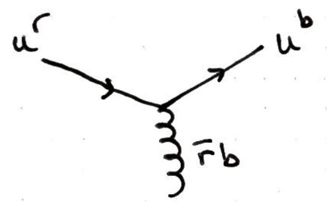
Quantum Chromodynamics

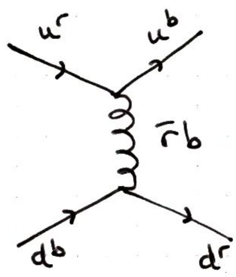
Gluons and Colour

Strong Gauge bosons (gluons) conserve flavour but can effect changes of colour of the quarks, similar to how weak gauge bosons effect changes of flavour.



A red up quark has been converted to a blue up quark

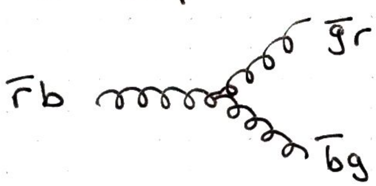
There are 6 colour changing gluons and 2 colour neutral gluons, making 8 gluons in total. In strong interaction processes, quarks exchange gluons and thus usually change colour:



This theory is called Quantum Chromodynamics

This may not be the right way to think of it, but I imagine it as: the d^b sends its b colour and a $\bar{r}$, so it becomes a d^r. The $\bar{r}$ cancels with r in u^r and the b causes it to be u^b.

Gluons can couple to each other with vertices:



Notice that total colour is conserved

We can also have coupling between 4 gluons like

For weak and EM interactions, the strengths of the couplings of gauge bosons to quarks and leptons are sufficiently small that we can calculate rates using perturbation theory. But it is too large for strong interactions in a nucleus, so we cannot calculate energy levels of nuclei!

Running coupling

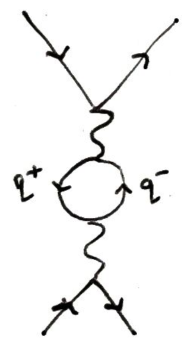
So what can we do with QCD then? Well, at sufficiently high energy / momentum scales, the effective strong coupling becomes small.

$$\alpha_s = \frac{g_s^2}{4\pi \varepsilon_0 \hbar c}$$

where g_s is coupling of gluons to quarks or coupling of gluons to each other.

So why does this become small at high energy scales?

"Negative screening": When an electric charge is probed by another electric charge, the exchanged virtual photon can create a particle-antiparticle pair which exist for a short time and then annihilate each other. If this happens enough times, the probed charge is surrounded by a cloud of created charged particles which "screen", i.e reduce the effective measured charge. As the energy-momentum scale increases, the probe penetrates further into the screen and so measured charge increases.



This is the Feynman diagram of the process of a particle antiparticle pair forming from a virtual photon and then annihilating. At low energy $\alpha = \frac{1}{137}$ but at higher energies $\alpha \simeq \frac{1}{129}$ [This is for EM]

In QCD, a cloud of gluons can be produced by the exchange of virtual gluons. These gluons interact with each other (unlike photons) so the diagram is:

The effect of this is that for larger energies, the effective coupling <u>decreases</u>.

The momentum scale dependence of the coupling is described in terms of a function β of momentum scale $Q \sim M_Z c$

$$\beta = \frac{d\alpha(Q)}{d(\ln Q^2)}$$

For electromagnetism β is positive such that α increases with Q

But for QCD:

$$\beta = \alpha_s^2 \beta_0 + O(\alpha_s^3)$$

where $\beta_0 = -\frac{1}{4\pi}\left(11 - \frac{2}{3} n_f\right)$ where n_f is number of active flavours.

For $Q < 2M_c c$: $n_f = 3$

$2M_c c < Q < 2M_b c$: $n_f = 4$

$Q > 2M_b c$: $n_f = 5$

The $-\frac{11}{4\pi}$ term comes from interaction of gluons with each other, which decreases coupling with increasing Q

The term proportional to n_f comes from creation of q-$\bar{q}$ pair by virtual gluon and increases coupling with increasing Q

We solve this differential equation to find:

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{(1 - \beta_0 \alpha_s(\mu) \ln(Q^2/\mu^2))}$$

neglecting higher order terms in α_s

where $\alpha_s(\mu)$ is α_s at some reference momentum scale μ, usually taken as $\mu = M_Z c$ since α_s for this was measured very accurately at LEP I to be $\alpha_s(M_Z c) = 0.12$.

