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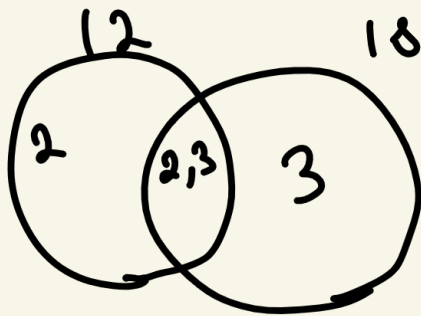
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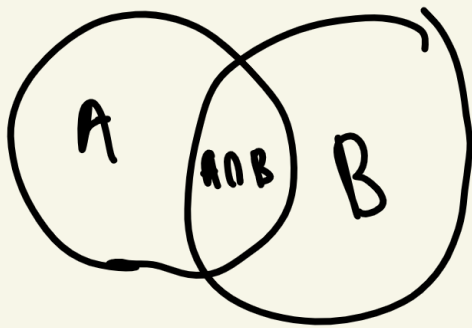


office hours  
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \text{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all  $x \in \mathbb{R}$ ,  $x \cdot 0 = 0$

and  $0 \cdot x = 0$

proof: Let  $x \in \mathbb{R}$

$$x + 0 = x$$

$$1 + 0 = 1 \quad \text{by identity}$$

$$0 + 0 = 0$$

$$x(1 + 0) = x \cdot 1$$

$$x(1 + 0) = x \quad \text{by identity}$$

$$x \cdot 1 + x \cdot 0 = x \quad \text{by distributivity}$$

$$x + x \cdot 0 = x \quad \text{by identity}$$


$$-x + (x + x \cdot 0) = -x + x$$

$$-x + x + x \cdot 0 = -x + x \quad \text{Associative}$$

$$0 + x \cdot 0 = 0 \quad \text{invertibility}$$

$$x \cdot 0 = 0 \quad \text{identity}$$

$$\text{Also, } 0 \cdot x = 0 \quad \text{for all}$$

$$x \in \mathbb{R}, x \cdot 0 = 0 \text{ and } 0 \cdot x = 0$$


January 7, 2026

Proposition 2: For all  $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let  $x, y \in \mathbb{R}$

Universal Generalization

To prove "for all  $x \in \mathbb{R}$ ,  $P(x)$ "

Let  $x \in \mathbb{R}$

\* Demonstrate  $P(x)$

Therefore, for all  $x \in \mathbb{R}$ ,  $P(x)$

$$-\square + \square = 0$$

$$-(xy) + xy = 0 \quad \text{by invertibility}$$

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x) \cdot y = 0 + \underbrace{(-x) \cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x) \cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x) \cdot y$$

$$1x = x \quad - (1x) = -x \quad (-1)x = -x$$

$$-(xy) + (x + (-x))y = (-x)y \quad \begin{array}{l} \text{Distrib.} \\ \text{and Ident.} \end{array}$$

$$-(xy) + 0 \cdot y = (-x)y \quad \text{Invertibility}$$

$$-(xy) + 0 = (-x)y \quad \text{Prop 1}$$

$$-(xy) = (-x)y \quad \text{by identity}$$

Therefore, for all  $x, y \in \mathbb{R}$   $(-x)y = -(xy)$

Proposition 3:

For all  $x, y \in \mathbb{R}$ ,  $(-x)(-y) = xy$

Proof: Let  $x, y \in \mathbb{R}$

$$-x + x = 0 \quad \text{Invertibility}$$

$$(-x + x)(-y) = 0 \cdot (-y)$$

$$(-x)(-y) + x(-y) = 0 \cdot (-y) \quad \text{dist.}$$

$$(-x)(-y) + x(-y) = 0 \quad \text{Prop 1}$$

$$(-x)(-y) + x(-y) + xy = 0 + xy$$

$$(-x)(-y) + x(-y) + xy = xy \quad (\text{Ident.})$$

$$(-x)(-y) + x(-y+y) = xy \quad \text{dist.}$$

$$(-x)(-y) + x \cdot 0 = xy \quad \text{invertib.}$$

$$(-x)(-y) + 0 = xy \quad \text{Prop 1}$$

$$(-x)(-y) = xy \quad \text{Identity}$$

$$\text{For all } x, y \in \mathbb{R} \quad (-x)(-y) = xy$$

January 9, 2026

My dog is yellow  
subject

X is yellow  $\nearrow$  open sentence

Examples:  $y = 2x + 1$   
 $x < 3$

Each variable has an allowable set of values called the

"universe of discourse" for variable

Example:  $X$  is wearing  $Y$

Quantified Statements:

Two of my cats are orange

predicate:  $X$  is orange

Universe of discourse for  $x$ :

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate:  $X$  is orange

universe:  $C$  = set of my cats

Notation:  $\forall x \in C, x \text{ is orange}$

Read: "for all values of  $x$  in  $C$ ,  
 $x$  is orange"



# Existential Quantified Statements

"Some of my cats are orange"

Notation:  $\exists x \in C, X \text{ is orange}$

Read: "for at least one value of  $x$  in  $C$ ,  $x$  is orange"

"there is a value of  $x$  in  $C$  where  $x$  is orange"

"there exists an  $x$  in  $C$  for which  $x$  is orange"

$\mathbb{N}$  natural nums  $\{1, 2, 3, 4, \dots\}$

$\mathbb{Z}$  integers  $\{\dots -4, -3, \dots 0, 1, 2, 3, \dots\}$

$\mathbb{Q}$  rational nums All fractions & integ.

