Alpha Decay

Alpha decay is the radioactive emission of a ^4_2He nucleus, a doubly magic, very stable nucleus. The daughter nucleus will thus have 2 protons and 4 nucleons fewer than the parent.

Kinematics

We define the "Q-value" Q of a decay as:

$$Q_\alpha = (\underbrace{M_P}_{\text{mass of parent}} - \underbrace{M_D}_{\text{mass of daughter}} - \underbrace{M_\alpha}_{\text{mass of } \alpha})c^2$$

$M_P - M_D - M_\alpha$ is usually estimated quite well from the liquid drop model.

The α particle emerges with an energy T_α, which is slightly below the value of Q_α because the daughter nucleus recoils in order to conserve momentum as the α particle leaves.

$$\therefore \quad Q_\alpha = T_\alpha + T_D$$

The momenta are:

$$\left.\begin{aligned} p_\alpha &= \sqrt{2M_\alpha T_\alpha} \\ p_D &= -\sqrt{2M_D T_D} \end{aligned}\right\} \text{ but we require } p_\alpha + p_D = 0$$

$$\Rightarrow \quad T_D = \frac{M_\alpha}{M_D} T_\alpha$$

$$\Rightarrow \quad T_D = \frac{4}{A} T_\alpha \quad \text{if we neglect binding energies}$$

so $\quad Q_\alpha - T_\alpha = \frac{4}{A} T_\alpha \qquad Q_\alpha = \left(\frac{4}{A} + 1\right) T_\alpha \quad \Rightarrow \quad Q_\alpha = \frac{4 + A}{A} T_\alpha$

so
$$\boxed{T_\alpha = \frac{A}{4 + A} Q_\alpha}$$

Experimentally we sometimes see the T_α being larger than predicted from the equation. This happens when the parent nucleus is itself the daughter of another parent. In this case, the nucleus about to undergo α decay is produced in an excited state which will decay through γ decay <u>before</u> the α decay. But in some cases (if the decay constant for α is large) the excited state can undergo α decay directly and the Q value for such is actually larger than we had in the first equation, hence why T_α is larger than we expect.

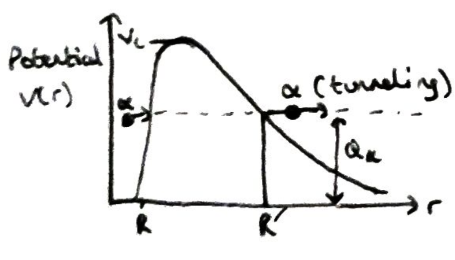
Decay Mechanism

So what exactly causes α decay and how does it work? 2 protons and 2 neutrons from the highest proton and neutron energy levels combine to form a α particle inside the nucleus, a "quasi bound-state", with energy $\sim Q_\alpha$ [we will henceforth neglect the recoil of the nucleus].

The α particle is bound by strong nuclear force. There is electrostatic repulsion between the α particle and rest of nucleus. These together form a potential barrier

$$V_c = \frac{2Ze^2}{4\pi\varepsilon_0 R}$$ Note this is just coulomb potential

The barrier extends from $r = R$ to $r = R'$ after which the α particle has enough energy to escape through Quantum Tunneling.



Now let's do some of the maths behind this.

For a square potential of height U_0 and width a, tunneling probability for a particle mass m, energy E is:

$$T = \exp\left(-2\sqrt{2m(U_0 - E)}\,\frac{a}{\hbar}\right)$$

You should remember how to get this from Quantum Physics in 2nd Year.

The value of T varies rapidly depending on its arguments so this is why the mean lifetime of α decay is a huge range from 10^{-7} secs to 10^{10} years.

But for non-square potential, i.e a potential $U = \frac{2Ze^2}{4\pi\varepsilon_0 r}$:

$$T = \exp\left(-\frac{2}{\hbar}\int_R^{R'} \sqrt{2m_\alpha\left(\frac{2Ze^2}{4\pi\varepsilon_0 r} - Q_\alpha\right)}\,dr\right)$$

We need to now multiply T by the number of times the α particle tries to escape, i.e the number of times it travels from centre to edge of nucleus and back. This is $\sim \frac{v}{2R}$ where $v = \sqrt{2Q_\alpha/m}$ is the velocity of α particle.

After all this, we arrive at:

$$\ln\lambda = f - g\frac{Z}{\sqrt{Q_\alpha}}$$

where $g = 2\sqrt{2}\,\pi\alpha\sqrt{m_\alpha c^2}$
$= 3.97\ \mathrm{MeV}^{1/2}$

$f = \ln\left(\frac{v}{2R}\right) + 8\sqrt{RZ\alpha m_\alpha c/\hbar}$
≈ 128

i.e $$\ln\lambda \approx 128 - (3.97\,\mathrm{MeV}^{1/2})\frac{Z}{\sqrt{Q_\alpha}}$$

This is a crude approximation but agrees with experiments reasonably well.