Adversarial Robustness

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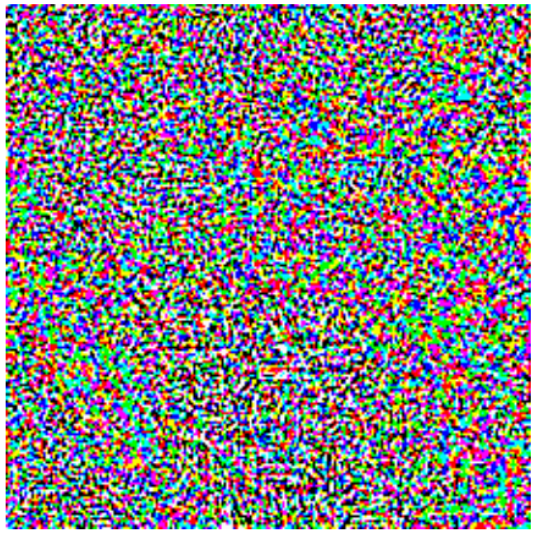
An Adversarial Example

$+ .007 \times$



$=$

"panda"
57.7% confidence

"gibbon"
99.3% confidence

? ? ?  ? ? ?

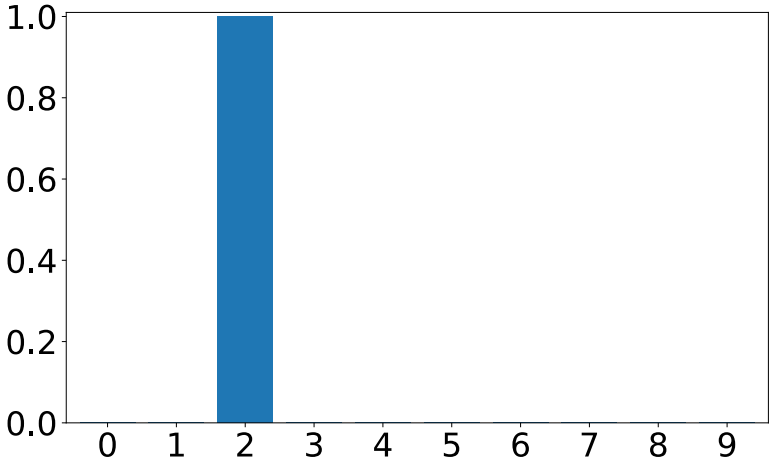
Revisit CNN for MNIST Classification

Neural Network for MNIST

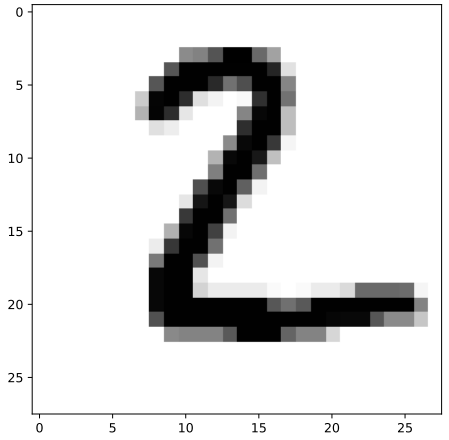
- Neural network: $\mathbf{p} = \mathbf{f}(\mathbf{x}; \mathbf{W})$

Trainable parameters of the neural network.

- The prediction --- a 10-dim vector.
- p_j indicates how likely $\mathbf{x}$ is the digit j.



28×28 input image.



CNN for MNIST

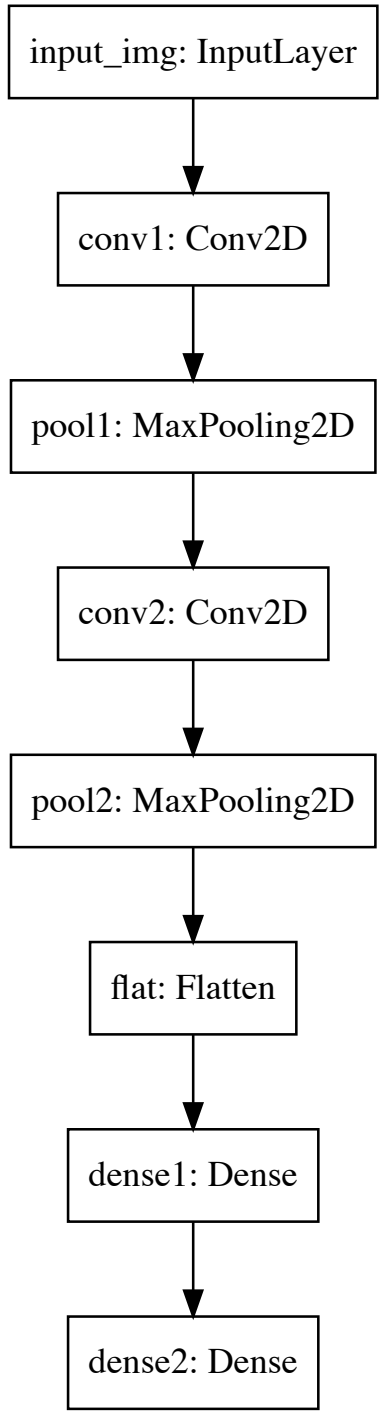
```
from keras.layers import Input, Conv2D, MaxPooling2D, Flatten, Dense
from keras import models



# input
input_img = Input(shape=(28, 28, 1), name='input_img')

# layers
conv1 = Conv2D(10, (5, 5), activation='relu', name='conv1')(input_img)
pool1 = MaxPooling2D((2, 2), name='pool1')(conv1)
conv2 = Conv2D(20, (5, 5), activation='relu', name='conv2')(pool1)
pool2 = MaxPooling2D((2, 2), name='pool2')(conv2)
flat = Flatten(name='flat')(pool2)
dense1 = Dense(100, activation='relu', name='dense1')(flat)
pred = Dense(10, activation='softmax', name='dense2')(dense1)

# model
model = models.Model(inputs=input_img, outputs=pred)
```



