GEBZE TECHNICAL UNIVERSITY

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRONICS ENGINEERING

MATH 214

NUMERICAL METHODS

PROJECT 3

Deadline: 14.12.2023

Name - Surname	
Student ID	

1) ***Scenario for Electronics Engineering Application:*** A specialized device operating in optical frequencies is sought to be designed for an electronic engineering application. To simulate the scattered electromagnetic waves from particle arrays used in this device, a need arises to model a two-dimensional representation of a single particle. The two-dimensional cross-section of this particle is represented by a periodic $\rho(\phi)$ function over $\phi \in [0,2\pi]$, defining a closed C curve on the xy-plane as follows:

$$\rho(\phi) = \left[\left(\left|\cos\frac{3\phi}{4}\right|\right)^8 + \left(\left|\sin\frac{3\phi}{4}\right|\right)^8\right]^{-\frac{1}{4}}$$

Also, as recalled from basic physics courses, $\vec{\boldsymbol{a}}_{\boldsymbol{x}}$ and $\vec{\boldsymbol{a}}_{\boldsymbol{y}}$ respectively represent unit vectors in the x and y directions. For the curve mentioned above, the tangent vector $\vec{\boldsymbol{l}}(\boldsymbol{\phi})$ at each ϕ is given by:

$$\vec{\boldsymbol{l}}(\boldsymbol{\phi}) = \frac{1}{L(\phi)}\left[\vec{\boldsymbol{a}}_{\boldsymbol{x}}\,\frac{dx(\phi)}{d\phi} + \vec{\boldsymbol{a}}_{\boldsymbol{y}}\,\frac{dy(\phi)}{d\phi}\right] \qquad L(\phi) = \sqrt{\left(\frac{dx(\phi)}{d\phi}\right)^2 + \left(\frac{dy(\phi)}{d\phi}\right)^2}$$

Accordingly,

a) Create ϕ in the range $[0,2\pi]$ with 360 points using the **`linspace`** command in the NumPy library in Python. Use the **`pyplot`** submodule in the Matplotlib library to plot the $\rho(\phi)$ function in *polar coordinates* with respect to ϕ. Then, using the expressions $x(\phi) = \rho(\phi)\cos\phi$ and $y(\phi) = \rho(\phi)\sin\phi$, plot the y function in *cartesian coordinates* with respect to x. Include the obtained graphics in your report and write down your observations.

b) Using forward difference, backward difference, three-point end point, and three-point midpoint formulas, implement Python functions that calculate numerically the x and y components of $\vec{\boldsymbol{l}}(\boldsymbol{\phi})$ and the value of $g(\phi) = \frac{d\rho(\phi)}{d\phi}$ for any desired small step size h and for all ϕ values in the $[0,2\pi]$ interval, divided into as many points as you want. Then, plot the calculated values of $\frac{dx(\phi)}{d\phi}$, $\frac{dy(\phi)}{d\phi}$, $\frac{d\rho(\phi)}{d\phi}$ and $L(\phi)$with respect to ϕ. Include the obtained graphics in your report.

c) Divide the $[0,2\pi]$ interval into any desired N subintervals. Then, determine the third-degree Hermite polynomial representing the estimated value of the $\rho(\phi)$ function for each interval. Overlay the obtained piecewise function plot and the given $\rho(\phi)$ function plot with respect to ϕ on the same graph and include them in your report. Observe how the results change as N increases, evaluate the relative error, and write your comments in the report.

d) Utilizing the piecewise function found with the third-degree Hermite polynomial interpolation in (c), repeat (b). Overlay each calculated function over the corresponding

version from (b) in the same graph and include it in your report. Then, compare the results obtained in (b) and (d), evaluate the relative error, and include the error graph in your report.

e) Repeat (c) using Cubic Spline interpolation instead of Hermite.

f) Using the results from (e), repeat (b). Overlay each calculated function over the corresponding version from (b) in the same graph and include it in your report. Then, compare the results obtained in (b) and (e), evaluate the relative error, and include the error graph in your report.

g) For given $f(\phi) = ln\big(\rho(\phi)\big)L(\phi)$ function, calculate the integral

$$\int_0^{2\pi} f(\phi)d\phi$$

using the Trapezoidal, Simpson, and Gaussian quadrature methods, respectively.

h) Review all the steps and write in detail what you have learned and your findings from this project assignment and provide your comments.

Instructions for Project

1) **This assignment cannot be shared anywhere without permission.**

2) **You may seek assistance from other sources to complete your project, but please avoid plagiarism (academic dishonesty). Include the sources you benefited from in the "References" section of your project report. If you are getting help from a Chegg homework solution, when referencing it, add the Chegg source code to the end of your report. If you are using an AI tool like ChatGPT, you should include screenshots of the conversation history in your report. Assignments meeting the following criteria listed below will be considered plagiarism and will receive a grade of 0.**
   - **Directly copied from another student's work,**
   - **Directly copied from online source codes,**
   - **Directly copied from Chegg homework solutions,**
   - **Directly copied from generated codes/solutions by AI tools,**
   - **Projects where the used sources are not properly referenced.**

   The necessary disciplinary actions, as outlined in the regulations, will be taken for the owner of the assignment.

3) **The project report should be prepared in detail in English on a Jupyter Notebook, and the code should be written in Python.**

4) **At the end of the report, a "References" section should be added, and references should be provided as follows:**

   [1] R. L. Burden, J. D. Faires, "Numerical Analysis", 9th Edition, Brooks/Cole, 2011.

   [2] Chegg, "Homework Solution", "https://www.chegg.com/.....", Access Date: 12.10.2023. (The quoted solutions and code are included in the report.)

[3] ChatGPT, "https://chat.openai.com". (The quoted solutions and code are included in the report.)

[4] Python Forum, "Task Calendar Problem", "https://python-forum.io/thread-40640.html", Access Date: 12.10.2023.

5) You must upload your project report and code, which includes .ipynb format notebook file and its PDF version (two files in total), to the assignment created on Microsoft Teams. Before uploading your files, please pay attention to the following file naming rules:

StudentID_Name_Surname_Project1_ MATH214.ipynb

StudentID_Name_Surname_Project1_ MATH214.pdf

Example: ***121024005065_Sebastian_Cozer_Project1_MATH214.ipynb***
Example: ***121024005065_Sebastian_Cozer_Project1_MATH214.pdf***

After uploading your project assignment files on Teams, you are required to print a copy of your report by adding the cover page from this file to the first page and submit your report to the research assistants by the deadline.

Good Luck!