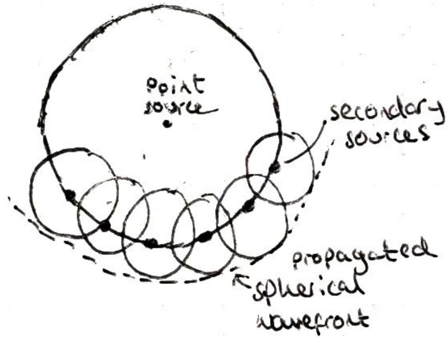
Huyger's Model of Wave Propagation

Huyger's principle is a way of diagrammatically representing wave propagation and can help us easily understand concepts like reflection or refraction. It works as follows:

- wavefronts propagate from an initial disturbance in all directions
- each point on the wavefront acts as a secondary source
- further wavefronts propagate from secondary sources
- Where wavefronts coincide, a new wavefront is formed
  ↳ i.e constructive interference forms one new larger wavefront

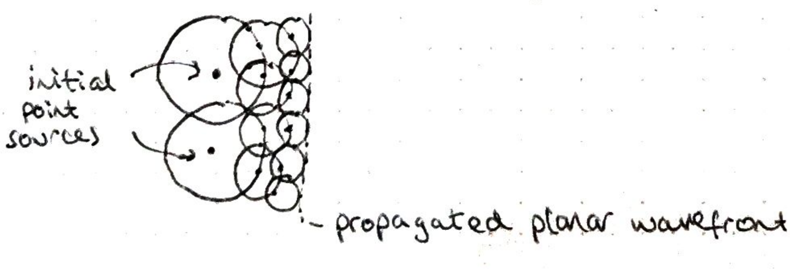
For a single point source:

A better diagram with an animation is available on the slides



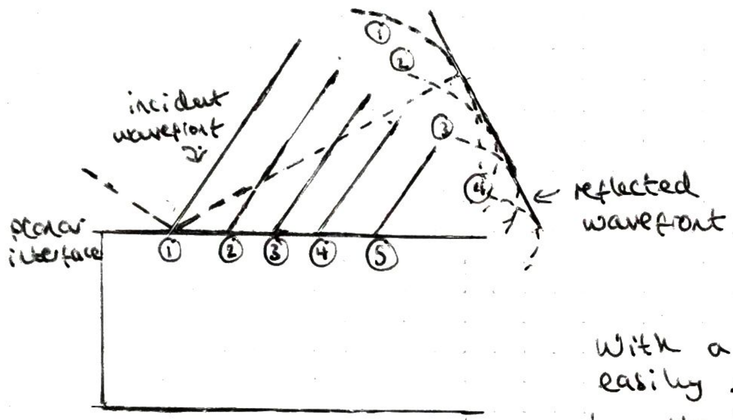
The initial wavefront propagates from the point source. We then imagine every point on this initial wavefront is a secondary source. We draw wavefronts propagating from each of the secondary sources. If we use enough secondary sources (theoretically an infinite number) the propagated wavefront becomes obvious. We can even continue further by imagining tertiary sources on all the wavefronts from the secondary sources.

If we use many point sources to start with (the more the better) we approximate a planar wavefront if the point sources are close together. A good animation for this is available in the slides. A basic diagram is:



Reflection at an Interface

A good animation is on the slides. In the diagram below, the planar wavefronts are solid lines and the construction lines are represented as a dashed line. At each point a planar wavefront touches the interface, we imagine that point to have a spherical wavefront propagating from it.

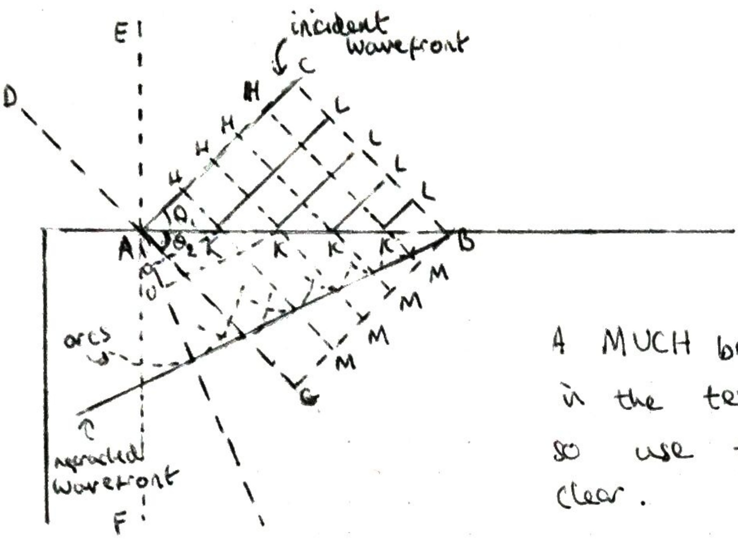


The arc of the spherical wavefronts originating from the point sources 1, 2, 3, 4 are labelled. As we can see, this makes a reflected wavefront.