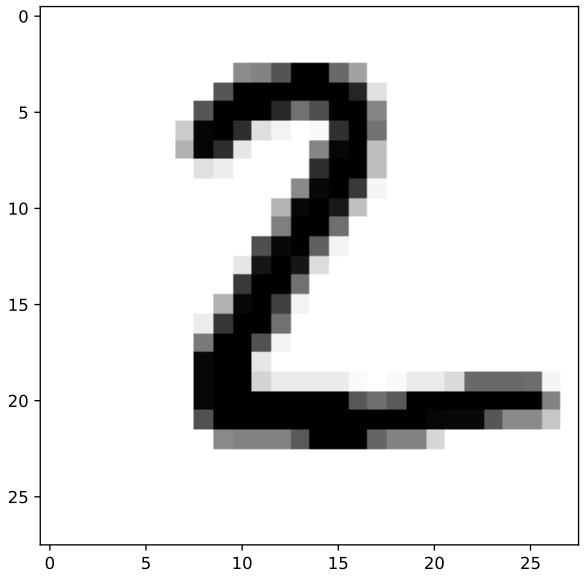
CNN for MNIST

```
model.compile(optimizer='RMSprop',
              loss='categorical_crossentropy',
              metrics=['accuracy'])
```

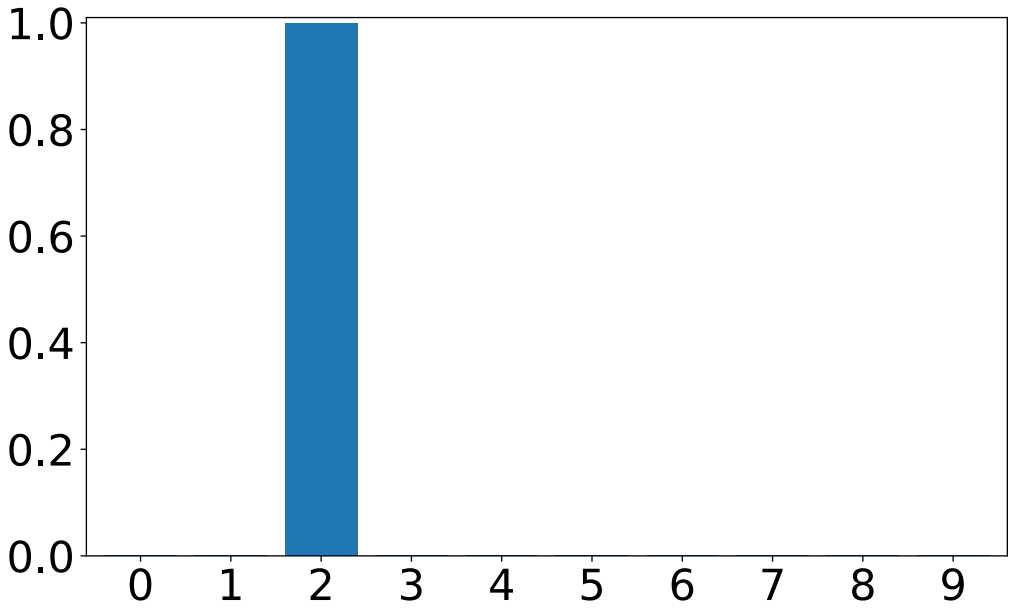
```
history = model.fit(
    x_train,
    y_train_vec,
    shuffle=True,
    epochs=10,
    batch_size=128,
    validation_data=(x_test, y_test_vec)
)
```

CNN for MNIST

- Training accuracy = 99.5%
- Validation accuracy = 99.1%



28×28 input image

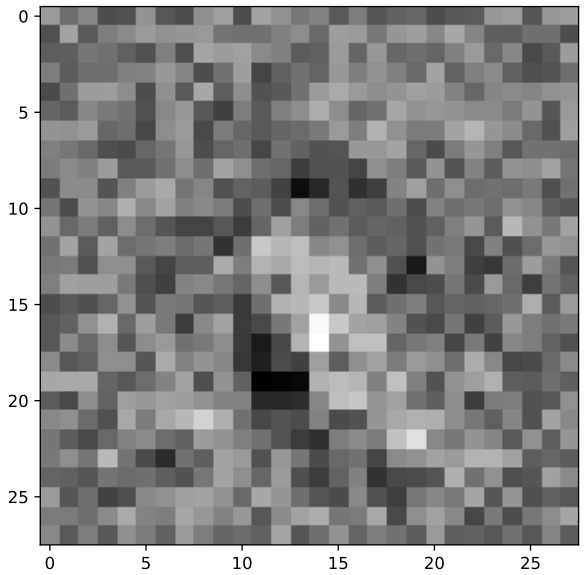


10-dim prediction vector

Optimization w.r.t. X

Fake Image

- Our trained CNN thinks the 28×28 input image is digit "**0**".

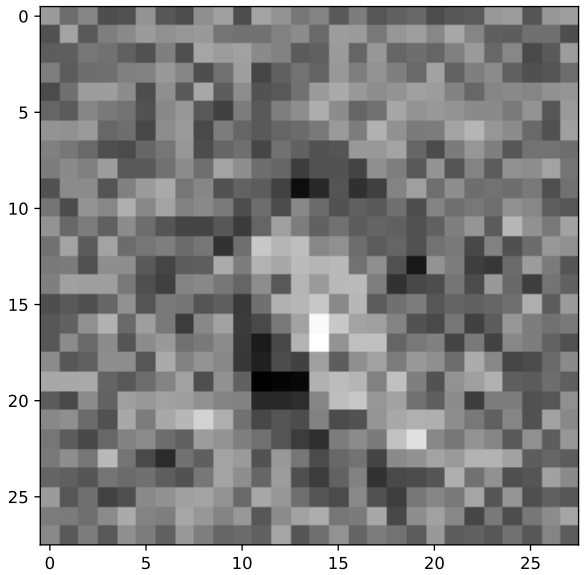


28×28 input image

10-dim prediction vector

Fake Image

Question: How is the fake image generated?



