Charge Independence and Isospin

The binding energies of two mirror nuclei ($N_1 = Z_2$, $Z_1 = N_2$) is almost the same since the only term in $B(A,Z)$ that is not invariant under this swap is the Coulomb term, but this term is very small since in the nucleus the strong force terms are dominant.

We also notice from experiments that the excited states are similar. (We expect ground state energies to be similar due to similar Binding energies, but the excited energies being similar is surprising!)

All this suggests that strong interactions are charge independent.

Let's consider a pair of mirror nuclei whose proton no. and neutron no. differ by 2, and the nuclide "between" them.

eg. ${}^6_2He$ ${}^6_4Be$ and the nuclide between them is ${}^6_3Li$

Each of the mirror nuclei has a closed shell of two protons and a closed shell of two neutrons, with ${}^6_2He$ having an unclosed shell of 2 neutrons and ${}^6_4Be$ having an unclosed shell of 2 protons. ${}^6_3Li$ has 1 proton and 1 neutron in outer shell.

From the principle of charge independence, we might expect all 3 nuclides to display same energy level structure. But this is not the case! Although ${}^6_3Li$ has states that are close to the mirror nuclei, there are also states that have no equivalent!

The reason for this is the Pauli exclusion principle. For ${}^6_2He$ and ${}^6_4Be$, the 2 protons or 2 neutrons in the outer shell have the same spin so they cannot be in the same state. But for ${}^6_3Li$ with 1 proton and 1 neutron in outer shell, this is not the case and there are states where the proton and neutron are in the same state.

Isospin

This is expressed more mathematically by introducing the concept of Isospin. If we have 2 electrons with z-component of spin set to $S_z = +\frac{1}{2}$ and $S_z = -\frac{1}{2}$, we can distinguish them by applying a non uniform magnetic field in z direction - the electrons will move in opposite directions. But in the absence of this external field the two cannot be distinguished.

Similerly, without EM interaction we cannot distinguish between a proton and a neutron. We therefore think of an imagined space (called an internal space) in which a nucleon has a property called "isospin", which is mathematically analogous to spin. The proton and neutron are now considered to be nucleons with different values of the third component of this isospin.

We assign $I_3 = +\frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the neutron. Thus, nucleons have isospin $\frac{1}{2}$. Just as angular momentum is conserved, isospin is conserved in any transition mediated by the the strong interactions.

The EM interactions couple to charge Q of particles. For nucleons this charge is related by: $Q = I_3 + \frac{1}{2}$ such that protons have $Q = +1$ and neutrons have $Q = 0$

Other particles are considered isospin multiplets, eg. the three pions π^+, π^-, π^0 which have roughly the same mass and 0 spin. The three behave roughly the same under strong interaction and so they form an isospin multiplet with $I = 1$ and $I_3 = +1, -1, 0$ (I_3 has 3 possibilities). For pions, $Q = I_3$. Members of an isospin multiplet have all the same properties with the exception of charge.

Just as two electrons can have total spin $S=0$ or $S=1$, two nucleons can have a total isospin $I=0$, or $I=1$

note, systems of n nucleons can have isospins up to $n/2$

The total wavefunction for 2 electrons is:

$$\Psi_{12} = \Psi(\underline{r}_1, \underline{r}_2)\, \chi(S_1, S_2)$$

where $\chi(S_1, S_2)$ is the spin part of the wavefunction.

For $S=1$:

$$\chi(S_1, S_2) = (\uparrow\uparrow), \quad S_z = +1$$

$$\chi(S_1, S_2) = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad S_z = 0$$

$$\chi(S_1, S_2) = (\downarrow\downarrow), \quad S_z = -1$$

which is symmetric under interchange of the two spins which means that (due to fermi statistics) the spatial part of the wavefunction must be antisymmetric under the interchange of the positions of the electrons:

$$\Psi(\underline{r}_1, \underline{r}_2) = -\Psi(\underline{r}_2, \underline{r}_1)$$

For $S=0$: $\chi(S_1, S_2) = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

which is antisymmetric under exchange of spins so is symmetric under exchange of positions:

$$\Psi(\underline{r}_1, \underline{r}_2) = \Psi(\underline{r}_2, \underline{r}_1)$$

Now let's consider the case of the two nucleons:

$$\Psi_{12} = \Psi(\underline{r}_1, \underline{r}_2)\, \chi_S(S_1, S_2)\, \chi_I(I_1, I_2)$$

where χ_I is isospin part of wavefunction. For total isospin $I=1$:

$$\chi_I(I_1, I_2) = (pp), \quad I_3 = +1$$

$$\chi_I(I_1, I_2) = \frac{1}{\sqrt{2}}(pn + np), \quad I_3 = 0$$

$$\chi_I(I_1, I_2) = (nn), \quad I_3 = -1$$

This is symmetric under interchange of isospins, so that the combined spatial and spin wavefunction must be antisymmetric under simultaneous interchange of two positions <u>and</u> two spins.

For $I = 0$: $\chi_1(I_1, I_2) = \frac{1}{\sqrt{2}}(pn - np)$

which is antisymmetric under interchange of spins so the combined spatial and spin wavefunction must be symmetric under simultaneous interchange of two positions <u>and</u> two spins.

Now let's return to ${}^6_2He$, ${}^6_4Be$ and ${}^6_3Li$. The closed shells of neutrons and protons have total $I = 0$ so we won't consider them when determining the isospin of the nuclei.

${}^6_2He$ has two neutrons in outer shell so it must have:
$I = 1$, $I_3 = -1$

${}^6_4Be$ has two protons in outer shell so it must have:
$I = 1$, $I_3 = +1$

Both are symmetric under interchange of isospins so the remaining part of wavefunction must be antisymmetric under simultaneous interchange of two positions and two spins.

But ${}^6_3Li$ has one proton and one neutron in outer shell so can either have $I = 1$ or $I = 0$. The strong interactions will give different energy levels depending on total isospin of outer shells. So two of the states for ${}^6_3Li$ can be identified as $I = 1$ states and match states for the other two nuclei but the others are $I = 0$ states which have no counterpart for the other two nuclei.

We can also deduce that ${}^6_2He$ and ${}^6_4Be$ have ground states of spin 0 and ${}^6_3Li$ has ground state of spin 1. Note this is spin, not isospin. We deduce these spins from the isospins:

For ground state wavefn, the orbital ang. mom. $l = 0$. The symmetry of the spatial part of the wavefn is given by $(-1)^l$ so we find $(-1)^l = 1 \Rightarrow$ symmetric. We know that the overall wavefn for the two nucleons in the outershell must be antisymmetric under interchange, so, because nucleons are fermions, it follows that the isospin part and spin part of the wavefn must have opposite symmetry.

So, since ${}^6_2He$ and ${}^6_4Be$ have ground states with $I=1$ (symmetric), the spin part of the wavefn. must be antisymmetric, so $S=0$

For ${}^6_3Li$ which from experiment is known to have ground state with $I=0$ (antisymmetric), the spin part of wavefn. must be symmetric, so $S=1$.