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January 8, 2026

Base 2, 8, 16

Binary  $(11111111)_2 \rightarrow (\quad)_{10}$

$$1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 255$$

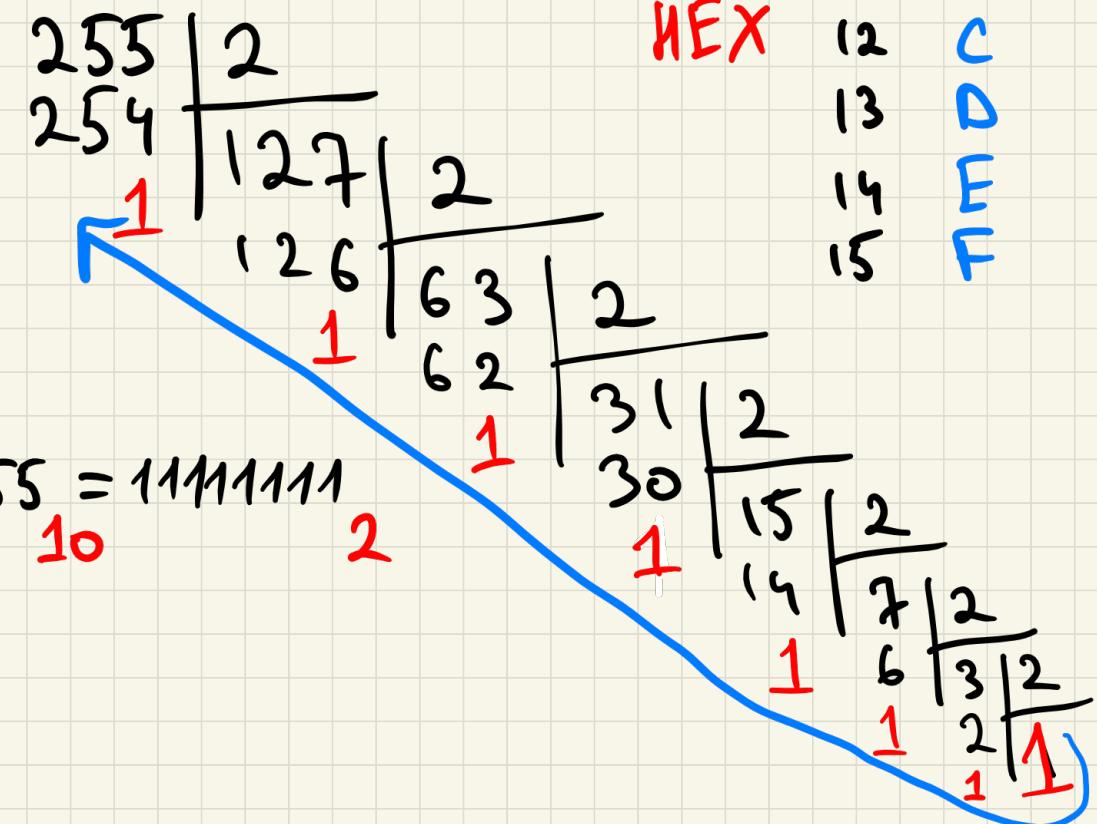
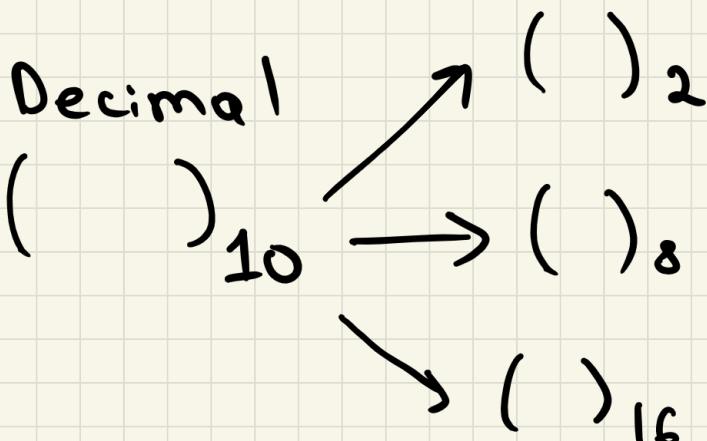
$$(377)_8 = 3 \cdot 8^2 + 7 \cdot 8^1 + 7 \cdot 8^0 = 255_{10}$$