28×28 input image

10-dim prediction vector

Generate Fake Image

Question: How is the fake image generated?

- Neural network: $\mathbf{p} = \mathbf{f}(\mathbf{x}; \mathbf{W})$.
- Previously, we fix $\mathbf{x}_1, \cdots, \mathbf{x}_n$ and solve

$$\mathbf{W}^\star = \underset{\mathbf{W}}{\operatorname{argmin}} \sum_{j=1}^{n} \operatorname{Dist}\left(\mathbf{y}_j, \mathbf{f}(\mathbf{x}_j; \mathbf{W})\right).$$

Generate Fake Image

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$$\mathbf{W}^\star = \underset{\mathbf{W}}{\operatorname{argmin}} \sum_{j=1}^{n} \operatorname{Dist}\big(\mathbf{y}_j, \ \mathbf{f}(\mathbf{x}_j; \mathbf{W})\big).$$

- To generate a fake image, we do the following:
  1. Set a fake target, e.g., $\tilde{\boldsymbol{y}} = [1, 0, 0, \cdots, 0]$.
  2. Fix the network parameters to $\mathbf{W}^\star$.
  3. Generate a fake image $\tilde{\mathbf{x}}$ by

$$\tilde{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \ \operatorname{Dist}(\tilde{\mathbf{y}}, \ \mathbf{f}(\mathbf{x}; \mathbf{W}^\star)).$$

Generate Fake Image

Step 1: Set a fake target.

```
import numpy as np
from keras import backend as K
from keras.layers import Input

j = 0 # the fake lable
y_tilde = np.zeros((1, 10))
y_tilde[0, j] = 1
print(y_tilde)

fake_target = Input(tensor=K.constant(y_tilde))
```

```
[[1. 0. 0. 0. 0. 0. 0. 0. 0. 0.]]
```

Generate Fake Image

Step 2: Fix the trained network's parameters, **W**.

```
from keras.layers import Input

model.trainable = False
input_img = Input(shape=(28, 28, 1))
pred = model(input_img)
```

Generate Fake Image

Step 3: Generate a fake image.

- Define the loss function and evaluate the gradient.
  - **`pred`**: the output of the pre-trained model.
  - **`fake_target`**: $[1, 0, 0, \cdots, 0]$.
  - Loss function: `CrossEntropy(pred, fake_target)`.

```
import keras
from keras import backend as K

loss = keras.metrics.binary_crossentropy(pred, fake_target)
grads = K.gradients(loss, [input_img])[0]
fetch_loss_and_grads = K.function([input_img], [loss, grads])
```

Generate Fake Image

Step 3: Generate a fake image.

initialization

gradient descent

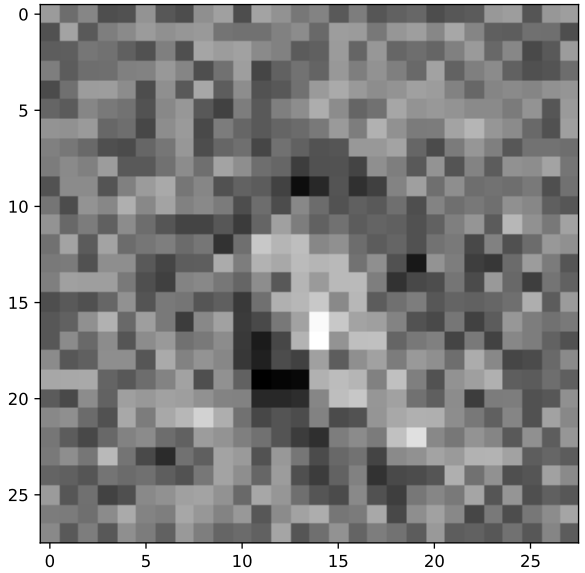
```
import numpy as np

fake_img = np.random.rand(1, 28, 28, 1)
learn_rate = 0.5

for i in range(10):
    l, g = fetch_loss_and_grads([fake_img])
    print('iter ' + str(i) + ': loss = ' + str(l))
    fake_img -= learn_rate * g
```

```
iter 0: loss = [3.039003]
iter 1: loss = [2.6458066]
iter 2: loss = [1.0068675]
iter 3: loss = [0.22192626]
iter 4: loss = [0.01752027]
iter 5: loss = [0.01353873]
iter 6: loss = [0.01130872]
iter 7: loss = [0.00976203]
iter 8: loss = [0.00869376]
iter 9: loss = [0.00787604]
```

