Homework 2

ENPM662: Introduction to Robot Modeling

Deadline: October 14th, 2022

Instructions:

1. Submit your assignment as your_directoryID.pdf

1.1 Composition of transforms

The world axes are fixed. Consider the following sequence of rotations and translations:

1. Rotate by ϕ about the world z-axis.
2. Translate by y along the current y-axis.
3. Rotate by θ about the current z-axis.
4. Rotate by ψ about the world x-axis.

Consider 4 × 4 homogeneous transformation matrices R_{angle} (with zero translation), $T_{distance}$ (with identity rotation). Write the matrix production equation using rotation matrices, R_{angle}, or translation matrices, $T_{distance}$, that will give the resulting pose of the frame and explain why you chose that order.

1.2 Modeling beyond rigid transformations

Assume that there exists a frame of reference on earth (F_e), which is centered at the earth's center with its Z axis through the north pole. Define t = 0 to be the exact moment of the summer solstice, and we define a fixed reference frame (F_f) to be coincident with the Earth's frame at time t = 0. Determine as a function of time the homogeneous transformation matrix that specifies the Earth's frame with respect to this fixed reference frame F_f (F_f is fixed in space). Assume that the orbit is elliptical with the sun at the center, and the earth completes one rotation in exactly 24 hours and one revolution in exactly 365 days. Feel free to use astronomical data for max and min distance between the sun and earth.

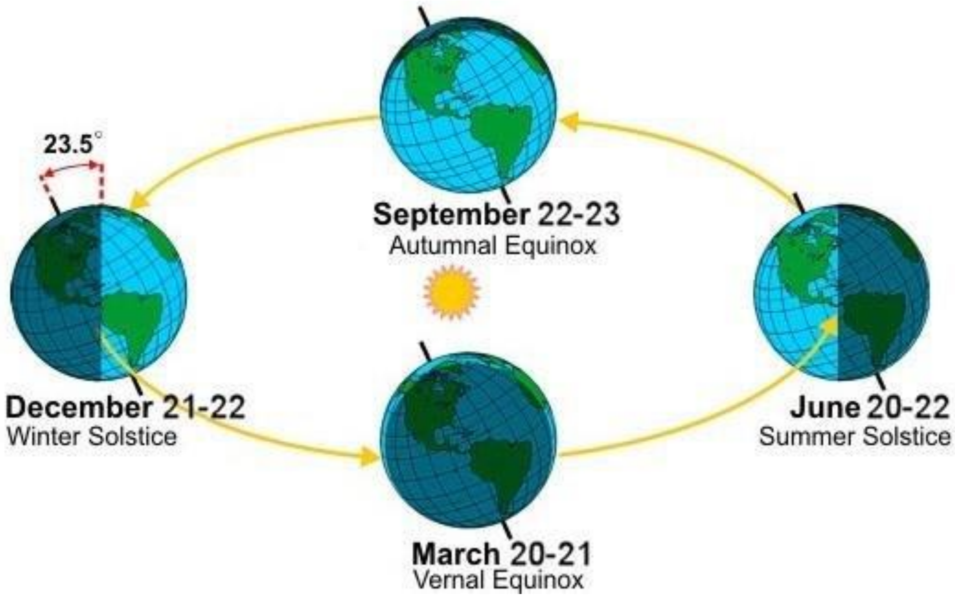

Figure 1: Motion of earth around the sun

1.2 Transform Estimation

Consider a cube shown in the following figure, with a side length of 1m. Find any such translations (x,y,z) and Euler angles (around X,Y,Z axis) which will result in a final position with EFGH plane on the cube coinciding with the XZ plane, and points F and G being on the Z and X axis respectively. Sketch your final cube orientation.

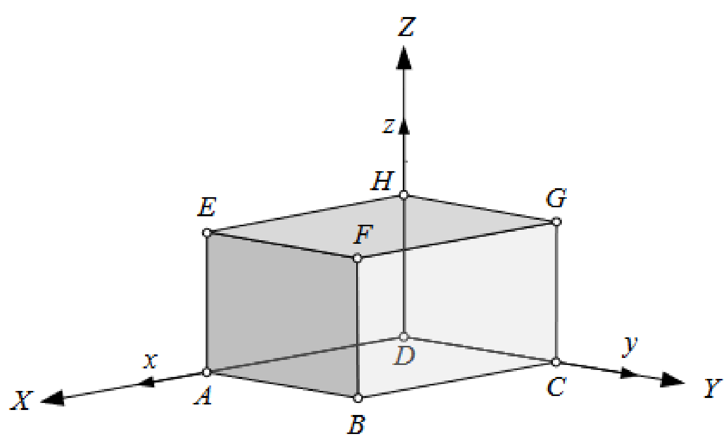

Figure 2: Cube with marked vertices

2.1 Trajectory Optimization

A satellite is revolving around earth, and gets instructions to move from a position X, Y, Z to X’, Y’, Z’ as shown in figure below. If its final orientation is $\psi g = 45^\circ$, $\theta g = 25^\circ$, $\phi g = 16^\circ$ and its maximum rate of turning around any axis is $\omega max = 2$ deg/s, plot its angular velocity trajectory such that it reaches the final orientation in the shortest amount of time. You can represent the trajectory as plots of six quantities ψ, θ, ϕ, ωx, ωy, ωz w.r.t time. Please describe your computations in the report.

Assumptions:

- The position X, Y, Z is aligned with the global frame of reference.
- ψ, θ, ϕ are consecutive rotations about the global X, Y, Z axes to obtain the satellite’s final orientation.
- ωx, ωy, ωz are angular velocities of the satellite about its local X, Y, Z axes.
- The satellite can change its angular velocities arbitrarily i.e., you can decide any profile for ωx, ωy , ωz (including initial and final values) as long as $|\omega x|, |\omega y |, |\omega z | \le \omega max$

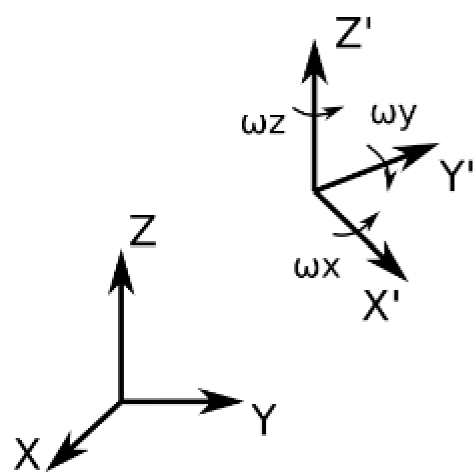

Figure 3: Illustration of angular speeds