$$(FF)_{16} \rightarrow (\quad)_{10}$$

base 16:

$$16^{\frac{1}{10}} \cdot F + 16^{\circ} \cdot F = 16 \cdot 15 + 15 = 255$$

10



$114_{10}$

$$\begin{array}{r} 114 \\ \hline 114 | 2 \\ 0 \quad 57 | 2 \\ 0 \quad 56 | 2 \\ 1 \quad 28 | 2 \\ 1 \quad 28 | 2 \end{array}$$

$1110010$

$2$

$$\begin{array}{r} 0 \quad 14 | 2 \\ 0 \quad 14 | 2 \\ 6 \quad 3 | 2 \\ 1 \quad 2 | 1 \\ 1 \end{array}$$

$$(0.12)_{10} \rightarrow (\quad)_2 = ( \quad .00011)_2$$

$$0.12 \times 2 = 0.24$$

$0$

just fraction

$$0.24 \times 2 = 0.48$$

$0$

$$0.48 \times 2 = 0.96$$

$0$

$$0.96 \times 2 = 1.92$$

$1$

$$0.92 \times 2 = 1.84$$

$1$

$$0.84 \times 2 = 1.68$$

$1$

Dec.	Binary	Dec.	Binary
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111
8	1000	Hex.	

## Binary addition

$$\begin{array}{r}
 + 01100 \\
 10001 \\
 \hline
 \boxed{11101}
 \end{array}$$

$$\begin{array}{r}
 1 \\
 + 1 0 \overset{1}{1} 1 0 \\
 \hline
 1 0 1 \overset{1}{1} 1 \\
 \hline
 1[0 1 1 0 1]
 \end{array}$$

$$\begin{aligned}
 (1)_2 + (1)_2 &= (10)_2 \\
 (10)_2 + (1)_2 &= (11)_2
 \end{aligned}$$

It is overflow, machine should know that it is overflow

## Binary subtraction

$$\begin{array}{cccc}
 -0 & -1 & -1 & +0 \\
 \hline
 0 & 1 & 0 & 1 \\
 \hline
 0 & 0 & 1 & 1
 \end{array}$$

$$\begin{array}{r}
 -10110 \\
 -10010 \\
 \hline
 00100
 \end{array}
 \quad
 \begin{array}{r}
 -10110 \\
 10011 \\
 \hline
 00011
 \end{array}$$

1's complement

$(10101111)_2$

logic operation

1's complement is inverse of digits

$(01010000)_2 + \underset{1}{1}$  1's comp.