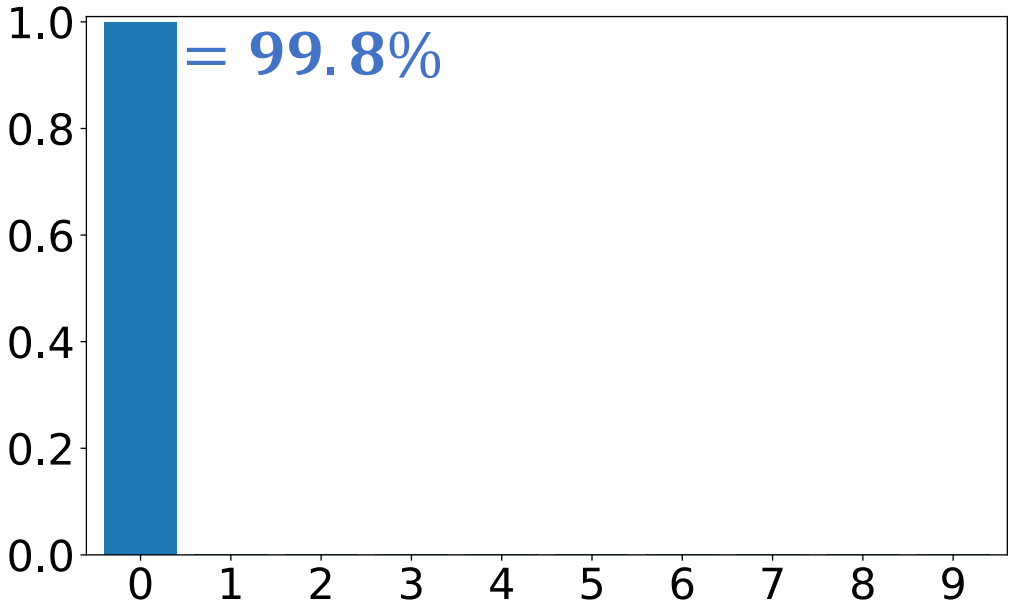
Generate Fake Image

```
print(model.predict(fake_img)[0])
```

```
[9.9776781e-01 4.1152799e-04 4.0594186e-04 8.5301363e-05 6.2220497e-05
 7.1304827e-04 2.1397672e-04 1.0672671e-04 1.9847257e-04 3.4956032e-05]
```



28×28 fake image



10-dim prediction vector

Untargeted Attack

Read the blog for details:

https://medium.com/onfido-tech/adversarial-attacks-and-defences-for-convolutional-neural-networks-66915ece52e7

Untargeted Attack

- $\mathbf{x}$: a real image.
- $\mathbf{y}$: true label.
- Maximize $\mathrm{Dist}(\mathbf{y}, \mathbf{f}(\mathbf{x}; \mathbf{W}^{\star}))$ w.r.t. $\mathbf{x}$.
  - Make the prediction, $\mathbf{f}(\mathbf{x}; \mathbf{W}^{\star})$, far from $\mathbf{y}$.
  - ➔ Wrong prediction.

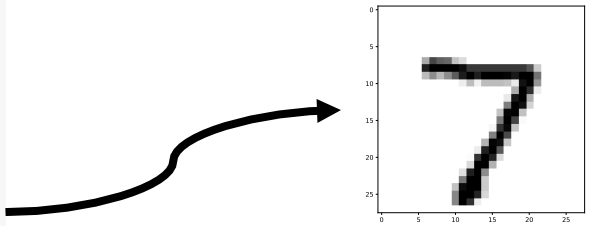
Untargeted Attack

- $\mathbf{x}$: a real image.
- $\mathbf{y}$: true label.
- Maximize $\text{Dist}(\mathbf{y}, \mathbf{f}(\mathbf{x}; \mathbf{W}^\star))$ w.r.t. $\mathbf{x}$.
  - Make the prediction, $\mathbf{f}(\mathbf{x}; \mathbf{W}^\star)$, far from $\mathbf{y}$.
  - ➔ Wrong prediction.
- Gradient ascent:
  - Repeat: $\mathbf{x} \leftarrow \mathbf{x} + \alpha \frac{\partial\, \text{dist}(\mathbf{y}, \mathbf{f}(\mathbf{x}; \mathbf{W}^\star))}{\partial \mathbf{x}}$.
  - Stop if the prediction, $\mathbf{f}(\mathbf{x}; \mathbf{W}^\star)$, is incorrect.

Untargeted Attack

Step 1: Get an image and its true target.

```
# get the i-th test sample
i = 0
digit = x_test[i].reshape((1, 28, 28, 1))
label = y_test[i]
print('The true label is ' + str(label))

# one-hot encode the label
y_true = np.zeros((1, 10))
y_true[0, label] = 1
print('One-hot encode: ' + str(y_true))
```



```
The true label is 7
One-hot encode: [[0. 0. 0. 0. 0. 0. 0. 1. 0. 0.]]
```

Untargeted Attack

Step 2: Fix the trained network's parameters.

```
from keras.layers import Input


input_img = Input(shape=(28, 28, 1))
true_target = Input(tensor=K.constant(y_true))
pred = model(input_img)
```

Untargeted Attack

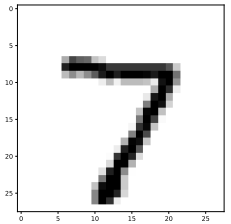
Step 3: Generate a fake image using gradient ascent.

```
import keras
from keras import backend as K

loss = keras.metrics.binary_crossentropy(pred, true_target)
grads = K.gradients(loss, [input_img])[0]
fetch_grads = K.function([input_img], [grads])
```

