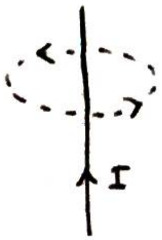
Ampere and Faraday's Laws

We know that moving electric charges, i.e currents, generate magnetic fields. The direction of the field generated by a current carrying wire can be found experimentally and remembered by using the Right-Hand-Thumb rule.



By pointing your right hand thumb in the direction of the current and observing the direction your fingers naturally curl into a fist, the direction of $\underline{B}$ can be found.

Biot-Savart Law

This is the magnetic equivalent of Coulomb's Law. In order to think about the field caused by a "point" of current (like a point charge for electric fields), we can say the minimum generator of a $\underline{B}$ field is a very short length of current carrying wire. The length of the wire is $\delta \underline{L}$.

$|\underline{B}|$ falls with $\frac{1}{|\underline{r}|^2}$ just like electric fields.

$|\underline{B}|$ depends on the current element $I|\delta \underline{L}|$

$$\therefore \quad \boxed{d\underline{B} = \frac{\mu_0}{4\pi} \frac{1}{|\underline{r}|^2} I \, d\underline{L} \times \hat{\underline{r}}}$$

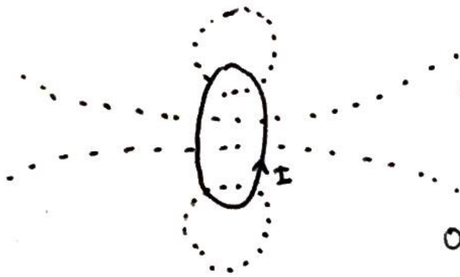
Here, the $\times \hat{\underline{r}}$ is to provide the right direction.

μ_0 is called the permeability of free space

$\mu_0 = 4\pi \times 10^{7} \ NA^{-2}$

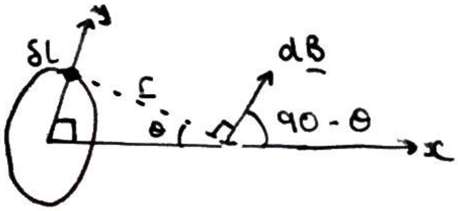
An example with a circular current loop:

Consider a loop of current and the magnetic field it generates:



The circular $\underline{B}$ fields generate on either side reinforce on the axis to create a field like the one drawn. On the axis itself, there are no y or z components to the field, only an x component.

To work out this x component of the field:



$|\underline{r}| = \sqrt{a^2 + x^2}$ where a is the radius of the loop

$\delta \underline{L} = a\, d\phi\, \hat{\underline{z}}$ it is an arclength perpendicular to $\hat{y}$ in $\hat{\underline{z}}$ direction

$$\delta B_x = |\delta \underline{B}| \cos\left(\frac{\pi}{2} - \theta\right)$$

$$= |\delta \underline{B}| \sin\theta = |\delta \underline{B}| \frac{a^2}{\sqrt{a^2 + x^2}}$$

To work out $|\delta \underline{B}|$, use Biot-Savart law:

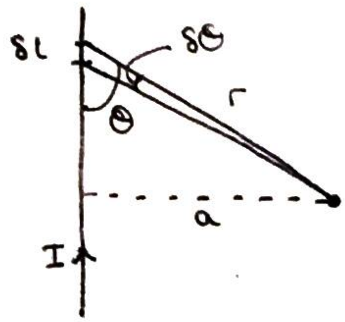
$$|\delta \underline{B}| = \frac{\mu_0}{4\pi} \underbrace{\frac{1}{(a^2 + x^2)}}_{|\underline{r}|^2} I \underbrace{a\, d\phi}_{\delta \underline{L}}$$

$$\delta B_x = \frac{\mu_0}{4\pi} \frac{I a^2}{(a^2 + x^2)^{3/2}} \delta\phi$$ so for total contribution to B_x of all the current elements, integrate over whole loop

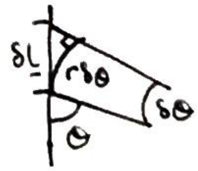
$$\therefore \quad B_x = \frac{\mu_0}{4\pi} \frac{I a^2}{(a^2 + x^2)^{3/2}} \int_0^{2\pi} d\phi = \underline{\underline{\frac{\mu_0}{2} \frac{I a^2}{(a^2 + x^2)^{3/2}}}}$$

