Gamma Decay

γ decay often occurs in conjunction with α or β decay when the daughter nucleus is created in an excited state and has to make one or more transitions to ground state, emitting a photon, emitting a photon whose energy is equivalent to the energy difference between initial and final nuclear state.

The lifetime is of the order 10^{-13} or 10^{-12} s so too short to be measured.

The transition amplitude (as in Atomic Physics) is proportional to the matrix element of the electric field between the initial and final wavefunctions of the nucleon that make the transition.
The Electric field has a space dependence: $E = E_0 e^{i\underline{k}\cdot\underline{r}}$
where $\underline{k}$ is the wavevector of the emitted photon.

For photons of energy 100 keV and nucleus of radius $\sim$ few fM, $\underline{k}\cdot\underline{r} \ll 1$ so we can expand to first order:

$$A \propto \int \Psi_f^*(\underline{r})\, \underline{k}\cdot\underline{r}\, \Psi_i(\underline{r})\, d^3\underline{r}$$

where Ψ_f and Ψ_i are initial and final wavefn of proton making transition

The proton transition is called the electric dipole transition.
The rate for such a transition is approximated by:

$$\lambda = 10^5 E_\gamma^3 A^{2/3}$$ where E_γ is energy of photon.

We get $A^{2/3}$ term since $\lambda \propto$ square of amplitude, and amplitude is proportional to nuclear radius, i.e $\propto A^{1/3}$

For the electric dipole matrix element to be non-zero, the initial and final nuclear spins must obey the selection rule:

$$\Delta I = 0, \pm 1 \quad (I = 0 \to I = 0 \text{ forbidden})$$

Also, since $\underline{r} \to -\underline{r}$ under parity reversal, we require initial and final state to be opposite parity, so orbital ang. mom. changes by one unit.

If the parity of initial and final states are the same, then the transition is still allowed but the photon carries away the ang. momentum by flipping the spin of the nucleon that makes the transition.

To do this, the magnetic moment of nucleon interacts with magnetic field component of EM wave of emitted photon.
This "magnetic dipole transition" amplitude is suppressed relative to amplitude for a typical electric dipole transition by a factor of $\frac{\hbar c}{M_p R} \sim 0.1$ for a nucleus of radius few fm

But sometimes, the photon can carry away more than one unit of angular momentum, i.e the photon can also carry away orbital ang. mom. relative to recoiling nucleus. So total ang. mom. change in a nuclear transition can take values:

$$|I_i - I_f| \leq L \leq |I_i + I_f|$$

(I_i, I_f: initial and final nuclear spins)

However, for each increase in L, there is a suppression in transition amplitude of kR, since these higher multipoles come from higher order terms in the expansion of $e^{i\vec{k}\cdot\vec{r}}$.
Even higher values of L have a further suppression factor and a transition will only proceed by the lowest allowed values of L, subject to selection rules for the parity difference between initial and final states:

$$\Delta P = (-1)^L \quad \text{for electric transitions}$$

$$\Delta P = (-1)^{L-1} \quad \text{for further suppressed magnetic transitions}$$

So from initial and final nuclear spins, we can determine the "multi-polarity" of transition and whether it is electric or magnetic

eg: $2^+ \to 1^-$ is E1
↑ since $2-1 = 1 = L$
↑ since $(-1)^L = -1$ and we have $+ \to -$ so E

$2^+ \to 1^+$ is M1
↑ since $2-1 = 1 = L$
↑ since $(-1)^L = -1$ but $+ \to +$ so we have to instead do $(-1)^{L-1} = 1$ so M

$3^+ \to 1^-$ is M2
↑ since $3-1 = 2 = L$
↑ since $(-1)^L = 1$ but $+ \to -$ so we have to instead do $(-1)^{L-1} = -1$ and $+ \to -$ so M

$3^+ \to 1^+$ is E2
↑ since $3-1 = 2 = L$
↑ since $(-1)^L = 1$ and $+ \to +$ so E

Usually, the EM transitions have too short a lifetime to measure. But in the Shell Model, we can have a very high spin excited state next to a low spin ground state, a transition that is permitted by a high multipolarity and thus proceeds slowly with a long lifetime, which can then be measured. These metastable excited states are "isomers" and regions in the Periodic Table where such elements are common are called "islands of isomers".

The Mössbauer Effect

In Atomic Physics we can excite atoms by bombarding them with photons of energy equal to energy difference between ground and excited states.

But this isn't possible with nuclei due to the small nuclear recoil. The energy of emitted photon E_γ is not equal to the excitation energy E_0, due to the recoil kinetic energy.

The photon has momentum E_γ/c so recoiling nucleus has kinetic energy

$$T = \frac{p_\gamma^2}{2M_N} = \frac{E_\gamma^2}{2M_N c^2}$$ where M_N is nuclear mass

The de-excitation energy E_0 is the sum of photon energy plus kinetic energy: $E_0 = E_\gamma + \frac{E_\gamma^2}{2M_N c^2}$ ↓ approximate solution

$$\Rightarrow E_\gamma = E_0\left(1 - \frac{E_0}{2M_N c^2}\right)$$

Furthermore, if we use bombard another nucleus with this photon with the hope of exciting it, we find that the target nucleus also recoils, absorbing some of the energy in its own rest frame.

$$E_0' = E_\gamma\left(1 - \frac{E_0}{2M_N c^2}\right) \approx E_0\left(1 - \frac{E_0}{M_N c^2}\right)$$

so E_0' is smaller than E_0 by a small amount.

For fast decaying excited states with lifetime $\tau \sim 10^{-12}$s:
line width $\Gamma = \frac{\hbar}{\tau} \sim 10^{-3}$ eV

So if the difference between E_0' and E_0 is the typical 0.1eV, the difference between the excitation energy and the energy the recoiling nucleus can absorb is much larger than the width of the photon, so absorption (↑ i.e excitation) is impossible.

The solution to this is the Mössbauer effect.

If the source and target nuclei are both fixed in a crystal lattice, the recoil momentum can be taken up by the entire crystal, making the recoil energy negligible.

This lets us measure widths of nuclear transitions very accurately.

If the source is stationary, most of the γ-rays are absorbed by target so intensity reaching detector is low.

If the source is moving, there is an increase in intensity reaching detector since due to doppler effect the γ-rays are just off-resonance. Line widths can be measured this way.

If the source is moving with velocity v:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where $\Delta\lambda$ is difference between wavelength of emitted photon (in rest frame of source) and wavelength of absorbed photon.

in terms of photon energies:

$$\frac{\Delta E}{E} = \frac{v}{c}$$

If at this velocity, absorption by absorber has fallen to $\sim\frac{1}{2}$ of its peak at $v=0$, then this ΔE corresponds to the half-width $\frac{1}{2}\Gamma$ of spectral line.

$$\therefore \quad \Gamma = 2E\,\frac{v_{1/2}}{c}$$

where $v_{1/2}$ is velocity for which absorption has fallen to $1/2$ its peak value.