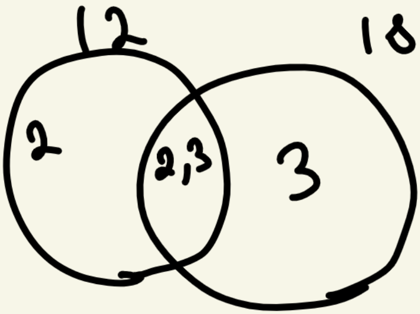
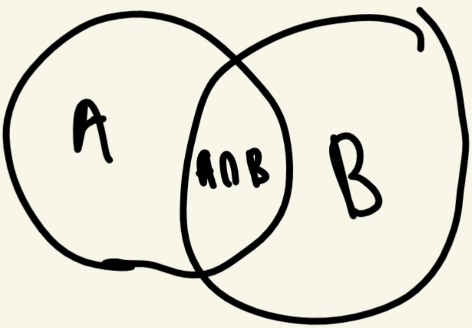
office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \operatorname{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$x+0=x$

$1+0=1$ by identity

$0+0=0$

$x(1+0) = x\cdot 1$

$x(1+0) = x$ by identity

$x\cdot 1 + x\cdot 0 = x$ by distributivity

$x + x\cdot 0 = x$ by identity

$-x+(x+x\cdot 0) = -x+x$

$-x+x+x\cdot 0 = -x+x$ Assosiativ.

$0 + x\cdot 0 = 0$ invertibility

$x\cdot 0 = 0$ identity

Also, $0\cdot x = 0$ for all

$x \in \mathbb{R}$, $x \cdot 0 = 0$ and $0 \cdot x = 0$

January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "For all $x \in R$, $P(x)$"

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, For all $x \in \mathbb{R}$, $P(x)$

$$-\square + \square = 0$$

$-(xy) + xy = 0$ by invertibility

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x)\cdot y = 0 + \underbrace{(-x)\cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x)\cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x)\cdot y$$

$$1x = x \qquad -(1x) = -x \qquad (-1)x = -x$$

$-(xy) + (x + (-x))y = (-x)y$ Distrib. and Ident.

$-(xy) + 0\cdot y = (-x)y$ Invertibility

$-(xy) + 0 = (-x)y$ Prop 1

$-(xy) = (-x)y$ by identity

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -(xy)$

Proposition 3:

For all $x, y \in \mathbb{R}$, $(-x)(-y) = xy$

Proof: Let $x, y \in \mathbb{R}$

$-x + x = 0$ Invertibility

$(-x + x)(-y) = 0 \cdot (-y)$

$(-x)(-y) + x(-y) = 0 \cdot (-y)$ dist.

$(-x)(-y) + x(-y) = 0$ Prop 1

$(-x)(-y) + x(-y) + xy = 0 + xy$

$(-x)(-y) + x(-y) + xy = xy$ (Ident.)

$(-x)(-y) + x(-y+y) = xy$ dist.

$(-x)(-y) + x \cdot 0 = xy$ invertib.

$(-x)(-y) + 0 = xy$ Prop 1

$(-x)(-y) = xy$ Identity

For all $x, y \in \mathbb{R}$ $(-x)(-y) = xy$

January 9, 2026

$\underbrace{\text{My dog}}_{\text{subject}}$ is yellow

X is yellow ↗ open sentence

Examples: $y = 2x + 1$

$x < 3$

Each variable has an allowable set of values called the "universe of discourse" for variable

Example: X is wearing Y

Quantified Statements:

Two of my cats are orange

predicate: X is orange

Universe of discourse for x:

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate: X is orange

universe: C = set of my cats

Notation: $\forall x \in C$, x is orange

Read: "for all values of X in C, X is orange"

Existential Quantified Statements

"Some of my cats are orange"

Notation: $\exists x \in C$, x is orange

Read: "for at least one value of x is C, x is orange"

"there is a value of x in C where x is orange"

" there exists an x in C for which x is orange"

$\mathbb{N}$ natural nums $\{1, 2, 3, 4 \ldots\}$

$\mathbb{Z}$ integers $\{\ldots -4, -3 \ldots 0, 1, 2, 3 \ldots\}$

$\mathbb{Q}$ rational nums All fraction & integ.

