1

$$\begin{cases} x+3y-z+t=0 \\ 3x-y+2z+t=0 \\ -x+5y-z+2t=0 \end{cases} = \left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 3 & -1 & 2 & 1 & 0 \\ -1 & 5 & -1 & 2 & 0 \end{array}\right]$$

$R_2 - 3R_1 \rightarrow R_2$
$R_3 + R_1 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & -10 & 5 & -2 & 0 \\ 0 & 8 & -2 & 3 & 0 \end{array}\right]$$

$-\frac{1}{10} R_2 \rightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{5} & 0 \\ 0 & 8 & -2 & 3 & 0 \end{array}\right]$$

$R_3 - 8R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{5} & 0 \\ 0 & 0 & 2 & 1.4 & 0 \end{array}\right]$$

$\frac{1}{2} R_3 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & 0.7 & 0 \end{array}\right]$$

$$\begin{cases} z+0.7t=0 \\ y-0.5z+0.2t=0 \\ x+3y-z+t=0 \end{cases} \quad \begin{array}{l} z=-0{,}7t \\ y=-0.55t \\ x=-0.05t \end{array} \quad \begin{bmatrix} -0.05t \\ -0.55t \\ -0.7t \\ t \end{bmatrix}$$

2)

$$\begin{cases} 2x + 6y = 5 \\ 4x + (k+15)y = \ell + 8 \end{cases} = \left[\begin{array}{cc|c} 2 & 6 & 5 \\ 4 & k+15 & \ell+8 \end{array}\right]$$

$\underline{R_2 - 2R_1}$

$$\left[\begin{array}{cc|c} 2 & 6 & 5 \\ 0 & k+15-12 & \ell+8-10 \end{array}\right] =$$

$$= \left[\begin{array}{cc|c} 2 & 6 & 5 \\ 0 & k+3 & \ell-2 \end{array}\right]$$

1. Unique solution: $k+3 \neq 0$
2. Infinite solutions: $k+3=0$, $\ell-2=0$
3. No solution: $k+3=0$, $\ell-2 \neq 0$

3)

$$B = \begin{bmatrix} -2 & -4 & 3 \\ 2 & 2 & 1 \\ -2 & 2 & 1 \end{bmatrix} \quad \text{Use } B^{-1} \text{ to } Bx = d$$

$$d = \begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 0 & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{10} & \frac{1}{10} & \frac{1}{5} \\ \frac{1}{5} & -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$$

$$x = \begin{bmatrix} 0 & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{10} & \frac{1}{10} & \frac{1}{5} \\ \frac{1}{5} & -\frac{3}{10} & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix}$$

$$x_1 = 0 \cdot 3 + \frac{1}{4} \cdot (-2) + \left(-\frac{1}{4}\right) \cdot (-4) = 0 - \frac{1}{2} + 1 = \frac{1}{2}$$

$$x_2 = -\frac{1}{10} \cdot 3 + \frac{1}{10} \cdot (-2) + \frac{1}{5} \cdot (-4) = -\frac{13}{10}$$

$$x_3 = \frac{1}{5} \cdot 3 + \left(-\frac{3}{10}\right) \cdot (-2) + \frac{1}{10} \cdot (-4) = \frac{4}{5}$$

$$x = \begin{bmatrix} \frac{1}{2} \\ -\frac{13}{10} \\ \frac{4}{5} \end{bmatrix}$$

4.

$$u = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \quad V = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad W = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{cases} a - b = 2 \\ a + 0 \cdot b = 3 \\ -2a + b = -5 \end{cases} \qquad \begin{aligned} b &= 1 \; ✓ \\ a &= 3 \; ✓ \\ -6 + 1 &= -5 \; ✓ \end{aligned}$$

w is a linear combination of u, V

$$w = 3u + V$$