$(01010001)$  2's complement

$$(0101)_2 = (5)_{10}$$

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$(r-1)$ 's r's complements

radix = base

$$r=10 \quad L=4 \quad N=2468 \quad r^4=10000$$

$$r^4 - 1 = 9999$$

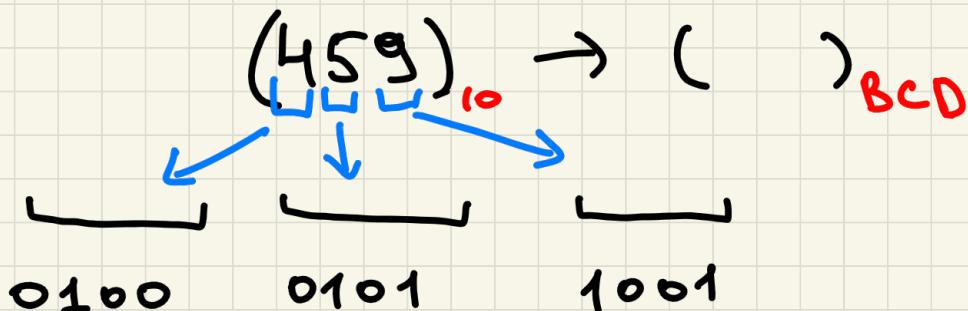
Therefore

$$\begin{array}{r} 9999 \\ - 2468 \\ \hline 7531 \end{array}$$

Decimal  $(185)_{10}$

Binary  $(10111001)_2$

Binary Coded Decimal BCD



BCD range

$$4_{10} + 8_{10} = 12_{10}$$

(0-9)

$$0100_2 + 1000_2 = 1100_2 \rightarrow 12_{10}$$

$$0110_2 \rightarrow 6_{10}$$

$$(0001\ 0010)_{BCD}$$

$$\begin{array}{r}
 + \quad 162 \\
 769 \\
 \hline
 931
 \end{array}$$

Binary Coded Decimal

0001	0110	0010
0111	0110	1001
<hr/>		
1000	1100	1011

doesn't exist in BCD  
 to solve add  $6_{10}$   
 more than 9

$$\begin{array}{r}
 0110 \quad 0110 \\
 \hline
 1001 \quad 0011 \quad 0001 \\
 9 \quad 3 \quad 1
 \end{array}$$

encoding  $\rightarrow$  public UTF-8  
 encryption  $\rightarrow$  special key

## BOOLEAN LOGIC

$A+B \rightarrow A \text{ or } B$

$A \cdot B \rightarrow A \text{ and } B$

$A', \bar{A} \rightarrow \text{NOT } A$

Inputs

A	B	A.B	A+B	A'	B'
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	1
1	1	1	1	0	0

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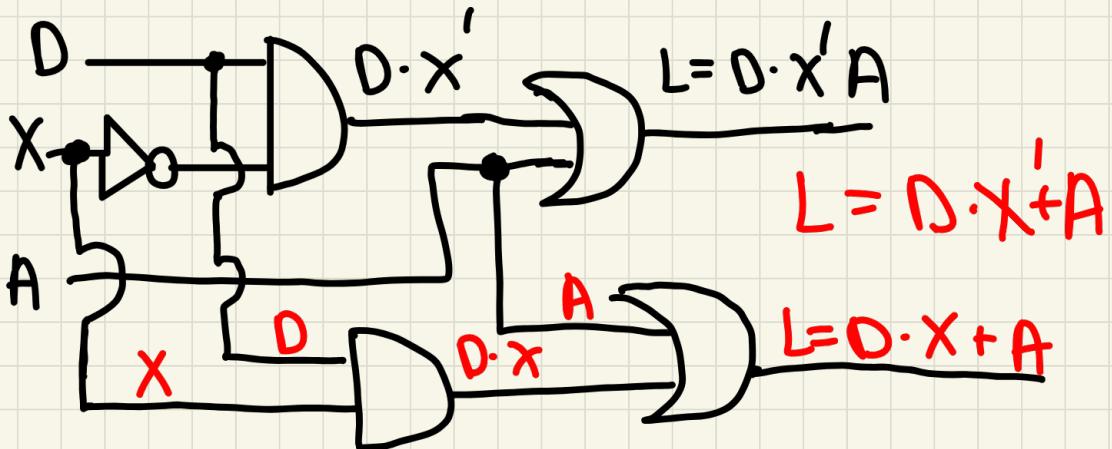
L = D · X' + A

output      ↑      ↑      inputs

Output

D	X	A	X'	D · X'	D · X' + A
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	0	0	0

0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	0	1



A	B	C	AB	$A'C$	BC	$AB + A'C$
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0

