Homework-3

ENPM662: Introduction to Robot Modelling

SAI DINESH GELAM

UID: 120167140

1.1) DH coordinate frames

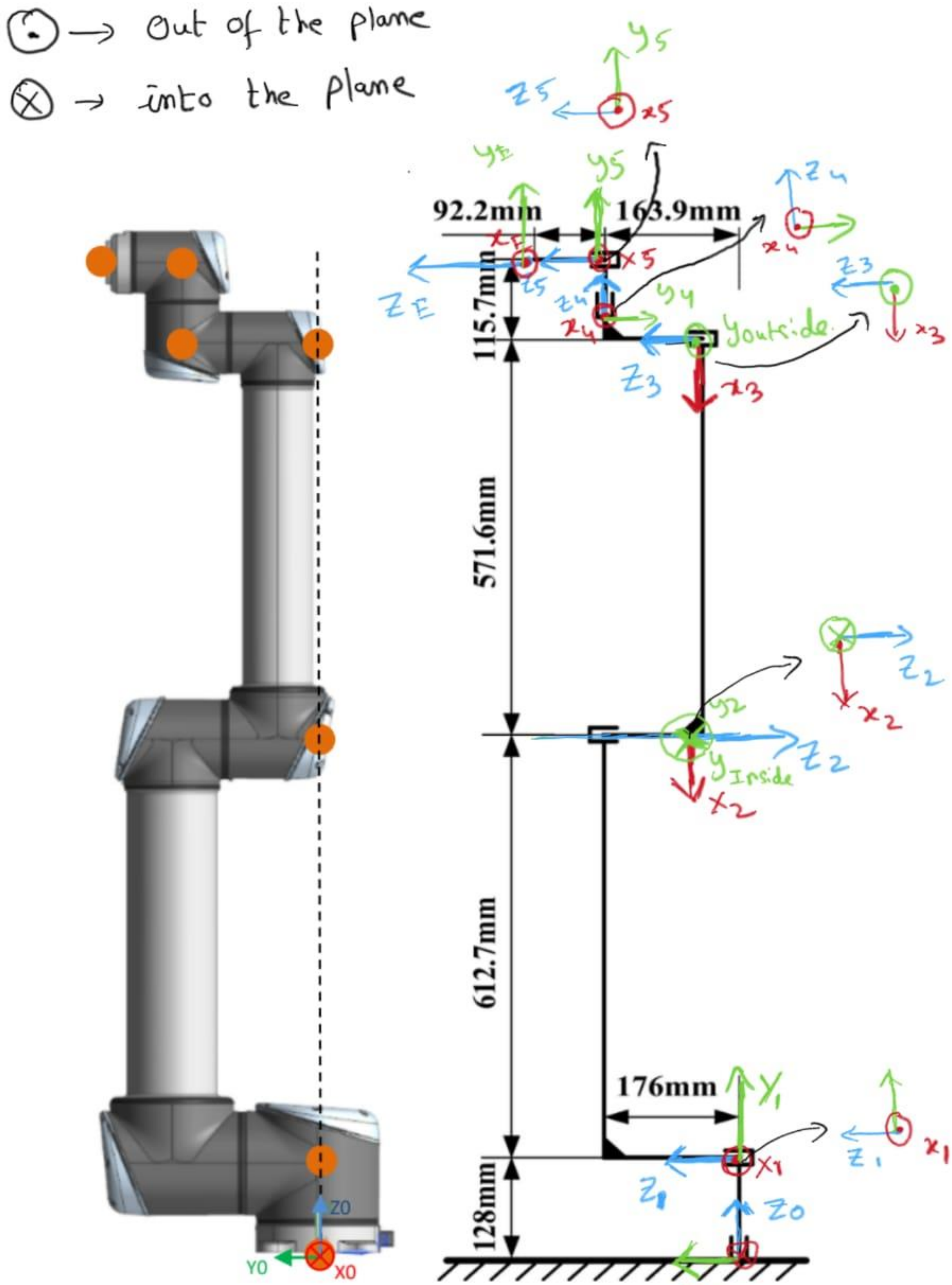


Fig. 2: UR10 - Home configuration

The following rules are used to fill the DH table.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. If joint i is prismatic, d_i is variable.

