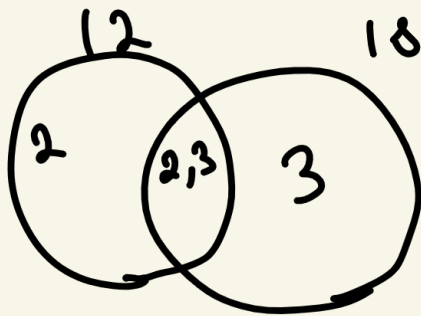
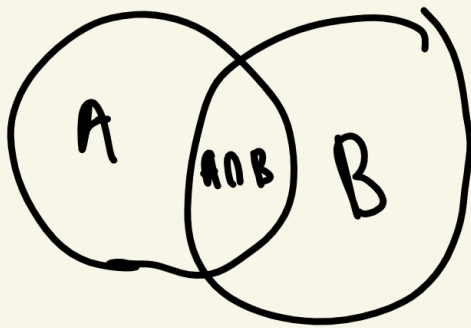


office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \text{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$$x + 0 = x$$

$$1 + 0 = 1 \quad \text{by identity}$$

$$0 + 0 = 0$$

$$x(1 + 0) = x \cdot 1$$

$$x(1 + 0) = x \quad \text{by identity}$$

$$x \cdot 1 + x \cdot 0 = x \quad \text{by distributivity}$$

$$x + x \cdot 0 = x \quad \text{by identity}$$


$$-x + (x + x \cdot 0) = -x + x$$

$$-x + x + x \cdot 0 = -x + x \quad \text{Associative}$$

$$0 + x \cdot 0 = 0 \quad \text{invertibility}$$

$$x \cdot 0 = 0 \quad \text{identity}$$

$$\text{Also, } 0 \cdot x = 0 \quad \text{for all}$$

$$x \in \mathbb{R}, x \cdot 0 = 0 \text{ and } 0 \cdot x = 0$$


January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "for all $x \in \mathbb{R}$, $P(x)$ "

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, for all $x \in \mathbb{R}$, $P(x)$

$$-\square + \square = 0$$

$$-(xy) + xy = 0 \quad \text{by invertibility}$$

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x) \cdot y = 0 + \underbrace{(-x) \cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x) \cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x) \cdot y$$

$$1x = x \quad - (1x) = -x \quad (-1)x = -x$$

$$-(xy) + (x + (-x))y = (-x)y \quad \begin{array}{l} \text{Distrib.} \\ \text{and Ident.} \end{array}$$

$$-(xy) + 0 \cdot y = (-x)y \quad \text{Invertibility}$$

$$-(xy) + 0 = (-x)y \quad \text{Prop 1}$$

$$-(xy) = (-x)y \quad \text{by identity}$$

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -(xy)$

Proposition 3:

For all $x, y \in \mathbb{R}$, $(-x)(-y) = xy$

Proof: Let $x, y \in \mathbb{R}$

$$-x + x = 0 \quad \text{Invertibility}$$

$$(-x + x)(-y) = 0 \cdot (-y)$$

$$(-x)(-y) + x(-y) = 0 \cdot (-y) \quad \text{dist.}$$

$$(-x)(-y) + x(-y) = 0 \quad \text{Prop 1}$$

$$(-x)(-y) + x(-y) + xy = 0 + xy$$

$$(-x)(-y) + x(-y) + xy = xy \quad (\text{Ident.})$$

$$(-x)(-y) + x(-y+y) = xy \quad \text{dist.}$$

$$(-x)(-y) + x \cdot 0 = xy \quad \text{invertib.}$$

$$(-x)(-y) + 0 = xy \quad \text{Prop 1}$$

$$(-x)(-y) = xy \quad \text{Identity}$$

$$\text{For all } x, y \in \mathbb{R} \quad (-x)(-y) = xy$$

January 9, 2026

My dog is yellow
subject

X is yellow \nearrow open sentence

Examples: $y = 2x + 1$
 $x < 3$

Each variable has an allowable set of values called the

"universe of discourse" for variable

Example: X is wearing Y

Quantified Statements:

Two of my cats are orange

predicate: X is orange

Universe of discourse for x :

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate: X is orange

universe: C = set of my cats

Notation: $\forall x \in C, x \text{ is orange}$

Read: "for all values of x in C ,
 x is orange"

Existential Quantified Statements

"Some of my cats are orange"

Notation: $\exists x \in C, X \text{ is orange}$

Read: "for at least one value of x in C , x is orange"

"there is a value of x in C where x is orange"

"there exists an x in C for which x is orange"

\mathbb{N} natural nums $\{1, 2, 3, 4, \dots\}$

\mathbb{Z} integers $\{\dots -4, -3, \dots 0, 1, 2, 3, \dots\}$

\mathbb{Q} rational nums All fractions & integ.