Experimental measurements of α_s over a large energy range agrees with this formula. For Q greater than few GeV, $\alpha_s(Q)$ would be small enough that a calculation using perturbation theory would be fairly useful. Inside the nucleus, the Q is lower than this so we cannot use perturbation theory and QCD is not very helpful.

The property of QCD that effective coupling decreases with increasing energy/momentum is called "asymptotic freedom".

Quark Confinement

Since gluons are massless, we might expect strong interactions to be long-range (like how EM is long range due to massless photons) but we know this is not the case. This is due to the fact that at large momentum, where we are probing short distances, effective coupling decreases. At large quark separations, the effective coupling increases and the binding gets stronger. Consider a meson, a quark-antiquark state with opposite colour bound by a "string" of gluons. As the quark-antiquark are pulled apart, binding (so tension of "string") gets stronger, until eventually it snaps, producing a quark at the end of the snapped string with the antiquark and an antiquark at the end of the string with the quark:

So now we have two mesons, both of which are colourless.

Thus, it is not possible to isolate a single quark!

We only observe colourless hadron states - either mesons which are a superposition of quark-antiquark pairs of opposite colours, or baryons which consist of three quarks but are antisymmetric under interchange of any two quark colours.

This is called quark confinement and its exact mechanism is not yet understood (2020) but numerical studies in QCD confirm that it does take place.

Quark - Antiquark Potential and Heavy Quark Bound States

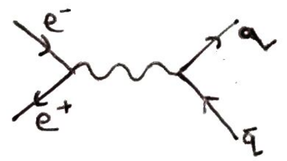
At momentum scales $Q \sim 2M_c c$, we find $\alpha_s \sim 0.3$ which is small enough to obtain energy levels for the J/ψ ($c-\bar{c}$ system) known as charmonium, by solving S.E. with a potential which contains a term that represents the confinement:

$$V(r) = \underbrace{-\frac{4}{3}}_{\text{associated with no. quark colours and no. gluons}} \underbrace{\frac{\alpha_s}{r}}_{\text{coulomb like potential}} + \underbrace{kr}_{\text{represents confinement, increases with } r}$$

We find $k = 0.85 \text{ GeV fm}^{-1}$. Using this potential and making corrections for relativity and spin orbit coupling, we can find the spectrum of the $c-\bar{c}$ system (J/ψ and excited states) and $b-\bar{b}$ system (Υ and its excited states) to high degree of accuracy.

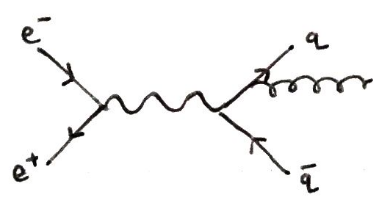
Three Jets in Electron-Positron Annihilation

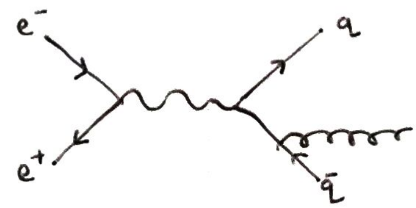
When we considered electron positron annihilation, we used the diagram:



so we said we see 2 jets of particles.

But we can also see 3 jets, since quarks interact with gluons. So we can have:





The gluon fragments into a hadron jet, hence the third jet. Since gluons can't be isolated, this was the first evidence that gluons exist and can couple to quarks.

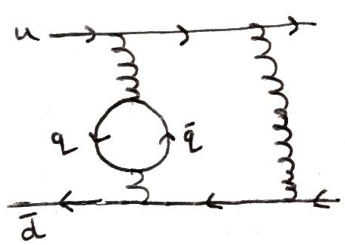
These 3 jet events are very rare compared to 2 jet events since at LEP energies, the running coupling is small, so the amplitude for the process is small since it contains a factor of g_s and thus the rate is suppressed as $\alpha_s \propto g_s^2$.

Four and Five jet events have also been observed and these are even rarer.

