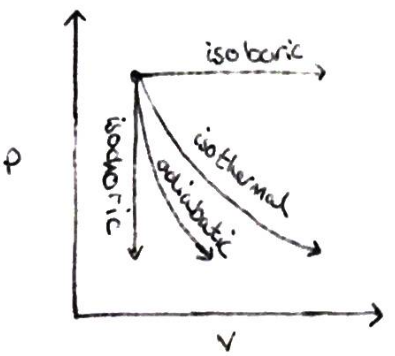
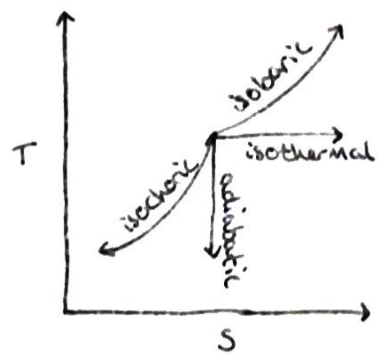
Ideal Reversible Processes

We have seen that it is sometimes convenient to model real systems as reversible processes to allow for the equations of state to be satisfied. This means, given any two of the four state variables: P, V, T, S, we can work out the others. Any reversible process can therefore be represented as a direct path on a P-V or T-S plane.

By holding one of the variables fixed, we can look at the changes in the other variables. There are different names for the different processes depending on which variable is fixed:





Notice that the adiabatic curve satisfies a $P \propto V^{-\gamma}$ relationship on the P-V diagram where γ is some constant. The isothermal curve is simply $P \propto V^{-1}$ as expected from the ideal gas equation when T is fixed.

So what is this γ factor we see in adiabatic processes? And why is the adiabatic process a vertical line on the T-S plane?

Adiabatic Processes

An adiabatic process is one that occurs without heat exchange with the environment. If the process is also reversible, then we can use Clausisus Entropy $dS = \frac{dQ}{T} \Rightarrow dQ = TdS$ so say that adiabatic processes are also isentropic since for $dQ = 0$, $dS = 0$. This is why adiabatic processes are a vertical line in the T-S plane. However, this is not true all the time so don't get muddled up! Adiabatic and isentropic aren't the same thing. For example, in Joule expansion, no heat is exchanged but it is certainly not isentropic.

Setting $dQ = 0$, we get $dU = dW = -PdV$:

$$-PdV = U \Rightarrow -PdV = C_v dT = C_v d\left(\frac{PV}{Nk_B}\right)$$

$$-PdV = \frac{C_v}{Nk_B} d(PV) = \frac{C_v}{Nk_B}(PdV + VdP)$$

$$\therefore -PdV = \frac{C_v}{Nk_B} PdV + \frac{C_v}{Nk_B} VdP \Rightarrow PdV\left(\frac{C_v}{Nk_B} + 1\right) = \frac{-C_v}{Nk_B} VdP$$

$$\therefore \frac{dP}{P} = \frac{C_v + Nk_B}{C_v} \cdot \frac{dV}{V}$$

$$\boxed{\frac{dP}{P} = \gamma \frac{dV}{V}}$$ where $\gamma = \frac{C_v + Nk_B}{C_v}$

we can use Mayer's Relation $C_p - C_v = R$ to get $\gamma = \frac{C_p - R + R}{C_v} \Rightarrow \boxed{\gamma = \frac{C_p}{C_v}}$

γ is the adiabatic index or the heat capacity ratio.

An ideal gas undergoing an adiabatic change will always satisfy

$$\boxed{PV^\gamma = \text{constant}}$$

This also gives us: $TV^{\gamma-1} = \text{const}$ and $P^{1-\gamma}T^{\gamma} = \text{const}$