Accelerators

High energy physics is the study of the fundamental forces of nature and particles that can be found at very high energies.
We need such high energies because we want to probe very short distances, distances that are small even compared with a typical nuclear radius, eg. $x \ll 1fm = 10^{-15}m$

From Heisenberg's uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta p \gg \frac{\hbar}{1fm} = 197 \; MeV/c$$

This is very large uncertainty in momenta

so the momenta of the particles must be even larger!
In fact, weak interaction have a range 2 orders of magnitude below this so we need energies of at least 100 GeV!
To achieve such high energies, we need an accelerator!

With an accelerator, we can accelerate incident particles to very high energies and collide them to produce particles with considerably higher mass than the incident particles. This is called inelastic scattering. If the final particles are the same as the initial particles, it is elastic scattering.
Elastic means that none of the incoming energy is used up in the production of other particles.

In elastic scattering, we talk about a differential cross-section (with respect to solid angle) which is the number of particles per incident flux in a given element of solid angle.
For inelastic scattering we can talk about the total cross-section for a particular process.

An example is electron-positron scattering at CERN:

$$e^+ + e^- \rightarrow W^+ + W^-$$

in which electron and positron annihilate each other and produce two W bosons, each with mass $80.4\ GeV/c^2$ so we would need total centre of mass energies of atleast over 160 GeV for this to take place. The cross section $\sigma(e^+ + e^- \rightarrow W^+ + W^-)$ is the total number of events per unit incident flux (i.e the number of W boson pairs produced divided by the number of particle scatterings per unit area)

Fixed Target Experiments vs Colliding Beams

The total energy of a projectile particle plus the target particle depends on the reference frame. For the production of high mass particles we care about the centre of mass frame.

Let's suppose the projectile and target particle are the same. So in the centre of mass frame, both particles have the same energy E_{CM}.

Let's construct a quantity: $s = \left(\sum_{i=1,2} E_i\right)^2 - \left(\sum_{i=1,2} p_i\right)^2 c^2$

In the centre of mass frame $p_1 = -p_2$ so the second term vanishes:

$s = 4E_{CM}^2$

The quantity s is a Lorentz invariant quantity.

In the frame in which the target particle is at rest, its energy is mc^2 and momentum is 0. The projectile particle has energy E_{lab} and momentum P_{lab}:

$$s = (E_{lab} + mc^2)^2 - P_{lab}^2 c^2 = E_{lab}^2 + m^2c^4 + 2mc^2 E_{lab} - P_{lab}^2 c^2$$
$$= 2m^2c^4 + 2mc^2 E_{lab}$$

if we equate the two s terms (since s is Lorentz invariant) and square root, we obtain an expression for $2E_{CM}$

$$\sqrt{s} = 2E_{CM} = \sqrt{2M^2c^4 + 2Mc^2 E_{lab}}$$

For non relativistic particles with Kinetic Energy $T \ll Mc^2$, $E_{lab} = Mc^2 + T$ so:

$$\sqrt{s} = 2E_{CM} = \sqrt{2M^2c^4 + 2Mc^2(Mc^2 + T)}$$

$$= \sqrt{2M^2c^4 + 2M^2c^4 + 2Mc^2T}$$

$$= \sqrt{4M^2c^4 + 2c^2T} = \sqrt{(2Mc^2 + T)^2}$$

$$\Rightarrow 2E_{CM} = 2Mc^2 + T$$

$$\Rightarrow E_{CM} = Mc^2 + \tfrac{1}{2}T \quad \text{as expected}$$

But for relativistic particle, centre of mass energy is considerably reduced. eg. for a proton with mass $1\,\mathrm{GeV}/c^2$, if we accelerate it to an energy of $100\,\mathrm{GeV}$, the total centre of mass energy is only $15\,\mathrm{GeV}$, so too small to produce a particle of mass $100\,\mathrm{GeV}/c^2$.

