Problem Set # 1

Due Date: Thursday Sept 21 in ELMS

Reading Assignment

1. Handout: Ordinary Differential Equations I (Posted on ELMS)
2. Handout: Ordinary Differential Equations II (Posted on ELMS)

Instructions:

- Please write clearly and show all your work. Credit will be given for all correct steps.
- The assignment will be submitted via ELMS.

Problem 1 (10pts): Find the values of α and β that make

$$dF(x,y) = \left(\frac{1}{x^2+2} + \frac{\alpha}{y}\right)dx + (xy^\beta + 1)dy$$

an **Exact Differential**. For these values solve the ODE $dF(x,y) = 0$.

Problem 2 (10pts): A series electric circuit contains a resistance R, a capacitance C and a battery supplying a time-varying electromotive force $V(t)$. The charge q on the capacitor therefore obeys the equation

$$R\frac{dq}{dt} + \frac{q}{C} = V(t)$$

Assuming that initially there is no charge on the capacitor, and given that $V(t) = V_0 \sin(\omega t)$, find the charge on the capacitor as a function of time.

Problem 3 (10pts): By finding an appropriate **Integrating Factor** solve the equation:

$$\frac{dy}{dx} = -\frac{2x^2 + y^2 + x}{xy}$$

Problem 4 (20pts): Solve the following equations using the **Method of Undetermined Coefficients** for the stated boundary conditions:

(a) $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 5f = 0, \quad \text{with } f(0) = 1, f'(0) = 0$

(b) $\dfrac{d^2f}{dt^2} + 2\dfrac{df}{dt} + 5f = e^{-t}\cos(3t), \quad \text{with } f(0) = 0, f'(0) = 0$

Problem 5 (10pts): Using the **Method of Laplace Transforms** solve the following differential equation:

$$y''(t) + y(t) = \sin(2t)$$

satisfying the initial conditions:

$$y(0) = 2,\ y'(0) = 1$$

Problem 6 (10pts): Find the general solution of the following third-order linear differential equation by using the **Method of Undetermined Coefficients**

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 30e^{-x}$$

Problem 7 (20pts): Use the **Method of Variation of Parameters** to find the solution of the following equations:

(a) $y'' - y = x^n$

(b) $y'' + y = \tan(x), \quad 0 < x < \pi/2$

Hint: $\int x^n e^{-x} dx = -e^{-x} n! \sum_{m=0}^{n} \frac{x^m}{m!}$