$\mathbb{R}$  real nums

$\mathbb{C}$  complex nums of real / imagin. numbers  
combination

1.  $\forall x \in \mathbb{N}$ ,  $0 \leq x$  True

2.  $\forall x \in \mathbb{R}$ ,  $0 < x^2$  False

3.  $\forall x \in \mathbb{R}$ ,  $\exists y \in \mathbb{R}$ ,  $x < y$  True

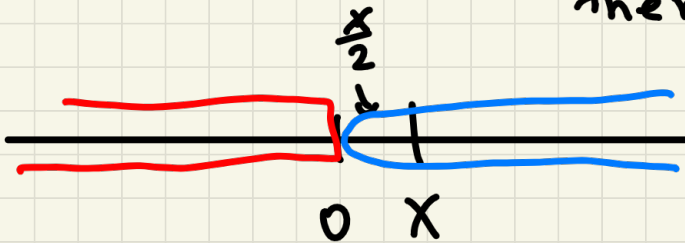
4.  $\exists y \in \mathbb{R}$ ,  $\forall x \in \mathbb{R}$ ,  $x < y$  False

5.  $\forall x \in \mathbb{Z}$ ,  $\exists y \in \mathbb{Z}$ ,  $y = 2x$  True

6.  $\forall x \in \mathbb{Z}$ ,  $\exists y \in \mathbb{Z}$ ,  $x = 2y$  False  
 $5 = 2y?$

7.  $\forall x \in \mathbb{Z}$ , If  $\exists y \in \mathbb{Z}$ ,  $x = 2y$  If its even then  
 then  $\exists q \in \mathbb{Z}$ ,  $x^2 = 2q$  True

8.  $\forall x \in \mathbb{R}$ , If  $\forall a \in (0, \infty)$ ,  $x \leq a$   
 then  $x \leq 0$



If its  $x \geq 0$ , but never less or equal to zero

True

9.  $\forall x \in \mathbb{R}$  If  $\forall a \in \mathbb{R}$ ,  $a \cdot x \leq 0$ , {0}  
 then  $\forall b \in \mathbb{R}$ ,  $0 \leq b \cdot x$  True

January 12, 2026

Proposition:  $0 < 1$

Proof by contradiction:

To prove a proposition  $P$

Assume  $\neg P$  (negation of  $P$ )

derive a contrad.  $Q$  and  $\neg Q$

Assume:  $0 \neq 1$

Since  $0 \neq 1$ , we have  $1 < 0$   
by trichotomy

$-1 + 1 < -1 + 0$  by monotonicity

$$0 < -1$$

then:

$0(-1) < (-1)(-1)$  by monotonicity

$0 < 1$  by Prop 1 & Prop 3

Now,  $0 < 1$  and  $0 \leq 1$ . This is contradiction

Therefore,  $0 < 1$

Proposition:  $1+1 \neq 1$

Proof:

Assume  $1+1 = 1$

$$1+1(-1) = 1+(-1)$$

$$1 = 0$$

But,  $1 \neq 0$  This is contradiction

Therefore,  $1+1 \neq 1$

Proposition:  $1+1 \neq 0$

Proof:

Assume  $1+1 = 0$

We know  $0 < 1$

then  $1+0 < \underline{1+1}$  by monotonicity

So,  $1 < 0$

this is a contradiction, since  $0 < 1$

Therefore,  $1+1 \neq 0$

Definition:

$$2 = 1+1 \quad 3 = 2+1 \quad 4 = 3+1 \quad 5 = 4+1$$

Example:

$$2 < 4$$

Proof:

$$0 < 1$$

$$\text{then } 1+0 < 1+1$$

$$\text{so } 1 < 2$$

by transitivity,  $0 < 2$

$$\text{then } 1+0 < 2+1$$

$$\text{so } 1 < 3$$

$$\text{then } 1+1 < 3+1, \text{ so } 2 < 4$$

January 14, 2026

Proposition:

$\forall x, y \in \mathbb{R}$ , if  $x < y$ , then  $-y < -x$

## Direct Proof

To prove If  $P$ , then  $Q$

Assume  $P$

\* Demonstrate  $Q$

Therefore, If  $P$ , then  $Q$

## Proof

Let  $x, y \in \mathbb{R}$

Assume  $x < y$

by monotonicity

$$-x + x + (-y) < -x + y + (-y)$$

$$0 + (-y) < -x + 0$$

$$-y < -x \quad \therefore$$

Therefore, If  $x < y$ , then  $-y < -x$

## Monotonicity (Negative Multiplication)

$\forall x, y, z \in \mathbb{R}$ , If  $x < y$  and  $z < 0$ ,  
then  $yz < xz$

Proof Let  $x, y, z \in \mathbb{R}$  target  $yz < xz$

Assume  $x < y$  and  $z < 0$

Since  $z < 0$ , we have  $0 < -z$   
 $-z + z < -z + 0$

Then  $x(-z) < y(-z)$

So,  $-xz < -yz$

$$xz + (-xz) + yz < xz + (-yz) + yz$$

$$0 + yz < xz + 0$$

$$yz < xz$$

Therefore, if  $x < y$  and  $z < 0$ ,

QED then  $yz < xz$

$\forall x, y \in \mathbb{R}$ , if  $x < y$ , then  $x+2 < y+3$

Proof Let  $x, y \in \mathbb{R}$

Assume  $x < y$

$$x+2 < y+3$$



$x+2 < y+2$  by monotonicity

Since  $2 < 3$ ,  $y+2 < y+3$  by monotonicity

$$x+2 < y+2 \text{ and } y+2 < y+3$$

By transitivity,  $x+2 < y+3$

Using transitivity to Prove  $A < B$

$$\textcircled{1} \quad A < C$$

$$\textcircled{2} \quad C < B$$

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$$A < B$$