Ideal Gases and Gas Laws

There are four gas laws, confirmed and derived empirically:

Boyle's Law: The volume of an ideal gas at fixed temperature is inversely proportional to its pressure.
$V \propto \frac{1}{P}$

Charles' Law: The volume of an ideal gas at fixed pressure is proportional to its temperature.
$V \propto T$

Gay-Lussac's Law: The pressure of an ideal gas kept at constant volume is proportional to its temperature.
$P \propto T$

Avogadro's Law: Equal volumes of different gases at the same pressure and temperature contain equally many particles.
$V \propto N$

These combine to give the ideal gas law:

$$\boxed{PV = Nk_B T}$$

where k_B is Boltzmann's constant, empirically found to be $k_B = 1.38 \times 10^{-23}\ \mathrm{J K^{-1}}$

Often, this is quoted as

$$\boxed{PV = n_M RT}$$

where n_M is the number of molecules in "moles" and R is the ideal gas constant:

$R = k_B N_A$ where N_A is a constant called Avogadro's number

A note on moles

Historically, the mass of atoms were unknown so it was hard to say exactly how many molecules were in a sample. The best one could do is compare one sample to another and say, for example, "this sample contains the number of molecules as 2g of hydrogen gas".

This is what we still do today! N_A is the number of molecules in 12g of Carbon-12, $N_A = 6.022 \times 10^{23}$

So the number of molecules in moles $\boxed{n_M = \frac{N}{N_A}}$

We can also say:

the number of molecules in moles $\boxed{n_M = \frac{M}{A_r}}$

where M is the mass of the sample

and A_r is the relative atomic mass.

Back to the topic

We can relate the ideal gas law to the kinetic theory of gases: $PV = Nk_BT \quad PV = \frac{2}{3}U$

$\therefore \quad \boxed{U = \frac{3}{2}Nk_BT}$ So the internal energy of a monoatomic ideal gas is a function of temperature only!

Going back to the first assumption in the Kinetic Theory of gases, that the internal energy is the total kinetic energy:

$$\boxed{\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT}$$