$\mathbb{R}$ real nums

$\mathbb{C}$ complex nums — combination of real / imagin. numbers

1. $\forall x \in \mathbb{N}, \boxed{0 \leq x}$ True

2. $\forall x \in \mathbb{R}, 0 < x^2$ False

3. $\forall x \in \mathbb{R}, \boxed{\exists y \in \mathbb{R}, x < y}$ True

4. $\exists y \in \mathbb{R}, \boxed{\forall x \in \mathbb{R}, x < y}$ False

5. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, y = 2x}$ True

6. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, x = 2y}$ False

$5 = 2y$?

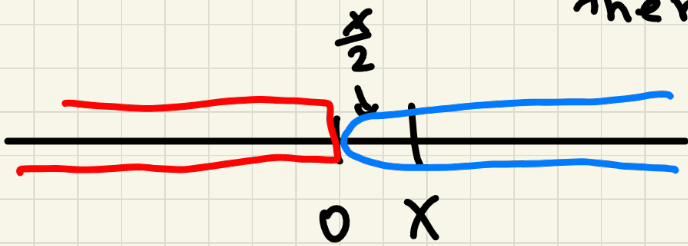
If its even then

7. $\forall x \in \mathbb{Z}$, if $\boxed{\exists y \in \mathbb{Z}, \ x = 2y}$

then $\boxed{\exists q \in \mathbb{Z}, \ x^2 = 2q}$ True

8. $\forall x \in \mathbb{R}$, if $\boxed{\forall a \in (0,\infty), \ x \le a}$

then $\boxed{x \le 0}$



If its $x \ge 0$, but never less or equal to zero

~~True~~

$\{0\}$

9. $\forall x \in \mathbb{R}$ if $\boxed{\forall a \in \mathbb{R}, \ a \cdot x \le 0}$,

then $\boxed{\forall b \in \mathbb{R}, \ 0 \le b \cdot x.}$ True

January 12, 2026

Proposition: $0 < 1$

Proof by contradiction:

To prove a proposition P

Assume $\neg P$ (negation of P)

derive a contrad. Q and $\neg Q$

Assume : $\boxed{0 \not< 1}$

Since $0 \neq 1$, we have $1 < 0$ by trichotomy

$-1 + 1 < -1 + 0$ by monotonicity

$0 < -1$

then:

$0(-1) < (-1)(-1)$ by monotonicity

$0 < 1$ by Prop 1 & Prop 3

Now, $0<1$ and $0 \not< 1$. This is contradiction

Therefore, $0 < 1$

Proposition: $1+1 \neq 1$

Proof:

Assume $1+1 = 1$

$$1+1(-1) = 1+(-1)$$

$$1 = 0$$

But, $1 \neq 0$ This is contradiction

Therefore, $1+1 \neq 1$

Proposition: $1+1 \neq 0$

Proof:

Assume $1+1=0$

We know $0<1$

then $1+0 < 1+1$ by monotonict.

So, $1 < 0$

this is a contradiction, since $0<1$

Therefore, $1+1 \neq 0$

Definition:

$2 = 1+1 \quad 3 = 2+1 \quad 4 = 3+1 \quad 5 = 4+1$

Example: $2 < 4$

Proof: $0 < 1$

then $1+0 < 1+1$

so $1 < 2$

by transitivity, $0 < 2$

then $1+0 < 2+1$

So $1 < 3$

then $1+1 < 3+1$, so $2 < 4$

$\boxed{\text{January 14, 2026}}$

Proposition:

$\forall\, x, y \in \mathbb{R}$, If $x < y$, then $-y < -x$

Direct Proof

To prove If P, then Q

Assume P

*Demonstrate Q

Therefore, If P, then Q

Proof

Let $x, y \in \mathbb{R}$

Assume $x < y$ by monoton

$$-x + x + (-y) < -x + y + (-y)$$

$$0 + (-y) < -x + 0$$

$$-y < -x \quad \therefore$$

Therefore, If $x < y$, then $-y < -x$

Monotonicity (Negative Multiplication)