α_i = the angle from z_{i-1} to z_i measured about x_i.

θ_i = the angle from x_{i-1} to x_i measured about z_{i-1}. If joint i is revolute, θ_i is variable.

1.2) D-H Table

links	$\boldsymbol{a_i}$	$\boldsymbol{\alpha_i}$	$\boldsymbol{d_i}$	$\boldsymbol{\theta_i}$
0-1	0	90	128	θ_0+180
1-2	-612.7	180	0	$\theta_1 - 90$
2-3	-571.6	180	0	θ_2
3-4	0	-90	163.9	$\theta_3 + 90$
4-5	0	90	115.7	θ_4
5-E	0	0	92.2	θ_5

1.3) Transformation matrix

Transformation matrix from link to link is given by.

$$T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\propto_i & \sin\theta_i \sin\propto_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\propto_i & -\cos\theta_i\sin\propto_i & a_i\sin\theta_i \\ 0 & \sin\propto_i & \cos\propto_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Applying the above matrix to every successive link to get transformation matrix

Note : lets take all the $\boldsymbol{\theta_i}$ = 0 for simplification.

Transformation matrix from frame 0 – 1

$${}^0_1T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T01

$$\begin{bmatrix} -\cos(\theta_0) & 0 & -\sin(\theta_0) & 0 \\ -\sin(\theta_0) & 0 & \cos(\theta_0) & 0 \\ 0 & 1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix from frame 1 – 2

$$ {}^{1}_{2}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 612.7 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T12

$$\begin{bmatrix} \sin(\theta_1) & -\cos(\theta_1) & 0 & -612.7\cdot\sin(\theta_1) \\ -\cos(\theta_1) & -\sin(\theta_1) & 0 & 612.7\cdot\cos(\theta_1) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix from frame 2 – 3

$$ {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & -571.6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T23

$$\begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 & -571.6\cdot\cos(\theta_2) \\ \sin(\theta_2) & -\cos(\theta_2) & 0 & -571.6\cdot\sin(\theta_2) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix from frame 3 – 4

$$ {}^{3}_{4}T = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T34

$$ \begin{bmatrix} -\sin(\theta_3) & 0 & -\cos(\theta_3) & 0 \\ \cos(\theta_3) & 0 & -\sin(\theta_3) & 0 \\ 0 & -1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix from frame 4 – 5

$$ {}^{4}_{5}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T45

$$ \begin{bmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & -\cos(\theta_4) & 0 \\ 0 & 1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix from frame 5 – E

$$ {}^{5}_{\mathrm{E}}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Actual matrix will be (when $\boldsymbol{\theta_i} \neq \mathbf{0}$)

T56

$$\begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.4) Final transformation matrix

To get the final orientation of the final end effector with respective to the base frame we need to post-multiple each and every above matrix

$$ {}^{0}_{\mathrm{E}}T = {}^{0}_{1}T * {}^{1}_{2}T * {}^{2}_{3}T * {}^{3}_{4}T * {}^{4}_{5}T * {}^{5}_{\mathrm{E}}T $$

We get the following matrix.

$$ {}^{0}_{\mathrm{E}}T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 256.1 \\ 0 & 1 & 0 & 1428 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

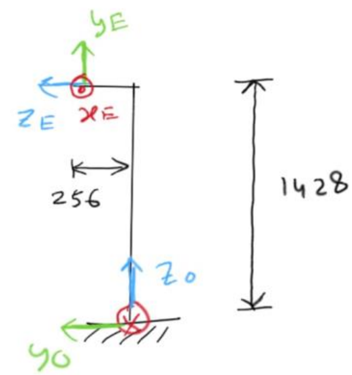
The actual matrix will be very big and cannot be pasted here, however it will be generated in the code which will be attached to the file.

1.5) Validation by geometrical method

Now let's validate the final matrix with some known orientations using a Geometrical approach.

