Parity, Charge Conjugation and CP

Intrinsic Parity –

Just like nuclear states, hadrons (bound by quarks and antiquarks) have parity, called intrinsic parity η. Under a parity inversion the wavefunction for a hadron acquires a factor η:

$$\hat{P}\psi_{\{p\}}(\underline{r}) = \psi_{\{p\}}(-\underline{r}) = \eta_{\{p\}}\psi_{\{p\}}(\underline{r})$$

where $\eta = \pm 1$, which means applying parity operator twice brings us necessarily back to original state.

Generally, light baryons have positive intrinsic parity. And since antiquarks have opposite parity to quarks, generally light antibaryons have negative intrinsic parity.

Generally, light mesons which are bound quark-antiquark pair have negative intrinsic parity. The lightest spin one mesons have 0 orbital ang. mom. and negative intrinsic parity.

For more massive particles, quarks can be in non-zero orbital angular momentum states so both baryons and mesons with higher mass can have either parity.

Parity is always conserved in strong interaction so:

$$\rho^0 \rightarrow \pi^+ + \pi^- \quad \text{is allowed}$$

This is because ρ^0 has spin 1. So we know the final state of pions must have $L=1$. The ρ has negative intrinsic parity and so do the two pions. The orbital ang. mom. $L=1$ so the parity of final state is $\eta_\pi^2(-1)^1 = -1$ so parity is conserved. Thus this is allowed.

On the other hand:

$$\rho^0 \to \pi^0 + \pi^0 \quad \text{is forbidden}$$

This is because two π^0's cannot be in a $L=1$ state so it is not possible for parity to be conserved. This is because the pions are bosons, so two identical pions will be symmetric under interchange but $L=1$ means that the wavefunction must be antisymmetric under interchange.

Unlike strong interactions, weak interactions do <u>not</u> conserve parity. An example of this is:

$$\underbrace{K^+}_{\eta=-1} \to \underbrace{\pi^+ + \pi^0}_{\eta=1}$$

Charge conjugation

Charge conjugation is the operation of replacing particles by their antiparticles. eg: $C\Psi\{p\} = \Psi\{\bar{p}\}$

Some mesons like π^0 are their own antiparticles, so in this case we show that we have done this operation with $\eta^c = \pm 1$: $C\Psi_{\pi^0} = \eta^c \Psi_{\pi^0}$

Note that two applications of C necessarily bring us back to the starting state.

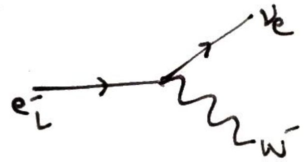
A photon has $\eta^c = -1$, because under charge conjugation electric charges switch sign and therefore so do $\underline{E}$ and $\underline{B}$ fields. But the photon itself is its own antiparticle.

Consider $\pi^0 \to \gamma + \gamma$ so to find the η^c of the LHS, i.e the π^0, we multiply $\eta^c_\gamma \times \eta^c_\gamma = 1$. So η_c of π^0 is $+\underline{1}$

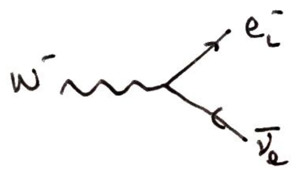
CP

Charge conjugation is conserved in strong and EM interaction but not by weak interaction. But the weak interactions are almost invariant under the combined effects of charge conjugation and parity inversion, called "CP".

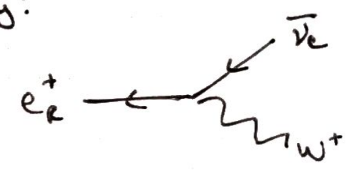
So weak interactions will allow a (highly relativistic) left-handed (negative helicity) electron to convert into a neutrino emitting a W^-



or a W^- can decay into a left-handed electron and antineutrino



similarly:



Note that in all these examples, the CP is constant.

$K^0 - \overline{K^0}$ oscillations

Now consider K^0 and $\overline{K^0}$ particles which has:

Parity inversion $P\psi_{K^0} = -\psi_{K^0}$

and Charge conjugation $C\psi_{K^0} = \psi_{\overline{K^0}}$

so it has CP: $CP\psi_{K^0} = -\psi_{\overline{K^0}}$ which is not in the form:

$$CP\,X = \lambda X$$

Therefore, ψ_{K^0} and $\psi_{\overline{K^0}}$ are not eigenstates of CP. But the energy eigenstates must be eigenstates of CP. These eigenstates of CP are:

$$\psi_{K_L} = \frac{1}{\sqrt{2}}(\psi_{K^0} + \psi_{\overline{K^0}}) \quad CP = 1$$

$$\psi_{K_S} = \frac{1}{\sqrt{2}}(\psi_{K^0} - \psi_{\overline{K^0}}) \quad CP = +1$$

The L and S stand for long and short which denote their relative lifetimes. These eigenstates are quantum superpositions of the K^0 and $\overline{K^0}$ states.