1	0	1	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	1	0	1	1	1

$$F(A, B, C) = (A+B)(A+C)$$

$$AA + AC + AB + BC \quad \text{Distributivity}$$

$$A + AC + AB + BC \quad A = AA$$

$$A(\underbrace{1+C+B}_1) + BC$$

$$A \cdot 1 + BC = \boxed{A + BC}$$

Sum of Products SOP

$$F(A, B, C) = A'B'C + ABC + A'BC'$$

Product of Sum

$$\begin{aligned}
 F(A, B, C) &= (A+B)(A+C) \\
 &= AA + AC + AB + BC \\
 &\quad \downarrow \quad \downarrow \\
 &= A(A+B+C) \\
 &= A(1+BC) \\
 &\quad \downarrow \\
 &= A + BC = A + BC
 \end{aligned}$$

Sum of Product terms

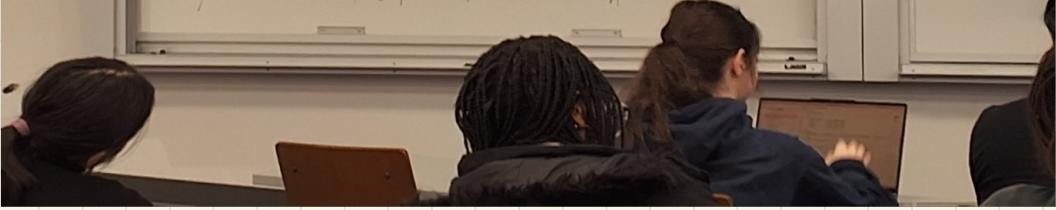
G =  $(A+B)(C+D+E)(F+H)$

Product of Sum terms

Pos

$D'$	$XV$	$A'$	$X'$	$D'X$	$D'X'A$	$D'X'A'$	$D'X'A''$
0	0	0	1	0	0	1	1
1	0	0	0	0	1	0	0
2	0	1	0	0	0	0	0
3	0	1	1	0	0	1	1
4	1	0	0	1	1	1	1
5	1	0	1	1	1	0	0
6	1	1	0	0	0	1	0
7	1	1	1	0	0	1	1

(D'X'A + D'X'A' + ...)



January 20, 2026

Q 1

$$\begin{aligned}
 y + x'z + xy' &= x + y + z \\
 &\underline{y + xy'} + x'z \\
 (y+x)(y+y') + x'z
 \end{aligned}$$

$$y + x + x'z$$

$$y + (x+x')(x+z)$$

$y + x + z = x + y + z$

**Q<sub>2</sub>**

$$x'y' + x'y + xy = x'y$$

$$x'(y' + y) + xy$$

$$x' + xy$$

$$(x' + x)(x' + y)$$

$$1 \cdot x' + y = \boxed{x'y}$$

**Q<sub>3</sub>**

$$x'y' + y'z + xz + xy + yz' \quad \text{13 operations}$$

$$x'y' + y'z(x + x') + xz + xy + yz'$$

$$\boxed{x'y'} + \boxed{xy'z} + \boxed{x'y'z} + xz + xy + yz'$$

$$x'y'(1+z) + xy'z + xz + xy + yz'$$

$$x'y' + \boxed{xy'z} + \boxed{xz} + xy + yz'$$

$$x'y' + xz(y' + 1) + xy + yz'$$

$$x'y' + xz + xy + yz'$$

$$x'y' + xz + xy(z+z') + yz'$$

$$x'y' + xz + xyz + xyz' + yz'$$

$$x'y' + xz + xyz + yz'(x+1)$$

$$x'y' + \underline{xz} + \underline{xyz} + yz'$$

$$x'y' + xz(1+z) + yz'$$

$$x'y' + xz + yz'$$

8 operations

SOP

$$F(a,b,c) = ab + bc + abc'$$

$$A(a,b,c) = \underbrace{abc}_\text{Minterms} + \underbrace{a'b'c'}_\text{Minterms} + \underbrace{abc'}_\text{Minterms}$$

