January 8, 2026

Base 2, 8, 16

Binary $(\overset{7}{1}\overset{6}{1}\overset{5}{1}\overset{4}{1}\overset{3}{1}\overset{2}{1}\overset{1}{1}\overset{0}{1})_2 \to (\quad)_{10}$

$1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 =$

$= 255_{10}$

$(\overset{2}{3}\overset{1}{7}\overset{0}{7})_8 = 3\cdot 8^2 + 7\cdot 8^1 + 7\cdot 8^0 = 255_{10}$

$(\overset{1}{F}\overset{0}{F})_{16} \to (\quad)_{10}$

base 16:

$16^{1} \cdot F + 16^{0} \cdot F = 16 \cdot 15 + 15 = 255_{10}$

Decimal

$(\quad)_{10} \rightarrow (\quad)_{2}$

$(\quad)_{10} \rightarrow (\quad)_{8}$

$(\quad)_{10} \rightarrow (\quad)_{16}$

HEX

10	A
11	B
12	C
13	D
14	E
15	F

$$
\begin{array}{r|l}
255 & 2 \\
254 & 127 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
127 & 2 \\
126 & 63 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
63 & 2 \\
62 & 31 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
31 & 2 \\
30 & 15 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
15 & 2 \\
14 & 7 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
7 & 2 \\
6 & 3 \\
1 & \\
\end{array}
\quad
\begin{array}{r|l}
3 & 2 \\
2 & 1 \\
1 & \\
\end{array}
$$

$255_{10} = 11111111_{2}$

114_{10}

$$\begin{array}{l}
114 \mid 2 \\
114 \mid 57 \mid 2 \\
0 \quad 56 \mid 28 \mid 2 \\
\quad 1 \quad 28 \mid 14 \mid 2 \\
\quad\quad 0 \quad 14 \mid 7 \mid 2 \\
\quad\quad\quad 0 \quad 6 \mid 3 \mid 2 \\
\quad\quad\quad\quad 1 \quad 2 \mid 1 \\
\quad\quad\quad\quad\quad 1
\end{array}$$

1110010_2

$(0.12)_{10} \to (\quad\quad)_2 = (\quad .000111)_2$

$0.12 \times 2 = \underbrace{0}.24$	0	just fraction
$0.24 \times 2 = \underbrace{0}.48$	0	
$0.48 \times 2 = \underbrace{0}.96$	0	
$0.96 \times 2 = 1\underbrace{.92}$	1	
$0.92 \times 2 = 1\underbrace{.84}$	1	
$0.84 \times 2 = 1.68$	1	

Dec.	Binary	Hex.	Dec.	Binary
1	0001		9	1001
2	0010	A	10	1010
3	0011	B	11	1011
4	0100	C	12	1100
5	0101	D	13	1101
6	0110	E	14	1110
7	0111	F	15	1111
8	1000			

Binary addition

$$
\begin{array}{r}
+\ 01100 \\
10001 \\
\hline
11101
\end{array}
$$

$$
\begin{array}{r}
\scriptstyle 1\;\;\;\;\;1\;\;\;1\;\;\;\;\;\;\;\;\;\; \\
+\;10110 \\
10111 \\
\hline
1\,\boxed{01101}
\end{array}
$$

$(1)_2+(1)_2=(10)_2$

$(10)_2+(1)_2=(11)_2$

It is overflow, machine should know that It is overflow

Binary subtraction

$$
\begin{array}{r} 0 \\ -\,0 \\ \hline 0 \end{array}
\qquad
\begin{array}{r} 1 \\ -\,1 \\ \hline 0 \end{array}
\qquad
\begin{array}{r} 1 \\ -\,0 \\ \hline 1 \end{array}
\qquad
\begin{array}{r} \scriptstyle 1\leftarrow\;\; \\ 0 \\ -\,1 \\ \hline 1 \end{array}
$$

$$
\begin{array}{r} 10110 \\ -\,10010 \\ \hline 00100 \end{array}
\qquad
\begin{array}{r} 10110 \\ -\,10011 \\ \hline 00011 \end{array}
$$

