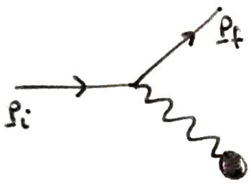
Fundamental Interactions of Nature

Interaction	Gauge Boson	Gauge Boson Mass	Interaction Range
Strong	Gluon	0	few fm
Weak	$W^{\pm}$, Z	$M_W = 80.4\,GeV/c^2$ $M_Z = 91.2\,GeV/c^2$	$\sim 10^{-3}$ fm
EM	Photon	0	long range
Gravity	Graviton	0	long-range

Relativistic Approach to Interactions

Let's consider Electromagnetic Interaction:



The potential of a charge e located at $\underline{r}$ due to another charge e' fixed at origin is

$$V(r) = \frac{ee'}{4\pi\varepsilon_0 r}$$

If the charge has momentum before interaction of $\underline{p}_i$, and momentum after interaction of $\underline{p}_f$, the initial and final wavefunctions are:

$$\psi_i \propto e^{i\underline{p}_i \cdot \underline{r}/\hbar}$$

$$\psi_f \propto e^{i\underline{p}_f \cdot \underline{r}/\hbar}$$

The amplitude of this transition is

$$A = \int \psi_f^* \, V(r)\, \psi_i \, d^3\underline{r} \;\propto\; ee' \int e^{i(\underline{p}_i - \underline{p}_f)\cdot \underline{r}/\hbar} \, \frac{1}{r} \, d^3\underline{r}$$

Notice that the integral is a Fourier Transform

$$\Rightarrow \quad A \propto \frac{ee'}{-|\underline{q}|^2} \qquad \text{where} \quad \underline{q} = \underline{p}_f - \underline{p}_i$$

For a relativistic particle, this is modified to:

$$A \propto \frac{ee'}{(q_0^2 - |\underline{q}|^2)} \qquad \text{where} \quad q_0 = \frac{E_f - E_i}{c}$$

In non relativistic case $\frac{E_f - E_i}{c} << |p_f - p_i|$ so q_0^2 is negligible.

The interpretation of this is that a photon which is the "carrier" of EM interactions, with energy cq_0 and momentum $\underline{q}$, is exchanged between the two charged particles. The electric charges e and e' measure the strengths of the coupling of the charged particles to the photon.

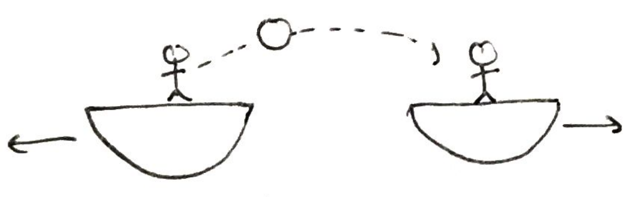
The quantity: $D(q_0, \underline{q}) = \frac{1}{q_0^2 - |\underline{q}|^2}$

is called the "propagator" and is the amplitude for the propagation of a photon with energy cq_0 and momentum $\underline{q}$.

Since $(E_f - E_i)^2 - |\underline{p}_f - \underline{p}_i|^2 c^2 = c^2(q_0^2 - |\underline{q}|^2)$ is a Lorentz invariant, we know that the propagator itself is Lorentz invariant.

In the relativistic approach, all interactions happen via the exchange of gauge bosons which carry the interaction between interacting particles. Particles only influence each other at a distance because gauge bosons are emitted by one of the particles and then absorbed by the other particle.

For a classical example, consider:



two people on two boats throwing a ball back and forth. Each boat experiences a repulsive force but there is no action at a distance, only an exchange of particles that carry momentum.

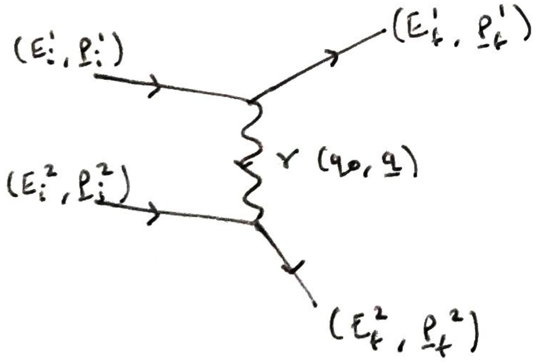
Virtual Particles

For a photon with energy cq_0 and momentum $\underline{q}$ where $q_0 = |\underline{q}|$, we would expect the propagator to diverge since we have $q_0^2 - |\underline{q}|^2$ on the denominator.
But Heisenberg's uncertainty principle tells us that in a sufficiently short period of time, there is uncertainty in the energy.
So if a particle lives for only a short period of time, $E^2 = p^2c^2 + m^2c^4$ no longer works, so for our photon $q_0 \neq |\underline{q}|$.

Such particles which are exchanged rapidly between other particles are called "virtual particles".
They are said to be "off mass-shell" since they don't obey the normal $E^2 = p^2c^2 + m^2c^4$

Feynman Diagrams

Let's try and calculate the scattering amplitude for the scattering of a particle of charge e_1 and incoming energy and momentum $(E_i^1, \underline{p}_i^1)$ against a particle of charge e_2 and incoming energy and momentum $(E_i^2, \underline{p}_i^2)$. We represent this as:



This is called a Feynman diagram.
We can obtain the amplitude by applying a set of Feynman rules.
The full set of rules also take into account the spins, but we will not consider those in this course.

