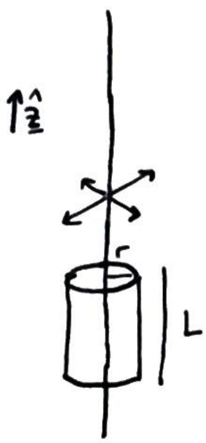
Gauss' Law

Flux is a measure of field lines leaving a surface. No matter what the shape of the surface is, the same number of field lines will leave the surface. So the only thing that matters is the charge generating the electric field. This is Gauss' Law:

$$\boxed{\int_{\text{surface}} \underline{E} \cdot d\underline{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}}$$

eg. Infinite Line of Charge with Charge Distribution $\lambda \, Cm^{-1}$



Due to symmetry, the electric field can only depend on $\underline{r}$. The angle of the field line or its position on the z axis will not affect it.

We choose our Gaussian surface to be a cylinder. The top and bottom faces of the cylinder have area vector $\underline{A}$ perpendicular to $\underline{E}$ so there will be no flux through them. The curved surface has area vector pointing radially.

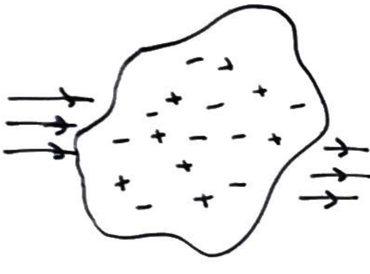
$$\text{Flux} = \int_{\text{surface}} \underline{E} \cdot d\underline{A} = |E(\underline{r})| \int_0^L 2\pi |\underline{r}||\underline{h}| \, d\underline{h} = E(\underline{r}) 2\pi |\underline{r}||\underline{L}|$$

Applying Gauss's Law: $\int \underline{E} \cdot d\underline{A} = \dfrac{q}{\varepsilon_0}$

$|E(\underline{r})| 2\pi |\underline{r}||\underline{L}| = \dfrac{q}{\varepsilon_0}$ but $q = \lambda |\underline{L}|$

$$\therefore |E(\underline{r})| = \frac{\lambda |\underline{L}|}{2\pi |\underline{r}||\underline{L}| \varepsilon_0} \quad \text{so} \quad \underline{\underline{\underline{E} = \frac{\lambda}{2\pi \varepsilon_0 |\underline{r}|} \hat{\underline{r}}}}$$

Charged conductors



In a neutral conductor, the charges are free to move around. When an electric field is applied, forces are generated on the free charges causing them to move.

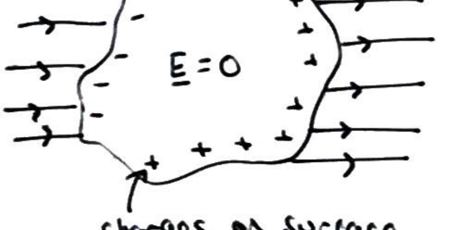
Eventually a steady state is reached where the particles have stopped moving and are in a position such that the force on them is 0. Since there is no force on any of the charges, the electric field $\underline{E}$ through the conductor is also 0. So where exactly in the conductor are the charges?

Let's put a gaussian surface inside the conductor. Applying gauss' Law:

$$\int \underline{E} \cdot d\underline{A} = \frac{q}{\varepsilon_0}$$ but $\underline{E}$ through the conductor is 0

$\therefore \frac{q}{\varepsilon_0} = 0$ and $q_{enclosed} = 0$. So there are no charges inside the conductor. They must all, therefore, be on the surface!

Note: the steady state looks something like:

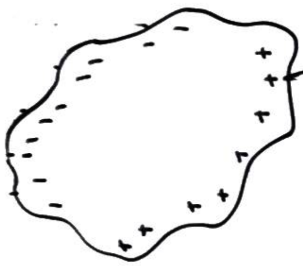


charges on surface of conductor

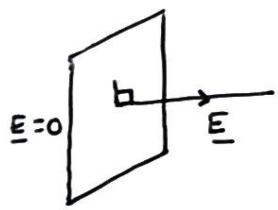
The field lines vanish into the negative charges and are regenerated by the positive charges. So we can see that there is no electric field through the conductor.

eg. computing the electric field on the surface of the conductor from earlier

At the surface of the conductor, the electric field being regenerated by the charges will be perpendicular to the surface. This is because if it wasn't, then there would be some component along the surface, causing the charge to move.

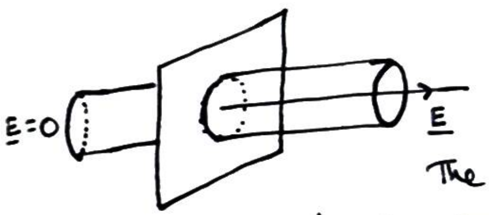


looking closely at the surface:



the small area element looks flat when it is really small.

Putting a gaussian surface (cylinder) through the area element:



The $\underline{E}$ through the cylinder end cap on the left is 0. There will be some $\underline{E}$ through the end cap on the right. The curved surface of the cylinder will have no $\underline{E}$ through it.

Since $\underline{E}$ is unvarying through the area element, and the cylinder only has one bit of area we need, we forego the integral:

$$\int \underline{E} \cdot d\underline{A} = |\underline{E}||\underline{A}| = \frac{q_{enclosed}}{\varepsilon_0} = \frac{|\underline{A}|\sigma}{\varepsilon_0}$$

where σ is the charge density on the surface

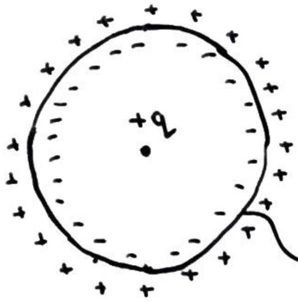
$$|\underline{E}||\underline{A}| = \frac{|\underline{A}|\sigma}{\varepsilon_0} \quad \text{so} \quad |\underline{E}| = \frac{\sigma}{\varepsilon_0}$$

but $\underline{E}$ is perpendicular to the surface so: $\underline{E} = \frac{\sigma}{\varepsilon_0}\hat{r}$

means perpendicular to surface

A surprising example

Let's put a charge inside a neutral conducting shell:



The negative charges are pulled inside the surface and become evenly distributed.
The positive charges are repelled outside the surface and become evenly distributed.
There will be no electric field through the shell itself since the field lines produced by +q will vanish into the negative charges

Therefore, the interior charge distribution is "hidden" from the outside and thus no matter how the +q charge is moved inside the sphere, the positive charges will always arrange themselves with spherical symmetry.