Collisions and Conservation of Momentum

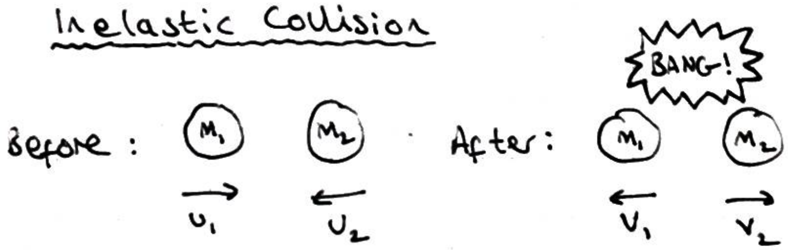
If all the kinetic energy before the collision is equivalent to all the kinetic energy after the collision, then the kinetic energy (and therefore momentum) is conserved. We can therefore call this collision elastic.

More commonly, some mechanical energy is nearly always lost as heat or light or sound. Therefore, kinetic energy and momentum is not conserved for the colliding objects. This is called an inelastic collision. If the colliding objects stick together after the collision, we say the collision is completely inelastic.

Elastic Collision

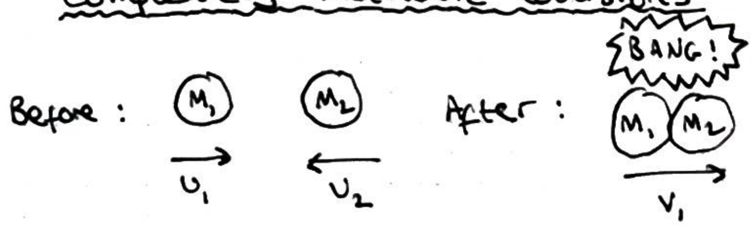
$$\frac{1}{2}m_1\underline{u}_1^2 + \frac{1}{2}m_2\underline{u}_2^2 = \frac{1}{2}m_1\underline{v}_1^2 + \frac{1}{2}m_2\underline{v}_2^2$$

Inelastic Collision



$$\frac{1}{2}m_1\underline{u}_1^2 + \frac{1}{2}m_2\underline{u}_2^2 \neq \frac{1}{2}m_1\underline{v}_1^2 + \frac{1}{2}m_2\underline{v}_2^2$$

Completely Inelastic Collisions

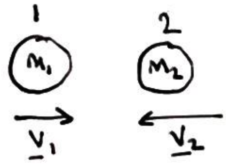


$$\frac{1}{2}m_1\underline{u}_1^2 + \frac{1}{2}m_2\underline{u}_2^2 \neq \frac{1}{2}(m_1+m_2)\underline{v}_1^2$$

A collision is defined as an isolated event in which one or more objects exert relatively strong forces on each other for a relatively short period of time with no external forces acting.

Conservation of momentum

Consider 2 isolated particles colliding elastically:



Newton's 3rd Law states that the force 1 exerts on 2 is equal and opposite to the force 2 exerts on 1.
Thus:

$$\underline{F}_{12} = -\underline{F}_{21}$$

Newton's 2nd Law states that the force exerted is proportional to the rate of change of linear momentum:
Thus:

$$\underline{F}_{12} = \frac{d}{dt}(m_1 \underline{v}_1)$$

$$\underline{F}_{21} = \frac{d}{dt}(m_2 \underline{v}_2)$$

We can therefore say:

$$\frac{d}{dt}(m_1 \underline{v}_1) = -\frac{d}{dt}(m_2 \underline{v}_2)$$

$$\frac{d}{dt}(m_1 \underline{v}_1 + m_2 \underline{v}_2) = 0$$

$$\frac{d}{dt}(p_1 + p_2) = 0$$

Since the total momentum does not change with time, we can say that momentum before is the same as momentum after. Therefore:

$$\boxed{\text{Total Linear Momentum in an isolated system is always conserved.}}$$

Impulse

Impulse is defined as force $\underline{F}$ multiplied by the time the force is exerted for. This can be expressed as an integral:

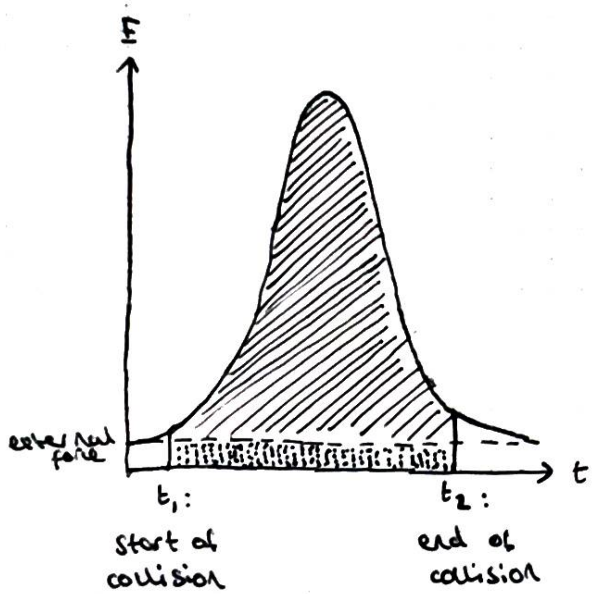
$$\boxed{\text{Impulse} = \int_{t_1}^{t_2} \underline{F} \cdot dt}$$

Note that Newton's 2nd law states that $\underline{F} = \frac{d\underline{p}}{dt}$. Therefore:

$$\int_{t_1}^{t_2} \underline{F} \cdot dt = \int_{t_1}^{t_2} \frac{d\underline{p}}{dt} \cdot dt = \int_{t_1}^{t_2} d\underline{p} = \underline{\underline{p_{t_2} - p_{t_1}}}$$

This is just change in momentum, showing that impulse can also be defined as the change in momentum during the time period the force is exerted.

We can represent this graphically:



Here, the collision is shown as a sudden pulse in a graph of force against time. The impulse is represented as the area under the curve.

Note that there is some small external force that acts a little like noise. This will be included in the area under the curve but it is relatively so small it is considered negligible.

The area under the curve is equivalent to the object's change in momentum.