Problem Set # 2

Due Date: Sunday Oct 8 in ELMS

Reading Assignment

1. Course Text (Robot Modeling and Control): Chapter 1, Appendix A, Appendix B
2. Handout: Matrices and Vector Spaces

Instructions:

- Please write clearly and show all your work. Credit will be given for all correct steps.
- The assignment will be submitted via ELMS.

Problem 1 (10pts): Evaluate the determinants

$$(a)\begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ 3 & -3 & 4 & -2 \\ -2 & 1 & -2 & 1 \end{vmatrix}, \quad (b)\begin{vmatrix} gc & ge & a+ge & gb+ge \\ 0 & b & b & b \\ c & e & e & b+e \\ a & b & b+f & b+d \end{vmatrix}$$

Problem 2 (10pts): Using the properties of determinants, solve with a minimum of calculations the following equations for x:

$$\begin{vmatrix} x & a & a & 1 \\ a & x & b & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{vmatrix} = 0$$

Problem 3 (10pts): Given a matrix

$$A = \begin{bmatrix} 1 & \gamma_1 & 0 \\ \gamma_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where γ_1 and γ_2 are non-zero complex numbers.

(a) Find the Eigenvalues and Eigenvectors of A

(b) Find the Conditions for the Eigenvalues to be Real

(c) Find the Conditions for the Eigenvectors to be Orthogonal

Problem 4 (10pts): Make an LU decomposition of the matrix

$$A = \begin{pmatrix} 2 & -3 & 1 & 3 \\ 1 & 4 & -3 & -3 \\ 5 & 3 & -1 & -1 \\ 3 & -6 & -3 & 1 \end{pmatrix}$$

Hence solve $Ax = b$ for (i) $b = \begin{pmatrix} -4 & 1 & 8 & -5 \end{pmatrix}^T$, (ii) $b = \begin{pmatrix} -10 & 0 & -3 & -24 \end{pmatrix}^T$
Deduce that $det(A) = -160$ and confirm this by direct calculation.

Problem 5 (10pts): Consider the following 3×3 matrix:

$$\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

(i) Is this matrix diagonalizable?

(ii) If yes, provide a diagonalization of this matrix.

Problem 6 (10pts): Let $A \in \mathcal{R}^{n\times n}$, $U \in \mathcal{R}^{n\times k}$, and $V \in \mathcal{R}^{n\times k}$ be given matrices. Suppose that A, $A + UV^T$, and $I + V^T A^{-1} U$ are non-singular matrices. Prove that

$$\left(A + UV^T\right)^{-1} = A^{-1} - A^{-1}U\left(I + V^T A^{-1} U\right)^{-1} V^T A^{-1}$$