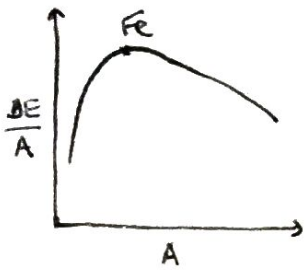
Nuclear Fusion

Let's recall the binding energies per nucleon of different elements:



So while heavier elements after iron have decreasing binding energy, elements lighter than iron have increasing binding energy with no. nucleons. The increase is also far more rapid than the decrease after iron.

Thus, much more energy can be released by fusion of two light nuclei into a heavier nuclei than in the case of fission.

Eg, let's consider the fusion of a deuteron and a hydrogen nucleus:

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$

The γ is emitted since Helium is made in an excited state.

The mass of ^2_1H is 3.34358×10^{-27} kg

^1_1H is 1.67262×10^{-27} kg

^3_2He is 5.00832×10^{-27} kg

Using $E = \Delta mc^2$ where Δm is mass difference between left and right sides of the chemical formula, we find $E = 4.4\,MeV$ which is carried off by the γ and the kinetic energy of the ^3_2He nucleus. Although this energy seems less than for fission, the energy per nucleon is much higher.

The fusion products usually have too few neutrons to form a stable nucleus so we get β^+ decay:

$$^1_1H + ^1_1H \rightarrow ^2_2He \rightarrow ^2_1H + e^+ + \nu$$

Fusion reactions also usually do not occur spontaneously because the nuclei repel each other due to coulomb interaction and must have enough energy to overcome the coulomb barrier to get close enough to fuse.

An estimate of the required energy for 2 protons to fuse is:

$$E = \frac{e^2}{4\pi\varepsilon_0 R} \qquad \text{where } R \text{ is nuclear radius}$$

if $R \sim 4\ \text{fm}$, then $E \sim 350\ \text{keV}$

In order for the protons to have this average energy associated with the degree of freedom corresponding to motion in the direction of the target proton, we would need a temperature T:

$$E = \frac{1}{2} kT \qquad \text{where } k \text{ is the Boltzman constant}$$

so $T \sim 4 \times 10^9\ \text{K}$ which is <u>really</u> hot.

In practice though, fusion can happen at lower temperatures such as in our Sun at $1.3 \times 10^7\ \text{K}$ because:

1) The temperature we determined was for the average energy per photon, but energies are distributed according to Maxwell Boltzmann distribution, which has a tail, and this tail means that there are some particles whose energy is much larger than average.

2) It is not necessary for the incident protons to have sufficient energy to overcome the coulomb barrier. They can also get through by quantum tunneling, provided the barrier height is not too high above the kinetic energy of the incoming particle.

Fusion is the source of energy in stars and it works in cycles, a series of stages of fusion in which the initial particles are protons but in subsequent stages fusion can happen between a fusion product and a proton. At the end of the cycle, all intermediate fusion products have disappeared, leaving only a stable fusion product, usually ^4_2He.

The most common such cycle is the proton cycle:

$$ {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu \qquad (\times 2)$$

$$ {}^1_1H + {}^2_1H \rightarrow {}^3_2He + \gamma \qquad (\times 2)$$

$$ {}^3_2He + {}^3_2He \rightarrow {}^4_2He + {}^1_1H + {}^1_1H$$

At the end of the cycle, there is no ${}^3_2He$ left. The overall process is:

$$4\,{}^1_1H \rightarrow {}^4_2H + 2e^+ + 2\nu + 2\gamma$$

The total energy released by this is 24.7 MeV.

Other cycles also occur within the sun:

$$ {}^{12}_6C + {}^1_1H \rightarrow {}^{13}_7N + \gamma$$

$$ {}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu$$

$$ {}^{13}_6C + {}^1_1H \rightarrow {}^{14}_7N + \gamma$$

$$ {}^{14}_7N + {}^1_1H \rightarrow {}^{15}_8O + \gamma$$

$$ {}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \nu$$

$$ {}^{15}_7N + {}^1_1H \rightarrow {}^{12}_6C + {}^4_2He$$

This is called the carbon cycle

Two steps shown above are β decays of fusion products.
The initial carbon is produced by the fusion of 3 ${}^4_2He$ nuclei.
The fusion process in the carbon cycle require more energy to overcome the coulomb barrier so is only likely at higher temperatures.
But at higher temperatures, this cycle is more likely than the proton cycle, because in the proton cycle we require 2 ${}^3_2He$ to fuse which is unlikely because the ${}^3_2He$ are produces from previous fusion processes and their density is low.
The sun is relatively cool for which the proton cycle dominates.

The produced γ-rays scatter around other charged particles in the sun, losing energy at each scattering and eventually "thermalize", i.e settle at an energy distribution which is the black-body distribution.