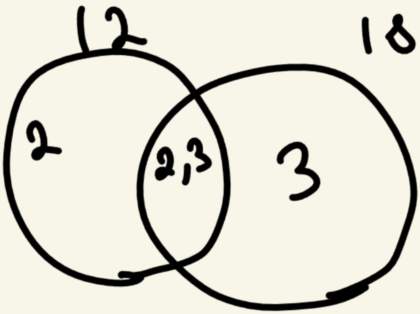
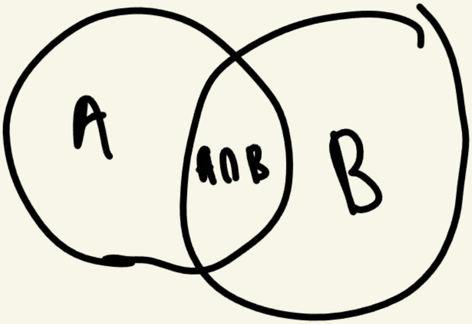
office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \operatorname{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$x + 0 = x$

$1 + 0 = \underline{1}$ by identity

$0 + 0 = 0$

$x(1+0) = x \cdot 1$

$x(1+0) = x$ by identity

$x \cdot 1 + x \cdot 0 = x$ by distributivity

$x + x \cdot 0 = x$ by identity

$-x + (x + x \cdot 0) = -x + x$

$-x + x + x \cdot 0 = -x + x$ Assosiativ.

$0 + x \cdot 0 = 0$ invertibility

$x \cdot 0 = 0$ identity

Also, $0 \cdot x = 0$ for all

$x \in \mathbb{R}$, $x \cdot 0 = 0$ and $0 \cdot x = 0$

January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "For all $x \in \mathbb{R}$, $P(x)$"

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, For all $x \in \mathbb{R}$, $P(x)$

$$-\square + \square = 0$$

$$-(xy) + xy = 0 \quad \text{by invertibility}$$

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x)\cdot y = 0 + \underbrace{(-x)\cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x)\cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x)\cdot y$$

$$1x = x \qquad -(1x) = -x \qquad (-1)x = -x$$

$$-(xy) + (x + (-x))y = (-x)y \quad \text{Distrib. and Ident.}$$

$$-(xy) + 0\cdot y = (-x)y \quad \text{Invertibility}$$

$$-(xy) + 0 = (-x)y \quad \text{Prop 1}$$

$-(xy) = (-x)y$ by identity

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -(xy)$

Proposition 3:

For all $x, y \in \mathbb{R}$, $(-x)(-y) = xy$

Proof: Let $x, y \in \mathbb{R}$

$-x + x = 0$ Invertibility

$(-x + x)(-y) = 0 \cdot (-y)$

$(-x)(-y) + x(-y) = 0 \cdot (-y)$ dist.

$(-x)(-y) + x(-y) = 0$ Prop 1

$(-x)(-y) + x(-y) + xy = 0 + xy$

$(-x)(-y) + x(-y) + xy = xy$ (Ident.)

$(-x)(-y) + x(-y+y) = xy$ dist.

$(-x)(-y) + x \cdot 0 = xy$ invertib.

$(-x)(-y) + 0 = xy$ Prop 1

$(-x)(-y) = xy$ Identity

For all $x, y \in \mathbb{R}$ $(-x)(-y) = xy$