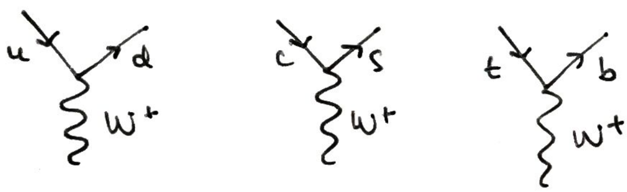
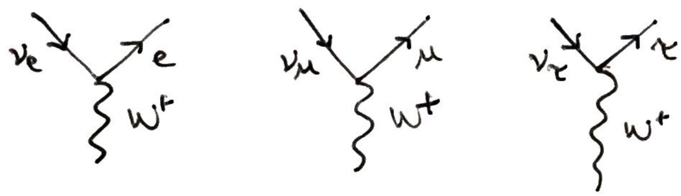
Weak Interactions

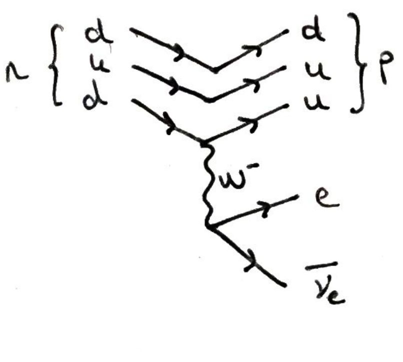
Weak interactions are mediated by $W^\pm$ or Z exchange.
With $W^\pm$ exchange, the flavours of the quarks interacting with the gauge boson can change. $W^\pm$ couples to quark pairs (u, d) (c, s) (t, b) with vertices:



It also couples to lepton pairs (ν_e, e) (ν_μ, μ) (ν_τ, τ)



Quark number and lepton number are conserved in all these interactions.
This is the process responsible for β-decay: a neutron decays into a proton because a d quark converts to a u quark, emitting a W^- which then decays into an electron and antineutrino:



The amplitude for this decay is

$$\frac{g_W^2}{(q^2 - M_W^2 c^2)}$$

where g_W is strength of coupling of W^- to the quarks or leptons.

$$q^2 = E_q^2/c^2 - |\underline{q}|^2$$

where $\underline{q}$ is momentum transferred between neutron and proton, E_q is energy transferred

This momentum is very small compared $M_W c$ so we can neglect it and say amplitude is proportional to:

$$\frac{g_W^2}{M_W^2 c^2}$$

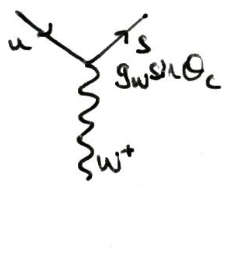
g_W is almost twice as large as electron charge e. But the weak interaction is so weak due to the large denominator.

In modern high energy accelerators we produce weak interaction processes where $|\underline{q}| \sim M_W c$ or even $|\underline{q}| \gg M_W c$ in which case the weak interactions are larger than EM and even comparable to strong interactions!

Cabibbo Theory

Particles with strange quarks cannot decay via strong interaction since this has to conserve flavour, but they can decay via weak interaction.

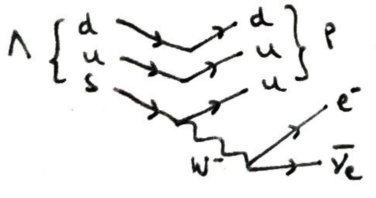
This is because the $W^{\pm}$ couples a u quark to a d quark and can also couple a u quark to a s quark so:



We have a vertex with coupling $g_W \sin\theta_c$.
(The u-d coupling is $g_W \cos\theta_c$)
θ_c is called the Cabibbo angle, and $\sin\theta_c \sim 0.22$

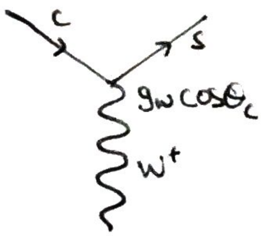
This coupling lets a strange hadron decay into a non-strange hadron (and sometimes even leptons!) eg:

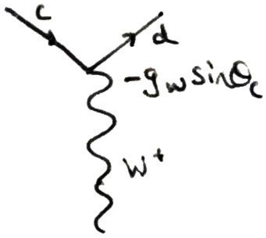
$$\Lambda \to p + e^- + \bar{\nu}_e$$



The c-quark can couple to the s quark with coupling $g_W \cos\theta_c$

The c quark can also couple to d-quark with coupling $-g_W \sin\theta_c$





so charm hadrons are more likely to decay into hadrons with strangeness since

$$g_W \cos\theta_c > -g_W \sin\theta_c$$

$$g_W \begin{pmatrix} d & s \end{pmatrix} \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

This is called the Cabbibo matrix and combines all the couplings.

