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1.

$$S = \{ \vec{x} \in \mathbb{R}^4 \mid \vec{x} \cdot \vec{n} = 0 \}$$

$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$$

$$x_1 + 0x_2 - x_3 + 2x_4 = 0$$

$$x_1 - x_3 + 2x_4 = 0$$

$$\dim(S) = 4 - 1 = 3$$

Answer: 3

2

$$S = \left\{ \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix} \mid a, b, c \in R \right\} \text{ of } R^3$$

$$\begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

basis for S :-

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

3.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 6 & 0 & 1 \end{array}\right] \quad R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -2 & 1 \end{array}\right] \quad \frac{1}{2}R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & \frac{1}{2} \end{array}\right] \quad R_1 - 2R_2 \rightarrow R_1$$

$$A^{-1} = \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & \frac{1}{2} \end{array}\right] = \begin{bmatrix} 3 & -1 \\ -1 & \frac{1}{2} \end{bmatrix}$$

$(A^{-1})^T = A^{-1}$, as A is symmetric

$(A^T)^{-1} = A^{-1}$, as A is symmetric.

4.

$$B = (A^2)^{-1}$$

$$BA^2 = I$$

$$BA^2A^{-1} = A^{-1}$$

$$BA = A^{-1}$$

Thus, BA is the inverse of A.

5.

$$B = \begin{bmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 6 & 2 & 6 & 9 \\ -2 & -4 & 1 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 3 & 6 & 2 & 6 & 9 & 0 \\ -2 & -4 & 1 & 1 & -1 & 0 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \to R_2 \\ R_3 + 2R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 3 & 7 & 7 & 0 \end{array}\right] \quad R_3 - 2R_2 \to R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & -2 & -2 & 0 \end{array}\right] \quad -\frac{1}{2} R_3 \to R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \quad R_2 - 3R_3 \to R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \quad R_1 - 3R_3 \to R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$x_1 + 2x_2 + x_3 + x_5 = 0$$

$$x_3 = 0$$

$$x_4 + x_5 = 0 \qquad x_4 = -x_5$$

$$x_2 = t \qquad x_5 = s$$

$$x_1 = -2t - s \qquad x_3 = 0 \qquad x_4 = -s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Null Space basis

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

Colum Space

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} \right\}$$

Row Space

$$\left\{ \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix} \right\}$$