Torque Control of Power-Assisted Bike via Planetary Gear Mechanism

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***Abstract*--This paper presents an electronically controlled continuously variable transmission (E-CVT) function for a power-assisted system designed for use in a bike and implemented by a single planetary gear and an electric motor. The kinematic analysis of the planetary gear is investigated based on the block diagram representation. Moreover, the design of feedforward controlled speed ratio of the power-assisted bike is coordinated with pedaling speed. The performance of the E-CVT power-assisted system is evaluated experimentally in this study.**

***Index Terms*—power-assisted bike, E-CVT system, planetary gear, EMD process**

I. Introduction

Considering the need for reducing transportation costs and pollution, as well as improving city mobility, power-assisted bikes (PABs) have gained more and more popularity in the recent past [1][2]. Without loss of generality, it is known that the PABs can offer a transportation alternative inside cities. A PAB, regarded as a conventional bicycle with power assistance, is only activated while pedaling, where the power assistance comes from an electric motor. The rider and the power assistance system in PAB share the driving effort such that the cyclist's fatigue and physical effort can be reduced compared to a standard bike [3]. A PAB system is endowed with a transmission control system governed by the combined management of the ride-ability and power assistance.

It is interesting to investigate the role of transmission system in bike. Human pedaling have a narrow preferred pedaling cadence; but to accommodate the feature of the undulating terrain or city traffic, it is required to change the gears often [4][5]. At the moment, the multi-sprocket and derailleur systems are the cheapest and most efficient transmissions for urban transportation. However, a suitable transmission requires well-adjustment to prevent failure of engagement, resulting in a falling hazard for the rider. Accordingly, an improvement in the transmission system of PABs is to achieve a fine and automatic regulation of the transmission ratio [4]. In this respect, a flexible transmission system to be considered is a so-called continuously variable transmission (CVT), by which the transmission regulation is smooth along entire ratio range while in motion [6]. In this study, a transmission system based on a planetary gear is employed to realize the electronically controlled continuously variable transmission (E-CVT). Due to its transmission feature of 2 inputs/1 output, this architecture effectively integrates the cyclist's pedaling power and the power assistance from the electric motor. For further transmission control, the kinematic characteristic is then analyzed via the block diagram representation [7], which is often adopted in the control community. The kinematic torque and speed relationships between the cyclist's pedaling and the motor are clearly determined.

Moreover, it should be noted that the power assistance strategy adopted in the PABs plays a crucial role. For a PAB with an E-CVT system, the power assistance from the electric motor is generated according to a predesigned regulation strategy, which is subjected to different riding conditions. General speaking, there are two kinds of assisted power methods for the practical PAB system, which are the constant-assisted power (CAP) method and the proportion-assisted power (PAP) method [8][9]. Although the CAP method is easier to be implemented, the assisted power is provided without considering the riding environment and the pedaling conditions such that it may directly impact the comfortability and safety to the cyclist. By contrast, based on the PAP method, the assisted ratio can be regulated with the pedaling cadence. While the pedaling cadence is lower than a pre-defined value, the power assistance is maintained at the maximum assisted ratio. Otherwise, the assisted ratio is being gradually decreased according to the increasing pedaling cadence. Different designs for PAP method are discussed in this study. The feasibility of the proposed E-CVT system based on planetary gear and the power assisted method are verified via the experimental study.

II. System Model Description

In this investigation, a planetary gear-based E-CVT system is utilized to simulate the performance for the PAB system. The system model is represented as a block diagram representation to characterize the pedaling behavior and the riding conditions. The kinematic relationships of the E-CVT system are also discussed in this section.

A. Modeling of Bike System

In this section, the dynamics of a conventional bike system is first considered as shown in Fig. 1. From the point of view of control, there are two components of interests: the road load and the transmission mechanism. Under the assumption that the bike system moves straightly, the balance of the force acting on the bike yields the road load equations as [11]:

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longitunal: $F_R - F_F - F_d - (Mg + mg)\sin\theta$
rear wheel: $T/N - B_w\omega - F_R R = I\alpha$ (1)
front wheel: $F_F R_{\text{Wheel}} - B_{\text{Wheel}}\omega_{\text{Wheel}} = I\alpha$

where the parameters utilized for the model description are defined in Table I. The subscript *i=F, R* denotes the front and rear wheels, respectively.

