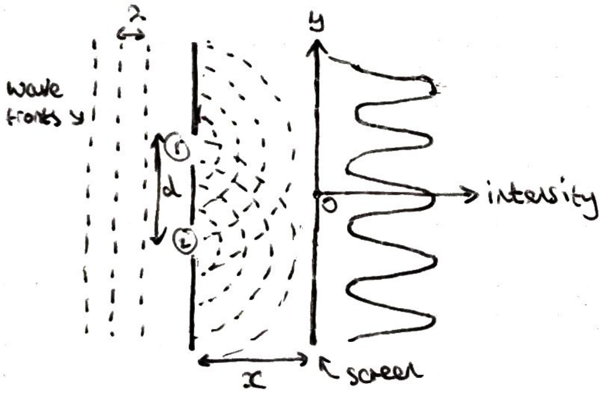
Interference

Interference is a consequence of the linearity of wavefunctions allowing new solutions to the wave equation to be formed from superpositions of existing solutions. We observe interference in various ways but in this section we will look at Young's Double Slit Experiment and The Michelson Interferometer.

Young's Double Slit Experiment

Young's experiment, if described very simply, consisted of light shone on two small slits from one side, in order to observe the interference pattern on the other side:



As you can see, when the planar wavefronts meet a slit, as they diffract through the slit they become spherical wavefronts. The spherical wavefronts from each of the slits will interfere with each other and we observe dark and light fringes on the string.

How do we express the intensity distribution mathematically?

We can think of spherical wavefronts originating from point sources at ① and ②, the slits. The propagated disturbance corresponds to the phasor sum of the waves from these point sources.

If the centres of the slits are at $y = \pm d/2$, we can write the amplitudes of the contribution arriving at a point X with coordinates x, y as:

$$\psi_{1,2} = \frac{\psi_0}{r_{1,2}} \exp(ikr_{1,2}) \quad \text{where } k = \frac{2\pi}{\lambda}$$

$r_{1,2}$ is distance from slits to x and is given by:

$$r_{1,2} = \sqrt{x^2 + (y \pm \frac{d}{2})^2}$$

The total disturbance is therefore:

$$\psi = \psi_1(x,y) + \psi_2(x,y)$$
$$= \frac{\psi_0}{r_1} \exp(ikr_1) + \frac{\psi_0}{r_2} \exp(ikr_2)$$

if d is very small, we can simplify $r_{1,2}$ with the binomial expansion: $r_{1,2} \approx r_0 (1 \mp \frac{d}{2r_0{}^2} y)$

$$\therefore \psi \approx \frac{\psi_0}{r_0} \exp(ikr_0) \left[\exp(ik \frac{d}{2r_0} y) + \exp(-ik \frac{d}{2r_0} y) \right]$$

$$= 2 \frac{\psi_0}{r_0} \exp(ikr_0) \cos(k \frac{d}{2r_0} y)$$

if we say $\sin\theta = \frac{y}{r_0}$, then

$$\psi = 2 \frac{\psi_0}{r_0} \exp(ikr_0) \cos(\frac{kd}{2} \sin\theta)$$

$\psi = A \cos(\frac{kd}{2} \sin\theta)$ where A is the amplitude $= \frac{2\psi_0}{r_0} \exp(ikr_0)$

so we get intensity as:

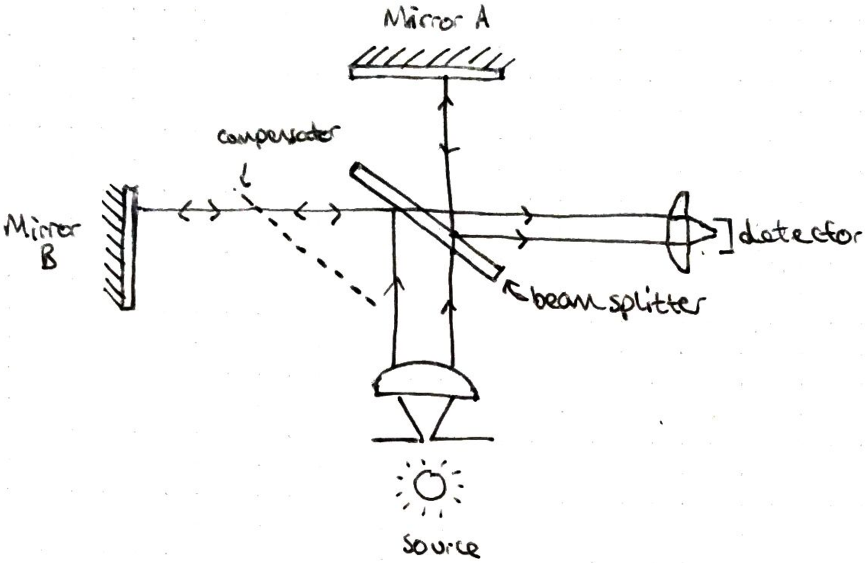
$$\underline{\underline{I(\theta) = I_0 \cos^2(\frac{kd}{2} \sin\theta)}}$$

Remember, intensity is the square of ψ. (not sure about this!)

Thus, we have found a mathematical expression for intensity.

The Michelson Interferometer

I'm sure by now you're well aware of the Michelson Interferometer:



This is the basic setup that should be familiar by now.

For monochromatic light (i.e single wavelength light), the electric field of the travelling light wave is:

$$E(x,t) = E_0 \cos(kx - \omega t) \quad \text{where } k = \frac{2\pi}{\lambda},\ \omega = ck$$

and x is distance travelled since leaving source.

If the beamsplitter reflects a fraction r of light and transmits a fraction t of light, we can write the light arriving at the detector from 1 mirror as:

$$E_{A,B}(t) = rt\, E_0 \cos(k(x_0 + 2x_{A,B}) - \omega t)$$

So the light arriving from both mirrors at the detector is:

$$\begin{aligned} E(t) &= E_A(t) + E_B(t) \\ &= rtE_0 \left[\cos(k(x_0 + 2x_A) - \omega t) + \cos(k(x_0 + 2x_B) - \omega t)\right] \\ &= 2rt\, E_0 \cos(k(x_0 + x_A + x_B) - \omega t)\cos(k(x_A - x_B)) \end{aligned}$$

since $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$$\therefore \quad E(t) = 2rt\, A\cos(k(x_A - x_B)) \quad \text{where } A = E_0\cos(k(x_0 + x_A + x_B) - \omega t)$$

This gives us $I(t) = 4r^2t^2 I_0 \cos^2(\kappa(x_A - x_B))$

letting $r^2 = R$, the intensity reflectivity of the beamsplitter

$t^2 = T$, the intensity transmissivity of the beamsplitter

$$I(t) = 4RTI_0 \cos^2(\kappa(x_A - x_B))$$

If the intensity is recorded as one of the mirrors is moved away from the beam splitter, we will observe a series of sinusoidal fringes (if the source is monochromatic) with κ related to fringe periodicity.