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1

$R^2 \to R^2$ defined by: $L(x) = L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 - x_2 \end{bmatrix}$
is linear mapping and

$R^3 \to R^2$ defined by $L'\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - 2x_3 \\ x_2^2 + 1 \end{bmatrix}$
is not linear mapping.

Addition: $L(\vec{u} + \vec{v}) = L(\vec{u}) + L(\vec{v})$
Homogenous: $L(C \cdot \vec{u}) = C \cdot L(\vec{u})$

Addition:

$$L(\vec{u} + \vec{v}) = L\left(\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}\right) = \begin{bmatrix} (u_1 + v_1) + 3(u_2 + v_2) \\ 2(u_1 + v_1) - (u_2 + v_2) \end{bmatrix}$$

$$L(\vec{u}) = \begin{bmatrix} u_1 + 3u_2 \\ 2u_1 - u_2 \end{bmatrix} \quad L(\vec{v}) = \begin{bmatrix} v_1 + 3v_2 \\ 2v_1 - v_2 \end{bmatrix}$$

$$L(u) + L(v) = \begin{bmatrix} u_1 + 3u_2 + v_1 + 3v_2 \\ 2u_1 - u_2 + 2v_1 - v_2 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 + 3(u_2 + v_2) \\ 2(u_1 + v_1) - (u_2 + v_2) \end{bmatrix}$$

$$L(c \cdot \vec{u}) = c \cdot \begin{bmatrix} u_1 + 3u_2 \\ 2u_1 - v_2 \end{bmatrix} = c \cdot L(u)$$

As homogenicty is justified $\boxed{\text{It is a linear mapping}}$

$$L'\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - 2x_3 \\ x_2^2 + 1 \end{bmatrix}$$

let $u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$$L'(u) = \begin{bmatrix} 1+2-2\cdot 0 \\ 2^2+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$L'(v) = \begin{bmatrix} 2+1-2\cdot 0 \\ 1^2+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$L'(u) + L'(v) = \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$$

$u+v = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ $L'(u+v) = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$

$$L'(u+v) = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$L'(u) + L'(v) = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$$

$$L'(u+v) \neq L'(u) + L'(v)$$

Therefore It is not linear mapping.

2.

$T: R^3 \to R^2$ Given:

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}2\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}5\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}7\\-3\end{bmatrix} \quad \text{find } T\left(\begin{bmatrix}7\\2\\2\end{bmatrix}\right)$$

$$T\left(\begin{bmatrix}7\\2\\2\end{bmatrix}\right)=7\cdot\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}5\\0\end{bmatrix}+2\begin{bmatrix}7\\-3\end{bmatrix}=$$

$$=\begin{bmatrix}14\\7\end{bmatrix}+\begin{bmatrix}10\\0\end{bmatrix}+\begin{bmatrix}14\\-6\end{bmatrix}=\begin{bmatrix}24+14\\7-6\end{bmatrix}=$$

$$=\begin{bmatrix}38\\1\end{bmatrix}$$

$$T\left(\begin{bmatrix}7\\2\\2\end{bmatrix}\right)=\begin{bmatrix}38\\1\end{bmatrix}$$

3. Given $L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 4x_1 - 7x_i \\ -3x_1 - 2x_2 \\ -4x_1 - 3x_2 \end{bmatrix}$

Find the standert matrix.

linear mapping $R^2 \to R^3$, so basis are $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

the co-domain of R^3, standart matrix is 3x2 matrix.

image of $e_1 = L(e_1) = L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 4\cdot 1 - 7\cdot 0 \\ -3\cdot 1 - 2\cdot 0 \\ -4\cdot 1 - 3\ 0 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -3 \\ 4 \end{bmatrix}$

image of $e_2 = L(e_2) = L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 4\cdot 0 - 7\cdot 1 \\ -3\cdot 0 - 2\cdot 1 \\ -4\cdot 0 - 3\cdot 1 \end{bmatrix}$

$= \begin{bmatrix} -7 \\ -2 \\ -3 \end{bmatrix}$ $A = \left(L(e_1)\ L(e_2)\right) = \begin{bmatrix} 4 & -7 \\ -3 & -2 \\ -4 & -3 \end{bmatrix}$

the standert matrit of linear mapping is:

$\begin{bmatrix} 4 & 7 \\ -3 & -2 \\ -4 & -3 \end{bmatrix}$

4.

