Constituent Quark Model

Quarks are spin-$\frac{1}{2}$ fundamental particles that make up all hadrons. Baryons are made up of 3 quarks, mesons are made up of a quark and an anti-quark.

Symbol	Flavour	Electric Charge (e)	Isospin	I_3	Mass (GeV/c^2)
u	up	$+2/3$	$1/2$	$+1/2$	≈ 0.33
d	down	$-1/3$	$1/2$	$-1/2$	≈ 0.33
c	charm	$+2/3$	0	0	≈ 1.5
s	strange	$-1/3$	0	0	≈ 0.5
t	top	$+2/3$	0	0	≈ 172
b	bottom	$-1/3$	0	0	≈ 4.5

Each of the quarks have a corresponding antiparticle which have the same mass but opposite I_3, electric charge and flavour.

Hadrons from u, d quarks and anti-quarks

Baryons:

Baryon	Quark Content	Spin	Isospin	I_3	Mass MeV/c^2
p	uud	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	938
n	udd	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	940
Δ^{++}	uuu	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	1230
Δ^{+}	uud	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	1230
Δ^{0}	udd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	1230
Δ^{-}	ddd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1230

Since baryons are made of 3 quarks, they can have either $\frac{1}{2}$ or $\frac{3}{2}$ total spin.

Notice that the particles with the same isospin (but different I_3) have similar mass. The differences are due to the EM interactions which distinguish members of the isospin multiplet with different electric charge. If we could switch off the EM interactions, we would see the masses being exactly equal.

The baryons with 3 u quarks or 3 d quarks only occur for spin $\frac{3}{2}$ which we will explain later.

Mesons:

Meson	Quark Content	Spin	Isospin	I_3	Mass MeV/c^2
π^+	$u\bar{d}$	0	1	+1	140
π^0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	0	1	0	135
π^-	$d\bar{u}$	0	1	-1	140
ρ^+	$u\bar{d}$	1	1	+1	770
ρ^0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	1	1	0	770
ρ^-	$d\bar{u}$	1	1	-1	770
ω	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	1	0	0	782

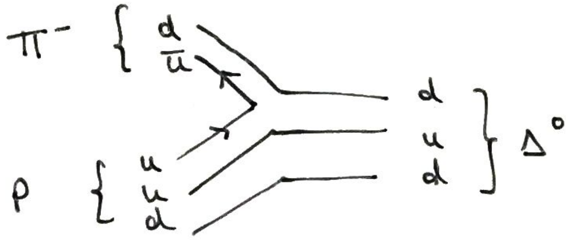
Spin $\frac{1}{2}$ quark and antiquark combine (when orbital ang. mom. is 0) to give mesons with spin 0 or spin-1

Neutral mesons are not pure $u\bar{u}$ or $d\bar{d}$ states but quantum superpositions of these. They also have $I_3 = 0$ and can be in $I = 1$ state in which case their masses are similar to those of their charged counterparts, or in $I = 0$ in which case their masses are somewhat different.

We previously stated that the strong interactions conserve flavour, so a d-quark cannot for example be converted to a s-quark, even though electric charge is the same.

But in a scattering process, a quark and antiquark of the same flavour can annihilate, giving energy which can be used to make a more massive particle.

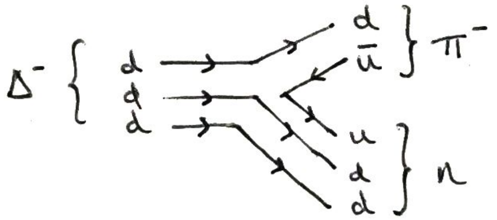
eg.



$\pi^- + p \rightarrow \Delta^0$

The Δ^0 is very short lived and appears as resonance in the $\pi^- p$ scattering cross section.

Now let's consider a decay process:



The u and $\bar{u}$ are created when th Δ^- decays, the u binds to $d\,d$ to make n and the $\bar{u}$ binds to d to mak π^-

$\Delta^- \rightarrow \pi^- + n$

Quark anti-quark pair creation is possible in any particle-particle scattering process provided there is sufficient energy.

So, for example, we can have the inelastic process:

$$\underset{(uud)}{p} + \underset{(uud)}{p} \longrightarrow \underset{(udd)}{n} + \underset{(udd)}{n} + \underset{(u\bar{d})}{\pi^+} + \underset{(u\bar{d})}{\pi^+}$$

Hadrons with s-quarks

The tables for Baryons and Mesons are in the typed notes (or google it!)

For historical reasons, s-quark is assigned strangeness -1, so baryons (depending on no. s quarks) have strangeness $0, -1, -2$ or -3.

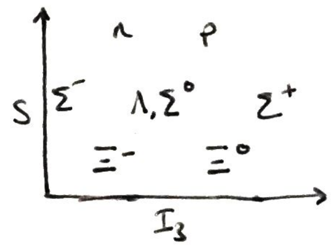
Likewise, the b quark has -1 bottom flavour, but the c quark has $+1$ charm and the t quark has $+1$ top flavour.

