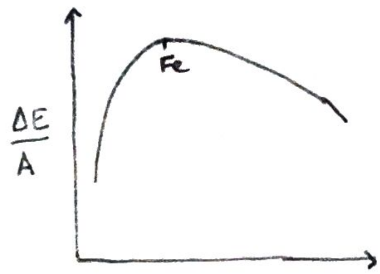
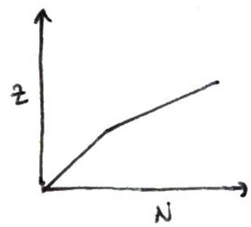
Nuclear Fission

Let's look at the binding energies per nucleon for different nuclei:



we notice that heavier elements after iron have a lower binding energy, meaning it is energetically favourable for a heavy nucleus to split into 2 fragments of smaller nuclei, thereby releasing energy which goes into the kinetic energy of the fragments.

This process is called Nuclear Fission.



If we recall the proton-neutron plot: we see that heavier elements prefer more neutrons than protons, so in the process of splitting into smaller nuclei, there will be some "spare" nuclei emitted, which will also take up some of the released energy. Usually 3 or 4 neutrons are emitted per fission reaction:

eg. $${}^{238}_{92}U \rightarrow {}^{145}_{57}La + {}^{90}_{35}Br + 3n$$

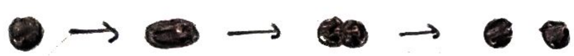
Notice that the split doesn't make 2 nuclei with roughly half the atomic mass each, but instead the atomic mass numbers are separated by ~50. This is normal, but it is not well-understood why.

$B(238, 92) = 1803\ MeV \quad B(145, 57) = 1198\ MeV$

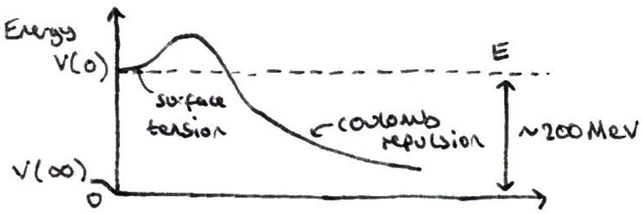
$B(90, 35) = 763\ MeV$

$1198 + 763 - 1803 = 158\ MeV$ of energy is released.

These kinds of fission reactions are very rare in nature since in order to split into 2 parts, the nucleus' shape is first deformed into an ellipse and then develop a "neck" before finally splitting.



In the deformed state, there are 2 forces acting on the nucleus. The surface energy (surface tension in liquid drop model) holding it together, and coulomb repulsion pushing it apart. Combined, these forces produce a potential barrier.



The fission fragments must undergo quantum tunnelling through this barrier.
The height of the barrier is ~6 MeV, similar to α decay

You will remember that for α decay:

$$T \sim \exp\left\{ -\frac{2}{\hbar} \int_R^{R'} \sqrt{2M\left(\frac{ZZ'}{4\pi\varepsilon_0 r} - Q\right)}\, dr \right\}$$

where M is mass of emitted particle. So we will see that if M is higher, T is smaller, i.e tunneling probability is lower for heavier fission products.
Since our products in the example are quite heavy, this probability is very low, so natural fission is unlikely.

Far more likely is induced fission. In this case, we bombard the parent nucleus with a neutron which, if absorbed by the parent nucleus, releases energy equal to the binding energy of the neutron in the form of vibrational energy. This energy is enough to overcome the potential barrier.

If the binding energy of the neutron is still insufficient, the potential barrier can still be overcome if the neutron had sufficient kinetic energy.
Eg. with $^{238}_{92}U$, the binding energy of a neutron is 1 MeV smaller than necessary, so we require a neutron with > 1 MeV kinetic energy.
But with $^{235}_{92}U$, there is one unpaired neutron which the extra neutron pairs with, and there is extra binding energy from pairing term, enough to induce fission.

Even though 3 - 4 neutrons are emitted in the fission reaction, the fragments still contain more neutrons than stable isobars so the fission fragments are usually unstable in beta decay. The fragments often β decay multiple times and these β-decay chains release further energy which carried away by the electrons and antineutrinos.

Note, since there is no coulomb repulsion, there is no common mechanism by which a fragment with too many neutrons can simply emit a neutron. The surface energy tends to keep the neutron bound, hence why β decay is the most common Mechanism to become stable.

But it does sometimes occur! eg. the fission fragment $^{90}_{35}Br$ decays into $^{90}_{36}Kr$ in its first β decay. But the Krypton is produced in an excited state with enough energy to overcome the surface energy, so $^{90}_{36}Kr \rightarrow {}^{89}_{36}Kr$ emitting a neutron directly. This is still unstable in β decay so continues to beta decay.

The neutrons produced in a fission reaction can be absorbed by another nucleus which then undergoes induced fission. This is the principle behind the "chain reaction".

Let K be the number of neutrons produced in a sample of fissile material at stage n divided by the number of neutrons produced at stage $n-1$. i.e. K depends on how many of the neutrons produced at stage $n-1$ are absorbed by a nucleus that can undergo fission.

Turn Over.

- if $k < 1$ the chain will simply fizzle out and the chain will stop very quickly.

- if $k > 1$ the chain reaction will grow until all the fissile material is used up - atomic bomb :(

  This is done by enriching the source to have a sufficiently large concentration of $^{235}_{92}U$. In a spherical sample, the value of k grows with the neutron absorption probability which grows with the radius of the sphere. The mass of uranium must thus exceed some "critical mass"

- if $k = 1$ we have a controlled reaction, as is needed in a nuclear reactor. The absorption of excess neutrons is handled by interspersing the uranium with cadmium or boron rods which absorb neutrons. The rods are moved in and out to keep k at one.