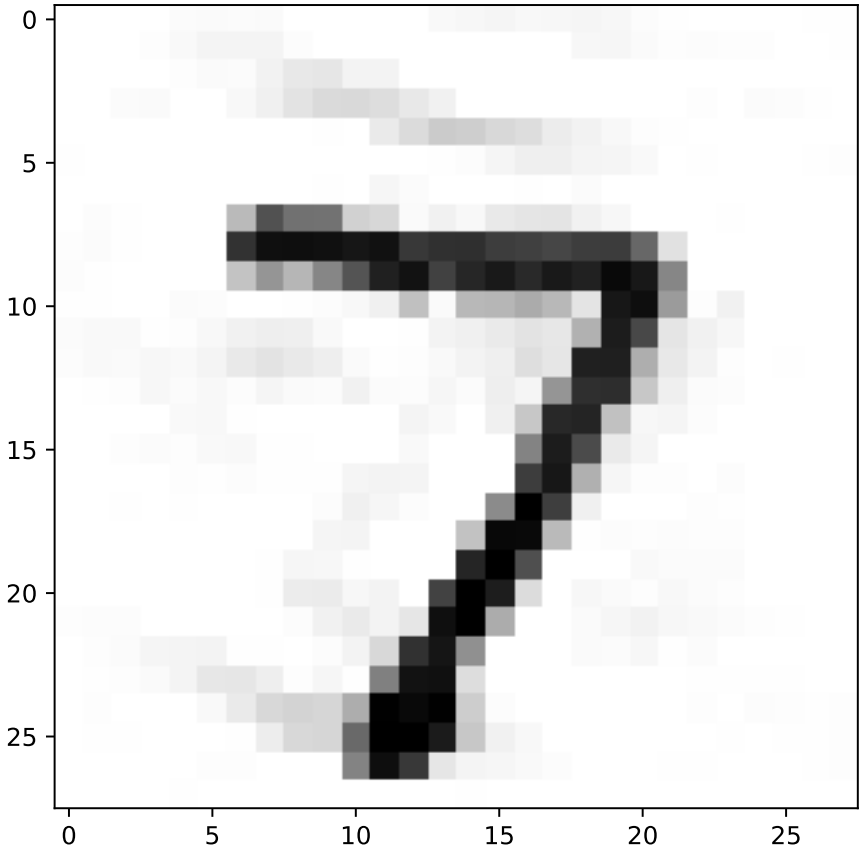
Untargeted Attack

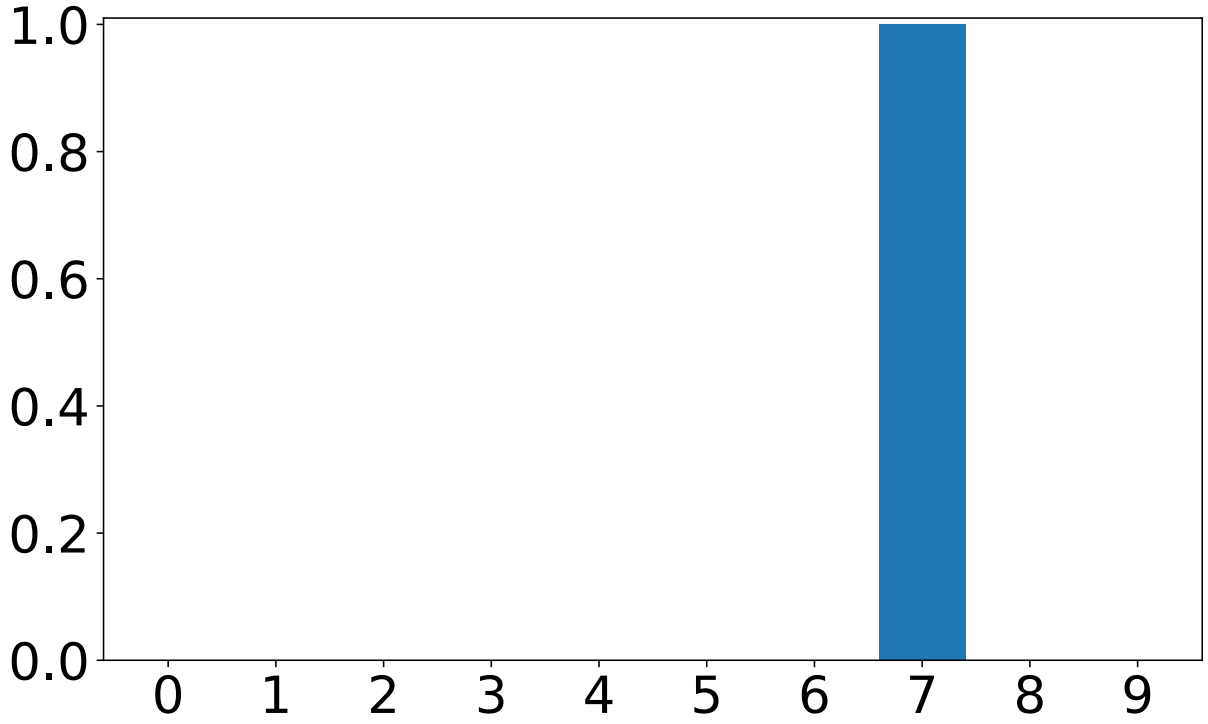
Step 3: Generate a fake image.

```
import numpy

fake_img = digit.copy() # initialize

for t in range(20):
    # gradient descent
    grad = fetch_grads([fake_img])[0]
    g = grad / numpy.max(numpy.abs(grad))
    fake_img += 0.05 * g
    # project to [0, 1]
    fake_img[fake_img < 0] = 0
    fake_img[fake_img > 1] = 1
```