Combing the road load equations, (1) can be rewritten as

$$\begin{aligned} T &= (2I + MR^2)\alpha + 2B_w\omega + F_d R + R(M+m)g\sin\theta \\ &= J_{\text{Bike}}\alpha + B_{\text{Bike}}\omega + T_{Load} \end{aligned} \tag{2}$$

where J_{Bike} and B_{Bike} are separately the equivalent moment of inertia and damping coefficient of the bike system. T_{Load} represents the external load torque including the effects of wind resistance, the frictional force, and the slope.

Moreover, an equivalent block diagram description for the bike system is given as shown in Fig. 2. It should be noticed that the pedaling torque inherently oscillates with the pedaling angle, which can be approximately expressed as:

$$T_{\text{Ped}} = F_{\text{Ped}} R_{\text{Crank}} \times \left|\sin(\theta_{\text{Ped}} + \theta^*)\right| \tag{3}$$

where F_{Ped} is the pedaling torque from the cyclist, θ_{Ped} and θ^* the pedaling and initial angle of the crank, and R_{Crank} the radius of the rank. From (3), it is clear to see that the pedaling torque oscillates with two peaks per crank revolution and causes a significant discomfort to the cyclist. Therefore, a transmission regulation based on E-CVT is proposed in this study in order to reduce the effect of oscillating pedaling torque.

TABLE I
PARAMETERS IN BIKE SYSTEMS

M, m	Mass of bike and cyclist, respectively
g	gravitational acceleration
R_{Wheel}	radius of wheel
B_w	damping coefficient of wheel
θ	slope angle
N	speed ratio of transmission system
T_{Ped}	pedaling torque
F_i, F_d	frictional force, wind resistance
τ_i	induced torque by frictional force
a, α	longitudinal and angular acceleration
ω_{Ped} , ω_{Wheel}	angular velocity of pedaling and wheel

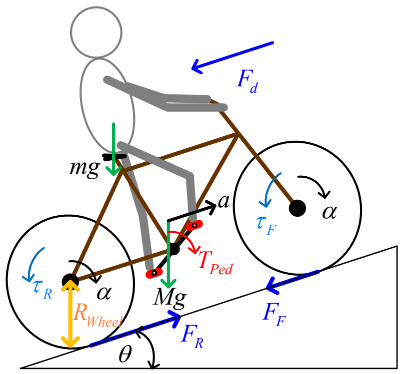


Fig. 1. Free-body diagram of bike system.

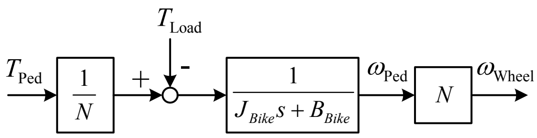


Fig. 2. Equivalent block diagram of bike system

B. Kinematic Analysis of E-CVT

Based on the 2-input/1 output transmission feature of the planetary gear, an E-CVT system can be realized by regulating the input port, which is connected to the electric motor. An illustrative configuration for the planetary gear utilized in this study is depicted in Fig. 3. Generally, there are 6 different input/output combinations of a planetary gear for different applications. Considering the requirement of a bike transmission system, the rotating direction of the input (the cyclist's pedaling) and the output (connected to the rear wheel) of the E-CVT system should be the same. The transmission should also be an increasing gear ratio such that increasing the input pedaling cadence results in an increased output through the E-CVT system. Accordingly, the carrier and the sun gear are determined as the one of the input ports (for pedaling) and the output port in this study, respectively. The ring gear is utilized as the other input port and connected with the electric motor. The ring gear regulates the speed ratio regulation of the E-CVT system.

