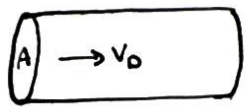
Circuits

In a wire, electroms will accelerate and decelerate when they hit an ion. Applying an electric field that is constant will help to achieve a steady state, i.e. one in which the electrons travel with constant mean drift velocity, $\underline{v}_D$

The current, I, is therefore defined as charge passing a given point per unit charge:

$$\boxed{I = \frac{q}{t} = n q v_D A}$$



where n is the density of charges and A is the area the charge is passing through.

Since the current changes with time, we define the instantaneous current as $I = \frac{dq}{dt}$

Since the current can also vary with space, we define the current density as $\underline{J} = n q \underline{v}_D$ $\quad I = \int \underline{J} \cdot d\underline{A}$

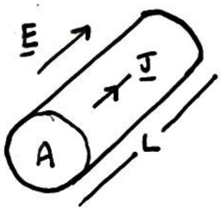
Think of this in the same way as flux.

Ohm's Law

In normal materials, the current density is proportional to the electric field $\underline{J} \propto \underline{E} \Rightarrow \underline{J} = \frac{\underline{E}}{\rho}$

where ρ is a property of the material defined as resistivity.

This relationship of proportionality is a microscopic version of Ohm's Law and we can compute a general version of it for resistors.



From this, we know that $\underline{J} = \frac{\underline{E}}{\rho}$

The potential difference, V, here is: $V = |\underline{E}| L$

The current, I, here is: $I = \underline{J} . \underline{A}$

Using these in the original formula: $\underline{J} = \frac{\underline{E}}{\rho}$

$$\frac{I}{A} = \frac{1}{\rho} \frac{V}{L} \qquad V = \frac{IL\rho}{A}$$

we can define a new quantity Resistance R as $R = \frac{L\rho}{A}$

$$\therefore \quad \boxed{V = IR}$$

Power

A charge q travelling through a potential difference V gains energy qV. In steady state all this energy goes to heating the components in the circuit.
The power P is defined as the energy in a component per unit time.

$$P = \frac{\text{energy}}{\text{time}} = \frac{dqV}{dt} = IV$$

So, Power $\boxed{P = IV = I^2 R = \frac{V^2}{R}}$

Circuitry Components

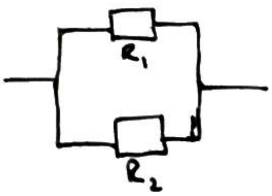
Wires: For the purposes of this course, we assume wires are perfect conductors with no line loss and all points along the wire uninterrupted by components have the same potential.

Resistors: Resistors follow Ohm's Law. The resistances add when they are connected "in series"

Resistors in series add linearly:

Total $R = R_1 + R_2$

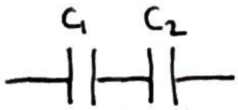
The inverse of the resistances add when the resistors are connected "in parallel"



Inverse of resistances add when parallel:

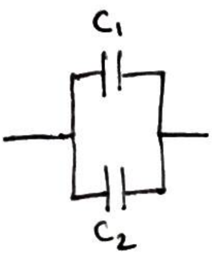
$$\frac{1}{\text{Total } R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Capacitors: We know that $C = Q/V$. The capacitances add in series and parallel opposite to the way they do for resistors.



Capacitors in series add the inverse of their capacitance

$$\frac{1}{\text{Total } C} = \frac{1}{C_1} + \frac{1}{C_2}$$

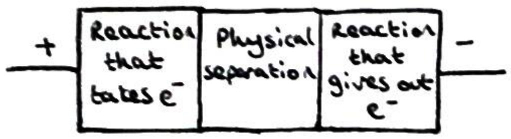


Capacitors in parallel add linearly:

Total $C = C_1 + C_2$

Batteries

If we take one chemical reaction that takes in excess electrons when reacting, and another chemical reaction that gives out excess electrons when reacting, we can make a battery. We separate these reactions physically so that if we connect them with wires, electrons will flow from one reaction to another.