The Ω^- has 3 s-quarks, so it has strangeness -3 and spin $\frac{3}{2}$.

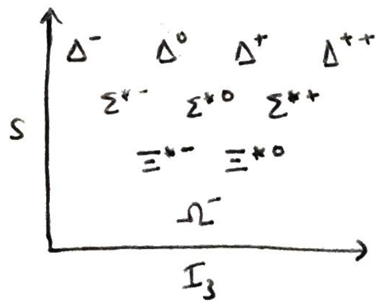
Eightfold Way

We can classify hadrons made up of u, d, s quarks and antiquarks by plotting particles with same spin on a plot of strangeness against I_3:

Spin-$\frac{1}{2}$ Baryons :



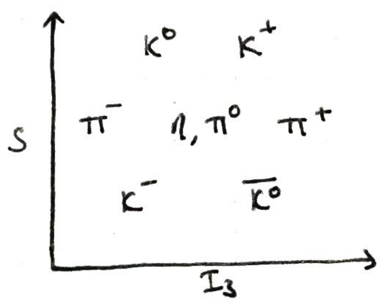
Spin-$\frac{3}{2}$ Baryons :



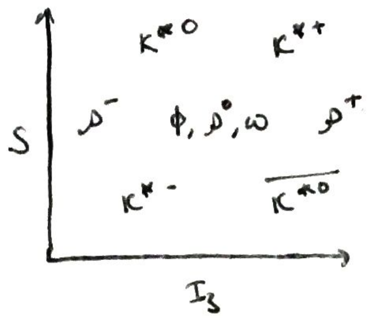
Notice that the rows contain the isospin multiplets

In the case of the row for $I=1$ there can also be states with $I=0$, $I_3=0$ such that the middle point can have 2 or more entries.

Spin-0 Mesons :



Spin-1 Mesons:

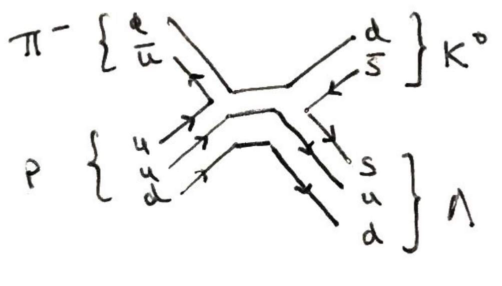


The meson multiplets contain the mesons and their antiparticles, whereas baryon multiplets have separate antiparticle multiplets.

Some mesons, like π^0, ρ^0, η are their own antiparticle because they are bound by quark-antiquark pair of same flavour so "reversing" the quarks to construct the antiparticle, we get the same configuration

Associated Proton Decay

S-quarks cannot be created or destroyed by the strong interactions, hence why strangeness is conserved in these interactions. One can only create a particle containing a strange quark if at the same time, there is a particle containing a $\bar{s}$ quark. An example of this is:



$\pi^- + p \to K^0 + \Lambda$

A u quark annihilates a $\bar{u}$ antiquark and a $s\,\bar{s}$ pair is created.

This reaction is only possible above a threshold energy. In the C of M frame, the lowest total energy of the incoming particles is the sum of the masses of Λ and K^0:

$$\sqrt{s} = E^{TOT}_{CM} = (M_\Lambda + M_{K^0})c^2$$

in a proton fixed target experiment, the pions must be accelerated to energy high enough that the E_{CoM} exceeds this value.

On the other hand:

$$\underset{(d\bar{u})}{\pi} + \underset{(duu)}{p} \rightarrow \underset{(dus)}{\Lambda} + \underset{(d\bar{d},\bar{u}u)}{\pi^0}$$

is forbidden since strangeness is not conserved.

We can make strange baryons by scattering charge kaons against nucleons. eg:

$$\underset{(s\bar{u})}{K^-} + \underset{(udd)}{n} \rightarrow \underset{(dus)}{\Lambda} + \underset{(\bar{u}d)}{\pi^-}$$

so we have made the strange baryon Λ

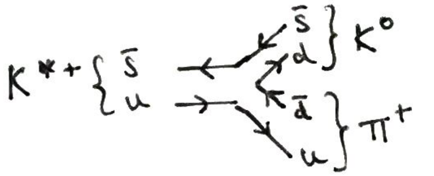
A strange baryon contains s-quarks but no $\bar{s}$ antiquarks, so scattering $K^+(\bar{s}u)$ against a neutron will not make a strange baryon.

Recent evidence has suggested that there is resonance in $K^+ - n$ scattering at $E_{CM} \sim 1.5\,GeV$, implying there is an unstable particle with mass $1.5\,GeV/c^2$. If confirmed, this would be a new type of particle called a pentaquark since it would have to be made of 4 quarks and 1 antiquark. Such a particle does not fit in with the usual quark model of hadrons.