$L: R^4 \to R^4$ defined by:

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} -3x_1 - 3x_2 + x_3 + 3x_4 \\ x_1 - x_2 - 5x_3 - 5x_4 \\ -4x_1 - x_2 - 5x_3 + x_4 \\ -4x_1 - 2x_2 + 2x_3 - 3x_4 \end{bmatrix}$$

determine injective or not.

$$A = \begin{bmatrix} -3 & -3 & 1 & 3 \\ 1 & -1 & -5 & -5 \\ -4 & -1 & -5 & 1 \\ -4 & -2 & 2 & -3 \end{bmatrix}$$

If the RREF for has has a pivot in each column, rank = 4

$Ker(L) = \{0\}$ L is injective.

$$\begin{bmatrix} -3 & -3 & 1 & 3 \\ 1 & -1 & -5 & -5 \\ -4 & -1 & -5 & 1 \\ -4 & -2 & 2 & -3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -1 & -5 & 5 \\ -3 & -3 & 1 & 3 \\ -4 & -1 & -5 & 1 \\ -4 & -2 & 2 & -3 \end{bmatrix} \begin{matrix} R_2 + 3R_1 \to R_2 \\ R_3 + 4R_1 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & -5 & 5 \\ 0 & -6 & -14 & -12 \\ 0 & -5 & -25 & -19 \\ -4 & -2 & 2 & -3 \end{bmatrix} \xrightarrow{4R_1 + R_4 \to R_4} \begin{bmatrix} 1 & -1 & -5 & 5 \\ 0 & -6 & -14 & -12 \\ 0 & -5 & -25 & -19 \\ 0 & -6 & -18 & -23 \end{bmatrix} -\frac{1}{6} R_2 \to R_2$$

$$\begin{bmatrix} 1 & -1 & -5 & 5 \\ 0 & 1 & \frac{7}{3} & 2 \\ 0 & -5 & -25 & -19 \\ 0 & -6 & -18 & -23 \end{bmatrix} \begin{matrix} R_2 + R_1 \to R_1 \\ 5R_2 + R_3 \to R_3 \\ R_4 + 6R_2 \to R_4 \end{matrix} \begin{bmatrix} 1 & 0 & -\frac{8}{3} & -3 \\ 0 & 1 & \frac{7}{3} & 2 \\ 0 & 0 & -\frac{10}{3} & -9 \\ 0 & 0 & -6 & -11 \end{bmatrix}$$

$-\frac{3}{10}R_3 \to R_3$ $\begin{bmatrix} 1 & 0 & -\frac{8}{3} & -3 \\ 0 & 1 & \frac{7}{3} & 2 \\ 0 & 0 & 1 & \frac{27}{10} \\ 0 & 0 & -6 & -11 \end{bmatrix}$ $\frac{8}{3}R_3 + R_1 \to R_1$ $6R_3 + R_4 \to R_4$

$-\frac{7}{3}R_3 + R_2 \to R_2$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

RREF is Identity matrix.
Each column has pivot.
Matrix is invertible and
transformation is
injective.

5.

$$A = \begin{pmatrix} 7 & 1 & -4 \\ 1 & -7 & -4 \\ -4 & -4 & -4 \end{pmatrix}$$

find characteristic polynomial of matrix

$\det(A - \lambda I)$

$$A - \lambda I = \begin{bmatrix} -7-\lambda & 1 & -4 \\ 1 & -7-\lambda & -4 \\ -4 & -4 & -4\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -7-\lambda & 1 & -4 \\ 1 & -7-\lambda & -4 \\ -4 & -4 & -4-\lambda \end{vmatrix}$$

$$-7-\lambda \begin{vmatrix} -7-\lambda & -4 \\ -4 & -4-\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & -4 \\ -4 & -4-\lambda \end{vmatrix} + (-4) \begin{vmatrix} 1 & -7-\lambda \\ -4 & -4 \end{vmatrix}$$

$= (-7-\lambda)(-4-\lambda) - (-4)(-4)$

$= (7+\lambda)(4+\lambda) - 16$

$= \lambda^2 + 11\lambda + 28 - 16 = \lambda^2 + 11\lambda + 12$

$= (-7-\lambda)(\lambda^2 + 11\lambda + 12)$

$$\begin{vmatrix} 1 & -4 \\ -4 & -4-\lambda \end{vmatrix} = 1 \cdot (-4-\lambda) - (-4)(-4)$$

$= -4 - \lambda - 16 = -\lambda - 20$

$(-1) \cdot (-\lambda - 20) = \lambda + 20$

$$\begin{vmatrix} 1 & \cancel{7}-\lambda \\ -4 & -4 \end{vmatrix} = 1\cdot(-4) - (-4)(-7-\lambda) =$$

$$= -32 + 4 - \lambda = -32 - 4\lambda$$

$$-4(-32-4\lambda) = 128 + 16\lambda$$

$$-(\lambda+7)(\lambda^2 + 11\lambda + 12) + (\lambda + 20) + (128 + 16)$$

$$= (\lambda^3 + 11\lambda^2 + 11\lambda + 7\lambda^2 + 77\lambda + 84$$

$$= -\lambda^3 - 18\lambda^2 - 88\lambda - 84 + \sqrt{} + 20 + 18 + 161$$

$$= 17\cancel{\lambda} + 148$$

$$- \lambda^3 - 18\lambda^2 - 72\lambda + 64$$

$$P(\lambda) = \lambda^3 + 18\lambda^2 + 72\lambda - 64$$

λ^2