We extend this to a 3x3 matrix since there are 3 generations of quarks:

$$g_W \begin{pmatrix} d & s & b \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{tb} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

This is the CKM (Cabbibo, Kobayashi, Maskawa) matrix

Although there are 9 parameters, only 4 are independent parameters.

We see to a good approximation $V_{ud} \approx V_{cs} \approx \cos\theta_c$

$V_{us} \approx -V_{cd} \approx \sin\theta_c$

Leptonic, Semi-Leptonic and Non-Leptonic Weak Decays

Decay of strange mesons can be:

Leptonic - final state is only leptons

Semi-leptonic - final state is both hadrons and leptons

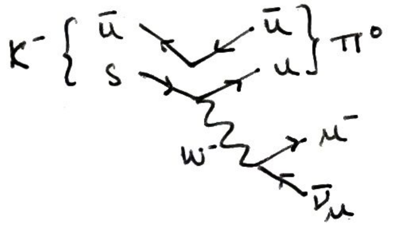
Non-leptonic - final state is only hadrons

For strange baryons, only semileptonic and nonleptonic decay is possible, because baryon number must be conserved. Lepton number is also conserved so a charged lepton must be accompanied by its antineutrino in the final state.

eg. for mesons: $K^- \to \mu^- + \bar{\nu}_\mu$

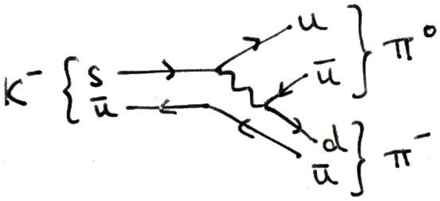
Leptonic decay

$K^- \to \mu^- + \bar{\nu}_\mu + \pi^0$



semi leptonic decay

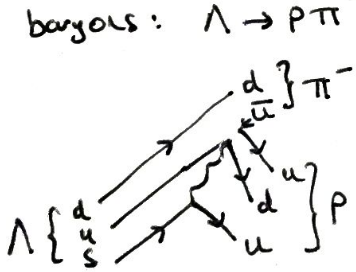
$K^- \to \pi^0 + \pi^-$



Non leptonic decay

(note $M_K > 2M_\pi$ so this is possible)

eg. for baryons: $\Lambda \to p\pi^-$



Non leptonic decay

(we already saw semi leptonic decay with $\Lambda \to p + e^- + \bar{\nu}_e$

Flavour Selection Rules in Weak Interactions

In the exchange of a single $W^\pm$ an s-quark can be converted into a non strange quark. It is not likely that two strange quarks would be converted into non-strange quarks in the same process. So we can make the selection rules:

$$\Delta S = \pm 1$$

So hadrons with strangeness -2 decay first weakly into a hadron with strangeness -1, which can then decay into non-strange hadrons.
The same selection rules apply for charm and bottom as well.

Parity Violation

Parity violation in β decay is because the $W^\pm$ tends to couple to quarks or leptons which are left-handed (negative helicity) i.e states in which the component of spin in the direction of motion is $-\frac{1}{2}\hbar$

$W^\pm$ always couple to left-handed neutrinos. For quarks and massive leptons, the $W^\pm$ can couple to positive helicity states but the coupling is suppressed by a factor of $\frac{mc^2}{E}$ where m is particle mass and E is its energy. The suppression is larger for relativistically moving particles.

For coupling of $W^\pm$ to antiquarks or antileptons, the $W^\pm$ always couples to right handed antineutrinos, and usually to positive helicity states, with suppressed coupling to left-handed antileptons or antiquarks.

Consider the leptonic decay of K^+:

$$K^+ \rightarrow \mu^+ + \nu_\mu$$

$$\nu_\mu \xleftarrow[LH]{} K^+ \xrightarrow[LH]{} \mu^+$$

In the rest frame of K^+, momentum is 0. So μ^+ and ν_μ must move in opposite directions.