Case-1:

The initial orientation (**$\boldsymbol{\theta}_{\mathrm{i}}$ =** 0 degrees), we get the following configuration



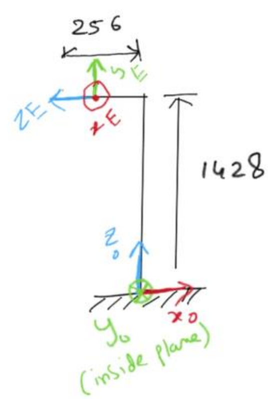
[x,y,z]=[0,256.1,1428.0]

$$ {}^{0}_{\mathrm{E}}T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 256.1 \\ 0 & 1 & 0 & 1428 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Case-2:

In the initial orientation lets rotate $\boldsymbol{\theta}_0$ 90 degrees, we get the following configuration

[x,y,z]=[-256.1,0,1428.0]

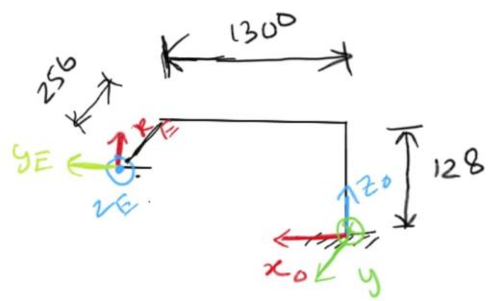


$$
{}^{0}_{E}T = \begin{bmatrix} 0 & 0 & -1 & -256.1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1428 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Case-3:

In the initial orientation lets rotate $\boldsymbol{\theta}_1$ by 90 degrees, we get the following configuration

[x,y,z]=[1300,256,128]

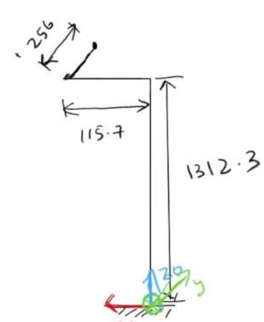


$$
{}^{0}_{E}T = \begin{bmatrix} 0 & 1 & 0 & 1300 \\ 0 & 0 & 1 & 256 \\ 1 & 0 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Case-4:

In the initial orientation lets rotate $\boldsymbol{\theta}_3$ by 90 degrees, we get the following configuration

[x,y,z]=[115.7,256.1,1312.3]

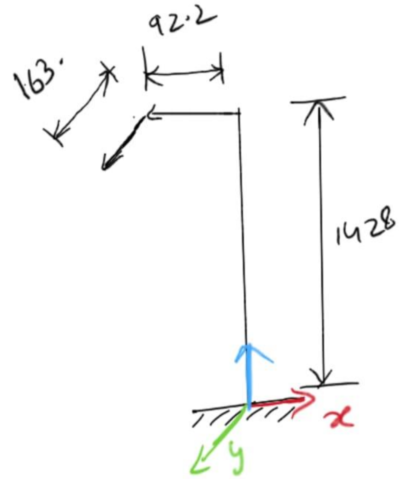


$$
{}^{0}_{\mathrm{E}}T = \begin{bmatrix} 0 & 1 & 0 & 115.7 \\ 0 & 0 & 1 & 256.1 \\ 1 & 0 & 0 & 1312.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Case-5:

In the initial orientation lets rotate $\boldsymbol{\theta}_4$ by 90 degrees, we get the following configuration

