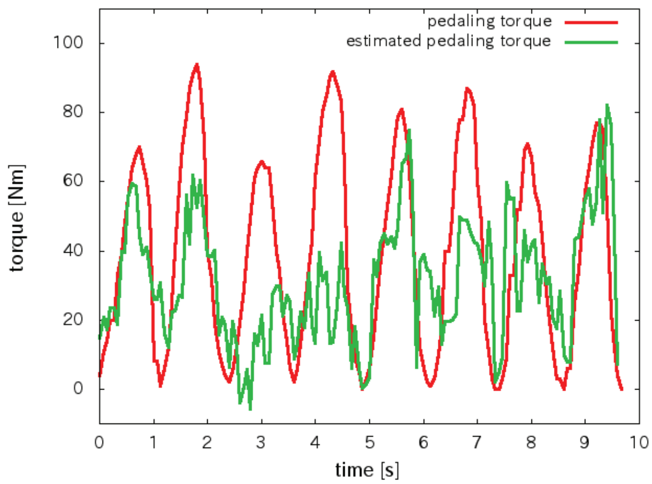


Fig. 12: Estimated pedaling torque without power assistance.

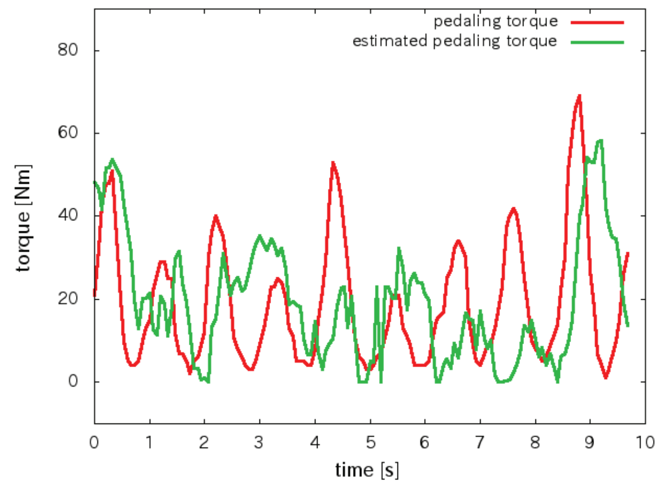


Fig. 13: Estimated pedaling torque with power assistance.

VII. Conclusion

This paper described the pedaling torque estimator and investigated the validity of the proposed method via simulations and experiments. We modeled the torque characteristics of the bicycle by the fundamental measurements. We found that the bicycle has the nonlinear characteristics. Finally, we were able to estimate in part the periodic pedaling torque by the experiment using the proposed method.

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