The torque and speed kinematic relationships of the transmission configuration in Fig. 3 are given as

$$\omega_S = \frac{2R_C}{R_S}\omega_C - \frac{R_R}{R_S}\omega_R \;\&\; T_S = \frac{R_S}{R_C}T_C - \frac{R_S}{R_R}T_R \tag{4}$$

where R_i , ω_i and T_i are the radius, speed, and torque of each gear component, respectively. The subscript *i=C, S, R, P* denotes separately the carrier, sun gear, ring gear, and planet gears. In Fig. 4(a), the pedaling torque and the torque assistance from the electric motor can be integrated at the output port based on the transmission feature of 2 inputs/1 output. Accordingly, the speed ratio of this proposed transmission system can then be regulated by controlling the torque generated from the motor, which is given as $T_S = \alpha_T(\omega_C)T_C$. The index α_T indicates the level of torque assistance and is related to the input pedaling cadence ω_C . In other words, the angular speed of the motor ω_R is regulated according to ω_C and a designed transmission index α_ω such that $\omega_R = \alpha_\omega(\omega_C)\omega_C$. The speed relationship in (4) is then rewritten as

$$\omega_S = \frac{2R_C - \alpha_\omega R_R}{R_S}\omega_C \tag{5}$$

Therefore, the continuously variable transmission can be achieved by regulating the transmission index α_ω.

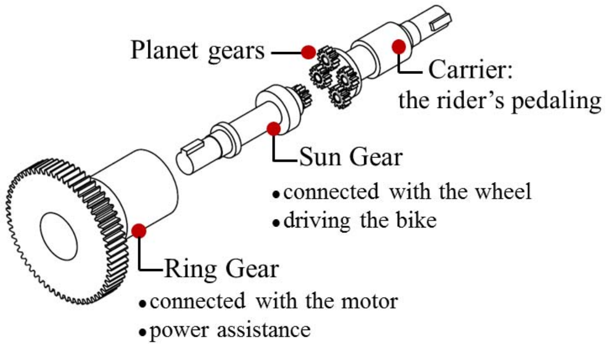
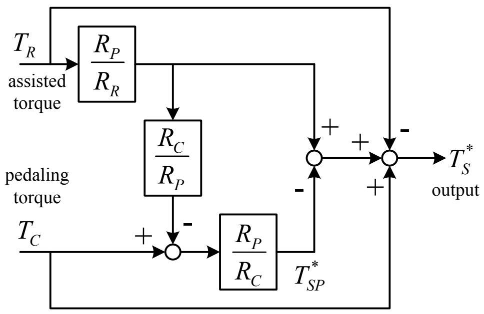
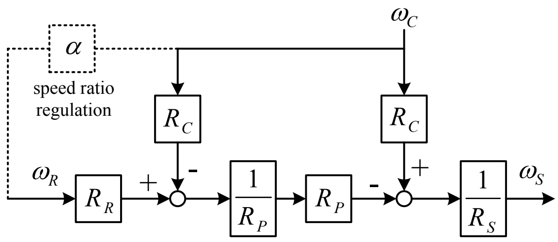


Fig. 3. Configuration of planetary gear-based E-CVT



(a) Output torque regulation of E-CVT



(b) Speed ratio regulation of E-CVT

Fig. 4. Transmission mechanism of planetary gear-based E-CVT

III. STRATEGY OF TRANSMISSION REGULATION

A planetary gear-based E-CVT system is developed in this study. The speed ratio can be regulated by an electric motor and a designed transmission index α_ω. According to the basic operating principle of CVT, the speed ratio should be regulated in accordance with the pedaling cadence. In other words, the power assistance from the E-CVT system would be regulated via controlling the speed ratio in this study such that α_ω varies with the different pedaling cadence.

The PAP method is proposed by Japan's industry and has been adopted by regulation. General speaking, the PAP method can be easily formulated by the linear piece-wise equation and the exponential equation as shown in Fig. 5(a) and (b), respectively, and the equation formulations are as follows.

