Nozimboev Saidjon 110186060

1.

$$u = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad w = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$$

a)

$$u \cdot w = 2 \cdot 4 + 1 \cdot 0 + 2 \cdot 3 = 8 + 0 + 6 = 14$$

~~$u \cdot (v + w) =$~~

$$u \cdot (v + w) = u \cdot \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} = 2 \cdot 5 + 1 \cdot 3 + 2 \cdot 4 = 21$$

b)

$$\|u\| = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

$$\|v\| = \sqrt{1 + 9 + 1} = \sqrt{11}$$

$$\|w\| = \sqrt{16 + 9} = \sqrt{25} = 5$$

c)

$$\frac{u}{\|u\|} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

d)

$$\|u\| \cdot \frac{v}{\|v\|} = \begin{bmatrix} \frac{3}{\sqrt{11}} \\ \frac{9}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \end{bmatrix}$$

e)

Triang: $\|w+v\| \leq \|w\| + \|v\|$

$\|w\| + \|v\| = 5 + \sqrt{11}$

$\|w+v\| = \sqrt{25+9+16} = \sqrt{50}$

$\sqrt{50} \leq 5 + \sim 3.3$ ✓

Schwartz: $|w \cdot v| \leq \|w\| \cdot \|v\|$

$2 \cdot 1 + 1 \cdot 3 + 2 \cdot 1 = 7 \Rightarrow w \cdot v$

$5 \cdot \sqrt{11} = 5\sqrt{11} \Rightarrow \|w\| \cdot \|v\|$

$7 \leq 5\sqrt{11}$ ✓

Triangl.

$\|w\| + \|v\| = 5 + \sqrt{11}$

$\|w+v\| = \sqrt{25+9+16} = \sqrt{50}$

$\sqrt{50} \leq 5 + \sim 3.3$ ✓

2) $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

a)

$1 \cdot 1 + 2 \cdot (-1) + 3 \cdot 3 = 1 - 2 + 9 = 8$ $u \cdot v$

$\sqrt{1+4+9} = \sqrt{14} \Rightarrow \|u\|$

$\sqrt{1+1+9} = \sqrt{11} \Rightarrow \|v\|$

$8 = \sqrt{14} \cdot \sqrt{11} \cdot \cos\theta$

$\frac{8}{\sqrt{14} \cdot \sqrt{11}} = \cos\theta$

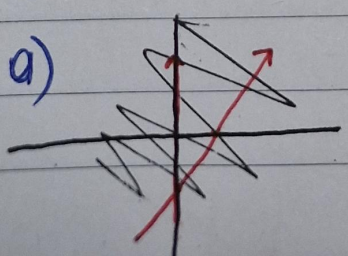
~~$\cos\theta = 0.6$~~

$\frac{8}{\sqrt{154}} = \cos\theta$ $\theta = \cos^{-1}\left(\frac{8}{\sqrt{154}}\right)$

$\theta = \sim 49.8$

b) No, because degree between u and v is $\sim 50^\circ$

3)

a) 

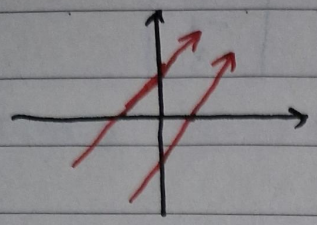
$a = 0$

$-2a = -1$

$0 = -1$

is not a linear combination

b)



$$\begin{cases} 1+(-2) = -1 \\ -2+3 = 1 \end{cases}$$

$$a + 2b = 0$$
$$-2a + 3b = 0$$

$$-2(2b) + 3b = 0$$
$$-4b + 3b = 0$$
$$b = 0 \qquad a = 0$$

c)

$$\begin{cases} a + 2b = 0 \\ -2a + 3b = -1 \end{cases}$$

$$-4b + 3b = -1$$

$$-b = -1$$
$$b = 1$$

$$a - 2 = 0$$
$$a = 2$$
$$b = 1$$

$$a \neq b \ \checkmark$$