Charge Density and Total Charge

Electrons are so numerous in objects that it is difficult to consider individual electrons. Instead, we treat the charge as continuous and not discrete. We therefore use the measurement Charge Density [$\lambda\, Cm^{-1}$, $\sigma\, Cm^{-2}$, $\rho\, Cm^{-3}$]

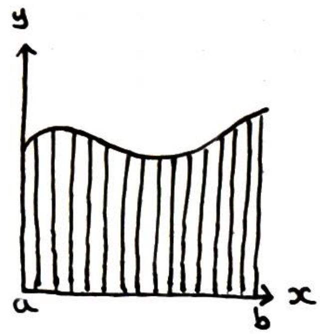
We can therefore work out total charge in a 3 dimensional object by multiplying charge density with volume.

$Q_{TOTAL} = \rho \times V$, assuming an even distribution of charge.

However, charge is often not evenly distributed in an object and therefore charge density varies through the object. In this case, it is necessary to divide the object into regions of constant charge density and then add up the charge in each region. For constantly varying charge density, the most accurate Total Charge is worked out by dividing the object into infinitesimally small regions.

A Note on Adding by Integration

The area under a curve can be calculated by adding up the areas of infinitely thin rectangles under the curve:



Area $= \sum_a^b f(x)\,\delta x$ Since the area can also be worked out normally through integration, we can say:

$$\boxed{\sum_a^b f(x)\,\delta x = \int_a^b f(x)\,dx}$$

An example with varying Charge Density

Consider a charged piece of string with 1 dimensional linear charge density $\lambda(x)\ \mathrm{Cm}^{-1}$ where $\lambda(x)$ is defined as:
$\lambda(x) = 4x^3$

It is necessary to split the string into infinitely many infinitesimally small regions of x and then sum them up. This can be written:

$$\text{Total Charge} = \lambda(0)\delta x + \lambda(\delta x)\delta x + \lambda(2\delta x)\delta x + \dots$$

$$= \sum_0^L \lambda(x)\delta x$$

We know that this is equivalent to:

$$\text{Total Charge} = \int_0^L \lambda(x)\,dx$$

$$= \int_0^L 4x^3\,dx$$

$$= \left[\frac{4x^4}{4}\right]_0^L$$

$$= \left[x^4\right]_0^L$$

$$= \underline{\underline{L^4}}$$