$$\alpha_{\omega\_\mathrm{LPW}} = \begin{cases} 1 & \omega_C \le \omega_\mathrm{L} \\ 1 - \dfrac{\omega_C - \omega_\mathrm{L}}{\omega_\mathrm{H} - \omega_\mathrm{L}} & \omega_\mathrm{L} < \omega_C \le \omega_\mathrm{H} \\ 0 & \omega_C > \omega_\mathrm{H} \end{cases} \tag{6}$$

$$\alpha_{\omega\_\mathrm{EXP}} = \begin{cases} 1 - e^{(\omega_C - \omega_\mathrm{H})/\omega_\mathrm{L}} & \omega_C \le \omega_\mathrm{H} \\ 0 & \omega_C > \omega_\mathrm{H} \end{cases} \tag{7}$$

where $\alpha_{\omega\_\mathrm{LPW}}$ and $\alpha_{\omega\_\mathrm{EXP}}$ denote the transmission ratios based on the linear piecewise and exponential approaches, respectively. ω_L and ω_H are separately the pre-defined indexes for the transmission regulation. For the purpose of cyclist's safety, the assisted power is regulated in both approaches when $\omega_C > \omega_\mathrm{H}$. However, the non-differentiable characteristic at the switch of different ranges of pedaling cadence would cause the sudden variation of angular acceleration, as shown in Fig. 6. In consequence, the non-smoothness would become more noticeable and cause discomfort to the cyclist.

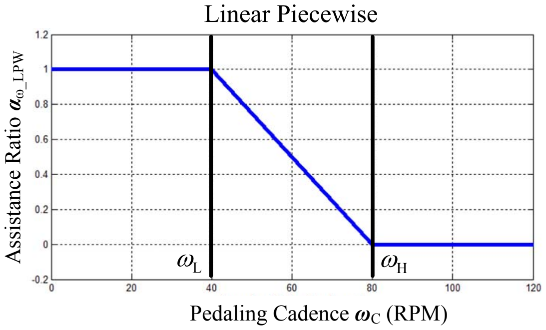
Therefore, a PAP method formulated by a S-shape function is proposed in this study, which is given as

$$\alpha_\omega = \begin{cases} 1 & \omega_C \le \omega_\mathrm{L} \\ S(\omega_C, \omega_\mathrm{L}, \omega_\mathrm{H}) & \omega_\mathrm{L} < \omega_C \le \omega_\mathrm{H} \\ 0 & \omega_C > \omega_\mathrm{H} \end{cases} \tag{8}$$

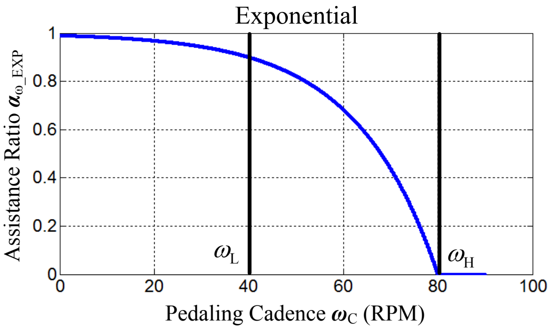
where $S(\bullet)$ is the S-shape function and determined as

$$\begin{aligned} & S(\omega_C, \omega_\mathrm{L}, \omega_\mathrm{H}) \\ &= \begin{cases} 1 - 2\left(\dfrac{\omega_C - \omega_\mathrm{H}}{\omega_\mathrm{H} - \omega_\mathrm{L}}\right)^2 & \omega_\mathrm{L} < \omega_C \le \dfrac{(\omega_\mathrm{L} + \omega_\mathrm{H})}{2} \\ 2\left(\dfrac{\omega_C - \omega_\mathrm{H}}{\omega_\mathrm{H} - \omega_\mathrm{L}}\right)^2 & \dfrac{(\omega_\mathrm{L} + \omega_\mathrm{H})}{2} < \omega_C \le \omega_\mathrm{H} \end{cases} \end{aligned} \tag{9}$$