Note, as $x \to 0$ $B_x = \frac{\mu_0 I}{2a}$ and as $x \to \infty$, $x \gg a$ so $B_x \sim \frac{\mu_0}{2} \frac{I a^2}{x^3}$

An example with an infinite straight current-carrying wire:



Using a polar coordinate system, the angle θ between $\delta \underline{L}$ and $\hat{\underline{r}}$ will be used as the coordinate.



$$\sin\theta = \frac{a}{r}$$

$$\therefore |\underline{r}| = \frac{a}{\sin\theta}$$

$$\sin\theta = \frac{r d\theta}{\delta \underline{L}}$$

$$\therefore |\delta \underline{L}| = \frac{r d\theta}{\sin\theta}$$

All of the $\delta \underline{L}$ pieces will generate a field into the page at this point (use right hand thumb rule to confirm). We can call this direction $\hat{\underline{z}}$.

Now, applying Biot-Savart law:

$$\delta \underline{B} = \frac{\mu_0}{4\pi} \frac{1}{r^2} I \, \delta \underline{L} \times \hat{\underline{r}}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^2} I |\delta \underline{L}| |\hat{\underline{r}}| \sin\theta \, \hat{\underline{z}}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^2} I |\delta \underline{L}| \sin\theta \, \hat{\underline{z}} = \frac{\mu_0}{4\pi} I \frac{\sin^2\theta}{a^2} \frac{r d\theta}{\sin\theta} \sin\theta \, \hat{\underline{z}}$$

$$= \frac{\mu_0}{4\pi} I \frac{\delta\theta}{a} \sin\theta \, \hat{\underline{z}}$$

Now sum over entire wire, i.e $0 < \theta < \pi$

$$\therefore \underline{B} = \frac{\mu_0}{4\pi} \frac{I}{a} \int_0^{\pi} \sin\theta \, d\theta \, \hat{\underline{z}}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{a} \left[-\cos\theta\right]_0^{\pi} \hat{\underline{z}}$$

So:

$$\underline{B} = \frac{\mu_0 I}{2\pi a} \hat{\underline{z}}$$

Forces between two wires

Using Fleming's left hand rule, we can see that the forces due to the magnetic fields cause I_1 and I_2 to be attracted to each other and I_3 and I_4 to be repelled from each other.

Wires with like currents attract Wires with unlike currents repel

Ampére's Law

This is the equivalent of Gauss' Law for magnetism. Consider an infinitely long straight wire:

Drawing a loop around this wire, we know from the previous question that the $\underline{B}$ field at the indicated point is $\frac{\mu_0 I}{2\pi r}\hat{\underline{\theta}}$. To find the field through the whole loop:

$$\oint \underline{B}.d\underline{l} = \frac{\mu_0 I}{2\pi r}\int_0^{2\pi r} d\underline{l} = \underline{\underline{\mu_0 I}}$$

We can deform the loop however we want and always get the same answer!

This is ampère's law:

$$\boxed{\oint \underline{B}.d\underline{l} = \mu_0 I_{enclosed}}$$

The same example with an infinite wire:

Draw an ampérian loop around the wire.
By symmetry, we know that the magnitude of $\underline{B}$ only depends on r and not θ.

$\therefore \quad \underline{B} = B(r)\,\underline{\hat{\theta}}$

Applying ampére's law:

$$\oint \underline{B} \cdot d\underline{L} = \mu_0 I$$

$$B(r) \int_0^{2\pi r} d\underline{L} = \mu_0 I$$

$$B(r) = \frac{\mu_0 I}{2\pi r} \quad \Rightarrow \quad \underline{B} = \frac{\mu_0 I}{2\pi r}\,\underline{\hat{\theta}}$$

An example with the field inside the wire:

cross section of wire

Draw an ampérian loop with radius r inside the wire. The same symmetry arguments as before apply.
Assume constant current density $= \frac{I}{\pi a^2}$

Using Ampére's Law:

$\oint \underline{B} \cdot d\underline{L} = \mu_0 I_{enc}$ but I_{enc} here is given by $\frac{I}{\pi a^2} \cdot \pi r^2$