$\forall x, y, z \in \mathbb{R}$, If $\boxed{x < y \text{ and } z < 0}$,

then $\boxed{yz < xz}$

Proof Let $x, y, z \in \mathbb{R}$ target $yz < xz$

Assume $x < y$ and $z < 0$

Since $z < 0$, we have $0 < -z$

$-z + z < -z + 0$

Then $x(-z) < y(-z)$

So, $-xz < -yz$

$$xz + (-xz) + yz < xz + (-yz) + yz$$

$$0 + yz < xz + 0$$

$$yz < xz$$

Therefore, If $x < y$ and $z < 0$,
QED then $yz < xz$

$\forall x, y \in \mathbb{R}$, If $x < y$, then $x + 2 < y + 3$

Proof Let $x, y \in \mathbb{R}$

Assume $x < y$

$x + 2 < y + 3$

$x+2 < y+2$ by monotonicity

Since $2 < 3$, $y+2 < y+3$ by monoton

$x+2 < y+2$ & $y+2 < y+3$

By transitivity, $x+2 < y+3$

Using transitivity to Prove $A < B$

$$\begin{array}{ll} 1 & A < C \\ 2 & C < B \\ \hline & A < B \end{array}$$

January 16, 2026

Rules of Negation

1. $\neg(\forall x \in U, P(x))$ is $\exists x \in U, \neg P(x)$

2. $\neg(\exists x \in U, P(x))$ is $\forall x \in U, \neg P(x)$

3. $\neg$ (P and Q) is $\neg$P or $\neg$Q

4. $\neg$ (P or Q) is $\neg$P and $\neg$Q

5. $\neg$ (If P, then Q) is P and $\neg$Q

$$\forall x \in \mathbb{R}, \text{ If } \overbrace{\boxed{\forall a \in (0,\infty), a \le x}}^{P}, \text{ then } \overbrace{\boxed{x \le 0}}^{Q}$$

Negation

$$\exists x \in \mathbb{R}, \overbrace{\boxed{\forall a \in (0,\infty), \ a \le x}}^{P} \text{ and } \overbrace{\boxed{0 < x}}^{Q}$$

1. $\forall_x \in \mathbb{Z}$, If $\boxed{\exists_a \in \mathbb{Z}, x = 2a+1}$ p, then $\boxed{\exists_b \in \mathbb{Z}, x = 3b}$ q

$\exists_x \in \mathbb{Z}$, $\exists_a \in \mathbb{Z}$, $x = 2a+1$ and $\forall_b \in \mathbb{Z}$, $x \neq 3b$ ✓

2. $\forall_x \in \mathbb{R}$, $\exists_y \in \mathbb{R}$, $x < y$

$\exists_x \in \mathbb{R}$, $\forall_y \in \mathbb{R}$, $x \geq y$ ✓

3. $\forall x, y \in \mathbb{R}$, If $0 < x < 1$ and $x + y = 1$, then $0 < y < 1$

✓

$\exists x, y \in \mathbb{R}$, $0 < x < 1$ and $x + y = 1$ and $y \leq 0$ or $1 \leq y$

1. $A < C$
2. $C < B$

$> A < B$

1. $\forall x, y \in \mathbb{R}$, $\boxed{\text{If } 0 < y < x}$, then $\boxed{y < 2x}$

Structure Let $x, y \in R$

Assume $0 < y < x$

$0 < y$ and $y < x$

then $0 < x$ $\quad x + 0 < x + x$

so $x < 2x$

$y < x < 2x$

So $\boxed{y < 2x}$

2. $\forall x, y \in \mathbb{R}$, If $x < 2 < y$, then $x + 2 < y^2$

$x < 2$ & $2 < y$

$2 \cdot 2 < 2 \cdot y$ so $4 < 2y$

Since $0 < 2 < y$ we have $0 < y$

$2 < y$ so $2(y) < y(y)$

$2y < y^2$

then $4 < y^2$

$x < 2$ so $x + 2 < 4$

$\boxed{\text{Now, } \quad x + 2 < 4}$

January 19, 2026

Proving Statements with Existential Qualifiers

Universal Generalization

To prove $\forall x \in \mathbb{R}, P(x$

Let $x \in \mathbb{R}$

* Show $P(x)$

Therefore, $\forall x \in \mathbb{R}, P(x)$

Existential Generalization

To prove $\exists x \in \mathbb{R}, P(x)$

Let $x = \boxed{}$ some specific value

* Demonstrate $P(x)$ for that specific value

Therefore, $\exists x \in \mathbb{R}, P(x)$

Example

$$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$$

proof: let $x \in \mathbb{R}$

let $y = x+1$

Since $0 < 1$, $x+0 < x+1$

then $x < y$

Therefore, $\exists y \in \mathbb{R}$, $x < y$

Example

$\forall x, y \in \mathbb{R}$, If $\boxed{x < y}$, then $\boxed{\exists z \in \mathbb{R}}$ $\boxed{x < z < y}$

