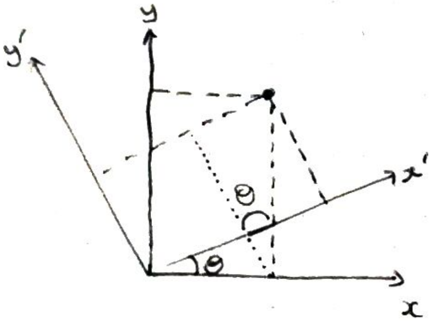
An Analogy to Rotations

It is helpful to think of Lorentz transformations as a generalisation of the idea of rotations.



Consider two coordinate systems, with one system rotated an angle θ about the origin relative to the other system.

The transformation is:

$$x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$

We can express this transformation in vector form:

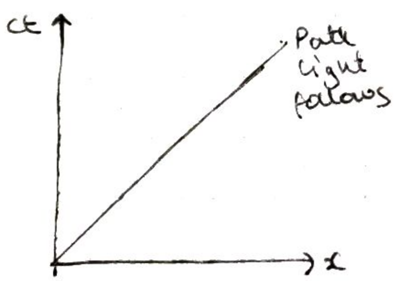
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

There is an invariant quantity, a property that is the same in both reference frames: the distance of a particle from the origin:

$$L^2 = x^2 + y^2 = x'^2 + y'^2$$

Now consider a Lorentz transformation in the same way. We can do something similar with Lorentz transformations.

Consider the $x - ct$ plane:



The transformation to a frame moving relative to this is:

$$ct' = \gamma ct - \frac{v}{c}\gamma x$$

when $ct' = 0 \Rightarrow ct = \frac{v}{c}x$

$$x' = \gamma x - \frac{v}{c}\gamma ct$$

when $x' = 0 \Rightarrow ct = \frac{c}{v}x$

The invariant in this case will be $\underline{\underline{ct^2 - |x|^2}}$

When we construct "4-vectors" in the next section, this will be the "length" of the 4-vector.

The constructed matrix will be:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma \\ -\frac{v}{c}\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$