Electromagnetic Interactions

Electromagnetic Decays

Consider the Σ^{*0} (mass 1385 MeV/c^2) particle. It would be possible for it to decay into a π^0 (mass 135 MeV/c^2) and a Λ (mass 1115 MeV/c^2) if we just think about flavour conservation, i.e the isospin is conserved as required in strong interaction (Σ^{*0} has $I=1$, Λ has $I=0$, π has $I=1$)

However, if we look at the masses, we see this is not possible. In such cases, decay can proceed via EM interactions:

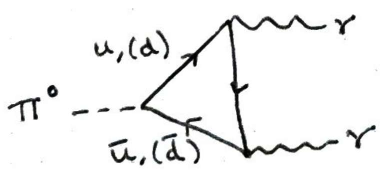
$$\underset{I=1}{\Sigma} \rightarrow \underset{I=0}{\Lambda} + \underset{I=0}{\gamma}$$

Note that EM interactions do not conserve isospin.
The EM coupling constant, e, is much smaller than the strong coupling constant, so the rates of EM decay are much smaller than rates of strong decay.

Another example of EM decay is $\pi^0 \rightarrow \gamma + \gamma$
[we cannot produce only one photon due to conservation of Energy and momentum]
We consider the π^0 as a superposition of u-$\bar{u}$ pair and d-$\bar{d}$ pair:

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$$

In either of the two states, the quark can annihilate against same flavour antiquark to produce two photons.



The feynman diagram for this process.

From Quantum Field Theory, we estimate π^0 lifetime as:

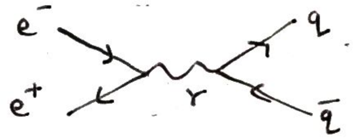
$$\tau = 7.6 \times 10^{-16}\,s$$

but we actually measure $8.4 \times 10^{-17}\,s$. The problem is that we haven't considered colour in the Feynman diagram. When we calculate decay amplitude, we must sum the contributions over u and d quarks of all three possible colours, which gives us a further factor 3 in decay amplitude, so a further factor 9 in decay rate.

Electron - Positron Annihilation

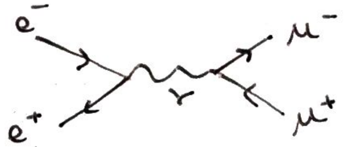
Another piece of evidence that quarks come in 3 colours is the process $e^+ + e^- \rightarrow$ hadrons

At the quark level:



where $\sqrt{s} << M_Z c^2$ so Z exchange neglected

Now consider $e^+ + e^- \rightarrow \mu^+ + \mu^-$



The only differences between these two is the couplings of the final states to the photon, i.e the electric charges of the quarks or muons.

So for quark flavour i with charge Q_i, the ratio of amplitudes is:

$$\frac{A(e^+e^- \rightarrow q_i\bar{q}_i)}{A(e^+e^- \rightarrow \mu^+\mu^-)} = Q_i$$

Recall the definition of amplitude from previous chapters:

$$A \propto \frac{ee'}{(q_0 - |\underline{q}|^2)}$$

where e is charge of final state particle
$\underline{q}$ is change of momentum

Let's now calculate the ratio of total cross-sections.
To do this, we square the amplitude and sum over all possible final state quarks that can be produced:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_i Q_i^2$$

The number we sum over depends on centre of mass energy $\sqrt{s}$.

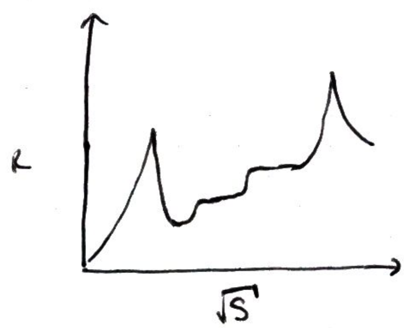
If $\sqrt{s} < 2M_c c^2$, only u d and s quarks are produced so:
M_c: mass of charm

$$R = 3(Q_u^2 + Q_d^2 + Q_s^2) = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = \underline{\underline{2}}$$

Note this factor of 3. We put this in to represent the 3 possible colours of every Q_i. This is only distinguishable at quark level, so is not needed for the amplitude.

If $2M_c c^2 < \sqrt{s} < 2M_b c^2$ then we add Q_c^2 to the list.
M_b: mass of bottom

For $\sqrt{s} > 2M_b c^2$ we include Q_b^2 in the list.
So we expect R to "jump" at certain points when we have enough $\sqrt{s}$ to produce another quark:



A better diagram is available anywhere online.

We observe some "jumps" (explained above) and some "resonances" (explained on next page).

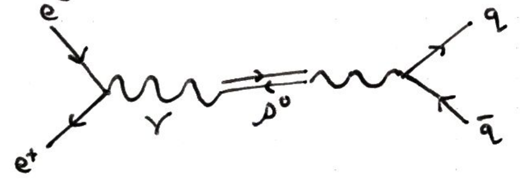
Fragmentation

We started the electron-positron annihilation section by considering electron-positron annihilation to produce hadrons. But we ended up looking at the production of quarks. The reason for this is that these produced quarks (which we don't observe directly) are converted into hadrons through interactions with gluons in a process we don't well understand. We call this process "fragmentation".

In C of M frame, the final state q and $\bar{q}$ move in opposite directions. The process of fragmentation leads to two narrow jets of particles in opposite directions.

Resonances

As shown in the diagram on the previous page, we see constant R between each energy threshold and a jump near each threshold. But we also see resonances wherever $\frac{\sqrt{s}}{c^2}$ is equal to the mass of a neutral, spin 1 particle that can couple directly to a photon.
We require spin 1 since there is a direct coupling between photon and resonant particle which must conserve ang. momentum:



At low energies these are mesons like ρ^0 with a quark antiquark pair of same flavour.

When $\sqrt{s}/c^2 = M_\rho$, the ρ propagator $\frac{1}{(s - M_\rho^2 c^4 + iM_\rho \Gamma c^2)}$ which gives rise to the resonance.
It is possible to create resonance of more massive particles at higher energies.