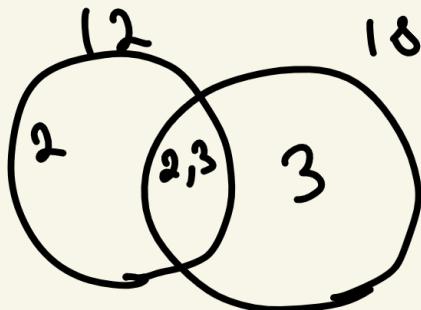
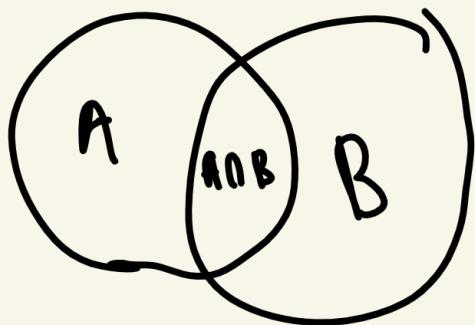



office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \text{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$$x+0=x$$

$$1+0=1 \quad \text{by identity}$$

$$0+0=0$$

$$x(1+0) = x \cdot 1$$

$$x(1+0) = x \quad \text{by identity}$$

$$x \cdot 1 + x \cdot 0 = x \quad \text{by distributivity}$$

$$x + x \cdot 0 = x \quad \text{by identity}$$

$$-x + (x+x \cdot 0) = -x+x$$

$$-x+x+x \cdot 0 = -x+x \quad \text{Associativ.}$$

$$0 + x \cdot 0 = 0 \quad \text{invertibility}$$

$$x \cdot 0 = 0 \quad \text{identity}$$

Also, $0 \cdot x = 0$ for all

$x \in \mathbb{R}$, $x \cdot 0 = 0$ and $0 \cdot x = 0$

January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "for all $x \in \mathbb{R}, P(x)"$

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, for all $x \in \mathbb{R}, P(x)$

$$-\square + \square = 0$$

$$-(xy) + xy = 0 \quad \text{by invertibility}$$

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x) \cdot y = 0 + \underbrace{(-x) \cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x) \cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x) \cdot y$$

$$1x = x \quad -(1x) = -x \quad (-1)x = -x$$

$$-(xy) + (x + (-x))y = (-x)y \quad \begin{matrix} \text{Distr.} \\ \text{and Identities} \end{matrix}$$

$$-(xy) + 0 \cdot y = (-x)y \quad \text{Invertibility}$$

$$-(xy) + 0 = (-x)y \quad \text{Prop 1}$$

$$-(xy) = (-x)y \text{ by identity}$$

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -xy$

Proposition 3:

$$\text{For all } x, y \in \mathbb{R}, (-x)(-y) = xy$$

Proof: Let $x, y \in \mathbb{R}$

$$-x + x = 0 \quad \text{Invertibility}$$

$$(-x + x)(-y) = 0 \cdot (-y)$$

$$(-x)(-y) + x(-y) = 0 \cdot (-y) \text{ dist.}$$

$$(-x)(-y) + x(-y) = 0 \quad \text{Prop 1}$$

$$(-x)(-y) + x(-y) + xy = 0 + xy$$

$$(-x)(-y) + x(-y) + xy = xy \quad (\text{Ident.})$$

$$(-x)(-y) + x(-y+y) = xy \quad \text{dist.}$$

$$(-x)(-y) + x \cdot 0 = xy \quad \text{invertib.}$$

$$(-x)(-y) + 0 = xy \quad \text{Prop 1}$$

$$(-x)(-y) = xy \quad \text{Identity}$$

For all $x, y \in \mathbb{R}$ $(-x)(-y) = xy$

January 9, 2026

My dog, is yellow
subject

X is yellow \nearrow open sentence

Examples: $y = 2x + 1$
 $x < 3$

Each variable has an allowable set of values called the

"universe of discourse" for variable

Example: X is wearing Y

Quantified Statements:

Two of my cats are orange

predicate: X is orange

Universe of discourse for X:

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate: X is orange

universe: C = set of my cats

Notation: $\forall x \in C, x \text{ is orange}$

Read: "for all values of X in C,
X is orange"

Existential Quantified Statements

"Some of my cats are orange"

Notation: $\exists x \in C, X \text{ is orange}$

Read: "for at least one value
of X in C , X is orange"

"there is a value of X
in C where X is orange"

"there exists an X in C
for which X is orange"

\mathbb{N} natural numbers $\{1, 2, 3, 4, \dots\}$

\mathbb{Z} integers $\{\dots -4, -3, \dots 0, 1, 2, 3, \dots\}$

\mathbb{Q} rational numbers All fractions & integers

\mathbb{R} real num's

combination

\mathbb{C} complex num's of real / imagin. numbers

1. $\forall x \in \mathbb{N}, \boxed{0 \leq x}$ True

2. $\forall x \in \mathbb{R}, \boxed{0 < x^2}$ False

3. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \boxed{x < y}$ True

4. $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, \boxed{x < y}$ False

5. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, \boxed{y = 2x}$ True

6. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, \boxed{x = 2y}$ False
 $5 = 2y ?$

7. $\forall x \in \mathbb{Z}$, If $\exists y \in \mathbb{Z}, x = 2y$

If its even then

then $\exists q \in \mathbb{Z}, x = 2q$ True

8. $\forall x \in \mathbb{R}$, If $\forall a \in (0, \infty), x \leq a$

then $x \leq 0$

If its $x \geq 0$, but never less or equal to zero

~~True~~

9. $\forall x \in \mathbb{R}$ If $\forall a \in \mathbb{R}, a \cdot x \leq 0$,

$\{0\}$

then $\forall b \in \mathbb{R}, 0 \leq b \cdot x$. True

January 12, 2026

Proposition: $0 < 1$

Proof by contradiction:

To prove a proposition P

Assume $\neg P$ (negation of P)



derive a contrad. Q and $\neg Q$

Assume : $0 \neq 1$

Since $0 \neq 1$, we have $1 > 0$

by trichotomy

$-1 + 1 < -1 + 0$ by monotonicity

$$0 < -1$$

then:

$0(-1) < (-1)(-1)$ by monotonicity

$0 < 1$ by Prop 1 & Prop 3

Now, $0 < 1$ and $0 \neq 1$. This is contradiction

Therefore, $0 < 1$

Proposition: $1+1 \neq 1$

Proof:

Assume $1+1 = 1$

$$1+1(-1) = 1 + (-1)$$

$$1 = 0$$

But, $1 \neq 0$ This is contradiction

Therefore, $1+1 \neq 1$

Proposition: $1+1 \neq 0$

Proof:

Assume $1+1 = 0$

We know $0 < 1$

then $1+0 < 1+1$ by monotonic.

So, $1 < 0$

this is a contradiction, since $0 < 1$

Therefore, $1+1 \neq 0$

Definition:

$$2 = 1+1 \quad 3 = 2+1 \quad 4 = 3+1 \quad 5 = 4+1$$

Example: $2 < 4$

Proof:

$$0 < 1$$

then $1+0 < 1+1$

so $1 < 2$

by transitivity, $0 < 2$

then $1+0 < 2+1$

so $1 < 3$

then $1+1 < 3+1$, so $2 < 4$

January 14, 2026

Proposition:

$\forall x, y \in \mathbb{R}$, if $x < y$, then $-y < -x$

Direct Proof

To prove If P , then Q

Assume P

* Demonstrate Q

Therefore, If P , then Q

Proof

Let $x, y \in \mathbb{R}$

Assume $x < y$

by monot

$$-x + x + (-y) < -x + y + (-y)$$

$$0 + (-y) < -x + 0$$

$$-y < -x \quad \therefore$$

Therefore, If $x < y$, then $-y < -x$

Monotonicity (Negative Multiplication)

$\forall x, y, z \in \mathbb{R}$, If $x < y$ and $z < 0$,

then $yz < xz$

Proof Let $x, y, z \in \mathbb{R}$ target
 $yz < xz$

Assume $x < y$ and $z < 0$

Since $z < 0$, we have $0 < -z$
 $-z + z < -z + 0$

Then $x(-z) < y(-z)$

So, $-xz < -yz$

$$xz + (-xz) + yz < xz + (-yz) + yz$$

$$0 + yz < xz + 0$$

$$yz < xz$$

Therefore, if $x < y$ and $z < 0$,

QED then $yz < xz$

$\forall x, y \in \mathbb{R}$, if $x < y$, then $x+2 < y+3$

Proof Let $x, y \in \mathbb{R}$

Assume $x < y$

$$\begin{array}{c} | \\ x+2 < y+3 \end{array}$$

$x+2 < y+2$ by monotonicity

Since $2 < 3$, $y+2 < y+3$ by monot.

$$x+2 < y+2 \quad \text{and} \quad y+2 < y+3$$

By transitivity, $x+2 < y+3$

Using transitivity to Prove $A < B$

① $A < C$

② $C < B$

$$A < B$$

January 16, 2026

Rules of Negation

1. $\neg(\forall x \in U, P(x))$ is $\exists x \in U, \neg P(x)$
2. $\neg(\exists x \in U, P(x))$ is $\forall x \notin U, \neg P(x)$
3. $\neg(P \text{ and } Q)$ is $\neg P \text{ or } \neg Q$
4. $\neg(P \text{ or } Q)$ is $\neg P \text{ and } \neg Q$
5. $\neg(\text{If } P, \text{then } Q)$ is $P \text{ and } \neg Q$

$\forall x \in \mathbb{R}, \text{If } \boxed{\forall a \in (0, \infty), a \leq x}, \text{ then } \boxed{x \leq 0}$

Negation

$\exists x \in \mathbb{R}, \boxed{\forall a \in (0, \infty), a \leq x} \text{ and } \boxed{0 < x}$

1. $\forall x \in \mathbb{Z}$, if $\exists a \in \mathbb{Z}, x = 2a+1$, then
 $\exists b \in \mathbb{Z}, x = 3b$

$\exists x \in \mathbb{Z}, \exists a \in \mathbb{Z}, x = 2a+1$ and
 $\forall b \in \mathbb{Z}, x \neq 3b$ ✓

2. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$

$\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x \geq y$ ✓

3. $\forall x, y \in \mathbb{R}$, if $0 < x < 1$ and
 $x+y=1$, then $0 < y < 1$

$\exists x, y \in \mathbb{R}, 0 < x < 1$ and $x+y=1$
and $y \leq 0$ or $1 \leq y$

1. $A < C > A < B$
2. $C < B$

1. $\forall x, y \in \mathbb{R}$, If $0 < y < x$, then $y < 2x$

Structure Let $x, y \in \mathbb{R}$

Assume $0 < y < x$

$0 < y$ and $y < x$

then $0 < x$ $x+0 < x+x$

so $x < 2x$

$y < x < 2x$

so $y < 2x$

2. $\forall x, y \in \mathbb{R}$, If $x < 2 < y$, then $x+2 < y^2$

$x < 2$ $2 < y$

$2 \cdot 2 < 2 \cdot y$ so $4 < 2y$

Since $0 < 2 < y$
we have $0 < y$ $2 < y$ so $2(y) < y(y)$

$2y < y^2$

then $4 < y^2$

$x < 2$ so $x+2 < 4$

Now, $x+2 < 4$