The feynman rules for EM interaction are:

- There is a factor of charge at each vertex between a charged particle and a photon.
- Energy and momentum are conserved at each vertex
- There is a factor of $D = \frac{1}{(q_0^2 - |\underline{q}|^2)}$ for the propagation of an internal gauge boson with energy cq_0 and momentum $\underline{q}$

For our example, due to conservation of E and $\underline{p}$ at each vertex:

$$q_0 = \frac{(E_f^1 - E_i^1)}{c} = \frac{(E_f^2 - E_i^2)}{c}$$

$$\underline{q} = \underline{p}_f^1 - \underline{p}_i^1 = \underline{p}_f^2 - \underline{p}_i^2$$

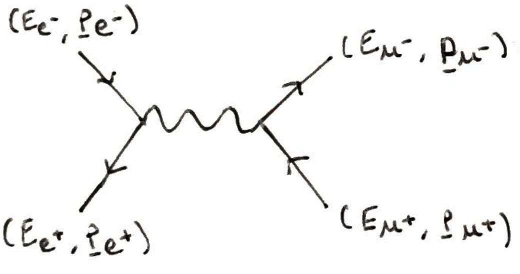
so amplitude is proportional to:

$$\frac{e_1 e_2}{(q_0 - |\underline{q}|^2)} = \underline{\underline{\frac{e_1 e_2}{\frac{(E_f^1 - E_i^1)^2}{c^2} - |\underline{p}_f^1 - \underline{p}_i^1|^2}}}$$

Now let's consider a particle and antiparticle annihilating:

$$e^+ + e^- \rightarrow \mu^+ + \mu^-$$

$\mu^\pm$ have same electric charge as $e^\pm$



Notice that the arrows for e^+ and μ^+ are drawn <u>opposite</u> to their directions of motion. This is convention since they are antiparticles.

in this case $q_0 = (E_{e^-} + E_{e^+})/c = (E_{\mu^-} + E_{\mu^+})/c$

$$\underline{q} = (\underline{p}_{e^-} + \underline{p}_{e^+}) = (\underline{p}_{\mu^-} + \underline{p}_{\mu^+})$$

so amplitude is proportional to $\dfrac{e^2}{\frac{(E_{e^-} + E_{e^+})}{c^2} - |\underline{p}_{e^-} + \underline{p}_{e^+}|^2}$

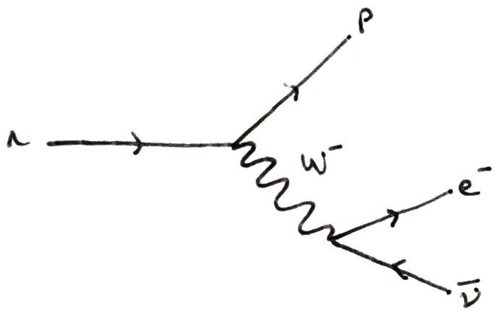
In the CoM frame of electron positron pair, $\underline{p}_{e^+} + \underline{p}_{e^-} = 0$ and $E_{e^-} + E_{e^+} = E_{CM} = \sqrt{s}$

so that we have amplitude $A \propto \frac{e^2}{s}$

Weak Interactions

Weak interactions have gauge bosons $W^{\pm}$ and Z, but we see that the W bosons are charged, which means that electric charge can be exchanged in the weak interaction. This is where β decay comes from.

The Feynman diagram for the process $n \to p + e^- + \bar{\nu}$ is:



The neutron emits a W^- so is converted to a proton. On the other end, the W^- decays into an electron and an antineutrino.

The equivalent of electric charge in weak interaction is coupling g_W which indicates the strength of the coupling of the weakly interacting particles to the W-bosons and is approximately twice the electron charge (the coupling to the neutral Z-boson is almost equal to this value).

The W^- has mass $M_W = 80.4\ GeV/c^2$ and for the propagation of a massive particle, the propagator is

$$D^W = \frac{1}{q_0^2 - |\underline{q}|^2 - M_W c^2}$$

The W boson is a virtual particle so D does not diverge.

The amplitude for the decay is proportional:

$$\frac{g_W^2}{q_0^2 - |\underline{q}|^2 - M_W^2 c^2}$$

If we take the non-relativistic limit, we may neglect q_0^2 compared with $|\underline{q}|^2$ and this amplitude can be viewed as the matrix element of a weak potential, V^{wk}, between the initial (neutron) state with momentum p_n and final (proton) state momentum p_p, with $q = p_p - p_n$, i.e:

$$\frac{g_w^2}{-|q|^2 - M_w^2 c^2} = \int e^{-ip_p \cdot r/\hbar} \, V^{wk}(r) \, e^{ip_n \cdot r/\hbar} \, d^3 r$$

where

$$V^{wk}(r) = \frac{g_w^2}{r} \exp(-M_w c r/\hbar)$$, called the Yukawa potential

As well as decreasing as $\frac{1}{r}$, this has an exponentially suppressed term for large values of r. The effective force therefore has range R where $R \sim \frac{\hbar}{M_w c}$, which is very small ($10^{-3}$ fm)

In the case of β decay, the momentum transferred is small compared to $M_w c$ so we can neglect it, giving an amplitude of $-\frac{g_w^2}{M_w^2 c^2}$

This is very small since M_w is so large, which is why weak interactions are so weak!

Strong Interaction

We may now think that an interaction with massless gauge bosons have range $\propto \frac{1}{r}$ but those with massive gauge bosons have range $\propto \frac{1}{M}$. But the strong interaction is an exception to this! Gluons are massless but they don't have infinite range $\propto \frac{1}{r}$.

This is due to quark confinement, discussed later.

The idea is that the coupling of strongly interacting particles to the gluons grows as distance between particles increases, so it is impossible to separate interacting particles to large distances.