Radioactivity

If a nuclide has a far higher binding energy than some of its neighbours, it is often energetically favourable for a nuclide with low binding energy (parent) to decay into one with higher binding energy (daughter) be releasing either an α particle, a β^+ or β^- particle and a neutrino or antineutrino.
Another source of radioactivity is when a nucleus in a metastable excited state decays into its ground state, emitting γ rays.

Decay Rates

We define the decay constant λ as the probability of a nucleus decaying in one second.
So for $N(t)$ nuclei, the expected no. decays per second is $\lambda N(t)$.

$$\Rightarrow \quad \frac{dN(t)}{dt} = -\lambda N(t)$$ solving this, we find:

$$\Rightarrow \quad N(t) = N_0 e^{-\lambda t}$$ where $N_0 = N(t=0)$

The mean time taken for the number of parent nuclei to fall to $1/e$ of initial value is called mean lifetime τ:

$$\tau = \frac{1}{\lambda}$$

Half-life is defined as $\tau_{1/2}$, the time taken for number of parent nuclei to fall to $1/2$ of initial value:

$$\tau_{1/2} = \frac{\ln 2}{\lambda} = \ln 2 \, \tau$$

Random Decay

The decay constant λ is only a probability so the number of decays per second is not precisely $\lambda N(t)$.

From general statistics, we know that if the expected number of events is a given period of time is ΔN, then the error on this number is $\sqrt{\Delta N}$, such that there is 68% probability that the "true" number of events lies in the range $\Delta N \pm \sqrt{\Delta N}$

No. decays per second is measured in Curies where one Curie is 3.7×10^{10} decays per second.
This is the no. decays per second of one gram of $^{226}_{88}Ra$

Carbon Dating

$^{14}_{6}C$ is created in the atmosphere by cosmic ray bombardment at the same rate that $^{14}_{6}C$ decays, so the global $^{14}_{6}C$ abundance is constant.

Living things contain carbon that is constantly being rejuvenated through processes like photosynthesis or eating plants / animals. But dead objects that contain carbon cannot rejuvenate their carbon supply as $^{14}_{6}$Carbon decays into $^{14}_{7}$Nitrogen through β-decay with halflife 5700 years.

Here, by measuring concentration of $^{14}_{6}C$ in fossil, we can estimate its age, using an estimate of how much $^{14}_{6}C$ we expect the living being would have had (around 1 part in 1.3×10^{12}) and how much we measure now.

Multi-Modal Decays

Sometimes a radioactive nucleus can decay through more than one way, and each way will have its own decay constant. An example is ${}^{212}_{83}Bi$:

$${}^{212}_{83}Bi \rightarrow {}^{208}_{81}Ti + \alpha \qquad \text{alpha decay} \quad \lambda_1$$

$${}^{212}_{83}Bi \rightarrow {}^{212}_{84}Po + \beta^- + \nu \qquad \beta^- \text{ decay} \quad \lambda_2$$

with total mean lifetime 536 seconds. and Ti : Po is 9 : 16

so $$\frac{dN(t)}{dt} = -\lambda_1 N(t) - \lambda_2 N(t)$$

$$\Rightarrow \quad N(t) = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

since $\tau = 536$: $\lambda_1 + \lambda_2 = \frac{1}{536} = 1.86 \times 10^{-3} \, s^{-1}$

and using ratio of abundances: $\frac{\lambda_1}{\lambda_2} = \frac{9}{16}$ combine these:

$$\lambda_1 = 6.8 \times 10^{-4} \, s^{-1}$$
$$\lambda_2 = 11.8 \times 10^{-3} \, s^{-1}$$

Decay Chains

We can form chains of decay in which the daughter nuclei from the first parent decays further.

eg.
$$\begin{tikzcd} {}^{210}_{83}Bi \arrow[r, "\beta"] \arrow[r, bend right, "1"'] & {}^{210}_{84}Po \arrow[r, "\alpha"] \arrow[r, bend right, "2"'] & {}^{206}_{82}Pb \end{tikzcd}$$

First let's consider 1 on next page.

(1) $$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$ as expected

$$\Rightarrow \quad N_1(t) = N_1(0)\, e^{-\lambda_1 t}$$ straightforward

Now let's consider (2)

(2) Here, as the Po decays into Pb, some of it is replenished by decay from Bi to Po. So:

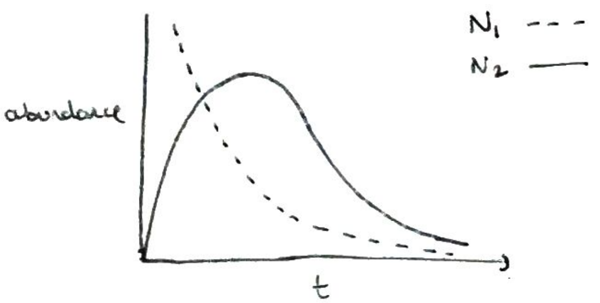
$$\frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_1(t)$$ subbing in eq from before:

$$\frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_1(0)\, e^{-\lambda_1 t}$$

This is an inhomogeneous differential equation whose solution with $N_2(0) = 0$ is given by:

$$N_2(t) = N_1(0) \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

which when plotted looks like:



As we can see, initially N_2 is replenished faster than it decays so we have an increase. But after some time N_1 is sufficiently depleted so that N_2 is not replenished fast enough, so it also decays.

It is possible to reach a secular equilibrium in which the quantities of daughter nuclei remain unchanged if:

$$\lambda_1 N_1 = \lambda_2 N_2$$

i.e N_2 continues to be replenished as fast as it decays.

Induced Radioactivity

We can bombard a non radioactive nuclide with neutrons or other particles to make it radioactive.

eg. bombard $^{23}_{11}Na$ with neutrons to make $^{24}_{11}Na$ which decays via β decay to $^{24}_{12}Mg$.

If we assume the rate at which the radioactive nuclide is being made is R, then:

$$\frac{dN(t)}{dt} = R - \lambda N(t)$$

if $N(0) = 0$, then: $N(t) = \frac{R}{\lambda}(1 - e^{-\lambda t})$

which starts at 0 and then grows so that asymptotically $R = \lambda N$ which is the equilibrium state in which we produce radioactive nuclide at the same rate at which they decay.