So what do we do?
We use colliding beams! Both the initial particles are accelerated and then stored in storage rings in which the particles move in opposite directions around the ring, with their high energies maintained due to a magnetic field.

At various points around the ring, the beams intersect and scattering happens. In this way, the lab frame <u>is</u> the centre of mass frame and the full energy delivered by the accelerator can be used to produce high-mass particles.

Luminosity ~

We define Luminosity $\mathcal{L}$ as the number of particle collisions per unit area per second. The number of events of a particular type which occur per second is the cross-section multiplied by the luminosity.

For example, let's recall the $e^+ + e^- \to W^+ + W^-$ collission.
$\sigma_{(e^+ + e^- \to W^+ + W^-)} = 15\,pb$ (where p is pico) and the luminosity is $10^{32}\ cm^{-2}s^{-1}$

so number of W boson pairs produced per second is:

$$\frac{dN_{W^+W^-}}{dt} = (15 \times 10^{-12} \times 10^{-28}) \times (10^{32} \times 10^{4})$$
$$= \underline{1.5 \times 10^{-3}}$$

luminosity converted to $m^{-2}s^{-1}$

this is σ converted to m^2

So if reaction rate is $R = \frac{dN}{dt}$, then $\underline{\underline{R = \sigma \times \mathcal{L}}}$

If we integrate luminosity over time such that $L = \int \mathcal{L}\, dt$ then the total number of events $N = L \times \sigma$

i.e. $\underline{\underline{N = \int \mathcal{L} \sigma\, dt}}$

$\mathcal{L}$ depends on the number of "bunches" in each particle beam n, and the revolution frequency f, and N_1 N_2 the number of particles in each bunch, and the beam cross section A

so $$\mathcal{L} = \frac{n f N_1 N_2}{A}$$

The cross section σ is a probability of a particular event occuring, so the actual number of events observed is a random distribution with that probability. So if a cross section predicts N events in a time period, the error $\sqrt{N}$!

So to be able to measure the above cross section for the $e^+ + e^- \to W^+ + W^-$ to an accuracy of 1% at CERN, it was necessary to collect 10 000 W pairs which took 3 months!

We pay a price for colliding beam experiments in terms of luminosity. In fixed target experiments, we make an estimate of luminosity based on the fact that the incident particles are travelling at almost the speed of light. So the luminosity is the number of protons in a column of the target of unit area and length c.

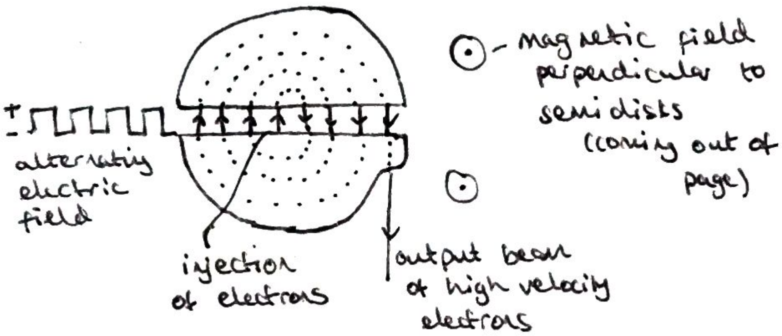
In colliding beams, it is necessary to focus incident beams as tightly as possible (using B fields) to maximise luminosity, but even with this, we cannot achieve similar luminosities to a fixed target experiment.

Types of Accelerators

Only stable charged particles can be accelerated and there are two general types of modern accelerators: Circular (cyclic) and Linear

Cyclotrons

The cyclotron ~~is~~ the prototype design for all circular accelerators is the cyclotron.



You should have seen cyclotrons before.

The device consists of two hollow metallic semidisks with a large magnetic field B applied normal to plane of the disks.
The particles move in a spiral and are accelerated in the gaps by an alternating electric field.