The K^+ has zero spin, so by conservation of ang. momentum, the two products must have opposite spin component in any one chosen direction. So they must have the <u>same</u> helicity. So $W^\pm$ couples to left-helicity anti muon μ^+, so coupling is suppressed by $\dfrac{M_\mu c^2}{E_\mu}$

Consider the decay mode:

$$K^+ \rightarrow e^+ + \nu_e$$

The same argument implies suppression of $\dfrac{M_e c^2}{E_e}$

but $M_\mu \gg M_e$, so the decay into positron is heavily suppressed with ratio of partial widths:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)}{\Gamma(K^+ \rightarrow e^+ \nu_e)} = \frac{M_\mu^2}{M_e^2} \approx 4 \times 10^4$$

which is close to experimental results.

Z-Boson Interactions

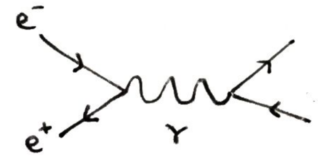
Weak interactions are also mediated by the neutral gauge-boson Z, which couples to both quarks and leptons but does not change flavour.

While the interactions of Z are similar to that of a photon, some differences are:

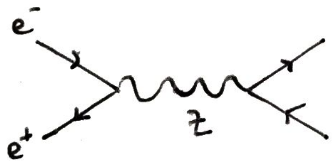
- Z couples to neutrinos, while photons do not since neutrinos have no electric charge.
- Z has a mass of 91.1 GeV/c^2 so interactions are short range, not long range like with photons
- Z has coupling of different strength to left-handed and right-handed quarks and leptons, so these interactions also violate parity

In any process where there can be photon exchange, there can also be Z exchange.

So:

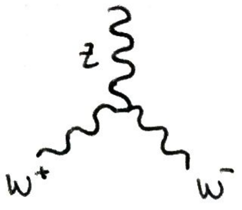


can also have:

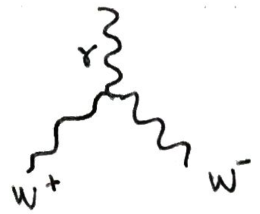


The first diagram has a propagator $1/s$ where $\sqrt{s}$ is centre of mass energy. Whereas the second diagram has propagator $\dfrac{1}{s - M_Z^2 c^4}$

The Z and photon can both couple to $W^\pm$ so we get interaction vertices:

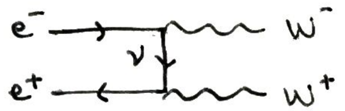


and

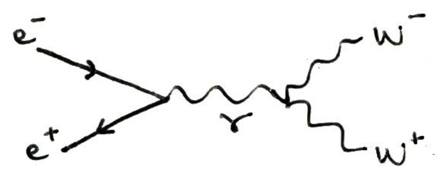


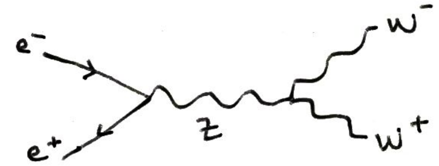
The coupling of Z and photon to $W^\pm$ was confirmed at the LEPII experiment at CERN, where they accelerated electrons to sufficient energies to produce W^+ and W^- in the final state.

From the coupling of W to electron and neutrino, the diagram is:



But we also have to consider the coupling of Z and photon to $W^\pm$ so we also have diagrams:





The standard model of weak and electromagnetic interactions gives a relation between weak coupling g_W, the electron charge e, and masses of Z and $W^\pm$:

$$\frac{M_W}{M_Z} = \cos\theta_W$$ where θ_W is weak mixing angle

$$e = g_W \sin\theta_W = g_W \sqrt{1 - \frac{M_W^2}{M_Z^2}}$$

So we can make order of magnitude estimates of the rates for weak processes at low energies.

At energies $\ll M_W c^2$, the amplitude for $W^\pm$ exchange process is proportional to

$$\frac{g_W^2}{4\pi\varepsilon_0 M_W^2 c^4}$$

so the rate is proportional to $\left(\frac{g_W^2}{4\pi\varepsilon_0 M_W^2 c^4}\right)^2$

For a weak decay rate, we want dimensions of inverse time. So we need to multiply this by something with dimensions

$$\frac{\text{Energy}^4}{\text{time}}.$$

The only quantity proportional to energy is Q value of decay, Q_β and to get time^{-1}, we divide by $\hbar$.

so $$\text{Rate} \sim \left(\frac{g_W^2}{4\pi\varepsilon_0 \hbar c M_W^2 c^4}\right)^2 \frac{Q_\beta^5}{\text{time}}$$

eg. for muon decay $Q_\beta \approx M_\mu c^2$

so $$\text{Rate } \frac{1}{\tau_\mu} = \frac{1}{768\pi^3}\left(\frac{g_W^2}{4\pi\varepsilon_0 \hbar c}\right)^2 \frac{M_\mu^4}{M_W^4} \frac{M_\mu c^2}{\hbar}$$

we know $$\frac{g_W^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2}{4\pi\varepsilon_0 \hbar c \sin^2\theta_W} = \frac{\alpha}{\sin^2\theta_W}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

so we can thus determine muon lifetime.

