Lab 1

Due: Friday, January 17, 2025 (11:59 PM)

Practice Questions. You do not need to submit solutions for these problems and during the lab the teaching assistants will solve these problems if requested.

1. What is $\log_4(1024)$? Show how you determine the result.
2. Given $\log_6(x) = 2$, solve for x. Show how you determine the result.
3. Find the tightest integer upper bound and lower bound for x if $x = \log_2(22)$. Justify your answer.
4. For question 3, write the upper bound and lower bound for x using interval notation.
5. Find answers for the following floor and ceiling functions.
   (a) $\lfloor 12.3 \rfloor = ?$
   (b) $\lceil -78.999 \rceil = ?$
   (c) $\lfloor -0.0001 \rfloor = ?$
   (d) $\lceil 10001.001 \rceil = ?$
6. Solve for x if $2x^2 - 3x - 2 = 0$ using the formula for the roots of a quadratic equation.
7. Solve the inequality $12 < -2x + 4$ and write it using interval notations for x.
8. Write these expressions without using division or exponentiation by using logarithm rules. Your answer will be another expression. You do not need to calculate a numeric answer.
   (a) $\log_3(12/17)$
   (b) $\log_5(3^4)$
9. Find the tightest integer upper bound and lower bound for x if $x = \log_2(52)$. Justify your answer.
10. Solve the inequality $5 \geq -10x + 20$ and write it using interval notation for x.
11. Find the tightest integer upper bound and lower bound for x if $x = \log_2(25)$. Justify your answer.
12. Simplify each of the following to a single power of 2:
    (a) $(2^4)^2$
    (b) $(2^4) \cdot (2^2)$
    (c) $(2^4)/(2^2)$
    (d) $(4^2)/(2^2)$
13. Given $\log_5(x) = 2$, solve for x. Show how you determine the result.

Marked Questions. Submit your answers for these problems to the submission form on Brightspace. The submission form has an equation editor for entering mathematical expressions. Each question has its own text box where you enter your answer, and images can be added into your answer if necessary.

1. What is $\log_{16}(4096)$? Show how you determine the result.
2. Given $\log_3(x) = 5$, solve for x. Show how you determine the result.
3. Find the tightest integer upper bound and lower bound for x if $x = \log_5(153)$. Justify your answer and write the upper bound and lower bound for x using interval notation.
4. For which real values x is $\log_2 |x|$ defined? Justify your answer.
5. Find answers for the following floor and ceiling functions.
   (a) $\lfloor 1.4 \rfloor = ?$
   (b) $\lceil 1.4 \rceil = ?$
   (c) $\lfloor -1.4 \rfloor = ?$
   (d) $\lceil -1.4 \rceil = ?$
6. Solve for x if $-2x^2 + x + 1 = 0$ using the formula for the roots of a quadratic equation.
7. Solve the inequality $3 \leq -4x + 5$ and write it using interval notation for x.
8. Is $\log_b(x) = \log_b(c) \cdot \log_c(x)$ a true identity? For this question, first try at least one example. If it is true then prove it; see the proofs in the basic math lecture for ideas.

   [Hint: let $m = \log_b(x)$ and $n = \log_b(c)$ and $p = \log_c(x)$. Then use the definition of logarithms and rules of exponents to show $m = np$.]
9. Prove the identity $\log_b(x/y) = \log_b(x) - \log_b(y)$. You can assume $b > 0$, $b \neq 1$, $x > 0$, and $y > 0$. See the proofs in the basic math lecture for ideas.
10. Simplify each of the following to a single power of 2, 3, or 4:
    (a) $(2^3)^4$
    (b) $(3^2) \cdot (3^4)$
    (c) $(4^3)/(4^2)$
    (d) $(8^2)/(2^2)$