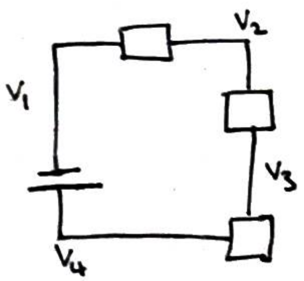


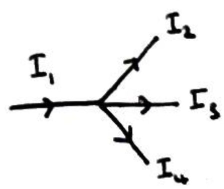
When the ends are not connected the reactions don't happen so the energy is chemically stored.

Kirchoff's Laws

Kirchoff's Current Law: The charge is conserved at any point in a circuit. In other words, the current into a node is equivalent to the current leaving the node.

Kirchoff's Voltage Law: The potential is single valued so the sum of the potential around a loop is 0.

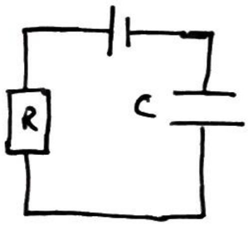




Using Kirchoff's Current Law: $I_1 = I_2 + I_3 + I_4$

Using Kirchoff's Voltage Law: $V_1 + V_2 + V_3 + V_4 = 0$

Charging a capacitor



We know that $V = IR$ and $V = Q/C$

So Voltage Source $= IR + Q/C$

$$V_S = IR + Q/C$$

Differentiating this with respect to time:

$$\frac{d}{dt} V_S = \frac{dI}{dt} R + \frac{dQ}{dt} \cdot \frac{1}{C}$$

but since the source voltage does not very with time:

$$\frac{dI}{dt} R + \frac{I}{C} = 0$$ so for a short period of time:

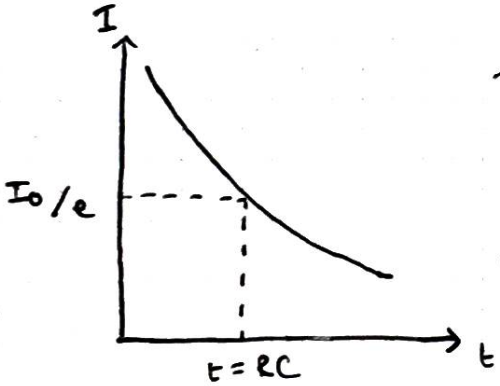
$$\frac{dI}{I} = \frac{-dt}{RC}$$ integrating both sides over a period of change in I:

$$\int_0^t \frac{-dt}{RC} = \int_{I_0}^{I} \frac{dI}{I}$$ where I_0 is the current when $t = 0$

$$-\frac{t}{RC} = \ln(I/I_0)$$

$$\therefore \boxed{I = I_0 e^{-t/RC}}$$

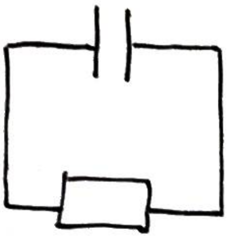
This shows that the current falls exponentially when charging a capacitor.



This is explained physically in the Capacitors chapter

RC here is called the characteristic time.

Discharging a Capacitor



we know that $V = IR$ and $V = Q/C$

so $IR + Q/C = 0$

$$\frac{dQ}{dt} R + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} R = -\frac{Q}{C}$$ which gives us $$\frac{dQ}{Q} = -\frac{dt}{RC}$$

Integrating both sides over a period of change in Q:

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{dt}{RC}$$ where Q_0 is the charge when $Q = 0$

$$\ln Q/Q_0 = -t/RC$$

$$\boxed{Q = Q_0 e^{-t/RC}}$$

So the charge on the plates drop exponentially with characteristic time RC as the capacitor discharges

This is because the force of repulsion from the plate on the charge is proportional to the amount of charge on the plate.

Notice that it is the same relationship for both charging and discharging the capacitor.