A charged particle with charge e moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ experiences a force: $\mathbf{F} = e\,\mathbf{v} \times \mathbf{B}$
which, when the magnetic field is perpendicular to plane of motion, always acts towards centre, giving rise to centripetal acceleration:

$$F = BeV = \frac{mv^2}{r}$$

The maximum energy that particles can acquire depends on the radius R for which the velocity has its max value V_{max}:

$$V_{max} = \frac{BeR}{m}$$

$$\Rightarrow \quad T_{max} = \frac{1}{2} m V_{max}^2 = \frac{B^2 e^2 R^2}{2m}$$

so we have the max energy possible from a cyclotron of radius R

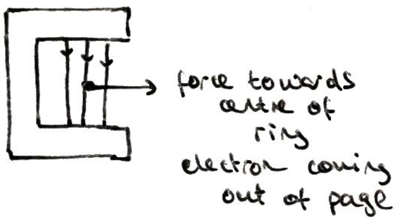
But this is only for non-relativistic particles! What about for energies where the particle becomes extremely relativistic?

Taking relativistic effects into account, the angular velocity ω is:

$$\omega = \sqrt{1 - v^2/c^2}\,\frac{Be}{M}$$

So now, as the particle accelerates, the frequency of the applied electric field must vary (such machines are called synchrocyclotrons) or the applied magnetic field must vary (or both) (such machines are called synchrotrons).

In a synchrotron, dipole magnets are used to keep particles in a circular orbit using $p = 0.3 \times B \times R$ (p in GeV/c, B in Tesla, R in meters), while quadrupole magnets are used to focus the beam.



Dipole Magnet


Quadrupole magnet

A limiting factor of synchrotron accelerators is Synchotron Radiation. A charged particle moving in a circular orbit is accelerating (even if the speed is constant) and therefore radiates.

The energy radiated per turn per particle is:

$$\Delta E = \frac{4\pi e^2 \beta^2 \gamma^4}{3R}$$

where e is the charge
β is $\frac{v}{c}$
γ is $1/\sqrt{1-\beta^2} = E/m$

$$\Rightarrow \Delta E \propto 1/m^4$$

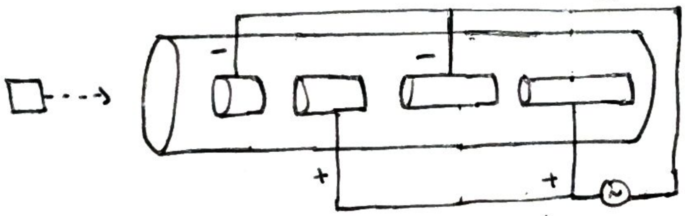
For relativistic electrons and protons of the same momentum the ratio of energy losses are very large for electrons versus protons:

$$\frac{\Delta E_e}{\Delta E_p} = \left(\frac{M_p}{M_e}\right)^4 \approx 10^{13}$$

Linear Accelerators (Linacs)

The energy loss due to synchrotron radiation can be avoided by using a linear accelerator.
Here, the particles are accelerated by means of an applied electric field along a long tube.

Proton linacs use a succession of drift tubes of increasing length to compensate for increasing velocity.



Particles always travel in a vacuum. There is no field inside the drift tubes. The external field between ends of tubes changes sign so the proton always sees -ve in front and +ve behind, causing acceleration. Proton linacs of 10 - 70m give energies of 30 - 200 MeV, and are usually used as injectors for higher energy machines.

Main Recent and Present Particle Accelerators (2020)

FermiLab: Tevatron is a synchrotron in which very high magnetic fields are achieved using superconducting electromagnets which are capable of maintaining large currents, thereby producing large B fields

CERN: LHC uses a specially designed magnetic field configuration, so two beams of protons moving in opposite direction around the same ring is possible.

DESY: Runs the HERA accelerator which accelerates proton to an energy of 820 GeV and electrons to 27 GeV. Only accelerator in which the initial particles are not the same or particle antiparticle pairs