Newtonian Mechanics

Any point particle can be defined by its mass and position vector. Vote that from here onwards, we will always use vectors for position which means we must assume an absolute time. i.e time is identical for all bodies, whether at rest or in motion. This was later challenged by Einstein's Theory of Relativity but at lower speeds (speeds NOT close to the speed of light) this is more or less true and will be a constant assumption in Newtonian Mechanics.

Newton's First Law

An isolated body (one on which no external force acts) moves with constant velocity.

This means that an object at rest will remain at rest and an object moving at constant velocity will remain at constant velocity, provided no external force is applied.

Newton's Second Law

We might know this from A-level Mechanics as $F = ma$, but this is not strictly accurate or specific enough. It is better defined as:

The rate of change of linear momentum, $\underline{p}$, of a body is equal to the force $\underline{F}$ acting on the body.

A note on differentiating a vector

In order to differentiate a vector, we split it into its x, y and z components and treat each of its components as a function of time.

$$\underline{a} = a_x(t)\,\hat{\underline{i}} + a_y(t)\,\hat{\underline{j}} + a_z(t)\,\hat{\underline{k}}$$

we the differentiate each of the components separately.

$$\frac{d\underline{a}}{dt} = \left[\frac{d}{dt} a_x(t)\right]\hat{\underline{i}} + \left[\frac{d}{dt} a_y(t)\right]\hat{\underline{j}} + \left[\frac{d}{dt} a_z(t)\right]\hat{\underline{k}}$$

Newton's Second Law (continued)

Newton's Second Law can be expressed mathematically as:

$\boxed{\underline{F} = \frac{d}{dt}\underline{p}}$ but density $\underline{\rho} = m\underline{v}$. so:

$\underline{F} = \frac{d}{dt} m\underline{v}$. Differentiating this, we get:

$$\underline{F} = m\left[\frac{d}{dt} v_x(t)\right]\hat{\underline{i}} + m\left[\frac{d}{dt} v_y(t)\right]\hat{\underline{j}} + m\left[\frac{d}{dt} v_z(t)\right]\hat{\underline{k}}$$

but, the derivative of velocity is the rate of change of velocity, which is the acceleration. We therefore have x, y and z components for the acceleration vector and can express this in the familiar:

$\boxed{\underline{F} = m\underline{a}}$ Note that this only works for constant mass.

Newton's Third Law

> Wherever two bodies A and B interact, the force F_1 that A exerts on B is equal to the force F_2 that B exerts on A.

This is essentially saying the old addage, " every action has an equal and opposite reaction."

F_1 and F_2 form what is known as an action-reaction pair.

Inertial Frames

An inertial reference frame is a reference frame that is not accelerating. In other words, it is a reference frame in which Newton's First Law holds true.

An example of this is juggling on the ground or in a train at constant velocity versus juggling in an accelerating train. The first two are easier because they are inertial reference frames and Newton's First Law is working.

Superposition

The force $\underline{F}$ in the second law is the vector sum of all the forces acting on the body.

$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 \dots + \underline{F}_n$. By substituting $\underline{F} = m\underline{a}$ and dividing by constant m, we can also get:

$$\underline{a} = \underline{a}_1 + \underline{a}_2 + \underline{a}_3 + \dots + \underline{a}_n$$