Problem Set # 4

Due Date: Tuesday Dec 5 @ 11.59pm in ELMS

Instructions:

- Please write your solutions clearly and show all your work.
- Please number all the pages of your submission.
- The assignment will be submitted online via ELMS.
- It should be noted that what the author of the textbook calls poles of the system are actually the roots of the characteristic equation $det(\lambda\mathbf{I} - \mathbf{A}) = 0$. In other words they are the eigen values of $\mathbf{A}$.

Reading Assignment

- Course Text (Robot Modeling and Control): **6.1, 6.6, Appendix C**

Problem 1: Consider the following state equation:

$$\dot{\mathbf{x}}(t) = \frac{1}{12}\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}\mathbf{x}(t) + e^{t/2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}u(t)$$

- Determine if the aforementioned system is controllable or not? Please show all your work.

Problem 2: Consider the following system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \gamma_1(t) & \gamma_2(t) \\ -\gamma_2(t) & \gamma_1(t) \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $\gamma_1(t)$ and $\gamma_2(t)$ are continuous functions of t. Compute the state transition matrix of the system and the solution to the state equation.

[SHV 6.18] Problem 3: One of the problems encountered in space applications of robots is the fact that the base of the robot cannot be anchored, that is, cannot be fixed in an inertial coordinate frame. Consider the idealized situation shown in the Figure below, consisting of an inertia J_1 connected to the rotor of a motor whose stator is connected to an inertia J_2.
For example, J_1 could represent the space shuttle robot arm and J_2 the inertia of the shuttle itself.

The simplified equations of motion are given by:

$$J_1\ddot{q}_1 = \tau$$

$$J_2\ddot{q}_2 = \tau$$

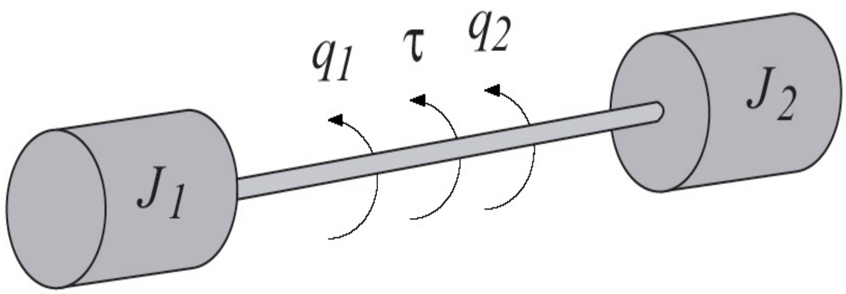

Figure 1: Coupled Inertias in Free Space

Write this system in state space form and show that it is uncontrollable. Discuss the implications of this and suggest possible solutions.

[SHV 6.19] Problem 4: Given the linear second-order system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & -3\\ 1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ -2\end{bmatrix} u$$

find a linear state feedback control $u = k_1x_1 + k_2x_2$ so that the closed loop system has poles at $s = -2, 2$

[SHV 6.20] Problem 5: Repeat the above problem if possible for the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u$$

Can the closed-loop poles be placed at -2?

Can this system be stabilized? Explain

The above system is said to be **stabilizable** which is a weaker notion than controllability

[SHV 6.21] Problem 6: Repeat the above for the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u$$

Problem 7: Show that if $y(t)$ is the output of a linear time-invariant system corresponding to input $u(t)$, then the output of the system corresponding to the input $\dot{u}(t)$ is given by $\dot{y}(t)$. Assume that the initial state is zero, $x(t_0) = 0$.

Problem 8: Find the solution of the following Linear-state equation

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 4 \\ 0 & -1 & 0 \end{bmatrix} \mathbf{x}(t) \,, \quad \mathbf{x}(t_0) = \mathbf{x}_0$$