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January 8, 2026

Base 2, 8, 16

$$\begin{array}{cccccccccccc} & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & & & & & \\ \text{Binary } (11111111) & \rightarrow & ( & & & & & & ) & & & & & \\ & & & & & & & & & & & & & \end{array}$$
$$1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$
$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 =$$
$$= 255$$

$$(377)_8 = 3 \cdot 8^2 + 7 \cdot 8^1 + 7 \cdot 8^0 = 255$$

$$(FF)_{16} \rightarrow ( )_{10}$$

base 16:

$$16^{\overset{1}{F}} + 16^{\circ F} = 16 \cdot 15 + 15 = 255_{10}$$

Decimal  $( )_{10}$   $\rightarrow$   $( )_2$   
 $( )_{10} \rightarrow ( )_8$   
 $( )_{10} \rightarrow ( )_{16}$

10  
11  
12  
13  
14  
15  
A  
B  
C  
D  
E  
F

HEX

$$\begin{array}{r}
 255 \mid 2 \\
 \hline
 254 \mid 127 \mid 2 \\
 \hline
 126 \mid 63 \mid 2 \\
 \hline
 62 \mid 31 \mid 2 \\
 \hline
 30 \mid 15 \mid 2 \\
 \hline
 14 \mid 7 \mid 2 \\
 \hline
 6 \mid 3 \mid 2 \\
 \hline
 2 \mid 1 \\
 \hline
 \end{array}$$

$255_{10} = 11111111_2$   
 (A blue arrow points from the bottom of the binary representation to the first remainder '1' in the division process.)

114<sub>10</sub>

$$\begin{array}{r|l} 114 & 2 \\ \hline 114 & 57 \\ \hline 0 & 56 \\ & 28 \\ & 14 \\ & 7 \\ & 3 \\ & 1 \end{array}$$

1110010<sub>2</sub>

$$(0.12)_{10} \rightarrow ( )_2 = ( .000111 )_2$$

$0.12 \times 2 = 0.24$	0	just fraction ↓
$0.24 \times 2 = 0.48$	0	
$0.48 \times 2 = 0.96$	0	
$0.96 \times 2 = 1.92$	1	
$0.92 \times 2 = 1.84$	1	
$0.84 \times 2 = 1.68$	1	

Dec.	Binary	Dec.	Binary
1	0001	9	1001
2	0010	10	1010
3	0011	11	1011
4	0100	12	1100
5	0101	13	1101
6	0110	14	1110
7	0111	15	1111
8	1000		

A
B
C
D
E
F
Hex.

Binary addition

$$\begin{array}{r}
 \times \quad 01100 \\
 \quad 10001 \\
 \hline
 \quad 11101
 \end{array}$$

$$\begin{array}{r}
 \overset{1}{+} \overset{1}{1} \overset{1}{0} \overset{1}{1} 1 0 \\
 \underline{10111} \\
 1[01101]
 \end{array}$$

$$(1)_2 + (1)_2 = (10)_2$$

$$(\overset{1}{1}0)_2 + (1)_2 = (11)_2$$

It is overflow, machine should know that it is overflow

## Binary subtraction

$$\begin{array}{r}
 \overset{0}{-} \overset{0}{0} \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 \overset{1}{-} \overset{1}{1} \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 \overset{1}{-} \overset{0}{0} \\
 \hline
 1
 \end{array}
 \quad
 \begin{array}{r}
 \overset{0}{-} \overset{1}{1} \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 -10110 \\
 \underline{10010} \\
 00100
 \end{array}$$

$$\begin{array}{r}
 -10110 \\
 \underline{10011} \\
 00011
 \end{array}$$

1's complement

logic operation

$$(10101111)_2$$

1's complement is inverse of digits

$$(01010000)_2 \text{ 1's comp.}$$

$$(01010001)_2 \text{ 2's complement}$$

$$(0101)_2 = (5)_{10}$$

January 13, 2026

$(r-1)$ 's  $r$ 's complements

radix = base

$$r=10 \quad L=4 \quad N=2468 \quad r^4=10000$$

$$r^4 - 1 = 9999$$

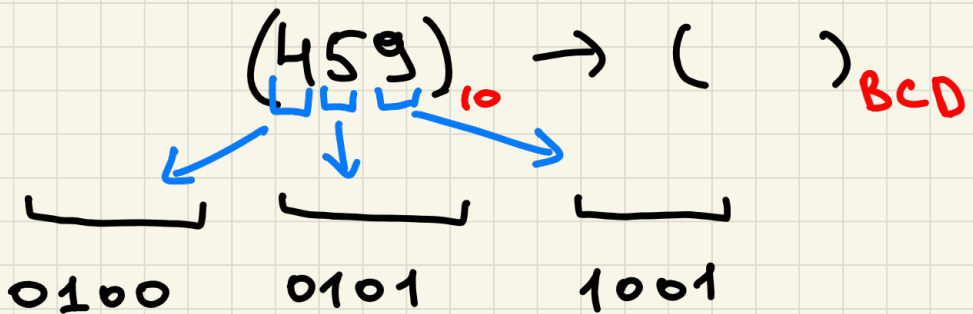
Therefore

$$\begin{array}{r} 9999 \\ - 2468 \\ \hline \underline{7531} \end{array}$$

Decimal  $(185)_{10}$

Binary  $(10111001)_2$

Binary Coded Decimal BCD



$$4_{10} + 8_{10} = 12_{10}$$

BCD range  
(0-9)

