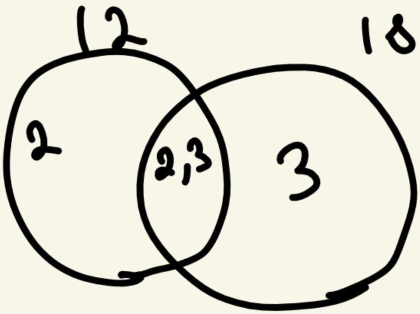
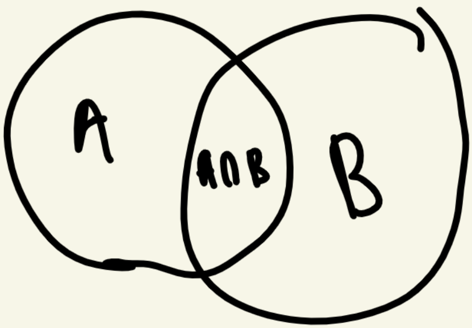
office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \operatorname{lcm}(a,b)$$



$$P(A \cup B) =$$
$$P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$x+0=x$

$1+0=\underline{1}$ by identity

$0+0=0$

$x(1+0) = x\cdot 1$

$x(1+0) = x$ by identity

$x\cdot 1 + x\cdot 0 = x$ by distributivity

$x + x\cdot 0 = x$ by identity

$-x * (x + x\cdot 0) = -x+x$

$-x + x + x\cdot 0 = -x+x$ Assosiativ.

$0 + x\cdot 0 = 0$ invertibility

$x\cdot 0 = 0$ identity

Also, $0\cdot x = 0$ for all

$x \in \mathbb{R}$, $x \cdot 0 = 0$ and $0 \cdot x = 0$

January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "For all $x \in \mathbb{R}$, $P(x)$"

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, for all $x \in \mathbb{R}$, $P(x)$

$$-\square + \square = 0$$

$-(xy) + xy = 0$ by invertibility

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x)\cdot y = 0 + \underbrace{(-x)\cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x)\cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x)\cdot y$$

$1x = x \qquad -(1x) = -x \qquad (-1)x = -x$

$-(xy) + (x + (-x))y = (-x)y$ Distrib. and Ident.

$-(xy) + 0\cdot y = (-x)y$ Invertibility

$-(xy) + 0 = (-x)y$ Prop 1

$-(xy) = (-x)y$ by identity

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -(xy)$

Proposition 3:

For all $x, y \in \mathbb{R}$, $(-x)(-y) = xy$

Proof: Let $x, y \in \mathbb{R}$

$-x + x = 0$ Invertibility

$(-x + x)(-y) = 0 \cdot (-y)$

$(-x)(-y) + x(-y) = 0 \cdot (-y)$ dist.

$(-x)(-y) + x(-y) = 0$ Prop 1

$(-x)(-y) + x(-y) + xy = 0 + xy$

$(-x)(-y) + x(-y) + xy = xy$ (Ident.)

$(-x)(-y) + x(-y+y) = xy$ dist.

$(-x)(-y) + x \cdot 0 = xy$ invertib.

$(-x)(-y) + 0 = xy$ Prop 1

$(-x)(-y) = xy$ Identity

For all $x, y \in \mathbb{R}$ $(-x)(-y) = xy$

January 9, 2026

$\underbrace{\text{My dog}}_{\text{subject}}$ is yellow

X is yellow $\nearrow$ open sentence

Examples: $y = 2x + 1$

$x < 3$

Each variable has an allowable set of values called the "universe of discourse" for variable

Example: X is wearing Y

Quantified Statements:

Two of my cats are orange

predicate: X is orange

Universe of discourse for x:

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate: X is orange

universe: C = set of my cats

Notation: $\forall x \in C$, x is orange

Read: "for all values of X in C, X is orange"

Existential Quantified Statements

"Some of my cats are orange"

Notation: $\exists x \in C$, X is orange

Read: "for at least one value of X is C, X is orange"

"there is a value of X in C where X is orange"

"there exists an X in C for which X is orange"

$\mathbb{N}$ natural nums $\{1, 2, 3, 4 \ldots\}$

$\mathbb{Z}$ integers $\{\ldots -4, -3 \ldots 0, 1, 2, 3 \ldots\}$

$\mathbb{Q}$ rational nums All fraction & integ.

$\mathbb{R}$ real nums

$\mathbb{C}$ complex nums combination of real / imagin. numbers

1. $\forall x \in \mathbb{N}, \boxed{0 \le x}$ True

2. $\forall x \in \mathbb{R}, 0 < x^2$ False

3. $\forall x \in \mathbb{R}, \boxed{\exists y \in \mathbb{R}, x < y}$ True

4. $\exists y \in \mathbb{R}, \boxed{\forall x \in \mathbb{R}, x < y}$ False

5. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, y = 2x}$ True

6. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, x = 2y}$ False

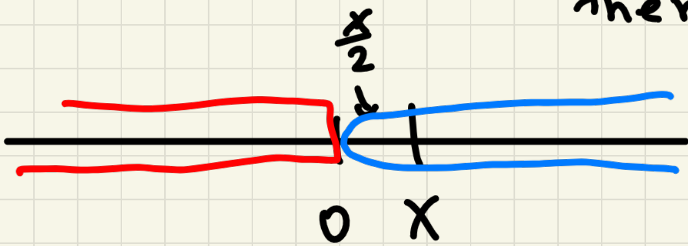
$5 = 2y$?

7. $\forall x \in \mathbb{Z}$, If $\boxed{\exists y \in \mathbb{Z}, \ x = 2y}$ If its even then

then $\boxed{\exists q \in \mathbb{Z}, \ x^2 = 2q}$ True

8. $\forall x \in R$, If $\boxed{\forall a \in (0,\infty), \ x \le a}$

then $\boxed{x \le 0}$



If its $x \geq 0$, but never less or equal to zero

True

$\{0\}$

9. $\forall x \in \mathbb{R}$ If $\boxed{\forall a \in \mathbb{R}, \ a \cdot x \le 0}$,

then $\boxed{\forall b \in \mathbb{R}, \ 0 \le b \cdot x.}$ True

January 12, 2026

Proposition: $0 < 1$

Proof by contradiction:

To prove a proposition P

Assume $\neg P$ (negation of P)

derive a contrad. Q and $\neg Q$

Assume : $\boxed{0 \not< 1}$

Since $0 \neq 1$, we have $1 < 0$

by trichotomy

$-1 + 1 < -1 + 0$ by monotonicity

$0 < -1$

then:

$0(-1) < (-1)(-1)$ by monotonicity

$0 < 1$ by Prop 1 & Prop 3

Now, $0<1$ and $0 \not< 1$. This is contradiction

Therefore, $0<1$

Proposition: $1+1 \neq 1$

Proof:

Assume $1+1=1$

$$1+1(-1) = 1+(-1)$$

$$1=0$$

But, $1 \neq 0$ This is contradiction

Therefore, $1+1 \neq 1$

Proposition: $1+1 \neq 0$

Proof:

Assume $1+1=0$

We know $0<1$

then $1+0 < \underline{1+1}$ by monotonict.

So, $1<0$

this is a contradiction, since $0<1$

Therefore, $1+1 \neq 0$

Definition:

$2 = 1+1 \quad 3 = 2+1 \quad 4 = 3+1 \quad 5 = 4+1$

Example: $2 < 4$

Proof: $0 < 1$

then $1+0 < 1+1$

so $1 < 2$

by transitivity, $0 < 2$

then $1+0 < 2+1$

So $1 < 3$

then $1+1 < 3+1$, so $2 < 4$