Canonical

(product terms)

K	a	b	c	A	A	Minterms SOP
0	0	0	0	$a'b'c'$	0	$x=1$
1	0	0	1	$a'b'c$	0	$x'=0$
2	0	1	0	$a'b c'$	0	
3	0	1	1	$a'b c$	1	
7	1	0	0	$a b'c'$	0	
5	1	0	1	$a b'c$	0	
6	1	1	0	$a b c'$	1	
7	1	1	1	$a b c$	1	

$$A(a,b,c) = \sum m(3,6,7) \rightarrow \text{minterms}$$

$\downarrow$   
sum of products

POS (product of sum)

$$G(a,b,c) = \underbrace{(a+b+c')} \cdot \underbrace{(a'b'+c')} \cdot \underbrace{(a+b+c)}$$

Maxterms:  $x=0$

POS  $x'=1$

	a	b	c	P	
0	0	0	0	$\bar{a} \bar{b} \bar{c}$	1
1	0	0	1	$\bar{a} \bar{b} c'$	1
2	0	1	0	$\bar{a} b' \bar{c}$	0
3	0	1	1	$\bar{a} b' c'$	0
4	1	0	0	$a' \bar{b} \bar{c}$	0
5	1	0	1	$a' \bar{b} c'$	0
6	1	1	0	$a' b' \bar{c}$	0
7	1	1	1	$a' b' c'$	1

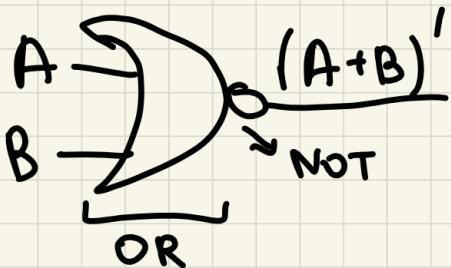
$$P(a,b,c) = \underbrace{\prod M}_{\text{POS}} (0, 1, 7) \underbrace{(0, 1, 7)}_{\text{maxterms}}$$

Always write Maxterms POS  $x=0$   $x'=1$

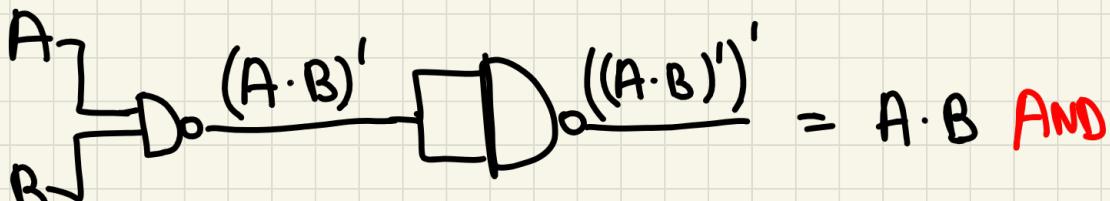
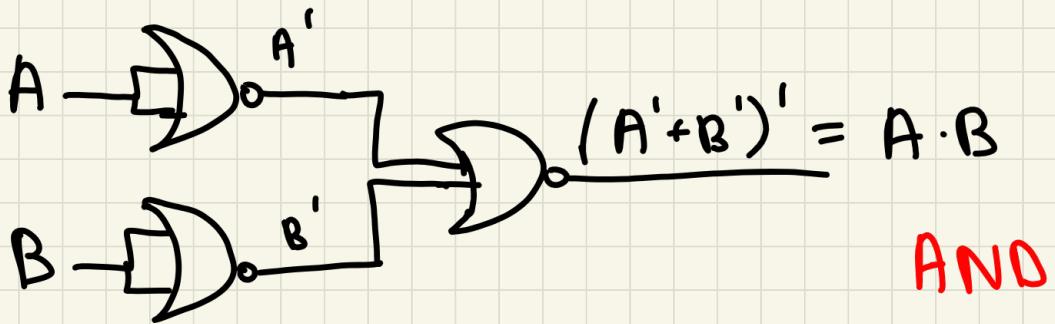
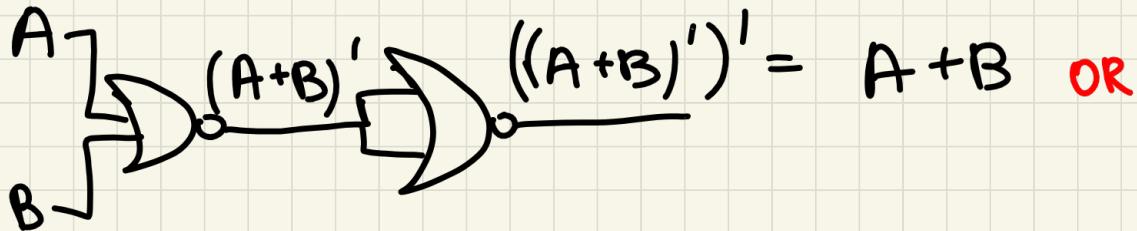
Minterms SOP  $x=1$   $x'=0$