$$0100_2 + 1000_2 = 1100_2 \rightarrow 12_{10}$$

$$0110_2 \rightarrow 6_{10}$$

$(\underline{0001} \underline{0010})_{BCD}$



$$\begin{array}{r}
 + 162 \\
 769 \\
 \hline
 931
 \end{array}$$

Binary Coded Decimal

$$\begin{array}{r}
 0001 \quad 0110 \quad 0010 \\
 0111 \quad 0110 \quad 1001 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1000 \quad \underline{1100} \quad \underline{1011} \\
 \text{doesn't exist in BCD} \quad \text{more than 9} \\
 \text{to solve add } 6_{10}
 \end{array}$$

$$\begin{array}{r}
 0110 \quad 0110 \\
 \hline
 \underline{1001} \quad \underline{0011} \quad \underline{0001} \\
 9 \quad 3 \quad 1
 \end{array}$$

encoding  $\rightarrow$  public UTF-8  
 encryption  $\rightarrow$  special key

## BOOLEAN LOGIC

$$A+B \rightarrow A \text{ or } B$$

$$A.B \rightarrow A \text{ and } B$$

$$A', \bar{A} \rightarrow \text{NOT } A$$

inputs

A	B	A.B	A+B	A'	B'
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	1
1	1	1	1	0	0

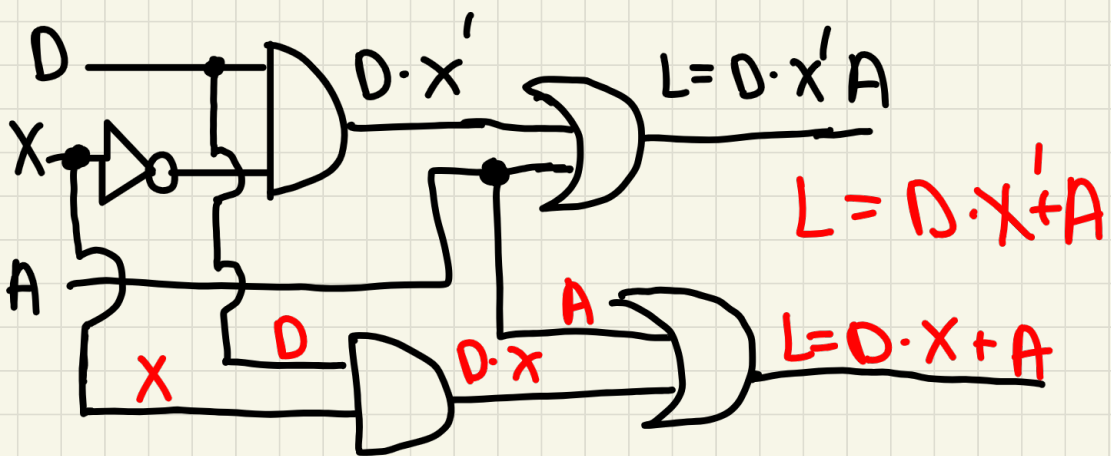
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$$L = \underbrace{D \cdot X'}_{\substack{\uparrow \\ \text{output}}} + \underbrace{A}_{\substack{\uparrow \\ \text{inputs}}}$$

D	X	A	X'	D · X'	D · X' + A
0	0	0	1	0	0
0	0	1	1	0	1
0	1	0	0	0	0

output

0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	0	1



A	B	C	AB	A'C	BC		AB + A'C
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	0	0	0	0
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0

1	0	1	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	1	0	1	1	1

$$F(A, B, C) = (A+B)(A+C)$$

$$AA + AC + AB + BC \quad \text{Distributivity}$$

$$A + AC + AB + BC \quad A = AA$$

$$A(\underbrace{1 + C + B}_1) + BC$$

$$A \cdot 1 + BC = A + BC$$

Sum of Products SOP

$$F(A, B, C) = A'B'C + ABC + A'BC'$$

Product of Sum

$$F(A,B,C) = (A+B)(A+C)$$

$$= (A+A) + A+C + AB+BC$$

$$= A + A + C + AB + BC$$

$$= A(1+C) + C + AB + BC$$

$$= A \cdot 1 + C = A + C$$

$$F = A + BC + DEF$$

Sum of Product terms

$$G = (A+B)(C+D+E)(F+H)$$

Product of Sum terms

(k)	D	X	A	X'	D X'	D X' + A		
0	0	0	0	1	0	0	←	$D'X'A$
1	0	0	1	0	0	1		
2	0	1	0	0	0	0		
3	0	1	1	0	0	1	←	$D'XA$
4	1	0	0	1	1	1	←	$DX'A'$
5	1	0	1	0	1	1	←	$DX'A$
6	1	1	0	0	0	0		
7	1	1	1	0	0	1	←	$DXA$

(D'X'A + D'XA + DX'A + DXA + ...)