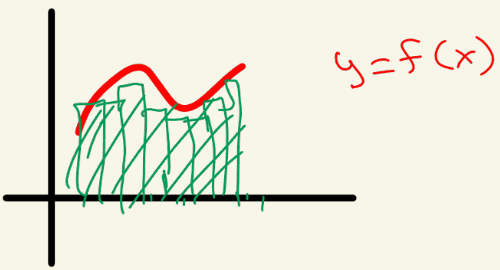
Jan 5, 2026

Part 1: Integrals
Part 2: Application of Integrals
Part 3: Sequences and Series

Definite Integrals



If $f(x) > 0$

$$\int_a^b f(x)\,dx = \text{area below the graph}$$

Conseptual Intepretations =
continuosly accumulation of quantity.

Indefinite Integrals $\int F(x)$

essentialy antiderivitives

$x^2 \rightarrow$ antider. $\frac{x^3}{3}$ so

the most general antider.

is $\frac{x^3}{3} + C$.

$\int f(x)\,dx = F(x) + C$ means

that most general antider.

of $f(x)$

$F'(x) = f(x)$ $\quad \int_0^1 x^2\,dx = \frac{1}{3}$

$\int x^2\,dx = \frac{x^3}{x} + c$

FTC: Foundamental Theory of Calculus

FTC: $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} - \frac{0}{3} = \frac{1}{3}$

this always works

January 7, 2026

$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

$\int (f'(x)g(x) + f(x)g'(x))\,dx = f(x)+g(x)+C$

$\int f'(x)+g(x)\,dx + \int f(x)g'(x) = f(x)g(x)+C$

$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)\,dx$

Integration by parts

$\int x^2\,dx - 7 = \frac{x^3}{3} + (C-7) = \int x^2\,dx$

1)

$\int x\cdot\cos x\,dx$ Integration by parts

Solution:

$$\int x\cdot \boxed{\cos x}\, dx = \int x \cdot \boxed{(\sin x)'}\, dx =$$

$$= x\cdot \sin x - \int x' \sin x \, dx = x\cdot \sin x -$$

$$- \int \sin x \, dx = x \cdot \sin x + \cos x + C$$

2)

$$\int 2^x \cdot x \, dx = \int \left(\frac{2^x}{\ln 2}\right)' \cdot x \, dx =$$

$$= \frac{2^x}{\ln 2} \cdot x - \int \frac{2^x}{\ln 2} \boxed{\overset{1}{x'}}\, dx =$$

$$= \frac{2^x \cdot x}{\ln 2} - \frac{1}{\ln 2} \int 2^x \, dx \rightarrow \frac{2^x}{\ln 2}$$

$$= \boxed{\frac{2^x \cdot x}{\ln 2} - \frac{2^x}{(\ln 2)^2}}$$

3) $\int x^2 e^x\,dx = \int x^2 \cdot (e^x)'\,dx =$

$x^2 e^x - \int (x^2)' \cdot e^x\,dx = x^2 e^x - 2\int$

$-2\int \underbrace{x \cdot e^x}\,dx = x^2 e^x - 2\int x \cdot (e^x)'\,dx$

$= x^2 \cdot e^x - 2\left(x \cdot e^x - \int \boxed{x'}^{\to 1} e^x\,dx\right) =$

$= x^2 \cdot e^x - 2 \cdot x \cdot e^x + 2\boxed{\int e^x\,dx}$ e^x

$= \boxed{x^2 e^x - 2x \cdot e^x + 2 \cdot e^x + C}$

4) $\int \ln x\,dx = \int \boxed{1}^{x'} \cdot \ln x\,dx =$

$= \int x' \ln x\,dx = x \cdot \ln x - \int x\,(\ln x)'\,dx$

$= x \cdot \ln x - \int x \cdot \frac{1}{x}\,dx = x \cdot \ln x - \int 1\,dx$

$= \boxed{x \cdot \ln x - x + c}$

5) $\int \cos^2 x \, dx = \int \cos x \cdot (\sin x)' \, dx$

$= \cos x \cdot \sin x - \int (\cos x)' \sin x \, dx$

$= \cos x \cdot \sin x + \boxed{\int \sin^2 x \, dx}$

$= \cos x \cdot \sin x + \int (1 - \cos^2 x) \, dx$

$= \cos x \cdot \sin x + \int 1 \, dx - \int \cos^2 x \, dx$

$= \cos x \cdot \sin x + x - \int \cos^2 x \, dx$

Solve for $\int \cos^2 x \, dx$:

$$\boxed{2\int \cos^2 x \, dx = \frac{1}{2}(\cos x \sin x + x) + C}$$

$u = x^2$ $\qquad$ $u = g(x)$

$\frac{du}{dx} = 2x$ $\qquad$ $g'(x)$

$du = 2x\,dx$ $\qquad$ $g'(x)\,dx$

$$\int \underbrace{f(x)}_{u}\underbrace{g'(x)}_{dv}\,dx = \underbrace{f(x)}_{u}\underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\underbrace{f'(x)}_{du}$$

$$\int u\,dv = uv - \int v \cdot du$$

1)

$$\int \underbrace{x}_{u} \cdot \underbrace{\cos x \, dx}_{dv} :$$

$u = x$ $\qquad$ $v = \sin x$

$dv = \cos x \cdot dx$

$du = dx$

$$\int \underbrace{x}_{u} \cdot \underbrace{\cos x \; dx}_{dv} = \int u \; dv = uv - \int v \, du$$

$$= x \cdot \sin x - \int \sin x \; dx = x \cdot \sin x - \cos x + C$$