1's complement

$(10101111)_2$ logic operation

1's compliment is inverse of digits

(01010000) 1's comp.
$+\ 1$

(01010001) 2's compliment

$(0101)_2 = (5)_{10}$

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(r-1)'s r's compliments

radix = base

$r = 10 \quad L = 4 \quad N = 2468 \quad r^4 = 10000$

$r^4 - 1 = 9999$

Therefore

$$\begin{array}{r} 9999 \\ -\ 2468 \\ \hline 7531 \end{array}$$

Decimal $(185)_{10}$

Binary $(10111001)_2$

Binary Coded Decimal BCD

$(459)_{10} \rightarrow (\quad)_{BCD}$

0100 0101 1001

BCD range (0-9)

$4_{10} + 8_{10} = 12_{10}$

$0100_2 + 1000_2 = 1100_2 \rightarrow 12_{10}$

$0110_2 \rightarrow 6_{10}$

$(\underbrace{0001}\ \underbrace{0010})_{BCD}$

Binary Coded Decimal

$$\begin{array}{rr} & 162 \\ + & 769 \\ \hline & 931 \end{array}$$

$$\begin{array}{rrr} 0001 & 0110 & 0010 \\ 0111 & 0110 & 1001 \\ \hline 1000 & \underbrace{1100} & \underbrace{1011} \end{array}$$

doesn't exist in BCD

more than 9

to solve add 6_{10}

$$\begin{array}{rrr} & 0110 & 0110 \\ \hline \underbrace{1001}_{9} & \underbrace{0011}_{3} & \underbrace{0001}_{1} \end{array}$$

encoding → public UTF-8

encryption → special key

BOOLEAN LOGIC

$A+B$ → A or B

$A.B$ → A and B

A', $\bar{A}$ → NOT A

inputs

A	B	$A.B$	$A+B$	A'	B'
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	1
1	1	1	1	0	0

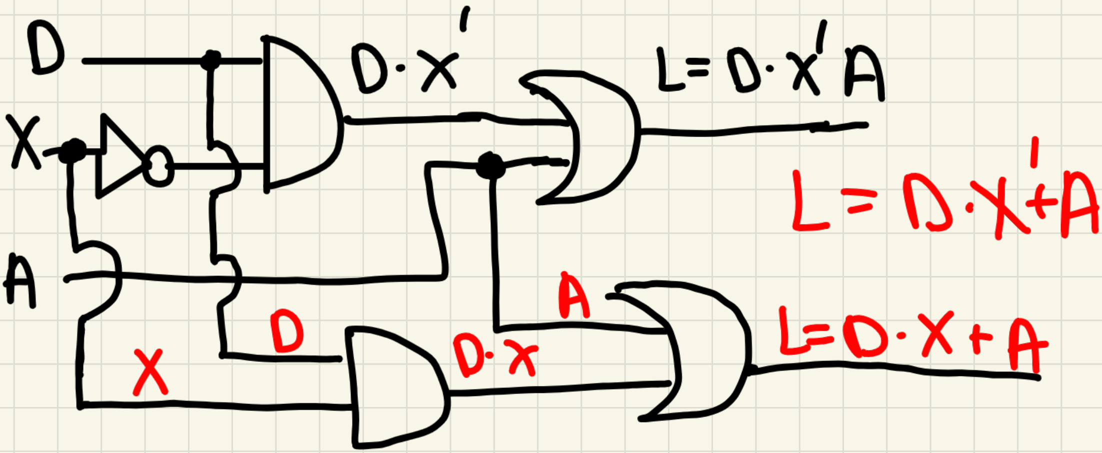
$\boxed{\text{January 15, 2026}}$

$$\underbrace{L}_{\text{output}} = \underbrace{D \cdot X' + A}_{\text{inputs}}$$

output

D	X	A	X'	$D \cdot X'$	$D \cdot X' + A$
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	0	0	0

