Heat Conduction

When allowed to, a system out of equilibrium will evolve spontaneously towards equilibrium. Usually these out-of-equilibrium systems have some thermodynamic properties that are not distributed evenly in space, i.e a higher T or P or N at one end of a container. Unless something prevents this, due to increasing entropy, diffusion will take place to reduce this imbalance. As a result, energy or matter (or even momentum) is displaced. This phenomena is called <u>Transport</u>.

All transport mechanisms follow the same form of transport equation:

$$\frac{d}{dt}(\text{stuff}) = (\text{transport coefficient}) \times \text{area} \times (-\text{gradient})$$

→ like a temp. gradient or something

eg. for heat: thermal conduction

$$\frac{dQ}{dt} = -K \times A \times \frac{\partial T}{\partial x}$$

→ temp. gradient

eg. for matter: diffusion

$$\frac{dN}{dt} = -D \times A \times \frac{\partial n}{\partial x}$$

→ gradient in number density where $n = N/V$

eg. for momentum: viscosity

$$\frac{dP_x}{dt} = -\eta \times A \times \frac{\partial v_x}{\partial y}$$

Focusing on Heat Transport:

The transport equation is called Fourier's Law:

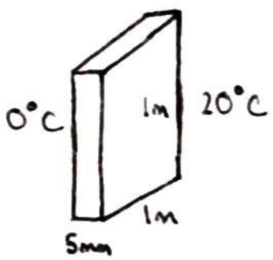
$$\boxed{\frac{dQ}{dt} = -KA\frac{\partial T}{\partial x}}$$

where K is thermal conductivity with units J/KSM or $\frac{W}{KM}$ or $\frac{W}{°CM}$

We can also define flux to be .

$$J_x = \frac{\dot{Q}}{A} = -K\frac{\partial T}{\partial x} \Rightarrow \underline{J} = -K\underline{\nabla}T$$

example: heat conduction through single-glazing window



K for glass $= 0.8\,Wm^{-1}K^{-1}$

We start by stating Fourier's Law:

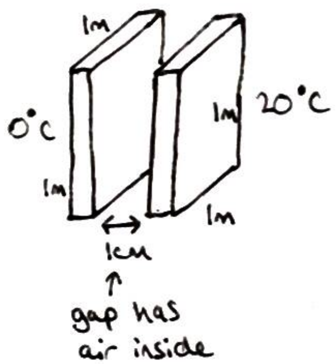
$$\frac{dQ}{dt} = -KA\frac{\partial T}{\partial x}$$

$$= -0.8 \times (1\times1) \times \left(\frac{20-0}{5\times10^{-3}}\right)$$

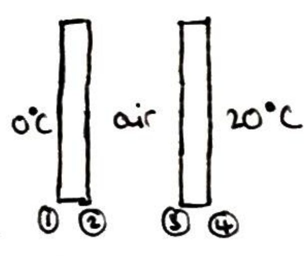
$$= \underline{\underline{-3200\,W}}$$

This was fairly simple. What about for double-glazing?

example: heat conduction through double-glazing



From a side on point of view:



We know the temperature at 1 and 4

$T_1 = 0°C$ $T_4 = 20°C$ but not at

3 and 4

Let's use Fourier's Law for each part:

For ① - ②: $\frac{dQ}{dt} = -K_{12} A \frac{\Delta T_{12}}{\Delta x_{12}}$

For ② - ③: $\frac{dQ}{dt} = -K_{23} A \frac{\Delta T_{23}}{\Delta x_{23}}$

For ③ - ④: $\frac{dQ}{dt} = -K_{34} A \frac{\Delta T_{34}}{\Delta x_{34}}$

Rearranging for Temperature:

$$\Delta T_{12} = T_2 - T_1 = -\frac{\dot{Q}}{A} \frac{\Delta x_{12}}{K_{12}}$$

$$\Delta T_{23} = T_3 - T_2 = -\frac{\dot{Q}}{A} \frac{\Delta x_{23}}{K_{23}}$$

$$\Delta T_{34} = T_4 - T_3 = -\frac{\dot{Q}}{A} \frac{\Delta x_{34}}{K_{34}}$$

Adding these: $T_4 - T_1 = -\frac{\dot{Q}}{A}\left[\frac{\Delta x_{12}}{K_{12}} + \frac{\Delta x_{23}}{K_{23}} + \frac{\Delta x_{34}}{K_{34}}\right]$

we can define $\frac{\Delta x}{K}$ to be "thermal resistance" R so:

$$\Delta T_{14} = -\frac{\dot{Q}}{A}\left[R_{12} + R_{23} + R_{34}\right]$$

$$\dot{Q} = -\frac{A}{R_{TOTAL}} \Delta T_{14}$$

$$= \underline{\underline{-46\,W}}$$

This is much lower than single-glazing so double-glazing really is much better.