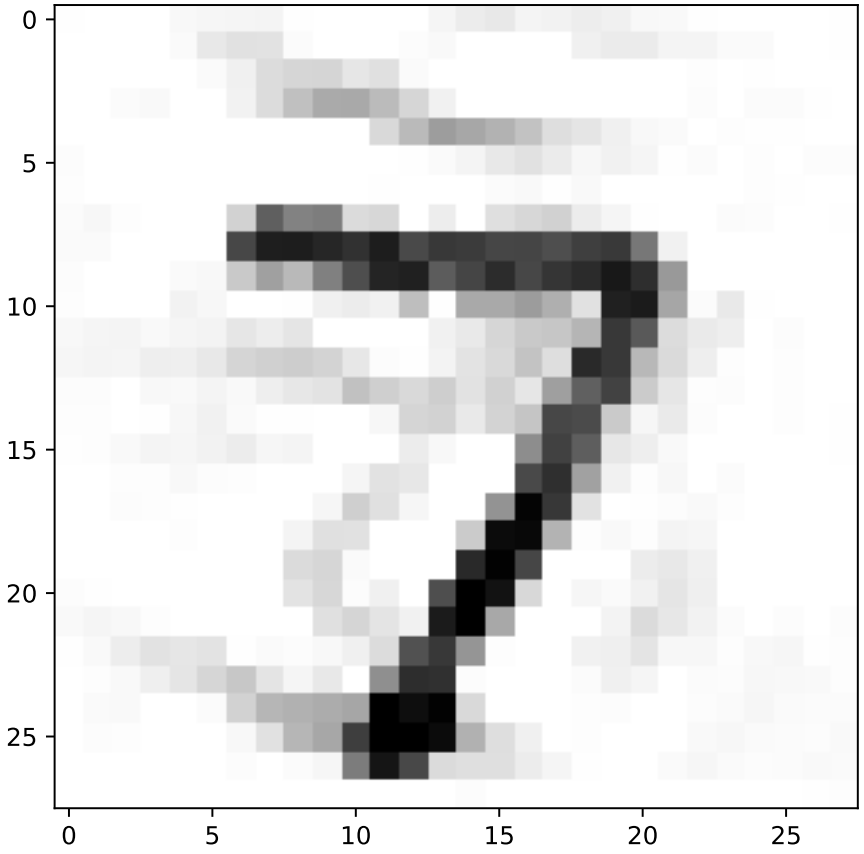


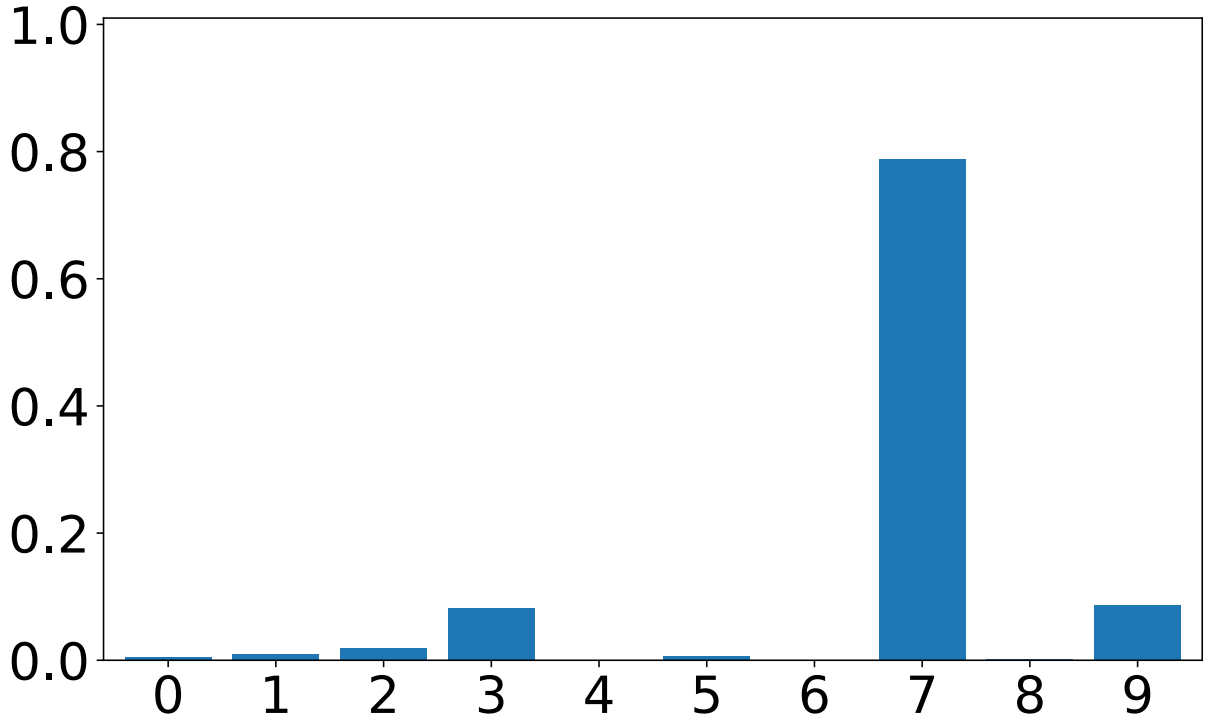
Results after 5 Steps



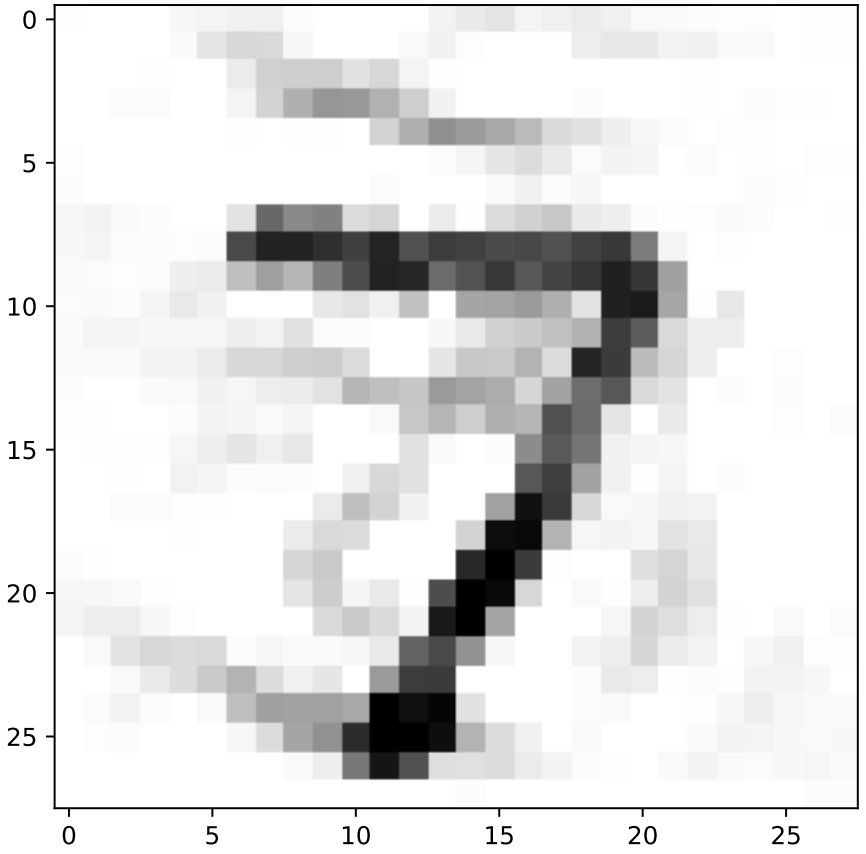


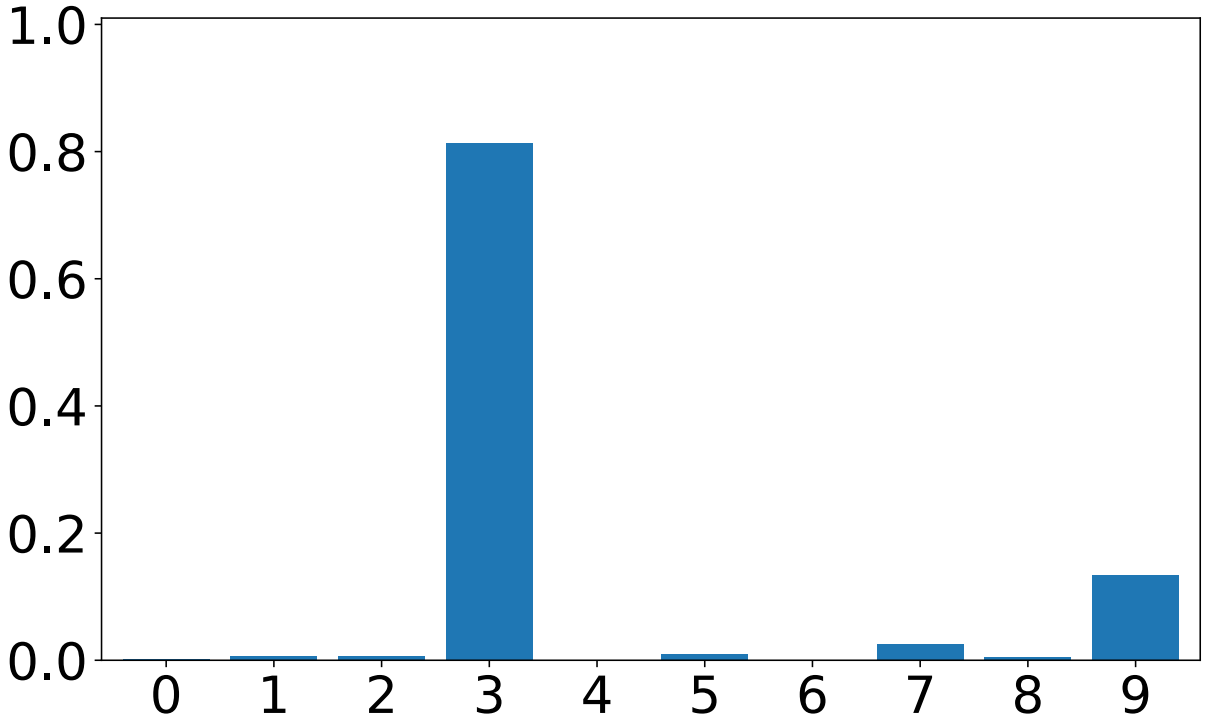
