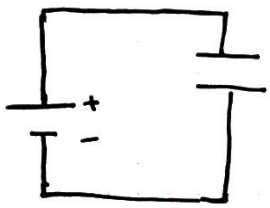
# Capacitors

Two parallel plates can be used to store energy (in the form of electric potential energy) by moving charges from one plate to the other.



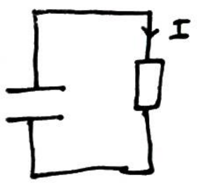

The battery generates a potential difference that causes charges in the plates to move from the negative terminal to the positive terminal. These charges (electrons) will therefore accumulate on the capacitor plates.

When the battery is disconnected, we are left with:


Electrons have accumulated on one plate, giving it a negative charge. Electrons have also left the other plate giving it a positive charge.

Since the charges want to move together, energy is stored in the plates as electric potential energy.

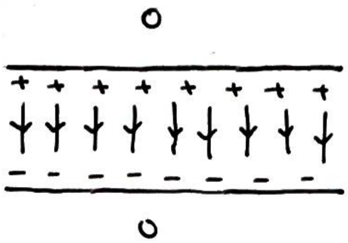
Currently, the charges have no way of moving together but reconnecting the plates allows them to move.




When a path is provided for the charges to move, they will move together. Thus, a current will flow through the resistor.

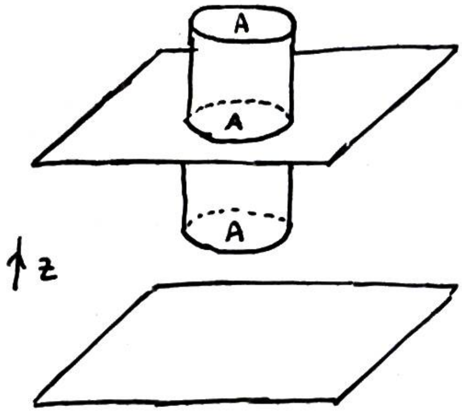
In this way, a capacitor is like a simple battery.

eg. Compute the electric field and potential difference between two charged parallel plates.




The electric field outside the plates is 0. This can be justified by remembering that $\underline{E}$ doesn't decrease with distance from an $\infty$ plate. So summing $\underline{E}$ from the positive and negative plates we get 0.

First, to compute $\underline{E}$ we need to use Gauss' Law. We select the gaussian surface to be a cylinder:




Using the same symmetry from the infinite charged plates in the electric fields chapter: $\underline{E} = E(z)\,\underline{\hat{z}}$

Using Gauss' Law:

$$\int \underline{E} \cdot d\underline{A} = \frac{q}{\varepsilon_0}$$

$$\therefore \quad |\underline{E}||d\underline{A}| = \frac{q}{\varepsilon_0} \qquad E(z)|\underline{A}| = \frac{\sigma|\underline{A}|}{\varepsilon_0}$$

$$\therefore \quad \underline{E} = -\frac{\sigma}{\varepsilon_0}\,\underline{\hat{z}}$$

Now to compute potential difference:

$$\phi = -\int_0^d \underline{E} \cdot d\underline{z} = -\int_0^d |\underline{E}||d\underline{z}| \cos\theta$$

where $\cos\theta = -1$ since $\underline{E}$ and $\underline{z}$ are in opposite directions

$$\therefore \phi = |\underline{E}|d$$

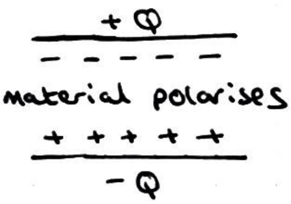
We can define a property Capacitance C as the amount of charge we can store on parallel plates for a given potential difference.

$$\text{Capacitance} = \frac{\text{charge}}{\text{Potential Difference}} \qquad C = \frac{Q}{V}$$

$$C = \frac{Q}{V} = \frac{\sigma A_{plate}}{|\underline{E}|\, d} = \frac{\sigma A_{plate}}{\sigma/\varepsilon_0 \; d}$$

$$\boxed{C = \frac{\varepsilon_0 A_{plate}}{d}}$$

We can therefore see that we can store more charge on the plates by increasing the area of the plates or decreasing the distance between them. It is also beneficial to put a dielectric material between the plates.




