Propagation of Light

There are different levels of sophistication for treating light. We choose the one that suits our purpose:

Hardest — Quantum Electrodynamic Model. This is a very complicated quantum field theory and isn't covered in detail on this course.

Maxwell's Equations for Physical Optics. This is a classical wave theory and is sufficient for studying the propagation of light but it fails with phenomena like the photoelectric effect and Compton Scattering.

Simplest — Geometric Optics. This neglects the wavelength of light and instead treats light like a perfect stream of particles. It is ideal for studying things like telescopes but only works on a macroscopic scale.

For a wave model of light, we can think of a light ray as the direction of energy flow.
For a particle model of light, we think of light rays as the direction of particle flow.
For wave models of light only, we can define a wavefront. A wavefront is a surface of all points in a wave disturbance that have the same phase. The ray can therefore be defined as being perpendicular to the wavefronts and in the direction of propagation of wavefronts.
Individual wavefronts propagate at phase speed $v = \frac{\omega}{k}$

Refractive Index

Rays change direction when they cross an interface between different media and in a medium whose properties change from point to point, the rays may follow a curved path. For a uniform medium, the rays are straight lines within the medium.

The reason for the change in direction has to do with the speed of light. The speed of light in a vacuum is found to be $c \simeq 3.0 \times 10^8 ms^{-1}$. When light travels through a medium like glass, its phase velocity is found to be less than c : $v < c$. This is not because the speed of light is itself changing. It is because glass is a dense collection of electrons and atomic nuclei that carry electric charges. This causes the light to scatter or become absorbed and re-emitted. The individual photons themselves are still travelling at c.

The net result of all of the interference of the original wave and new waves (scattered and re-emitted) is oscillations with the same frequency but with an additional phase shift which accumulates in proportion to the distance travelled by the wave. In a plane wave, phase is proportional to $|\underline{k}|$. So increasing phase increases $|\underline{k}|$. Since $|\underline{v}| = \frac{\omega}{|\underline{k}|}$, this means the phase speed is lower. We define refractive index, n, to be : $\boxed{n = \frac{c}{v}}$ which also gives $\boxed{\lambda_{medium} = \frac{\lambda_{vacuum}}{n}}$

Huygen's construction

Huygen's construction is a wave-like way to understand reflection and refraction of waves. These were previously understood well with the ray picture of optics.

The two steps to Huygen's Construction are:

1) Every point of the wavefront is a source of secondary spherical "wavelets" which spread out at wave speed.
2) The wavefront at a later time is the surface tangent to all the wavelets in the forward direction, called the "envelope" of the wavelets.

For example, in 2D:

This is a very useful method to model the wave propagation of light but is hardly a complete wave theory of light. However, the principle is physically appealing and was extended by Fresnel to explain diffraction.

Kirchoff later showed that the assumptions made by this construction are consistent with the wave equation: for example, the reason there is no need to consider the wavelets going 'backwards' is because of the interference given by the principle of superposition.

Fermat's Principle

Fermat originally proposed that the path taken by a light ray between 2 points is the one that takes the least time. This principle leads us to the laws of reflection and refraction.

However, we now know that this is not true in all scenarios. So the modern version defines a quantity called "the optical path length" and states that for the actual path, the optical path length is stationary.

So what is the optical path length? Consider a point from P to P' with s being the distance along the path. If we say the refractive index is a function of s, then we define optical path length as:

$$\boxed{\text{optical path length} = \int_P^{P'} n(s)\, ds}$$

For an infinitesimal segment of the path, time taken is given by:

$dt = \dfrac{ds}{v}$ but $v = \dfrac{c}{n}$ so:

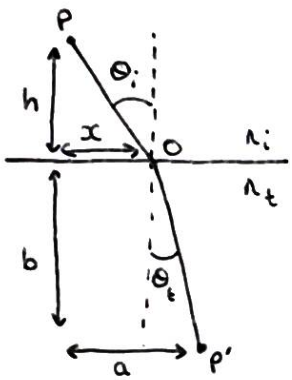
$$\boxed{dt = \frac{n\,ds}{c}}$$

So time taken is optical path length divided by c. In many cases, the path taken by light <u>is</u> the one with the smallest optical path length. But to account for the scenarios where it isn't, we say that the chosen path is the one for which the optical path length is stationary.

Reflection and Refraction

We can derive the laws of reflection and refraction from Fermat's Principle. We will first start with Snell's Law:

Consider a light ray moving from a point P in a region where refractive index is n_i to a point P' in a region where refractive index is n_t. The boundary between the regions is at point O.



The optical path length is given by:

$$\text{Optical path length} = \int_P^O n_i \, ds + \int_O^{P'} n_t \, ds$$

$$l = n_i |\underline{r}_O - \underline{r}_P| + n_t |\underline{r}_{P'} - \underline{r}_O|$$

$$l(x) = n_i (x^2 + h^2)^{1/2} + n_t ((a-x)^2 + b^2)^{1/2}$$

We know from Fermat's Principle that the optical path length must be stationary.

$$\frac{dl}{dx} = 0 \quad \therefore \quad \frac{n_i x}{(x^2+h^2)^{1/2}} - \frac{n_t(a-x)}{((a-x)^2+b^2)^{1/2}} = 0$$

giving: $$\frac{n_i x}{(x^2+h^2)^{1/2}} = \frac{n_t (a-x)}{((a-x)^2+b^2)^{1/2}}$$

but trig ratios tell us $$\frac{x}{\sqrt{x^2+h^2}} = \sin(\theta_i)$$

and $$\frac{a-x}{\sqrt{(a-x)^2+b^2}} = \sin(\theta_t)$$

This gives us Snell's Law: $\boxed{n_i \sin\theta_i = n_t \sin\theta_t}$

The same argument can be repeated with P' being in the same medium as P to obtain the law of reflection:

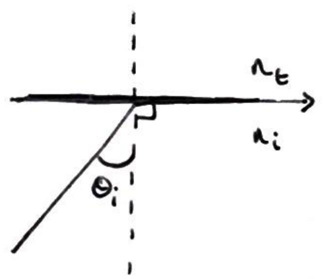
$$\boxed{\begin{array}{l} \sin\theta_i = \sin\theta_r \\ \text{or} \quad \theta_i = \theta_r \end{array}}$$

Total Internal Reflection

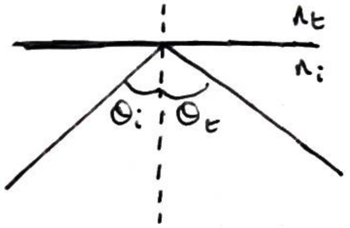
From Snell's Law : $n_t \sin\theta_t = n_i \sin\theta_i$, we can rearrange for

$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i \quad \Rightarrow \quad \theta_t = \arcsin\left[\frac{n_i}{n_t}\sin\theta_i\right]$$

if $n_i > n_t$, then there is no real solution for θ_t when $\sin\theta_i > \frac{n_t}{n_i}$ since then $\frac{n_i}{n_t}\sin\theta_i > 1$ and this does not have an inverse sine. So what does the light ray do? Total Internal Reflection.



The critical angle $\theta_c = \theta_i$ when $\theta_t = 90°$. This is the minimum angle for total internal reflection to occur. At this angle, the wave propagates down the surface of the boundary.



If $\theta_i > \theta_c$, the the ray is completely reflected back into the medium. This is total internal reflection.