Proof:

let $x, y \in \mathbb{R}$

Assume $x < y$

let $z = \frac{x+y}{2}$

Since $x < y$

then $x+x < y+x$

$2x < x+y$

$\frac{1}{2}(2x) < \frac{1}{2}(x+y)$

So, $x < z$

Also, since $x<y$, $x+y < y+y$

then $x+y < 2y$

so $2^{-1}(x+y) < 2^{-1}\,2y$

Now $\frac{x+y}{2} < y$, so $z<y$

Therefore, $\exists z \in \mathbb{R},\ x<z<y$

Example

$\boxed{\forall x \in \mathbb{R}}$, If $\boxed{2<x}$, then

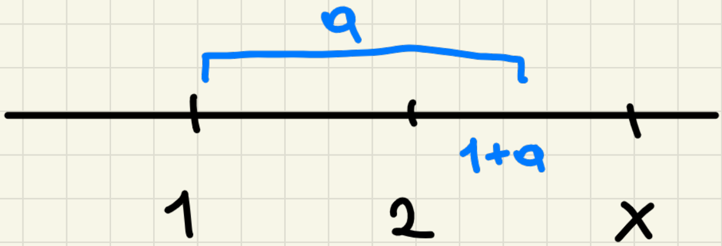
$\exists a \in \mathbb{R},\ 1<a$ and $1+a<x$

Proof

Let $x \in \mathbb{R}$

Assume $2 < x$ $\quad$ $1 < a$

Let $a = x/2$



$$1 + a = \frac{2 + x}{2}$$

$$a = \frac{x}{2}$$

Since $2 < x$, $\frac{1}{2}(2) < \frac{x}{2}$

So, $1 < a$

Then, $1 + a < a + a$

So, $1 + a < 2a$

$1 + a < 2\left(\frac{x}{2}\right)$ so $1 + a < x$

Therefore $1 < a$ and $1 + a < x$

So, $\exists a \in \mathbb{R}$, $1 < a$ and $1 + a < x$

Example

$\forall x, y \in \mathbb{R}$, If $0 < x < 1$ and $0 < y < 1$,

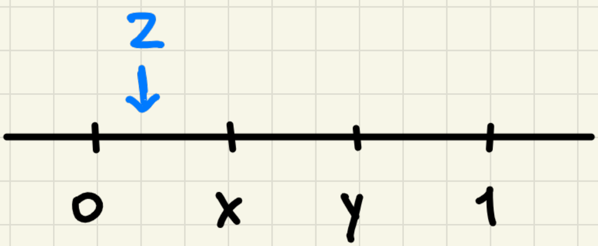
then $\exists z \in \mathbb{R}$, $0 < z < x$ and $0 < z < y$

Proof

let $x, y \in \mathbb{R}$

Assume $0 < x < 1$ and $0 < y < 1$

let $z = x \cdot y$



TBC

January 21, 2026

$\wedge$ "and" $\qquad$ $\vee$ "or" $\qquad$ $\mathbb{B} = \{T, F\}$

$\wedge$	T	F
T	T	F
F	F	F

$\vee$	T	F
T	T	F
F	T	F

Inclusive OR - includes the possibility of both statements being true

$\neg$ "not" $\quad \neg F = T \qquad \neg T = F$

$x \Rightarrow y$ statement or binary operation

In COMP 1000 $x \Rightarrow y$ is the same as $\neg x \vee y$ but not MATH 1020

In $\mathbb{R}$ $\quad x(y+z) = xy + xz$

in $\mathbb{B}$ boolean

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$$

$$x \wedge (x \vee y) = x$$

$$x \vee (x \wedge y) = x$$

$x \Rightarrow y$ on $\mathbb{B}$

behaves analogous to $x \leq y$ on $\mathbb{R}$

$$x \wedge T = x \qquad x \vee F = x$$

$$x \vee T = T \qquad x \wedge F = F$$