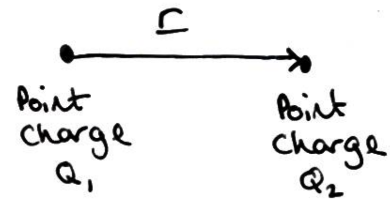
Coulomb's Law

The force exerted by Q_1 on Q_2 is:

$$\boxed{\underline{F}_{1 on 2} = \frac{k Q_1 Q_2}{|\underline{r}|^2}\,\hat{\underline{r}}}$$



Note that force is a vector. This is important. The direction of the force vector is determined by the type of charge Q_1 and Q_2 are as well as the unit vector $\hat{\underline{r}}$ from Q_1 to Q_2. It is also important that for force Q_1 on Q_2, the vector $\hat{\underline{r}}$ is from Q_1 to Q_2.

$k = \frac{1}{4\pi\varepsilon_0}$ where ε_0 is the permitivitty of free space.

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

Vectors in this course will always have 3 components, giving the force a magnitude in the x, y and z directions:

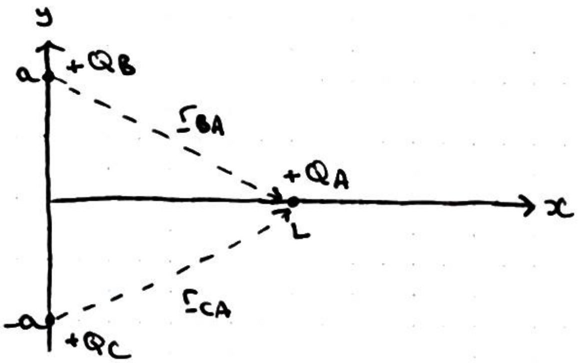
$$\boxed{\underline{F} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = F_x \hat{\underline{i}} + F_y \hat{\underline{j}} + F_z \hat{\underline{k}}}$$

For this reason, we must never write things such as:

$\underline{F} = 6$ or $\frac{1}{\underline{F}}$ as these have no meaning for vectors.

In multiple charge systems, if a charge is being acted on by the forces of many other charges, the individual forces add vectorially. Note, a charge never experiences a force on itself due to its own charge.

An example of a multiple charge system



There are two forces acting on Q_A: the force of Q_B on Q_A and the force of Q_C on Q_A. These forces will add vectorially.

$$\underline{F} = \frac{kQ_1Q_2}{|\underline{r}|^2}\,\hat{\underline{r}} = \frac{kQ_BQ_A}{|\underline{r}_{BA}|^2}\,\hat{\underline{r}}_{BA} + \frac{kQ_CQ_A}{|\underline{r}_{CA}|^2}\,\hat{\underline{r}}_{CA}$$

$$= kQ^2\left[\frac{\hat{\underline{r}}_{BA}}{|\underline{r}_{BA}|^2} + \frac{\hat{\underline{r}}_{CA}}{|\underline{r}_{CA}|^2}\right]$$

Note that since $\hat{\underline{r}} = \frac{\underline{r}}{|\underline{r}|}$, this can be written:

$$= kQ^2\left[\frac{\underline{r}_{BA}}{|\underline{r}_{BA}|^3} + \frac{\underline{r}_{CA}}{|\underline{r}_{CA}|^3}\right]$$

where $\underline{r}_{BA} = \begin{bmatrix} L \\ -a \\ 0 \end{bmatrix}$ and $\underline{r}_{CA} = \begin{bmatrix} L \\ a \\ 0 \end{bmatrix}$.

We can use pythagoras for $|\underline{r}_{BA}| = |\underline{r}_{CA}| = \sqrt{a^2+L^2}$

Since $|\underline{r}_{BA}|^3 = |\underline{r}_{CA}|^3$, we can take this out of the brackets:

$$\underline{F} = \frac{kQ^2}{(a^2+L^2)^{3/2}}\left[\begin{pmatrix} L \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} L \\ a \\ 0 \end{pmatrix}\right]$$

$$\underline{F} = \frac{kQ^2}{(a^2+L^2)^{3/2}}\begin{bmatrix} 2L \\ 0 \\ 0 \end{bmatrix} = \frac{kQ^2}{(a^2+L^2)^{3/2}}\,\hat{\underline{L}}$$

This answer intuitively makes sense as there should be no net force in the y or z direction