Lab 2

Due: Friday, January 24, 2025 (11:59 PM)

Practice Questions. You do not need to submit solutions for these problems and during the lab the teaching assistants will solve these problems if requested.

1. Let p, q, and r be the following propositions:

   p: You have the flu.

   q: You miss the midterm exam.

   r: You pass the course.

   Using the above propositions, write these compound propositions in English sentences:

   (a) $p \rightarrow \neg r$

   (b) $\neg p \wedge q \wedge r$

   (c) $\neg q \leftrightarrow r$

2. Which of these are propositions?

   (a) Do not pass go.

   (b) There is no green bird in BC.

   (c) $4 + z = 7$

   (d) Jupiter is made of water.

3. Construct a truth table for the following compound propositions:

   (a) $(p \vee q) \wedge \neg r$

   (b) $(p \wedge q) \leftrightarrow \neg r$

4. Show using a truth table that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology. Justify your answer.

5. Use a truth table to verify the associative law $(p \vee q) \vee r \equiv p \vee (q \vee r)$. Justify your answer.

6. Give a proof that $\neg p \rightarrow \neg q$ and $q \rightarrow p$ are logically equivalent without using truth tables.

   You can assume the "common logical equivalences" from the formula sheet and rule C1: that $\phi \rightarrow \psi$ is logically equivalent to $\neg\phi \vee \psi$ for all logical expressions ϕ and ψ.

   Each time you use an equivalence rule provide the abbreviation (e.g., DM$\wedge$) or name of the rule.

7. Give a proof that $p \vee (\neg p \wedge q)$ and $p \vee q$ are logically equivalent using the "common logical equivalences" from the formula sheet.

   For each step provide the abbreviation (e.g., DM$\wedge$) or name of the rule that you used.

Marked Questions. Submit your answers for these problems to the submission form on Brightspace. The submission form has an equation editor for entering mathematical expressions. Each question has its own text box where you enter your answer, and images can be added into your answer if necessary.

1. Let p be the proposition "You have a car", q be the proposition "You miss the game", and r be the proposition "You play golf". Express the following as an English sentence: $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

2. Construct a truth table for the following compound proposition: $\neg p \leftrightarrow \neg q$.

3. Construct a truth table for the following compound proposition: $r \rightarrow (p \wedge \neg q)$.

4. Show using a truth table that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology. Justify your answer.

5. Use a truth table to verify $p \wedge (p \vee q) \equiv p$. Justify your answer.

6. Give a proof that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent **without** using truth tables.

   You can assume the "common logical equivalences" from the formula sheet and rule C1: that $\phi \rightarrow \psi$ is logically equivalent to $\neg\phi \vee \psi$ for all logical expressions ϕ and ψ.

   Each time you use an equivalence rule provide the abbreviation (e.g., DM$\wedge$) or name of the rule.

   Note: You cannot use rule C6 in your proof (this is the rule you are trying to prove). You can only use the "common logical equivalences" on the top of page 3 of the formula sheet and rule C1.

7. Prove that $(\neg q \wedge (\neg p \rightarrow q)) \rightarrow p$ is a tautology using any of the propositional equivalence rules from page 3 of the formula sheet. Each line of your proof should include an equivalent logical formula and a reason (give the abbreviation or name of the equivalence rule used).