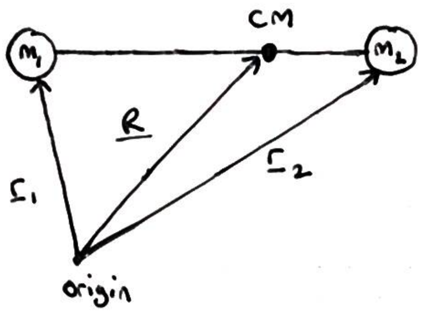
Centre of Mass

If we have a system of particles:



CM is the centre of mass of the two particles. $\underline{r}_1$ and $\underline{r}_2$ are the position vectors of the particle with mass m_1 and m_2 relative to the origin.

From this, we can calculate $\underline{R}$, the position vector of CM relative to the origin:

$$\boxed{\underline{R} = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2}}$$

We can also set the origin at one of the points to reduce some of the variables in the equation.

Differentiating this with respect to time:

$$\frac{d\underline{R}}{dt} = \frac{m_1 \underline{v}_1 + m_2 \underline{v}_2}{m_1 + m_2} = \frac{\underline{p}_1 + \underline{p}_2}{m_1 + m_2}$$

if $\underline{p}_1 + \underline{p}_2$ (linear momentum) is conserved, then the centre of mass moves with constant velocity.

The equation for centre of mass holds true for many particles as well:

$$\boxed{\underline{R} = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2 + \ldots + m_i \underline{r}_i}{\underline{m}_1 + \underline{m}_2 + \ldots \underline{m}_i}}$$

Centre of Mass moving at constant Velocity -

If there are N particles, the centre of mass is given by:

$$\underline{R} = \frac{\sum_1^N m_i \underline{r}_i}{\sum_1^N m_i}$$

Considering only the i^{th} particle, we can use Newton's 2 Law to give:

$$\underline{F}_i = m_i \underline{a}_i = m_i \frac{d^2 \underline{r}_i}{dt^2} = \frac{d^2 m_i \underline{r}_i}{dt^2}$$

Now for the total force:

$$\underline{F}_{total} = \sum_1^N \underline{F}_i = \sum_1^N \frac{d^2 m_i \underline{r}_i}{dt^2} = \frac{d^2}{dt^2}\left(\sum_1^N m_i \underline{r}_i\right)$$

$$= \frac{d^2}{dt^2}\left(\sum_1^N m_i \underline{r}_i\right) \times \frac{\text{total mass}}{\text{total mass}}$$

where total mass $= \sum_1^N m_i$

$$= \sum_1^N m_i \frac{d^2}{dt^2} \underbrace{\left(\frac{\sum_1^N m_i \underline{r}_i}{\sum_1^N m_i}\right)}_{\text{but this is } \underline{R}}$$

$$\therefore \boxed{\underline{F}_{total} = M \frac{d^2}{dt^2} \underline{R}}$$

where M is total mass

So regular Newtonian mechanics still apply to multiple particles focused on the centre of mass.