Differential Form of Maxwell's Equations

We derive the differential forms of these equations from two theorems:

Gauss' Theorem:

$$\boxed{\int_S \underline{F} \cdot d\underline{A} = \int \underline{\nabla} \cdot \underline{F} \, dV}$$

Stokes' Theorem:

$$\boxed{\int_S \underline{F} \cdot d\underline{L} = \int_S (\underline{\nabla} \times \underline{F}) \cdot d\underline{A}}$$

These are explained in greater detail in the Electromagnetism notes.

These give us the equations:

Gauss' Law:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\varepsilon_0}}$$

No Magnetic Monopoles:

$$\boxed{\underline{\nabla} \cdot \underline{B} = 0}$$

Faraday's Law:

$$\boxed{\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}}$$

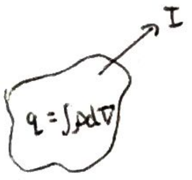
Ampere Maxwell Law:

$$\boxed{\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} + \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}}$$

We can also obtain the conservation of charge equation in differential form. Shown on the next page.

Conservation of Charge

Consider a volume. The charge inside the volume is equivalent to the charge flowing into the volume (since we cannot create charge inside). So the charge flowing out will be the change in charge within the volume. i.e, if there is no change in charge inside the volume, the charge flowing out will be the same as the charge flowing in.



So $$\int_S \underline{J}\cdot d\underline{A} = -\int \frac{\partial \rho}{\partial t}\, dV$$

Using Gauss' theorem:

$$\boxed{\underline{\nabla}\cdot\underline{J} = -\frac{\partial \rho}{\partial t}}$$

This is the equation for charge and current conservation.