Heat Capacity

Predictions of the equipartition theorem are verified experimentally by measuring the heat capacity of a body. Heat capacity is a measure of the change of internal energy with respect to temperature. These measurements are done by carefully measuring how much energy is fed into a system while measuring the change in energy of the system. For ideal monoatomic gases, we find heat capacity to be:

$$\boxed{C_V = \frac{dU}{dT} = \frac{3}{2} N k_B}$$

where C_V is the heat capacity at constant volume.

Since C_V scales with N, it is an extensive quantity. Experimentally, it is easier to use specific heat capacity, which is defined as heat capacity per mole:

$$c = \frac{C_V}{n_m} = \frac{3}{2}\frac{N}{n_m} k_B = \frac{3}{2} N_A k_B \quad \text{since} \quad n_m = \frac{N}{N_A}$$

so we can say specific heat capacity is

$$\boxed{c = \frac{3}{2} N_A k_B = \frac{3}{2} R}$$

This is for monoatomic gases only and is a very surprising result!

This suggests that all monoatomic gases have the same temperature-independent specific heat capacity at constant volume! This has been verified to a high precision!

This should also extend with equipartition theorem so polyatomic gases have $c = \frac{f}{2} R$ but this doesn't always work because of the failures of equipartition theorem.

We can also define specific heat capacity as heat capacity per unit mass. This is very confusing since this is a different value to heat capacity per mole even though it's called the same thing! So, to avoid confusion, we will use c^* for heat capacity per unit mass.

If $c = \frac{C_V}{n_M}$ $(J\,K^{-1}\,mol^{-1})$

$$\boxed{c^* = \frac{C_V}{m} \quad (J\,K^{-1}\,kg^{-1})}$$

we can relate the two using $c = c^* \frac{m}{n_M}$

c^* is a very useful quantity since it lets us derive the equation:

$$C_V = \frac{dU}{dT} \quad \text{so:} \quad dU = C_V dt \Rightarrow \Delta U = C_V \Delta T$$

but $C_V = mc^*$ so $\boxed{\Delta U = mc^* \Delta T}$

similarly: $\boxed{\Delta U = n_M c \Delta T}$

example: what is the final temperature if a 2kg piece of lead at 200°C is inserted in a container with 10kg of water at 50°C? $c^*_{lead} = 128\,J\,kg^{-1}\,K^{-1}$ $c^*_{water} = 4186\,J\,kg^{-1}\,K^{-1}$

As the temperature of lead falls, temperature of water will rise.

so $\Delta U_1 = -\Delta U_2$

$$m_1 c_1^* (T_1 - T_f) = -m_2 c_2^* (T_2 - T_f) \Rightarrow T_f = \frac{m_1 c_1^* T_1 + m_2 c_2^* T_2}{m_1 c_1^* + m_2 c_2^*} = 50.9°C$$