The Liquid Drop Model

Before we begin, we need to clarify some nomenclature:

- Nucleon: A proton or neutron
- Atomic Number Z: the number of protons in nucleus
- Atomic Mass Number A: the number of nucleons in nucleus
- Nucleide: A nucleus with a specified value of A and Z, written ${}^{A}_{Z}\{Ch\}$ where Ch is a chemical symbol
- Isotope: Nucleus with given atomic number but different atomic mass
- Isotone: Nucleus with given no. neutrons but different atomic number
- Isobar: Nucleus with given atomic mass no. A but different no. protons
- Mirror Nuclei: Two nuclei with odd A in which number of protons in one is the number of neutrons in the other and vice versa.

Binding Energy

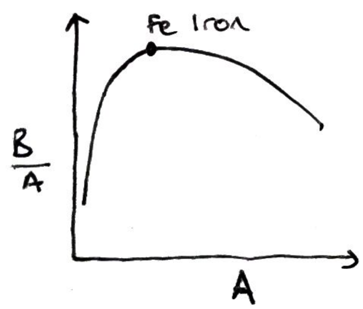
The mass of a nucleide is given by

$$M_N = Zm_p + (A-Z)m_n - B(A,Z)/c^2$$

where $B(A,Z)$ is the binding energy, which is due to strong force.

Note we have done $\frac{E}{c^2}$ to convert Binding energy to a mass

$B(A,Z)/c^2$ is often called the mass defect.



The Binding energy per nucleon increases until Fe and decreases from there.

So how do we calculate Binding Energy?

Semi-Empirical Mass Formula

The formula for Binding Energy is:

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(Z-N)^2}{A} + \frac{((-1)^Z + (-1)^N)}{2} \frac{a_P}{A^{1/2}}$$

① ② ③ ④ ⑤

We will discuss each of these separately, thinking of the nucleus as a liquid drop.

① Volume Term

Each nucleon interacts through strong force with its nearest neighbours so we get a term proportional to A contributing to binding energy : $a_V A$

② Surface Term

Nucleons at surface of liquid drop interact only with those inside, so there is a decrease in binding energy proportional to surface area of drop : $-a_S A^{2/3}$

③ Coulomb Term

The coulomb repulsion between protons pushes back against the strong force, decreasing the binding energy. Since each proton repels all other protons, the repulsion is proportional to Z^2 and is inversely proportional to nuclear radius which we previously saw was $\propto A^{1/3}$: $-a_C \frac{Z^2}{A^{1/3}}$

④ Asymmetry Term

A quantum effect that arises due to Pauli exclusion principle. Each energy state can only be occupied by 2 protons or 2 neutrons. So if $Z = N$, then there is no problem but if we replace one neutron by a proton, the proton would occupy a higher energy state, since the others are occupied. The spacing between energy levels is inversely proportional to volume of nucleus (and thus A) : $-a_A \frac{(Z-N)^2}{A}$

⑤ Pairing Term

Experimentally we see that 2 protons or 2 neutrons bind more strongly than 1 proton and 1 neutron.

To account for this, we need a term that:

- if both protons and neutrons are even, we add
- if both protons and neutrons are even, we subtract
- if one is odd and the other is even, we do nothing

Bohr and Mottelson also showed this term is proportional to $1/A^{1/2}$

so: $$\frac{((-1)^Z + (-1)^N)}{2} \frac{a_P}{A^{1/2}}$$

So we have 5 terms in our completed formula!

From fitting to measured binding energies, we find:

$a_V = 15.56$ MeV

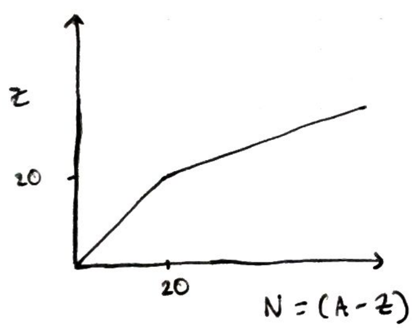
$a_S = 17.23$ MeV

$a_C = 0.697$ MeV

$a_A = 23.285$ MeV

$a_P = 12.0$ MeV

Empirically, we find that up to $N \sim 20$, the stable isotopes have $N \sim Z$ but above this $N \sim 1.5Z$:



Qualitatively, this is because of the coulomb term. Protons bind less tightly than neutrons since they repel each other. So it is energetically favourable to have more neutrons than protons. Below a certain limit, the asymmetry effect beats the coulomb effect, and thus equal numbers of protons and neutrons are favoured.