January 22

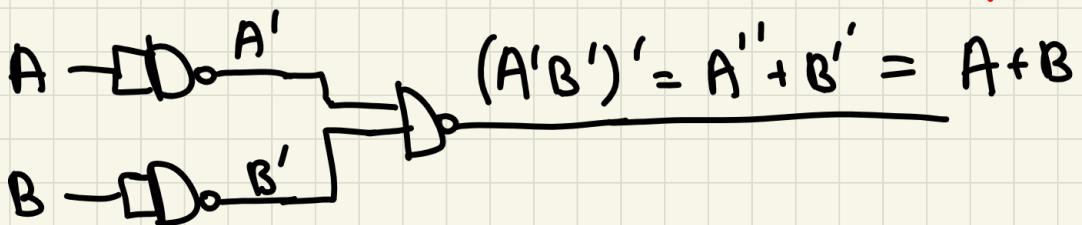
NOR Gate - Universal Gate means function completeness (OR, AND, NOT)



NOR



OR



$$F(a, b, c) = a'b'c + bc' + ac' \quad (\text{Standard})$$

missing a      missing b

$$F(a, b, c) = a'b'c + \cancel{abc'} + a'bc' + \cancel{abc'} + \cancel{ab'c'}$$

$$F(a, b, c) = a'b'c + a'bc' + abc' + ab'c'$$

SOP Canonical form

$$F(a, b, c) = (a+b+c) \cdot (a+b) \cdot (a+c') \quad (\text{Standard})$$

needs c      needs B

~~$$(a+b+c) \cdot (a+b+c) \cdot (a+b+c') \cdot (a+b+c') \cdot (a+b'+c)$$~~

$$(a+b+c) (a+b+c') (a+b'+c')$$

POS Canonical form

1-variable map

x	f(x)
0	f(0)
1	f(1)

## 2 variable maps

$x$	$y$	$f(x)$
0	0	$f(0)$
0	1	$f(1)$
1	0	$f(2)$
1	1	$f(3)$

0	00	01
1	10	11

## 2 variable maps

$x$	$y$	$z$	$f(x)$
0	0	0	$f(0)$
0	0	1	$f(1)$
0	1	0	$f(2)$
0	1	1	$f(3)$
1	0	0	$f(4)$
1	0	1	$f(5)$
1	1	0	$f(6)$
1	1	1	$f(7)$

gray codes

	$x'y'z'$	$x'y'z$	$x'y'z$	$x'y'z'$
0	$x^2x'$ 000	$x'y'z$ 001	$x'y'z$ 011	$x'y'z'$ 010
1	100 4	101 5	111 7	110 6