But what exactly is a jet? It depends on how big an opening angle constitutes a single jet, parametrised by variable y_{cut} which is a measure of max fraction of total energy that can be contained in a single jet. Perturbative QCD can be used to calculate the no. jets as a function of y_{cut} and we find the predictions match data very closely.

Sea Quarks and Gluon Content of Hadrons

Quarks inside hadrons are bound together by exchanging gluons. Thus, hadrons will have quarks and gluons inside them. These gluons can produce q-$\bar{q}$ pairs which exist for a short time and annihilate quickly. The inside of a π^- may thus look like:



So inside a hadron there are the main quarks, called valence quarks, which determine quantum numbers (flavour) of hadron, and "sea" quarks which are a cloud of q-$\bar{q}$ pairs created by exchanged gluons.

Parton Distribution Functions

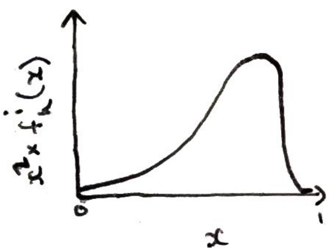
We call all quarks, antiquarks and gluons: Partons.

Consider any relativistically moving hadron. Some fraction x of momentum (which we call Bjorken-x) will be carried by a parton of each possible type.

We define "Parton Distribution Function" as

$f_h^i(x)$ which is the probability that a fraction x of momentum is carried by a parton of type i.

We can't calculate these functions with QCD, so we use experimental data to calculate them.



example parton dist. function.

Factorisation

If the energy/momentum scale of the process Q is large enough such that $\alpha_s(Q)$ is sufficiently small, we can use perturbative QCD to calculate cross-sections at parton level.

Let us denote the differential cross section for 2 partons of types i and j to go into 2 other partons with momentum p_T transverse to direction of incoming partons as:

$\dfrac{d\hat{\sigma}(\hat{s})}{dp_T}$ where $\hat{s}$ is the Centre of Mass energy of the incoming partons.

But this isn't a process we can really observe since we can't isolate the individual partons.

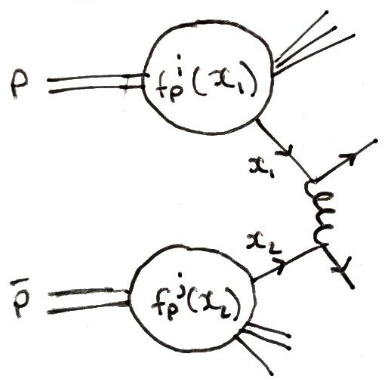
Instead, we can only observe interactions in which the initial states are hadrons, eg. proton - antiproton. So in order to obtain the diff. cross. sec. for something like this, we need to invoke factorisation theorem.

Consider a parton of type i with fraction of momentum x_1 from the proton and another parton of type j with fraction of momentum x_2 from the antiproton. Since energy $E \approx |\underline{p}|c$ for relativistically moving particles, the centre of mass energy of the two partons is $\hat{S} = x_1 x_2 S$ where $\sqrt{S}$ is centre of mass of proton - antiproton.

The Factorisation Theorem tells us, if $f_p^i(x_1)$ and $f_{\bar{p}}^j(x_2)$ are the parton distribution functions, then the contribution to the proton antiproton differential cross section is:

$$\int_0^1 \int_0^1 f_p^i(x_1)\, f_{\bar{p}}^j(x_2)\, \frac{d\hat{\sigma}(x_1 x_2 S)}{dP_T}\, dx_1\, dx_2$$

So if parton level scattering is quark - quark scattering, then the diagram is:



The total differential cross section is obtained by summing over all possible parton types in the proton - antiproton (q, $\bar{q}$, gluons).

So we obtain total proton - proton diff. cross. sec:

$$\frac{d\sigma_{p\bar{p}}(S)}{dP_T} = \sum_{i,j} \int_0^1 \int_0^1 f_p^i(x_1) f_{\bar{p}}^j(x_2)\, \frac{d\hat{\sigma}(x_1 x_2 S)}{dP_T}\, dx_1\, dx_2$$

QCD calculations based on the factorisation theorem agree with experiments!

Note:// only a single parton from each hadron takes place in parton scattering process. The other partons finally fragment into hadrons which are moving in the same direction as incoming protons.