0	1	1	0	0	1	
1	0	0	1	1	1	
1	0	1	1	1	1	
1	1	0	0	0	0	
1	1	1	0	0	1	



A	B	C	AB	A'C	BC		AB+A'C
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	0	0	0	0
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0

1	0	1	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	1	0	1	1	1

$$F(A,B,C) = (A+B)(A+C)$$

$$AA + AC + AB + BC$$ Distributivity

$$A + AC + AB + BC$$ $A = AA$

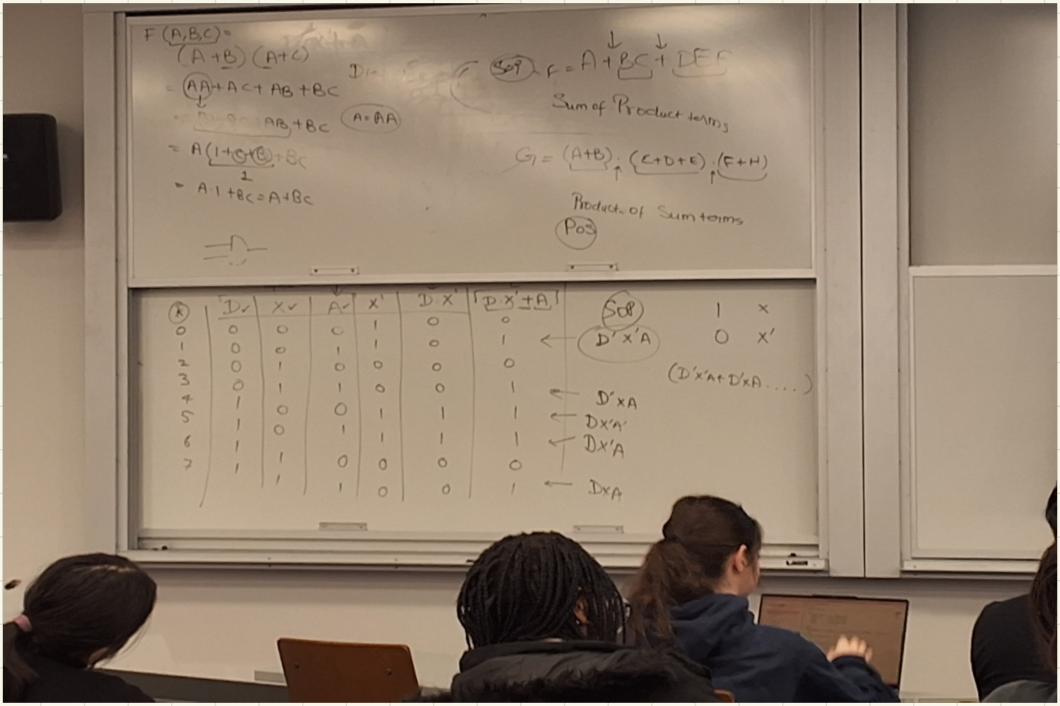
$$A\underbrace{(1 + C + B)}_{1} + BC$$

$$A \cdot 1 + BC = \boxed{A + BC}$$

Sum of Products SOP

$$F(A,B,C) = A'B'C + ABC + A'BC'$$

Product of Sum



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Q 1

$$y + x'z + xy' = x + y + z$$

$$\underbrace{y + xy'} + x'z$$

$$(y+x)(y+y') + x'z$$

$$y + x + x'z$$

$$y + (x+x')(x+z)$$

$$\boxed{y + x + z = x+y+z}$$

Q2

$$x'y' + x'y + xy = x'y$$

$$x'(y' + y) + xy$$

$$x' + xy$$

$$(x' + x)(x' + y)$$

$$1 \cdot x' + y = \boxed{x'y}$$

Q3

$$x'y' + y'z + xz + xy + yz'$$ 13 operations

$$x'y' + y'z(x + x') + xz + xy + yz'$$

$$\underbrace{x'y'}_{} + xy'z + \underbrace{x'y'}_{}z + xz + xy + yz'$$

$$x'y'(1 + z) + xy'z + xz + xy + yz'$$

$$x'y' + \underbrace{xy'z}_{} + \underbrace{xz}_{} + xy + yz'$$