Results after 10 Steps



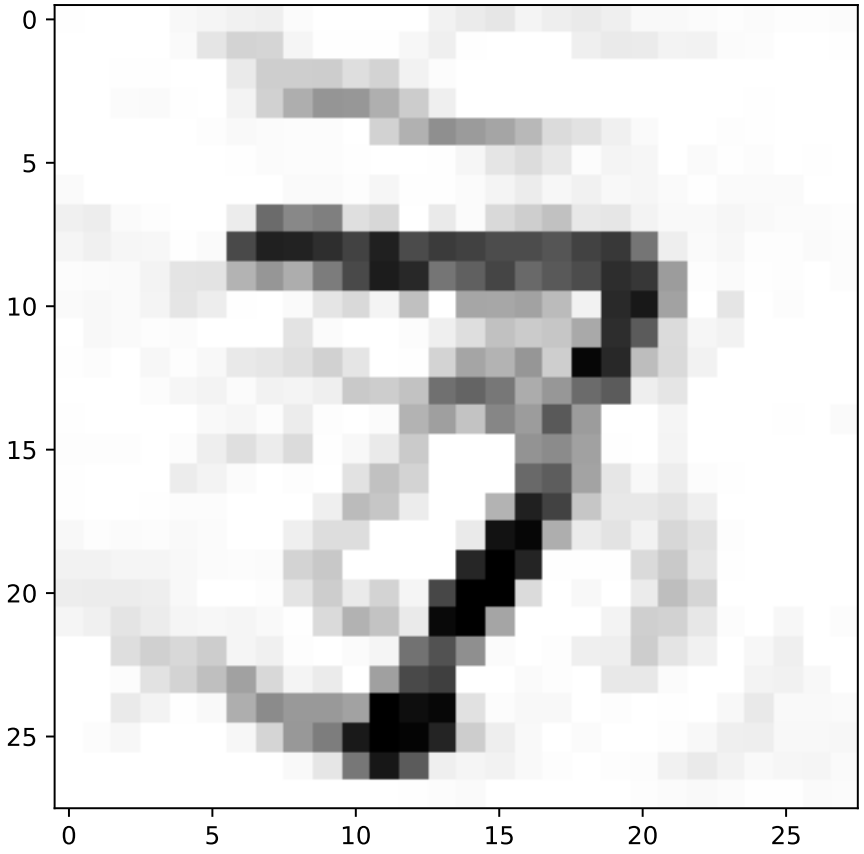


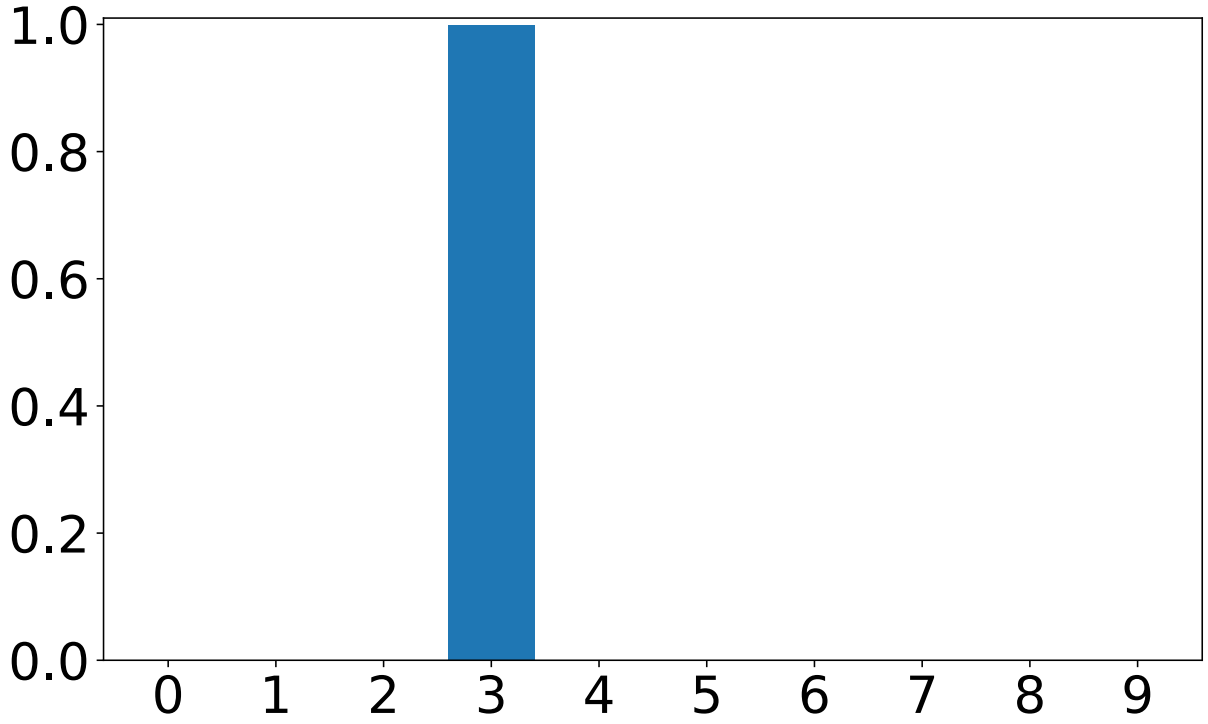
Results after 15 Steps





Results after 20 Steps