$$SOP \quad x = 1 \\ x' = 0$$

$$f(x, y, z) = \sum m (1, 3, 5, 7)$$

$$x'y'(z+z')$$

$$x'y'(1) = x'y'$$

$$(2^n) = 1, 2, 4, 8, 16$$

0	0	1	1	0
1	0	1	1	0

Group of 1's  
4

$$x'y'z + x'yz + xyz' + xyz$$
$$z(x'y' + x'y + xy' + xy)$$

$$z \left( x' \underbrace{(y'+y)}_1 + x \underbrace{(y'+y)}_1 \right)$$

$$z \underbrace{(x' + x)}_1$$

z

Shorter way to calculate

# 4 variable map

w x y z | f(w, x, y, z)

16 rows

w \ x	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0100 4	0101 5	0111 7	0110 6
11	1100 12	1101 13	1111 15	1110 14
10	1000 8	1001 9	1011 11	1010 10

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$$F(a, b, c) = \underbrace{a \cdot b \cdot c' + ab}_{\text{and}} + \underbrace{c}_{\text{OR}}$$

x		1	1
		1	1
	1	1	

$$SOP = x'y + z$$

2 implicant

a' \ b'c'	b'c'	b'c	b'c'	
a' \ 00	0	1	1	0
a \ 1	1	0	1	0

ab'c'  $\downarrow$  a'c  $\downarrow$  bc

$$F(a, b, c) = \sum m(1, 3, 4, 7)$$

$$F(a, b, c) = ab'c' + a'c + bc$$


---

a' \ bc	b'c'	b'c	bc	bc'
a' \ 1	1 <sub>0</sub>	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>
a \ 0	1 <sub>4</sub>	1 <sub>5</sub>	1 <sub>6</sub>	0

$$a'(b'c' + b'c + bc + bc') = a'$$

$a' + c$

$wx$	$y'z'$	$y'z$	$yz$	$yz'$
$w'x'$	1 0	0 1	0 3	1 2
$w'x$	1 4	1 5	0 7	0 6
$wx$	1 2	1 8	1 5	0 14
$wx'$	1 8	0 9	1 10	1 11

Annotations: A green bracket groups columns 1, 2, and 3. A red bracket groups columns 4 and 5. A green bracket groups rows 1, 2, and 3. A red bracket groups rows 4 and 5. A green bracket groups rows 1, 2, and 3. A red bracket groups rows 4 and 5. A green bracket groups columns 1, 2, and 3. A red bracket groups columns 4 and 5. A green bracket groups rows 1, 2, and 3. A red bracket groups rows 4 and 5. A green bracket groups columns 1, 2, and 3. A red bracket groups columns 4 and 5. A green bracket groups rows 1, 2, and 3. A red bracket groups rows 4 and 5.

Grouping:

groupsize 8, 4, 2, 1

$$\begin{aligned}
 F(w, x, y, z) = \\
 = (0, 2, 4, 5, 8, 10, 11, \\
 12, 13, 15)
 \end{aligned}$$

$$F(w, x, y, z) = 1, 2, 3, 4, 5$$

1	0	0	1
1	1	0	0
1	1	1	0
1	0	1	1

January 29, 2026

February 3, 2026

$$F(A, B, C, D) = \sum m(1, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15)$$

AB	CD	00	01	11	10
00	0	1	1	3	2
01	4	1	5	7	6
11	12	1	13	15	14
10	8	1	9	11	10

$$F(A, B, C, D) = \sum m(1, 5, 6, 7, 11, 12, 13, 15)$$

AB	CD	00	01	11	10
00	0	1	1	3	2
01	4	1	5	7	6
11	6	1	13	15	14
10	8	9	11	1	10

Prime:  $BD, A'C'D, A'BC, ABC', ACD$

# Combinational LOGIC Gates

## Half adder

$$\begin{array}{r} A \\ + B \\ \hline \text{Sum} \end{array}$$

2 inputs : A, B  
2 outputs : S, C

*inputs*

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = A'B + AB'$$

$$C = AB$$