$$x'y' + xz(y' + 1) + xy + yz'$$

$$x'y' + xz + xy + yz'$$

$$x'y' + xz + xy(z+z') + yz'$$

$$x'y' + xz + xyz + xyz' + yz'$$

$$x'y' + xz + xyz + yz'(x+1)$$

$$x'y' + \underline{xz} + \underline{xyz} + yz'$$

$$x'y' + xz(1+z) + yz'$$

$$\boxed{x'y' + xz + yz'} \qquad \text{8 operations}$$

SOP

$$F(a,b,c) = a + bc + abc'$$

$$A(a,b,c) = \underbrace{abc} + \underbrace{a'bc} + \underbrace{abc'} \qquad \text{Canonical}$$

Minterms (product terms)

Minterms

SOP

$x = 1$

$x' = 0$

K	a	b	c	A	A
0	0	0	0	$a'b'c'$	0
1	0	0	1	$a'b'c$	0
2	0	1	0	$a'bc'$	0
3	0	1	1	$a'bc$	1
7	1	0	0	$ab'c'$	0
5	1	0	1	$ab'c$	0
6	1	1	0	abc'	1
7	1	1	1	abc	1

$$A(a,b,c) = \sum m(3,6,7) \rightarrow \text{minterms}$$

sum of products

POS (product of sum)

$$G(a,b,c) = (a+b+c') \cdot (a'b'+c') \cdot (a+b+c)$$

Maxterms: $x = 0$

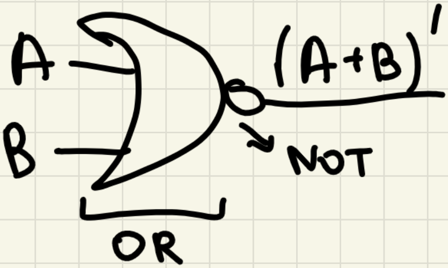
POS $x' = 1$

	a	b	c	P	
0	0	0	0	abc	1
1	0	0	1	abc'	1
2	0	1	0	$ab'c$	0
3	0	1	1	$ab'c'$	0
4	1	0	0	$a'bc$	0
5	1	0	1	$a'bc'$	0
6	1	1	0	$a'b'c$	0
7	1	1	1	$a'b'c'$	1

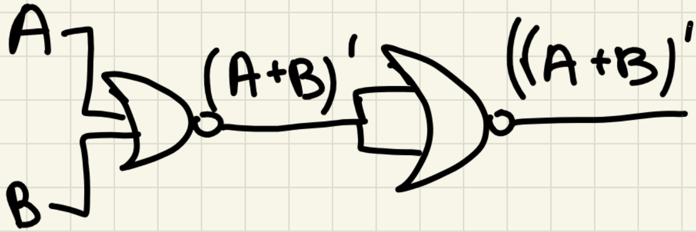
$$P(a,b,c) = \underbrace{\Pi M}_{\text{POS}} \underbrace{(0, 1, 7)}_{\text{maxterms}}$$