The decay of particles with s quarks or $\bar{s}$ antiquarks can proceed via the strong interaction [and thus will be very rapid, meaning large width] if and only if the decay products have a total strangeness equal to the strangeness of the decaying particle, i.e strangeness is conserved. The decaying particle must also have mass greater than combined mass of decay products.

An example is

$$\underset{(u\bar{s})}{K^{*+}} \rightarrow \underset{(d\bar{s})}{K^0} + \underset{(u\bar{d})}{\pi^+}$$



A $d\,\bar{d}$ quark antiquark pair has been created, but initial and final states have strangeness 1 so it is conserved.

$M_{K^{*+}} > M_{K^0} + M_{\pi^+}$ so there is enough energy in the decaying K^{*+} to produce the products.

The decay proceeds via strong interaction so K^{*+} has very short lifespan, so it is only seen as a resonance in the C of M frame of the product system.

The lighter strange particles do not have energy to decay into other strange particles, so they decay via weak interactions and have a much longer lifetime.

We can combine associated production and decay to have an event such as:

$$\pi^+ + \Lambda \underset{\text{associated production}}{\rightarrow} K^{*+} + \Lambda \underset{\text{decay}}{\rightarrow} K^0 + \pi^+ + \Lambda$$

The observed particles are the K^0, π^+ and Λ but if we look at energies and momenta of K^0 and π^+ and construct

$$P^2_{K\pi} = (E_{K^0} + E_{\pi^+})^2/c^2 - (\underline{p}_{K^0} + \underline{p}_{\pi^+})^2 ,$$ then

then we notice a resonance peak at

$$P_{K\pi} = 842 \text{ MeV}/c$$

indicating at that momenta K^* is produced for a very short time.

Heavy Flavours

The c, b, t quarks are much heavier quarks and there are hadrons that contain these quarks. Only hadrons that contain one c quark or one b quark or their antiquarks have been observed. It is believed a hadron with a t-quark would be too unstable to form a bound state.

There are also bound states of $c\bar{c}$ and $b\bar{b}$, which are neutral. These heavy quarks were first observed by observing these neutral bound states.

Quark Colour

We encounter one problem with the quark model when it comes to baryons, which we can see with Δ^{++}, Δ^-, Ω^- which are bound states of 3 quarks of same flavour. For these low-mass, the orbital angular momentum is 0 and so the spatial parts of the wavefunction for these baryons is symmetric under interchange of the position of two of these identical flavour quarks.

But the total wavefunction for a baryon must be odd since it has half-odd-integer spin. But these baryons have spin $\frac{3}{2}$ which means that the spin part of the wavefunction is symmetric since it requires all three quarks to have $S_z = +\frac{1}{2}$. This is a problem since symmetric spin part and symmetric spatial part means that the total wavefunction is symmetric.

We can solve this by assuming that quarks come in three possible "colour" states - R, G or B. The antisymmetry of the baryon is thus restored by the assumption that the baryon wavefunction is antisymmetric under interchange of two colours. If a baryon is composed of three quarks with flavours f_1, f_2, f_3, they are each assigned a colour so f_1^R or f_1^G or f_1^B. The colour antisymmetric wavefunction is written:

$$\frac{1}{\sqrt{6}}\left(|f_1^R f_2^G f_3^B\rangle + |f_1^B f_2^R f_3^G\rangle + |f_1^G f_2^B f_3^R\rangle - |f_1^B f_2^G f_3^R\rangle - |f_1^R f_2^B f_3^G\rangle - |f_1^G f_2^R f_3^B\rangle\right)$$

So we see that interchanging colours causes a change in sign. So to have a totally asymmetric wavefunction (including the colour part), the spin and spatial part must be symmetric so that a particle in which all three quarks have the same flavour (and zero orbital ang. mom.) must be symmetric under interchange of two spins - and this is the spin $\frac{3}{2}$ state.

A state of three different colours that is antisymmetric under the interchange of any two of the colours is called a "colour singlet" state - which we can think of as a colourless state.

The quarks themselves are considered "colour triplets" since they can be in any one of the three colour states.

we assume that all physically observable particles (i.e all hadrons) are colour singlets. So we cannot isolate individual quarks and observe them.

This is called "quark confinement" and it is why strong interactions are short range even though the exchange particle (gluon) is massless. You can't pull a quark too far away from the other quarks or antiquarks in the hadron to which it is bound.

For mesons, we also require the meson to be a colour singlet so the quarks and antiquarks bind in such a way to enable this. In the case of quark antiquark state, this means that the wavefunction is a superposition of R with $\bar{R}$, G with $\bar{G}$ and B with $\bar{B}$. So, for example, π^+ is:

$$|\pi^+\rangle = \frac{1}{\sqrt{3}}\left(|u^R \bar{d}^{\bar{R}}\rangle + |u^G \bar{d}^{\bar{G}}\rangle + |u^B \bar{d}^{\bar{B}}\rangle\right)$$

If we require that a quark of a given colour binds with an antiquark of the same colour, then we can achieve the same colourless property for mesons. Then we have to average over all the colours by taking a superposition of all three possible colour pairs.