The allowed non-leptonic decays are:

$K_L \to \pi^0 + \pi^0 + \pi^0$ $K_L \to \pi^0 + \pi^+ + \pi^-$ we need 3 pions to make $CP = -1$

$K_S \to \pi^0 + \pi^0$ $C_S \to \pi^+ + \pi^-$ we need 2 pions to make $CP = +1$

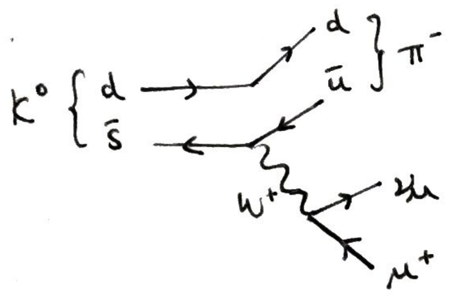
K_L has a longer lifetime than K_S since the Q value for decay into 2 pions is larger than that for decay into 3 pions.

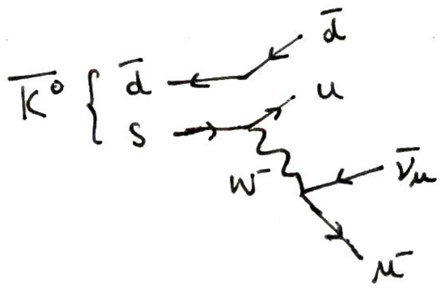
We distinguish K^0 from $\overline{K^0}$ by looking at their semi-lepton decay modes

$$K^0 \to \pi^- + \mu^+ + \nu_\mu$$

$$\overline{K^0} \to \pi^+ + \mu^- + \overline{\nu_\mu}$$

The diagrams are shown overleaf





The decay products are used to distinguish K^0 from $\overline{K^0}$

If at time $t=0$, we have pure K^0 state, we can write this as a superposition of ψ_S and ψ_L:

$$\psi_{K^0}(t=0) = \frac{1}{\sqrt{2}}\left(\psi_{K_L} + \psi_{K_S}\right)$$

It is important to note that K_L and K_S have different masses $\left(\frac{\Delta M}{M} = 7\times10^{-15}\right)$ and therefore have different energies, which means their wavefunctions have different frequencies.

So suppose we use the SE to obtain the time dependence of the wave fn, which at time $t=0$ is a pure K^0 state. We thus obtain a wavefunction that contains oscillations between the wave fn for K^0 and the wave fn for $\overline{K^0}$.

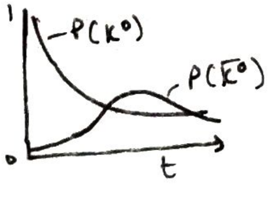
Thus, if at a later time t the particle decays semileptonically, we can write the probability of observing K^0 decay and the probability of observing $\overline{K^0}$ decay as:

$$P(K^0) = A(t) + B(t)\cos\left(\Delta M c^2 t/\hbar\right)$$

$$P(\overline{K^0}) = A(t) - B(t)\cos\left(\Delta M c^2 t/\hbar\right)$$

where $\Delta M = M_{K_L} - M_{K_S}$

So as time progresses, there are oscillations between K^0 and $\overline{K^0}$ states.



This has been observed experimentally.

This is an effect called Quantum Interference.

Although everything we studied in this chapter is still generally accepted (2020), in 1964 it was observed that a K_L could decay into two pions, which would be an example of Charge Parity Invariance violation in Weak Interactions!

Summary of Conservation Laws

Baryon Number $\begin{cases} \text{baryons} = +1 \\ \text{antibaryons} = -1 \\ \text{mesons} = 0 \\ \text{leptons} = 0 \end{cases}$

Lepton Number $\begin{cases} \text{electron number}: e^-, \nu_e = +1 \quad e^+, \bar{\nu}_e = -1 \\ \text{muon number}: \mu^-, \nu_\mu = +1 \quad \mu^+, \bar{\nu}_\mu = -1 \\ \tau \text{ number}: \tau^-, \nu_\tau = +1, \quad \tau^+, \bar{\nu}_\tau = -1 \end{cases}$

	Strong	EM	Weak
Baryon Number	✓	✓	✓
Lepton Number	✓	✓	✓
Ang. Momentum	✓	✓	✓
Isospin	✓	X	X
Flavour	✓	✓	X
Parity	✓	✓	X
Charge Conjugation	✓	✓	X
CP	✓	✓	almost?