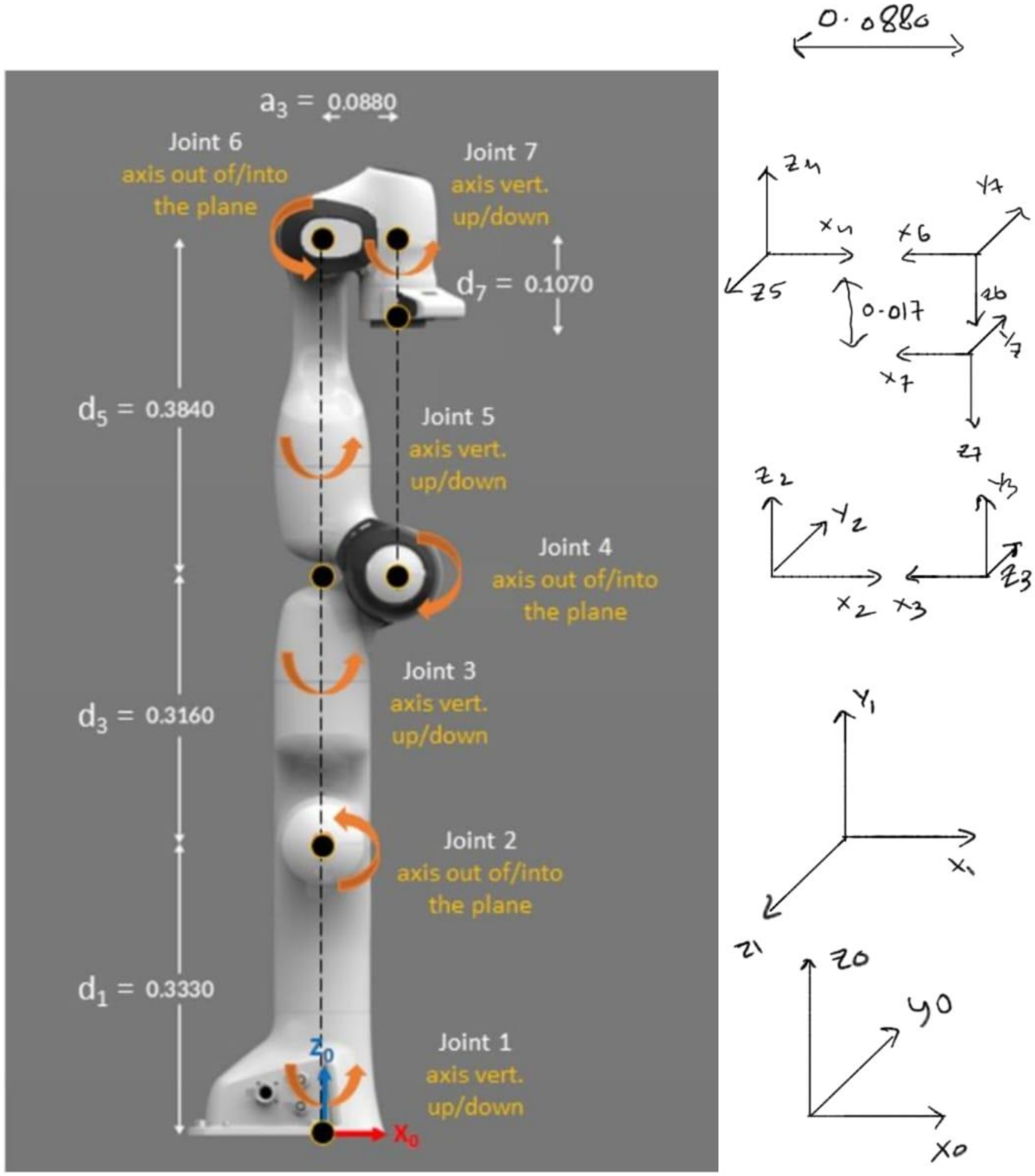
[x,y,z]=[-92.2,163.9,1428]



$$
{}^{0}_{\mathrm{E}}T = \begin{bmatrix} 0 & 0 & -1 & -92.2 \\ -1 & 0 & 0 & 163.9 \\ 0 & 1 & 0 & 1428 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

2) Position Kinematics- KUKA

2.1) D-H Coordinate frames:



2.2) DH parameters

DH parameters for the above-mentioned frames are:

Link	a_i	α_i	d_i	θ_i
0-1	0	90	0.33330	$\theta 1$
1-2	0	-90	0	$\theta 2$
2-3	0.088	90	0.3160	θ_3 +180
3-4	0.088	90	0	θ_4 +180
4-5	0	90	0.3840	θ_5
5-6	-0.088	-90	0	θ_6+180
6-7	0	0	0.107	θ_7