The Higgs Mechanism

Another particle predicted by the standard model is the Higgs Boson, which was discovered in 2012.

The prediction arises from the idea that there is a field, called the Higgs Field ϕ which has a constant non-zero value everywhere in space. The constant value is called vacuum expectation value $\langle\phi\rangle$

In the absence of this field, all particles would be massless, and would travel with velocity c. They are slowed down by their interaction with the field, and thus acquire a mass M:

$$M = \frac{1}{2} \frac{g_H}{\sqrt{\varepsilon_0 \hbar c}} \langle\phi\rangle$$

where g_H is the coupling of the particle to the Higgs field. The denominator is there to give correct dimensions.

The Higgs field couples to $W^{\pm}$ with coupling g_W to give:

$$M_W = \frac{1}{2} \frac{g_W}{\sqrt{\varepsilon_0 \hbar c}} \langle\phi\rangle$$

we can sub in $g_W = \frac{e}{\sin\theta_W}$ where $\cos\theta_W = \frac{M_W}{M_Z}$

$$M_W = 80.4 \text{ GeV}/c^2$$
$$M_Z = 91.2 \text{ GeV}/c^2$$

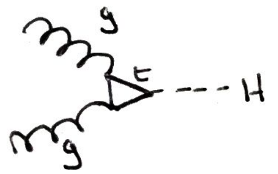
to find: $\langle\phi\rangle = 250 \text{ GeV}/c^2$

Other particles couple to the Higgs field with couplings that are proportional to their mass.

Quanta of the Higgs field are the Higgs boson. With a high confidence level, this particle is confirmed to have the properties:

1) It has spin zero.
2) It couples to $W^{\pm}$ and Z which are consequently massive
3) It does not directly couple to photons so it is uncharged
4) It does not directly couple to gluons (which are massless) so it does not take part in the strong interactions.
5) Its coupling to massive particles is proportional to particle mass
6) Its mass is measured to be $\sim 125\ GeV/c^2$

Gluon fusion is the main production process of the Higgs boson:

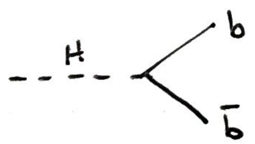


The higgs boson can interact with gluons via virtual massive quarks, eg. top quarks.

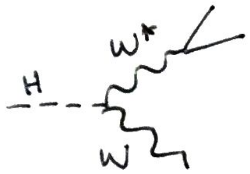
Higgs boson decay is dominated by the most massive particles since coupling to particles is proportional to the mass.

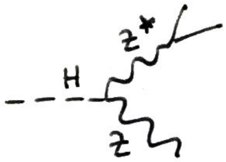
The t-quark has mass $175\ GeV/c^2$ so the higgs boson cannot decay into t-$\bar{t}$ pairs.

The next most massive quark is the b-quark and the Higgs boson primarily decays into a b-$\bar{b}$ pair:

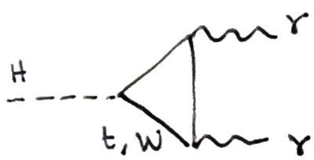


The Higgs boson doesn't have enough energy to decay into real W^+W^- pair (or two real Z particles), however it can decay into one real and one virtual W (or Z), denoted W^* (and Z^*), followed by decay into fermion antifermion pair, as shown below:

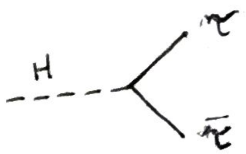




Higgs Boson can decay into photon pair via interactions with virtual top quark and W-boson:



Higgs Boson can also decay into $\tau\bar{\tau}$ pair, which is the dominant leptonic channel, since τ-lepton is the most massive lepton.



Higgs boson discovery was not based on process with highest decay rate, but on the one with best signal-noise ratio. One of the clearest signatures comes from the $H \to \gamma\gamma$ process, even though the decay rate is quite low.

Another important signature is based on $H \to ZZ^* \to 4$ leptons.