Boundary Conditions

Unlike continuity conditions which are constraints on the wave due to the system, boundary conditions are constraints we apply directly. For example, in the case of a guitar string, we fix the ends of the string, applying the constraint that at these ports $\psi = 0$.

Let's consider a guitar string. This will have infinite standing wave solutions $\psi(x,t)$ caused by the superposition of $\psi_+(x,t) = \psi_+(u_+)$ and the reflected wave $\psi_-(x,t) = \psi_-(u_-)$ where:

$u_+ = t - x/v_p \quad u_- = t + x/v_p$. Let's impose some boundary conditions. We know that at $x=0$ and $x=L$, the string is fixed so:

$\psi(0,t) = \psi(L,t) = 0$

$\Rightarrow \psi_+(t) + \psi_-(t) = 0 \longrightarrow \psi_-(t) = -\psi_+(t)$ ①

$\Rightarrow \psi_+(t - L/v_p) + \psi_-(t + L/v_p) = 0 \rightarrow \psi_+(t - L/v_p) = -\psi_-(t + L/v_p)$

sub into:

which leaves us with:

$\psi_+(t - L/v_p) = \psi_+(t + L/v_p)$ Putting this into the general solution:

$\psi(x,t) = \psi_+(t - x/v_p) - \psi_+(t + x/v_p)$ if we let $t' = t - L/v_p$:

$\psi_+(t') = \psi_+(t' + \frac{2L}{v_p})$ i.e ψ_+ is periodic, repeating every $\Delta t = \frac{2L}{v_p}$

This is the time taken for a travelling wave to make be reflected at each end and return to its starting point.

if $\Delta t = \frac{2L}{v_p}$, we find the harmonic frequencies at $\frac{n}{\Delta t}$

so $f = \frac{n v_p}{2L}$