Always write Maxterms POS $x' = 1$, $x = 0$

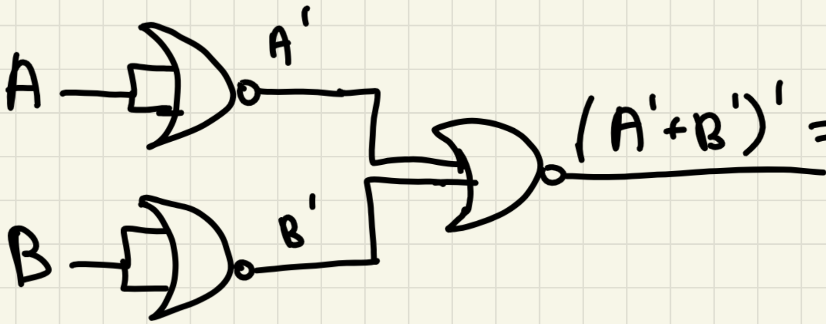
Minterms SOP $x = 1$, $x' = 0$



NOR

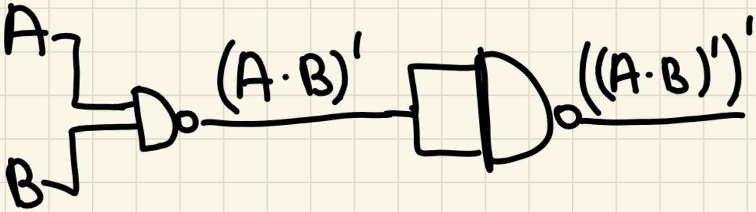


$((A+B)')' = A+B$ OR



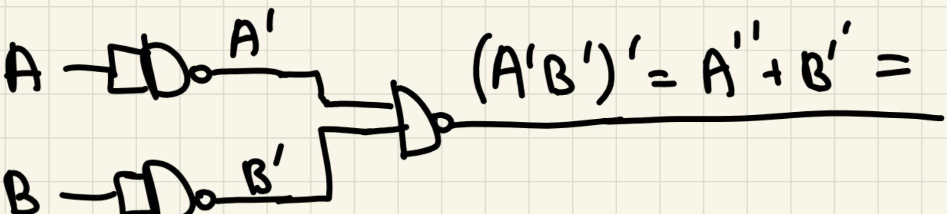
$(A'+B')' = A \cdot B$

AND



$= A \cdot B$ AND

OR



$(A'B')' = A'' + B'' = A+B$

$$F(a,b,c) = a'b'c + bc' + ac' \quad \text{(Standart)}$$

missing a   missing b

$$F(a,b,c) = a'b'c + \cancel{abc'} + a'bc' + + \underset{ab'c'}{\underline{abc'}} +$$

$$F(a,b,c) = a'b'c + a'bc' + abc' + ab'c'$$

SOP Canonic i. form

$$F(a,b,c) = (a+b+c).(a+b)\cdot(a+c') \quad \text{(standart)}$$

needs c   needs B

$$\cancel{(a+b+c)}.(a+b+c)\cdot\cancel{(a+b+c')}\cdot(a+b+c').(a+b'+c')$$

$$(a+b+c)(a+b+c')(a+b'+c')$$

POS Canonical form

1- variable map

x	f(x)
0	f(0)
1	f(1)

2 variable maps

x	y	$f(x)$
0	0	$f(0)$
0	1	$f(1)$
1	0	$f(2)$
1	1	$f(3)$

	0	1
0	00	01
1	10	11

2 variable maps

x	y	z	$f(x)$
0	0	0	$f(0)$
0	0	1	$f(1)$
0	1	0	$f(2)$
0	1	1	$f(3)$
1	0	0	$f(4)$
1	0	1	$f(5)$
1	1	0	$f(6)$
1	1	1	$f(7)$

$x \backslash yz$	00	01	11	10
0	$x'z'x'$ 000 0	$x'y'z$ 001 1	$x'yz$ 011 3	$x'yz'$ 010 2
1	100 4	101 5	111 7	110 6

→ gray codes

$$x'y'(z+z')$$

$$x'y'(1) = x'y'$$

SOP | $x = 1$, $x' = 0$

$$f(x,y,z) = \Sigma m(1,3,5,7)$$

0	0_0	1_1	1_3	0_2
1	0_4	1_5	1_7	0_6

Group of 1s
4

$$(2^n) = 1, 2, 4, 8, 16$$

$$x'y'z + x'yz + xyz' + xyz$$

$$z(x'y' + x'y + xy' + xy)$$

$$z(x'\underbrace{(y'+y)}_{1} + x\underbrace{(y'+y)}_{1})$$

$$z\underbrace{(x' + x)}_{1}$$

$$z$$

Shorter way to calculate

4 variable map

w x y z | f(w, x, y, z)

16 rows

wx \ yz	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0100 4	0101 5	0111 7	0110 6
11	1100 12	1101 13	1111 15	1110 14
10	1000 8	1001 9	1011 11	1010 10