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1.

$$a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad d = \begin{bmatrix} 2 \\ 7 \\ 6 \end{bmatrix}$$

a)

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & -1 & 7 \\ 2 & 4 & 0 & 6 \end{bmatrix} 2R_1 - R_2 \to R_2$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -3 & 3 \\ 2 & 4 & 0 & 6 \end{bmatrix} 2R_1 - R_3 \to R_3 = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

$$\frac{2}{3} R_2 + R_3 \to R_3 = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The ronk of the matrix is 2, which is less then the number of vectors. Therefore vectors a, b, c, d are not linearly independent.

Answer: vectors are linearly dependant.

b)

The rank is 2, which means only two of these vectors form a linearly independent subset. From echelon form It can be seen that a and c are linearly independent.

$$\{a c\} = \left\{ \begin{bmatrix} 7 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

2.

Assume that $\{v_1, v_2 \dots v_k\}$ is linearly independent set, which means that the only solution

$$c_1 v_1 + c_2 v_2 + \dots c_k v_k = 0$$

trivial solution

$$c_1 = c_2 = \dots = c_k = 0$$

Now suppose that $\{v_1, v_2 \dots v_k\}$ is not linearly independent. This means that there exist scalers $d_1, d_2 \dots d_{k-1}$ not all zero, such that:

$$d_1 v_1 + d_2 v_2 \dots + d_{k-1} v_{k-1} = 0$$

extend by $d_k = 0$

$$d_1 v_1 + d_2 v_2 \dots + d_{k-1} \cdot v_{k-1} \cdot 0 \cdot v_k = 0$$

Since ~~the~~ at least one of $d_1$, $d_2 \dots d_{k-1}$ is non zero, this contradicts to assumption $\{v_1, v_2 \dots v_k\}$ is linearly independent. Thus, assumption that $\{v_1 \dots v_{k-1}\}$ must be false.

The subset $\{v_1, v_2 \dots v_{k-1}\}$ must be lin. independent

3.

a)

$$S_1 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y, z \in R, x + z = 1 \right\}$$

The zero vector in $R^3$ is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
For this vector to be in $S_1$
It must satisfy $x + z = 1$
$0 + 0 = 0$, $0 \neq 1$, so the zero vector is not in $S_1$

b)

$$S_2 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y, z \in R, x > 1, y + z = 0 \right\}$$

Check for zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
It must satisfy $x > 1$ and $y + z =$
0 is not greater than 1,
so the zero vector is not in $S_2$.

a) $S_1$ is not a subspace of $R^3$
b) $S_2$ is not a subspace of $R^3$

4.

$3\times3$ metrix. A

1. The zero matrix is in the set:

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since It is symmetric ($0^T = 0$) It belongs to the set.

2. Set is closed under addition.

$$A^t = A \quad , \quad B^t = B$$

Their sum is: $(A+B)^t = A^t + B^t = A+B$.
Since the sum of these two symmetric matrices, is also symmetric, It is closed under add

3. set is under scaler multiplication

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix} \text{ multiply by } c = \begin{bmatrix} ca & cd & ce \\ cd & cb & cf \\ ce & cf & cc \end{bmatrix}$$

$(CA)^T = CA^T = CA$, multiplication is symmetr

Ans: The set of all $3\times3$ symmetric matrices is the subspace of $M_{3\times3}$

5.

$$B = \begin{bmatrix} -2 & 1 & 1 & -1 \\ 4 & -1 & -1 & 2 \\ 4 & 0 & 0 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix} \quad \text{find } a$$

$$\left[\begin{array}{cccc|c} -2 & 1 & 1 & -1 & 1 \\ 4 & -1 & -1 & 2 & 1 \\ 4 & 0 & 0 & 2 & a \end{array}\right] \quad R_2 + 2R_1 \rightarrow R_2$$

$$= \left[\begin{array}{cccc|c} -2 & 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 4 & 0 & 0 & 2 & a \end{array}\right] \quad R_3 + 2R_1 \rightarrow R_3$$

$$= \left[\begin{array}{cccc|c} -2 & 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 2 & 2 & 0 & a+2 \end{array}\right] \quad R_3 - 2R_2 \rightarrow R_3$$

$$a+2-6$$

$$= \left[\begin{array}{cccc|c} -2 & 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & a-4 \end{array}\right]$$

$$0 = a - 4$$

$$\underline{4 = a}$$

Vector b is in the column space if $a = 4$