Thermal Radiation

Experimental Facts

Consider a cavity of volume V in a reservoir in thermal equilibrium at a temperature T. The walls allow for thermal exchange between the cavity and the reservoir.

A little hole in the cavity allows an experimental study of the thermal radiation inside the cavity. Indicate with:

$$u = \frac{U}{V}$$

The energy density of the thermal radiation inside the cavity.

Define u_λ, the sepectral energy density in terms of the radiation wavelength,

$$u = \int_0^\infty u_\lambda \, d\lambda \qquad \text{since} \qquad u_\lambda = \frac{du}{d\lambda}$$

Some experimental facts:

- Cavity Approximates a black body
- Total energy density : $u(T) = AT^4$ where $A = \frac{4\sigma}{c}$
  where σ is stefan boltzman constant.
- The form of u_λ cannot be predicted by thermodynamics since A is not determined in TD

The Classical Result: Rayleigh Jeans Law

Rayleigh Jeans law is:

$$u_\lambda = \frac{8\pi}{\lambda^4} k_B T$$

$$u = \int_{\lambda_{min}}^{\infty} u_\lambda \, d\lambda$$

This is derived from classical physics

This agrees with experimental data for increasingly long λ but tends to ∞ for lower λ_{min}. This is not possible and is called the ultraviolet catastrophe!

The Planck Solution: A QM Approach

To solve this, we need a QM approach. We treat the EM wave as a Quantum Harmonic Oscillator, so energy becomes quantised.

$E_n = \hbar\omega(n + \frac{1}{2})$ Note: $\hbar\omega = 2\pi\nu\hbar = h\nu$

$\langle \varepsilon \rangle = \hbar\omega(\frac{1}{2} + \langle n \rangle)$ where mean value of energy level n is:

$\langle n \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}$ This is called the Planck Distribution.

$\langle \varepsilon \rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$ ignore temperature independent term $\frac{\hbar\omega}{2}$

Substitute into known equation $u_k = k_B T \frac{k^2}{\pi^2}$ with $\omega = ck$:

$$u_\lambda = \frac{8\pi}{\lambda^3} \frac{hc}{e^{\beta hc/\lambda} - 1}$$

This is the Planck Spectrum Formula

We can find the wavelength where this is a maximum to be:

$$\lambda_{max} \approx \frac{2.9 \times 10^{-3}}{T}$$

This is Wien's Displacement Law

If we integrate u_λ over λ, we obtain the total energy density:

$$u = \int_0^\infty \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\beta hc/\lambda} - 1} \, d\lambda$$ introduce $x = \frac{\beta hc}{\lambda}$

so $d\lambda = \frac{-\beta hc}{x^2} dx$

$$u = -\int_\infty^0 \frac{\beta hc}{x^2} \frac{8\pi hc x^5}{(\beta hc)^5} \frac{1}{e^x - 1} \, dx$$

$$= \frac{8\pi (k_B T)^4}{(hc)^3} I$$ where $I = \int_0^\infty \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$

recalling $h = 2\pi\hbar$, we find:

$$\underline{u = \frac{\pi^2}{15} \frac{(k_B T)^4}{(\hbar c)^3}}$$ which works! Solving the ultraviolet catastrophe!

Thermal Radiation as a Gas of Photons

So why did we remove the ground state term $\frac{\hbar\omega}{2}$?

This is because when Planck did this, QM hadn't been invented. He naturally started counting from 0 instead of the non-zero ground state term!

If we include this term, in the integration, we have an infinite result $\int_0^\infty \frac{\hbar\omega}{2} d\lambda = \left[\frac{\hbar\omega\lambda}{2}\right]_0^\infty = \infty$

We assume we can remove this by redefining the 0 to energy, since we only care about energy differences anyway!

EM radiation can be considered a collection of particles each with energy $\hbar\omega$.

An EM wave in energy level n of a QHO can be considered as n photons each of energy $\hbar\omega$.

This explains the discrete nature of energy since no. particles is discrete!