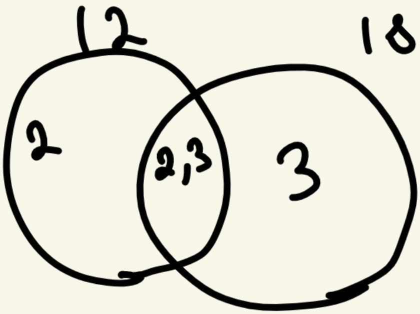
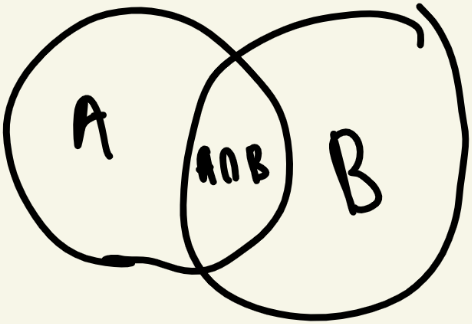
office hours
Erie 3125

January 5, 2026



$$\frac{ab}{\gcd(a,b)} = \operatorname{lcm}(a,b)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proposition 1:

For all $x \in \mathbb{R}$, $x \cdot 0 = 0$

and $0 \cdot x = 0$

proof: Let $x \in \mathbb{R}$

$x+0=x$

$1+0=\underline{1}$ by identity

$0+0=0$

$x(1+0) = x\cdot 1$

$x(1+0) = x$ by identity

$x\cdot 1 + x\cdot 0 = x$ by distributivity

$x + x\cdot 0 = x$ by identity

$-x + (x+x\cdot 0) = -x+x$

$-x+x + x\cdot 0 = -x+x$ Assosiativ.

$0 + x\cdot 0 = 0$ invertibility

$x\cdot 0 = 0$ identity

Also, $0\cdot x = 0$ for all

$x \in \mathbb{R}$, $x \cdot 0 = 0$ and $0 \cdot x = 0$

January 7, 2026

Proposition 2: For all $x, y \in \mathbb{R}$

$$(-x)y = -(xy)$$

proof: Let $x, y \in \mathbb{R}$

Universal Generalization

To prove "For all $x \in R$, $P(x)$"

Let $x \in \mathbb{R}$

* Demonstrate $P(x)$

Therefore, For all $x \in \mathbb{R}$, $P(x)$

$$-\boxed{} + \boxed{} = 0$$

$$-(xy) + xy = 0$$ by invertibility

$$\underbrace{-(xy)}_{\text{keep}} + xy + (-x)\cdot y = 0 + \underbrace{(-x)\cdot y}_{\text{keep}}$$

$$-(xy) + xy + (-xy) = 0 + (-x)\cdot y$$

$$-(xy) + xy + (-1)(xy) = 0 + (-x)\cdot y$$

$1x = x \qquad -(1x) = -x \qquad (-1)x = -x$

$$-(xy) + (x + (-x))y = (-x)y$$ Distrib. and Ident.

$$-(xy) + 0\cdot y = (-x)y$$ Invertibility

$$-(xy) + 0 = (-x)y$$ Prop 1

$-(xy) = (-x)y$ by identity

Therefore, for all $x, y \in \mathbb{R}$ $(-x)y = -(xy)$

Proposition 3:

For all $x, y \in \mathbb{R}$, $(-x)(-y) = xy$

Proof: Let $x, y \in \mathbb{R}$

$-x + x = 0$ Invertibility

$(-x + x)(-y) = 0 \cdot (-y)$

$(-x)(-y) + x(-y) = 0 \cdot (-y)$ dist.

$(-x)(-y) + x(-y) = 0$ Prop 1

$(-x)(-y) + x(-y) + xy = 0 + xy$

$(-x)(-y) + x(-y) + xy = xy$ (Ident.)

$(-x)(-y) + x(-y+y) = xy$ dist.

$(-x)(-y) + x \cdot 0 = xy$ invertib.

$(-x)(-y) + 0 = xy$ Prop 1

$(-x)(-y) = xy$ Identity

For all $x, y \in \mathbb{R}$ $(-x)(-y) = xy$

January 9, 2026

$\underbrace{\text{My dog}}_{\text{subject}}$ is yellow

X is yellow ↗ open sentence

Examples: $y = 2x + 1$

$x < 3$

Each variable has an allowable set of values called the "universe of discourse" for variable

Example: X is wearing Y

Quantified Statements:

Two of my cats are orange

predicate: X is orange

Universe of discourse for x:

The set of all my cats

Universal Quantified Statements

"All my cats are orange"

predicate: X is orange

universe: C = set of my cats

Notation: $\forall x \in C$, x is orange

Read: "for all values of X in C,

x is orange"

Existential Quantified Statements

"Some of my cats are orange"

Notation: $\exists x \in C$, X is orange

Read: "for at least one value of X is C, X is orange"

"there is a value of X in C where X is orange"

"there exists an X in C for which X is orange"

$\mathbb{N}$ natural nums $\{1, 2, 3, 4 \ldots\}$

$\mathbb{Z}$ integers $\{\ldots -4, -3 \ldots 0, 1, 2, 3 \ldots\}$

$\mathbb{Q}$ rational nums All fraction & integ.

$\mathbb{R}$ real nums

combination of real / imagin. numbers

$\mathbb{C}$ complex nums

1. $\forall x \in \mathbb{N}, \boxed{0 \leq x}$ True

2. $\forall x \in \mathbb{R}, 0 < x^2$ False

3. $\forall x \in \mathbb{R}, \boxed{\exists y \in \mathbb{R}, x < y}$ True

4. $\exists y \in \mathbb{R}, \boxed{\forall x \in \mathbb{R}, x < y}$ False

5. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, y = 2x}$ True

6. $\forall x \in \mathbb{Z}, \boxed{\exists y \in \mathbb{Z}, x = 2y}$ False

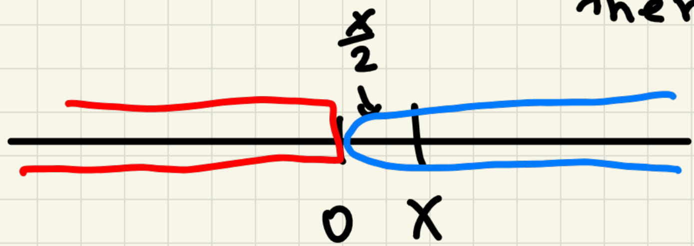
$5 = 2y$?

7. $\forall x \in \mathbb{Z}$, If $\boxed{\exists y \in \mathbb{Z},\ x = 2y}$ If its even then

then $\boxed{\exists q \in \mathbb{Z},\ x^2 = 2q}$ True

8. $\forall x \in \mathbb{R}$, If $\boxed{\forall a \in (0, \infty),\ x \leq a}$

then $\boxed{x \leq 0}$



If its $x \geq 0$, but never less or equal to zero

True

{0}

9. $\forall x \in \mathbb{R}$ If $\boxed{\forall a \in \mathbb{R},\ a \cdot x \leq 0}$,

then $\boxed{\forall b \in \mathbb{R},\ 0 \leq b \cdot x.}$ True