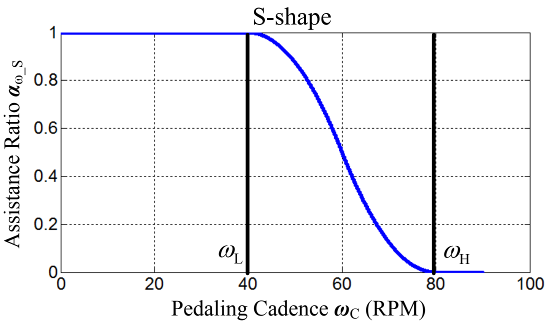
While the input velocity ω_C is lower than ω_L, the motor regulates the ring gear as $\omega_R = \omega_C$ such that the E-CVT system is operated at the maximum assistance ratio $\alpha_\omega = 1$. Otherwise, α_ω gradually decreases with the determined S-shape function while ω_c increases in the range $\omega_\mathrm{L} < \omega_C \le \omega_\mathrm{H}$. The assistance will be suspended while ω_C is over ω_H to avoid the over assistance. Because the S-shaped power assisted method is a continuous differentiable function, the speed ratio can be smoothly regulated along entire transmission ratio range as shown in Fig. 6.



(a) Linear piecewise approach



(b) Exponential approach



(c) S-shape approach

Fig. 5. Different approaches of PAP method

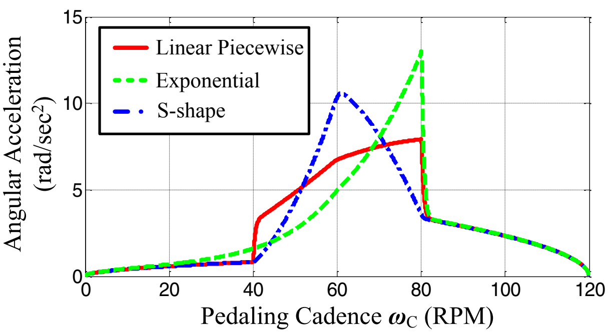


Fig. 6. Angular accelerations in different PAP approaches

IV. Experimental Results

The feasibility of the proposed planetary gear based E-CVT and the transmission strategy are evaluated by a simulated pedaling operation in experiment, as depicted in Fig. 7, where the pedaling behavior is realized by a pedaling mechanism connected to a planetary gear by a belt. In this proposed test platform for the E-CVT system, the two electric motors are utilized separately to regulate the speed ratio regulation and simulate external loads. Moreover, the torque sensor is equipped to measure the simulated load torque. The pedaling cadence is calculated based on the information of encoders on the electric motors.

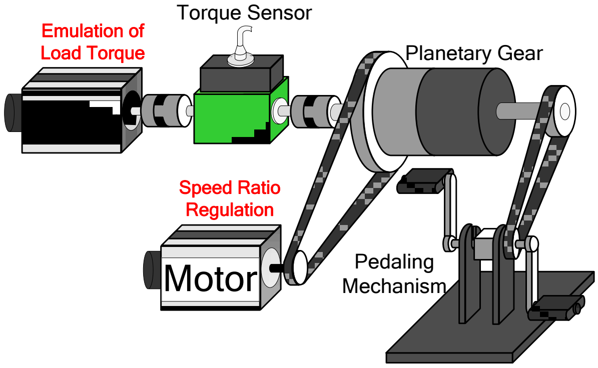
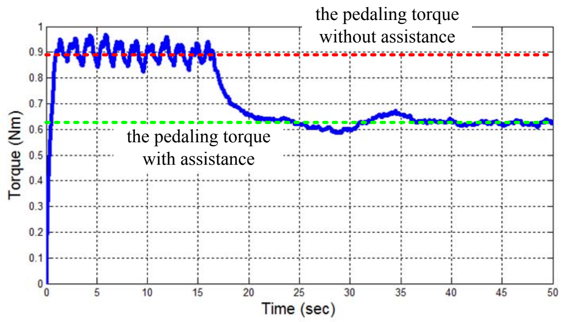
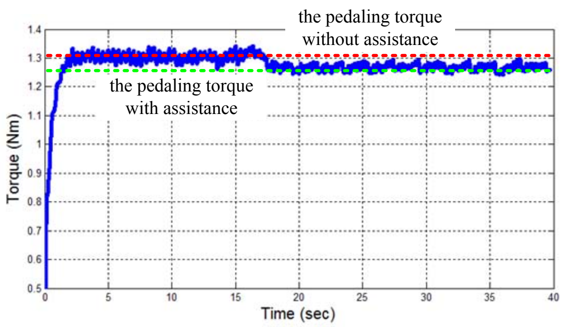