The dielectric material will polarise in the electric field which will reduce the electric field. This is like changing the value of the permitivitty of free space.

$$|\underline{E}| = \frac{\sigma}{\varepsilon_0 \varepsilon}$$ relative permativitty

So $$C = \frac{\varepsilon \varepsilon_0 A_{plate}}{d}$$

Different materials have different permitivities and so choosing one with a high $\varepsilon$ will increase the total charge we can store on the plates.

# Charging a Capacitor

As more and more charge builds up on a capacitor plate, it becomes harder and harder to add more. This is because the charge we are trying to add is repelled by the charges already accumulated on the plate.

Energy needed to move a charged particle across a potential difference = charge × potential difference

$$\left[ \text{Energy} = QV \right]$$

but V grows as more charge builds up!

To get the total energy needed to charge a capacitor, we can sum the energy needed to add tiny charges $\delta q$:

$$\delta U = V \delta q \qquad \therefore \int dU = \int V dq$$

$$U = \int_i^f V dq = \int_0^Q \frac{q}{C} dq$$

$$\boxed{\text{Energy needed to charge capacitor} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2}$$

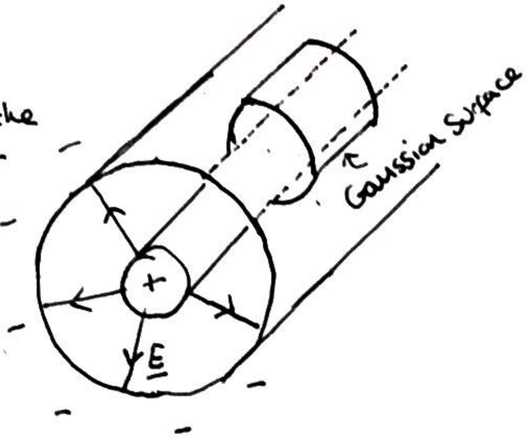
So where is this energy actually stored? In the electric field between the plates:

$$\text{Energy per unit volume} = \frac{1}{2}CV^2 / Ad \qquad \text{but } C = \frac{\varepsilon_0 A}{d} \text{ and } V = Ed$$

$$\boxed{\text{Energy per unit volume} = \frac{1}{2}\varepsilon_0 E^2}$$

# Cylindrical Capacitors

let the radius of the inner cylinder be $R_1$ and the outer cylinder be $R_2$




In order to find $\underline{E}$, the electric field, we have to use Gauss' Law. We choose the gaussian surface to be a cylinder around the inner cable.

To find $\underline{E}$: $\int \underline{E} \cdot d\underline{A} = \frac{q}{\varepsilon_0}$ so $|\underline{E}|\,|2\pi r l| = \frac{q}{\varepsilon_0}$

$$|\underline{E}| = \frac{2\pi R_1 l \sigma}{\varepsilon_0} \times \frac{1}{2\pi r l} = \frac{R_1 \sigma}{r \varepsilon_0}$$

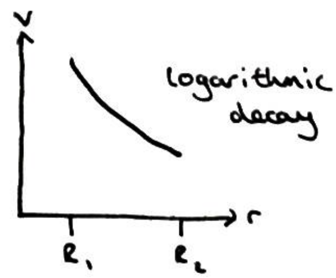
$$\therefore \underline{E} = \frac{R_1 \sigma}{r \varepsilon_0} \hat{\underline{r}}$$

To find Potential Difference V: $\Delta \phi = -\int_{R_1}^{R_2} \underline{E} \cdot d\underline{r}$ This is the radial path from $R_1$ to $R_2$

$$= -\int_{R_1}^{R_2} \frac{R_1 \sigma}{\varepsilon_0} \frac{dr}{r}$$

$$= -\frac{R_1 \sigma}{\varepsilon_0} \ln \frac{R_2}{R_1}$$

This would look like:




To find capacitance C: $C = Q/V$

$$C = \frac{\sigma 2\pi R_1 L_{total}}{R_1 \sigma / \varepsilon_0 \ln \frac{R_2}{R_1}} = \frac{2\pi \varepsilon_0 L_{total}}{\ln R_2/R_1}$$

This cable is an example of a coaxial cable.

where $L_{total}$ is the length of the cable.