With a little geometry, we can easily show the angle between the direction of propagation and the normal of the planar interface is equal for both the incident wavefront and the reflected wavefront.

Note, the arcs are drawn such that the radius of the arc is the distance of the arc from its point source. The radius is defined such that the [sum of the radius and the distance of the secondary source from the initial incident wavefront] is the same for each of the arcs. This is a little confusing so look at the diagram in the textbook or on the slides if it isn't clear.

Refraction at an Interface



This time, we don't draw the secondary wavefronts in the same way as for reflection.
We need to draw arcs that define paths of the same time rather than distance.

A MUCH better diagram is available in the textbook and on the slides so use that one if this isn't clear.

By considering triangles AKH and AKO (which should be symmetric about the interface although its not obvious in the diagram), and noting that the lengths AO and HK must be inversely proportional to the corresponding wave speed, we can say:

$$\frac{1}{v_1} HK = \frac{1}{v_2} AO$$

Using a little trigonometric magic, we can say:

$$\frac{1}{v_1} AK \sin(\theta_1) = \frac{1}{v_2} AK \sin\theta_2$$

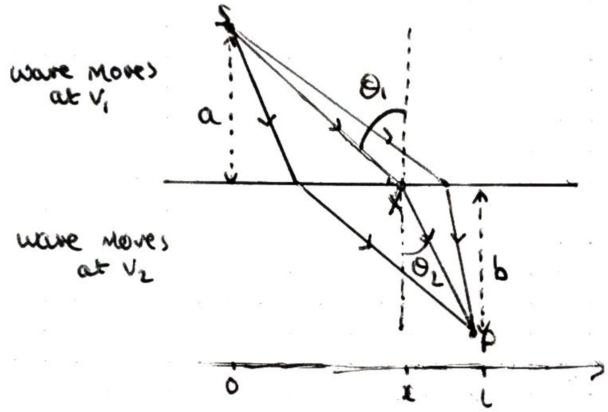
$$\frac{1}{v_1} \sin(\theta_1) = \frac{1}{v_2} \sin(\theta_2) \quad \text{defining } n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Thus we have derived Snell's law which we all know and love!

Fermat's Principle of Least Time

Fermat's Principle states "When travelling between two points, a wave follows the path that takes the least time"
The diagram below represents this:



Which path will the light take between S and P? Fermat's Principle tells us it will be the one that takes the least time. If X is the point the ray reaches the interface:

We can say the length of SX is $l_s = \sqrt{a^2 + x^2}$
the length of XP is $l_p = \sqrt{b^2 + (l-x)^2}$
The total time taken is given by $\tau = \frac{l_s}{v_1} + \frac{l_s}{v_2}$

$$\tau = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{b^2+(l-x)^2}}{v_2}$$ we need to find the minimum of this

$$\frac{d\tau}{dx} = \frac{\frac{1}{2}(a^2+x^2)^{-1/2} \cdot 2x}{v_1} + \frac{\frac{1}{2}(b^2+(l-x)^2)^{-1/2} \cdot (2(l-x)(-1))}{v_2}$$

$$= \frac{x(a^2+x^2)^{-1/2}}{v_1} - \frac{(l-x)(b^2+(l-x)^2)^{-1/2}}{v_2}$$

$$= \frac{1}{v_1} \underbrace{\frac{x}{\sqrt{a^2+x^2}}}_{\sin(\theta_1)} - \frac{1}{v_2} \underbrace{\frac{l-x}{\sqrt{b^2+(l-x)^2}}}_{\sin\theta_2}$$ = 0 for minimum

letting $n = \frac{c}{v}$:

$$n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$

$$\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

We have thus extracted a known true result. So Fermat's Principle tells us it's the middle path!