\mathbb{R} real nums

\mathbb{C} complex nums of real / imagin. numbers
combination

1. $\forall x \in \mathbb{N}$, $0 \leq x$ True

2. $\forall x \in \mathbb{R}$, $0 < x^2$ False

3. $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$, $x < y$ True

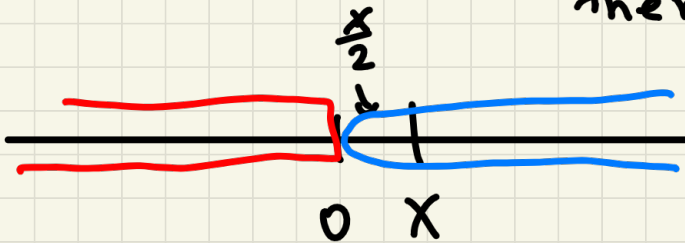
4. $\exists y \in \mathbb{R}$, $\forall x \in \mathbb{R}$, $x < y$ False

5. $\forall x \in \mathbb{Z}$, $\exists y \in \mathbb{Z}$, $y = 2x$ True

6. $\forall x \in \mathbb{Z}$, $\exists y \in \mathbb{Z}$, $x = 2y$ False
 $5 = 2y?$

7. $\forall x \in \mathbb{Z}$, If $\exists y \in \mathbb{Z}$, $x = 2y$ If its even then
 then $\exists q \in \mathbb{Z}$, $x^2 = 2q$ True

8. $\forall x \in \mathbb{R}$, If $\forall a \in (0, \infty)$, $x \leq a$
 then $x \leq 0$



If its $x \geq 0$, but never less or equal to zero

True

9. $\forall x \in \mathbb{R}$ If $\forall a \in \mathbb{R}$, $a \cdot x \leq 0$, {0}
 then $\forall b \in \mathbb{R}$, $0 \leq b \cdot x$ True

January 12, 2026

Proposition: $0 < 1$

Proof by contradiction:

To prove a proposition P

Assume $\neg P$ (negation of P)

derive a contrad. Q and $\neg Q$

Assume: $0 \neq 1$

Since $0 \neq 1$, we have $1 < 0$
by trichotomy

$-1 + 1 < -1 + 0$ by monotonicity

$$0 < -1$$

then:

$0(-1) < (-1)(-1)$ by monotonicity

$0 < 1$ by Prop 1 & Prop 3

Now, $0 < 1$ and $0 \leq 1$. This is contradiction

Therefore, $0 < 1$

Proposition: $1+1 \neq 1$

Proof:

Assume $1+1 = 1$

$$1+1(-1) = 1+(-1)$$

$$1 = 0$$

But, $1 \neq 0$ This is contradiction

Therefore, $1+1 \neq 1$

Proposition: $1+1 \neq 0$

Proof:

Assume $1+1 = 0$

We know $0 < 1$

then $1+0 < \underline{1+1}$ by monotonicity

So, $1 < 0$

this is a contradiction, since $0 < 1$

Therefore, $1+1 \neq 0$

Definition:

$$2 = 1+1 \quad 3 = 2+1 \quad 4 = 3+1 \quad 5 = 4+1$$

Example:

$$2 < 4$$

Proof:

$$0 < 1$$

$$\text{then } 1+0 < 1+1$$

$$\text{so } 1 < 2$$

by transitivity, $0 < 2$

$$\text{then } 1+0 < 2+1$$

$$\text{so } 1 < 3$$

$$\text{then } 1+1 < 3+1, \text{ so } 2 < 4$$

January 14, 2026

Proposition:

$\forall x, y \in \mathbb{R}$, if $x < y$, then $-y < -x$

Direct Proof

To prove If P , then Q

Assume P

* Demonstrate Q

Therefore, If P , then Q

Proof

Let $x, y \in \mathbb{R}$

Assume $x < y$

by monotonicity

$$-x + x + (-y) < -x + y + (-y)$$

$$0 + (-y) < -x + 0$$

$$-y < -x \quad \therefore$$

Therefore, If $x < y$, then $-y < -x$

Monotonicity (Negative Multiplication)

$\forall x, y, z \in \mathbb{R}$, If $x < y$ and $z < 0$,
then $yz < xz$

Proof Let $x, y, z \in \mathbb{R}$ target $yz < xz$

Assume $x < y$ and $z < 0$

Since $z < 0$, we have $0 < -z$
 $-z + z < -z + 0$

Then $x(-z) < y(-z)$

So, $-xz < -yz$

$$xz + (-xz) + yz < xz + (-yz) + yz$$

$$0 + yz < xz + 0$$

$$yz < xz$$

Therefore, if $x < y$ and $z < 0$,

QED then $yz < xz$

$\forall x, y \in \mathbb{R}$, if $x < y$, then $x+2 < y+3$

Proof Let $x, y \in \mathbb{R}$

Assume $x < y$

$$x+2 < y+3$$

$x+2 < y+2$ by monotonicity

Since $2 < 3$, $y+2 < y+3$ by monotonicity

$$x+2 < y+2 \text{ \& } y+2 < y+3$$

By transitivity, $x+2 < y+3$

Using transitivity to Prove $A < B$

$$\textcircled{1} \quad A < C$$

$$\textcircled{2} \quad C < B$$

$$A < B$$

January 16, 2026

Rules of Negation

1. $\neg(\forall x \in U, P(x))$ is $\exists x \in U, \neg P(x)$
2. $\neg(\exists x \in U, P(x))$ is $\forall x \in U, \neg P(x)$
3. $\neg(P \text{ and } Q)$ is $\neg P \text{ or } \neg Q$
4. $\neg(P \text{ or } Q)$ is $\neg P \text{ and } \neg Q$
5. $\neg(\text{If } P, \text{ then } Q)$ is $P \text{ and } \neg Q$

$\forall x \in \mathbb{R}, \text{ If } \overset{P}{\boxed{\forall a \in (0, \infty), a \leq x}}, \text{ then } \overset{Q}{\boxed{x \leq 0}}$

Negation

$\exists x \in \mathbb{R}, \overset{P}{\boxed{\forall a \in (0, \infty), a \leq x}} \text{ and } \overset{Q}{\boxed{0 < x}}$

1. $\forall x \in \mathbb{Z}$, If $\exists a \in \mathbb{Z}, x = 2a + 1$, then
 $\exists b \in \mathbb{Z}, x = 3b$ 2

$\exists x \in \mathbb{Z}, \exists a \in \mathbb{Z}, x = 2a + 1$ and
 $\forall b \in \mathbb{Z}, x \neq 3b$ ✓

2. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$

$\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x \geq y$ ✓

3. $\forall x, y \in \mathbb{R}$, If $0 < x < 1$ and
 $x + y = 1$, then $0 < y < 1$

$\exists x, y \in \mathbb{R}, 0 < x < 1$ and $x + y = 1$ ✓
and $y \leq 0$ or $1 \leq y$

1. $A < C$
2. $C < B > A < B$

1. $\forall x, y \in \mathbb{R}$, If $0 < y < x$, then $y < 2x$

Structure Let $x, y \in \mathbb{R}$

Assume $0 < y < x$

$$0 < y \text{ and } y < x$$

$$\text{then } 0 < x \quad x+0 < x+x$$

$$\text{so } x < 2x$$

$$y < x < 2x$$

So

$$y < 2x$$

2. $\forall x, y \in \mathbb{R}$, If $x < 2 < y$, then $x+2 < y^2$

$$x < 2 \text{ and } 2 < y$$

$$2 \cdot 2 < 2 \cdot y \text{ so } 4 < 2y$$

Since $0 < 2 < y$
we have $0 < y$

$$2 < y \text{ so } 2(y) < y(y)$$

$$2y < y^2$$

$$\text{then } 4 < y^2$$

$$x < 2 \text{ so } x+2 < 4$$

$$\text{Now, } x+2 < 4$$