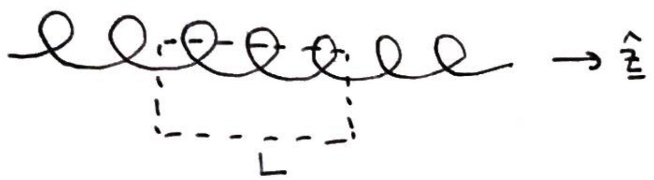
$$\therefore \; B(r) \int_0^{2\pi r} dL = \mu_0 \frac{I}{\pi a^2} \pi r^2$$

$$B(r) = \mu_0 \frac{I}{\pi a^2} \pi r^2 \cdot \frac{1}{2\pi r}$$

$$\underline{B} = \mu_0 \frac{Ir}{2\pi a^2}\,\underline{\hat{\theta}}$$

An example with a solenoid :

Neighbouring current loops will reinforce $\underline{B}$ along their axes. As the number of loops $\rightarrow \infty$, the reinforced $B_{inside} >> B_{outside}$ so we assume that $\underline{B}$ is linear and restricted to the inside of the solenoid.



Draw an ampérian loop as shown. Only the parts of the loop inside the solenoid will have a field along them. This means only the part of the loop parallel to the axis will have a field along them.

Applying Ampére's Law:

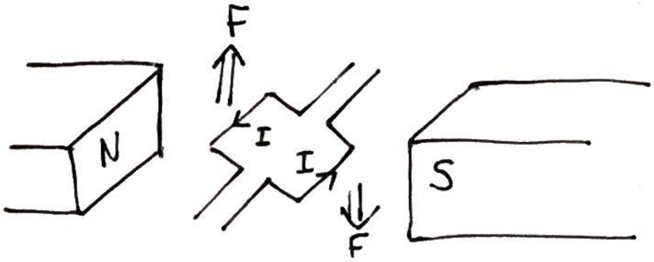
$$\oint \underline{B} \cdot dl = \mu_0 I$$

$|\underline{B}| L = \mu_0 I N$ where N is the number of loops

$$\therefore \underline{B} = \frac{\mu_0 I N}{L} \hat{\underline{z}} \Rightarrow \underline{B} = \mu_0 I n \hat{\underline{z}}$$

where n is number of loops per unit length

Magnetic Induction

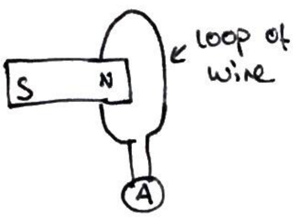


When a current is put through the loop, the moving charges experience a force. $\underline{F} = q\,\underline{v} \times \underline{B}$

The force is in opposite directions on the sides of the loop so the loop spins.

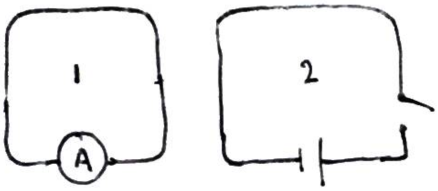
However, if we turn off the current and manually turn the loop, we see that a current flows.

eg



As the magnet is moved in and out, a current is generated in the loop. The faster the motion, the more current that is generated. The direction of the current depends on the direction of motion.

eg.



We see a current in 1 as the current in 2 grows or falls. If the current is stable and unchanging in 2, no current is generated in 1

We see that a current (or emf) is induced by a changing magnetic field. The key, as it turns out, is the change in magnetic flux:

$$\Phi_B = \int \underline{B} \cdot d\underline{A}$$

This is Faraday's Law:

$$\boxed{\text{emf induced} = -\frac{d\Phi}{dt}}$$

The reason for the - sign is another law called Lenz's law.

$\frac{d\Phi}{dt}$ is the rate of change of magnetic flux.

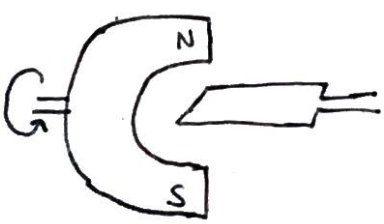
> Lenz's law states that the induced emf is a such a direction as to oppose the change that produced it.

This basically means that if we look at the magnetic field created by the induced emf, it will be in a direction that opposes the change that produced it. i.e, it will reinforce a decreasing magnetic field or oppose an increasing one.

Also, since emf is a potential difference $\Delta\Phi = -\int \underline{E} . d\underline{L}$, we can write Faraday's Law as:

$$\boxed{-\int \underline{E} . d\underline{L} = -\frac{d\Phi}{dt}}$$

AC Power Generation



By turning the magnet using any power source, we generate an AC emf in the coil

emf induced $= V_0 \sin(\omega t)$

Alternatively, the coil can be spun for the same effect