Probability : Basics

This section covers some basic probability theory you really should know by now!

For a discrete random variable X with values X_i for $i = 1 \ldots N$, each with probability P_i :

$$\sum_i^N P_i = 1$$

The expected value of a function $f(X)$ is given by:

$$\langle f \rangle = \sum_i^N f(X_i) P_i$$

The mean of the discrete random variable itself is:

$$\boxed{\langle x \rangle = \sum_i^N X_i P_i}$$

The variance of the discrete random variable is:

$$\sigma_x^2 = \sum_i P_i (X_i - \langle x \rangle)^2$$

$$\Rightarrow \boxed{\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2}$$

Standard deviation is square root of variance

We can extend these definitions to continuous functions by replacing the discrete sum with an integral:

$$\langle x \rangle = \int_{-\infty}^{\infty} x P(x) \, dx$$

Basic Combinatorics

consider N distinguishable objects, say the positive integers from 1 to N. If we pick 5 numbers without replacement, we would have a sequence, eg:

1 7 3 9 4

How many different sequences are there?

Since there is no replacement after every pick, the number of objects to pick from decreases after every pick.

So the number is $N(N-1)(N-2)(N-3)(N-4) = \frac{N!}{(N-5)!}$

In fact, if we pick n numbers from a sample of N, the number of sequences is $\underline{\underline{\frac{N!}{(N-n)!}}}$

If we do not care about the order (i.e $17394 \equiv 14397$) then the number of sequences is even smaller $\underline{\underline{\frac{N!}{n!(N-n)!}}}$

We write this as "N choose n":

$$\boxed{\frac{N!}{n!(N-n)!} = \binom{N}{n}}$$

where $\binom{N}{n}$ is called the binomial coefficient

Binomial Distribution

Consider a discrete random variable X that can only take two values X_1 or X_2, with probabilities p and $(1-p)$ respectively.

If we consider N trials, then the probability of n events giving value X_1 is given by:

$$\boxed{P_n = \binom{N}{n} p^n (1-p)^{N-n}}$$

This is the binomial distribution.

The mean is given by:

$$\boxed{\langle n \rangle = Np}$$

The variance is given by:

$$\boxed{\sigma_n^2 = Np(1-p)}$$