Fig. 7. Experimental test platform for E-CVT system

Firstly, the power assistance from the proposed E-CVT system is examined by comparing the pedaling torque with and without the assisted torque as shown in Fig. 8. In Fig. 8(a), the output port of E-CVT system is maintained at a lower speed as 40rpm, in which a higher assistance ratio $\alpha_{\omega} = 1$ is regulated according to the predesigned strategy. The pedaling torque can be reduced from 0.9Nm to 0.63Nm with the assistance from the electric motor. A similar effect of power assistance can also be found in Fig. 8(b), where the output port of the E-CVT system is operated at a higher speed 150rpm. It should be noted that the assistance ratio decreases while the pedaling cadence increases in this study. Thus, the reduction in pedaling torque is smaller than in lower cadence.



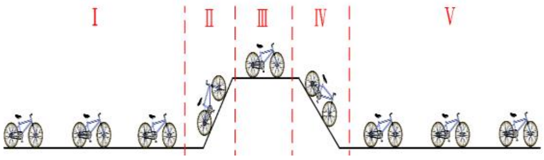
(a) Lower output speed: 40rpm



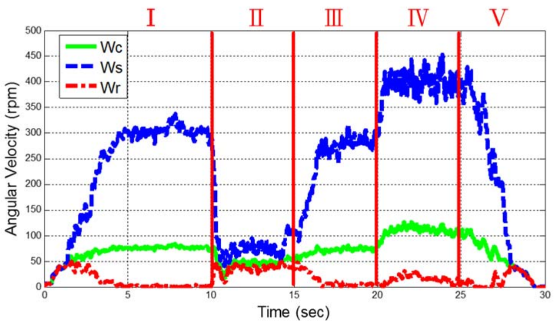
(a) Higher output speed: 150rpm

Fig. 8. Power assistance from the E-CVT

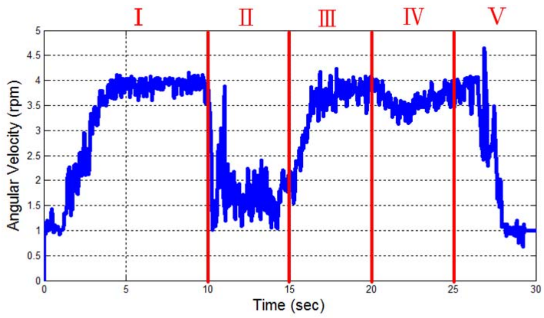
Moreover, this experimental study includes five different riding situations: from start to high pedaling speed, steep uphill riding, smooth riding, steep downhill riding, and decelerating to stop as shown in Fig. 9(a). The experimental results are shown in Fig. 9(b) and (c). In Fig. 9(b), the ring gear rotates at a higher angular velocity to assist the starting and decelerating to zero for maintaining the highest speed ratio. In uphill riding, the electric motor, which is connected to the ring gear of the E-CVT system, generates the assistance based on the proposed modified transmission strategy. In downhill riding, the ring gear rotates as an additional load to decrease the speed ratio for maintaining the highest pedaling speed as predetermined. The results confirm that the proposed E-CVT system and the transmission strategy achieve comfortable, efficient and safe riding.



(a) Emulated riding situations



(b) Angular velocities of each gear component of E-CVT system



(c) Speed ratio regulation

Fig. 9. Bike Emulation of different road situations

V. CONCLUSION

This paper presented an E-CVT architecture based on a planetary gear mechanism for power-assisted bikes. The block diagram representation is adopted to easily analyze the bike behavior and the kinematic characteristics of the E-CVT system. Moreover, the regulation strategy of the E-CVT system ensures that the power assistance can be regulated according to the pedaling cadence. Moreover, the effect caused by different PAP approaches are discussed and a S-shape based PAP method is then proposed in this study. The transmission regulation would consequently be smooth along the entire ratio range while in motion. The simulations of different riding situations were also given in this study to verify the feasibility of the proposed E-CVT system and its power assistance for bikes.

ACKNOWLEDGMENT

This work is supported by the Ministry of Science and Technology (MOST) of Taiwan, under grant number MOST 105-2218-E-006-008.

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