Beta Decay

In β decay, either an electron or positron is emitted:

$${}^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + e^- + \bar{\nu}$$

$${}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + e^+ + \nu$$

Note the atomic mass is unchanged so the parent and daughter are isobars.

The electron/positron is created when a neutron/proton in the nucleus turns into a proton/neutron:

$$n \rightarrow p + e^- + \bar{\nu}$$

The neutron's mass actually exceeds the sum of the masses of the proton and electron so a free neutron can undergo this decay with lifetime 11 minutes.

But inside the nucleus, this isn't always energetically allowed. This is because of the difference in binding energies between parent and daughter. When $n \rightarrow p$, the coulomb repulsion between nucleons increases, decreasing the binding energy [recall the $B(A,Z)$ formula]. It also decreases due to the pairing term in the formula.

β^--decay is allowed if $M(Z,A) > M(Z+1,A) + M_e$

i.e parent nuclei mass > daughter nuclei mass + electron mass.

and for β^+ decay: $M(Z,A) > M(Z-1,A) + M_e$

For nuclei with even A:

Due to the pairing term in $B(A,Z)$:

- odd proton odd neutron nuclei are unstable in β decay
- even proton even neutron nuclei are unstable in β decay IF no. neutrons is too high or too low.

For nuclei with odd A, either the no. protons or no. neutrons must be odd, so the pairing term no longer matters, as there will always be one unpaired nucleon. So, to determine stability in β decay, we have to consider the symmetry term (which likes $Z = N$) and the coulomb term (which likes fewer protons)

So for a given odd A, there is only one isobar stable in A, with some Z_A, which can be calculated with:

$$Z_A = A\frac{2a_A + (M_n - M_p - M_e)c^2/2}{4a_A + a_C A^{2/3}}$$

where a_A and a_C are terms from the $B(A, Z)$ formula

if $Z > Z_A$, there are too many protons for stability so β^+ happens

if $Z < Z_A$, there are too few protons for stability so β^- happens

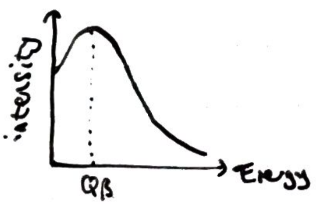
Neutrinos

As with α decay, we define Q-value as:

$$Q_\beta = (M_P - M_D - M_e)c^2$$

we can neglect recoil of daughter nucleus here since $M_D \gg M_e$

we would expect Q_β = kinetic energy of electron, but we actually see a spectrum with peak at Q_β:



So we are clearly missing a piece of the puzzle. The number of spin-$\frac{1}{2}$ nucleons in parent is the same as that in the daughter, so the difference in spins between parent and daughter must be an integer.

But on the right hand side of equation, we also have an electron with extra spin-half, which seems to violate conservation of angular momentum.

The solution is a massless spin-$\frac{1}{2}$ particle with no charge: neutrinos! This allows for angular momentum to be conserved, and the Q_β value is sum of kinetic energies of electron and neutrino.

Electrons and neutrinos are called leptons (which means they do not interact under strong force). Electrons and neutrinos have lepton no. 1 and positrons and antineutrinos have lepton number -1. Since lepton number must be conserved, we must have a total of 0 being produced, hence why electrons (positrons) and antineutrinos (neutrinos) are made together.

So why do we think that neutrinos are massless?
If neutrinos are massless, they can have arbitrarily small energy so electron energy can go up to Q-value. If they have some mass m_ν, their lowest energy is $m_\nu c^2$ and the electron energy spectrum drops off sharply at end point.

We now know that neutrinos do have mass, but we will ignore this for our purposes.

Electron capture

Nuclei that can undergo β^+ decay can also decay through electron capture.

$$e^- + {}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + \nu$$

Here, an electron from one of the innershells can be absorbed into the nucleus, converting a proton to a neutron. The energy is entirely carried away by the neutrino which is usually undetected since neutrinos interact so weakly with matter.

Parity Violation

CS Wu discovered that if she kept a cobalt sample in a magnetic field such that the spin of the cobalt pointed along the direction of the magnetic field, the electrons emerging after β^- decay mostly came out in the opposite direction from the applied magnetic field.

|| The experiment used: ${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}$

If we write $\underline{S}$ for spin of parent nuclei and $\underline{p}_e$ for momentum of electron, then average value of $\underline{S} \cdot \underline{p}_e$ must have been negative for this to be the case. The antineutrinos must be emitted in direction of magnetic field so that momentum can balance, such that $\underline{S} \cdot \underline{p}_{\bar{\nu}}$ was positive.

Under the parity operations we studied in Atomic Physics:

$$\underline{r} \rightarrow -\underline{r}$$

$$\underline{p} \rightarrow -\underline{p}$$

but $\underline{L} \rightarrow \underline{L}$ since $\underline{L} = \underline{r} \times \underline{p}$

$\underline{S} \rightarrow \underline{S}$ since it is an <u>internal</u> ang. mom.

$$\therefore \quad \underline{S} \cdot \underline{p}_e \rightarrow -\underline{S} \cdot \underline{p}_e$$

which is surprising! Since $\underline{S} \cdot \underline{p}_e$ has a non-zero expectation value, the mechanism of β-decay violates parity conservation!

i.e. if we were in a mirror-world in which all the directions were reversed, spin would still point in the same direction but the electron would now emerge in the direction of the magnetic field!

The spin of the parent is 5 in this case, and the spin of the daughter is 4. So in order to compensate for the "lost" angular momentum the antineutrinos and electrons have their spins in direction of magnetic field. Antineutrinos have a spin component $+\frac{1}{2}$

in their direction of motion and the electrons have spin component $-\frac{1}{2}$ in their direction of motion.

The sign of component of spin in direction of motion is called helicity of the particle.

Neutrinos always have negative helicity, antineutrinos always have positive. Electrons can have either negative or positive helicity.

However, electrons emitted in β decay usually have negative helicity (positrons would have positive helicity).

So the mechanism responsible for β-decay ("weak interaction") distinguish between positive and negative helicity, and therefore violate parity.