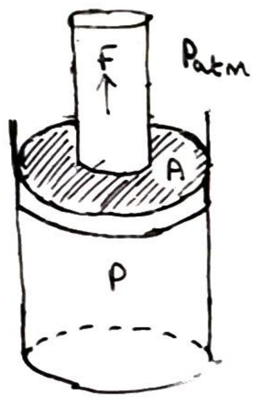
Work

In mechanics we are used to defining work as:

$W = \int \underline{F} \cdot d\underline{x}$ so how does this apply to gases?

A gas in a container can do work if the volume of the container is allowed to change.

Consider a piston whose volume is allowed to change in a controlled way as the gas expands:



If the piston moves by an infinitesimal amount dy so that the change in volume is $dV = A\,dy$, the infinitesimal amount of work done by the gas is

$W = F\,dy$ where $F = (P - P_{atm})A$

$W = (P - P_{atm})A\,dy = (P - P_{atm})\,dV$

If we want to write a general statement, we can use P as net pressure (imagine there is no atmospheric pressure).

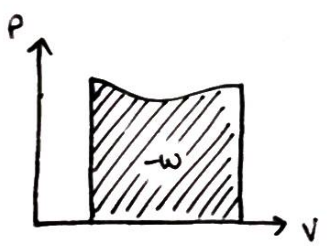
We will also define W as work done <u>on</u> the gas so:

$dW = -P\,dV$

$$\boxed{W = -\int P\,dV}$$

It is vital to note that this is work done <u>on</u> the gas, hence the minus sign!

In a PV diagram:



The negative of the area under the graph is the work done <u>on</u> the gas example : Consider an isothermal process (constant temperature) in which the gas is at initial Pressure P_i and initial volume V_i and isothermally expands to a final volume V_f. What is the work done on the gas?

The internal energy of the gas remains constant throughout this process so $PV = \frac{2}{3}U = P_i V_i$

$$PV = P_i V_i \quad \therefore \quad P = \frac{P_i V_i}{V}$$

$$W = -\int P\,dV = -\int_{V_i}^{V_f} \frac{P_i V_i}{V}\, dV$$

$$= -P_i V_i \left[\ln V\right]_{V_i}^{V_f}$$

$$= -P_i V_i \{\ln V_f - \ln V_i\}$$

$$= P_i V_i \{\ln V_i - \ln V_f)$$

$$= \underline{\underline{P_i V_i \ln \frac{V_i}{V_f}}}$$

The important thing here was to notice that internal energy doesn't change, allowing us to construct a function of volume for P.