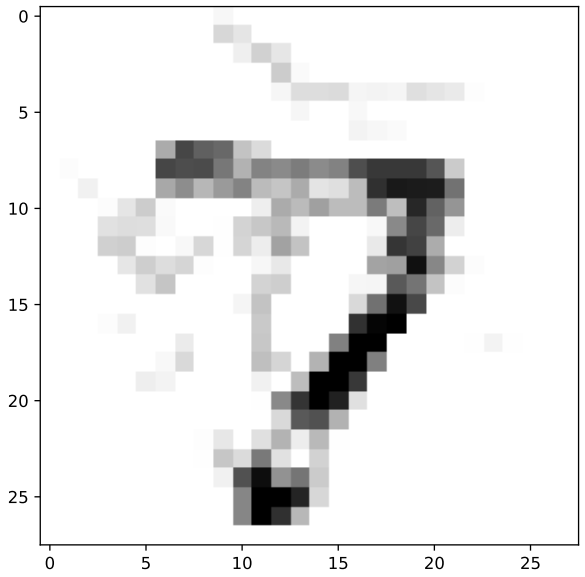




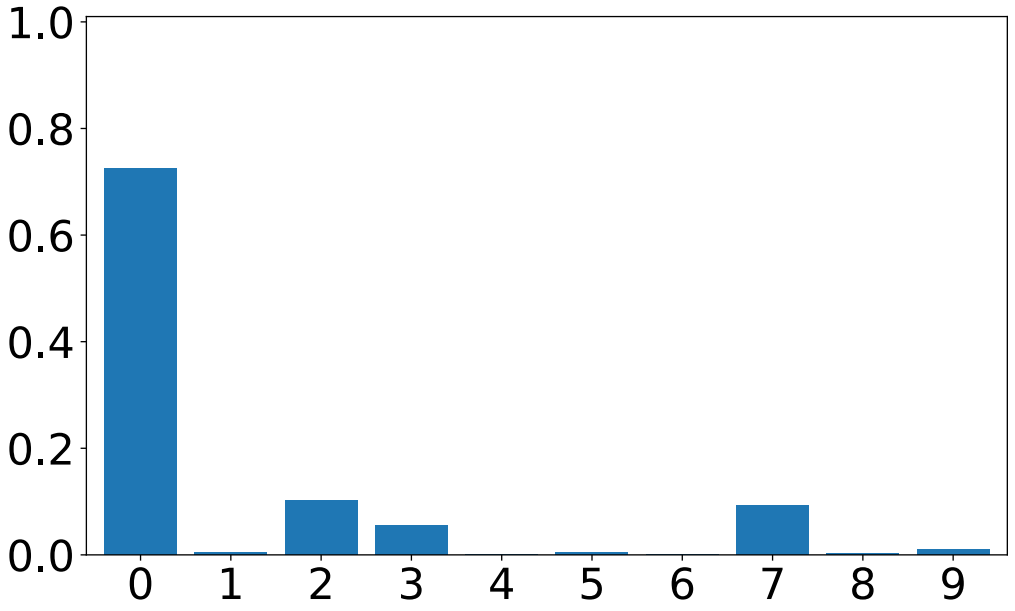
Targeted Attack

Targeted Adversarial Example

- Our trained CNN thinks the 28×28 input image is digit "9".



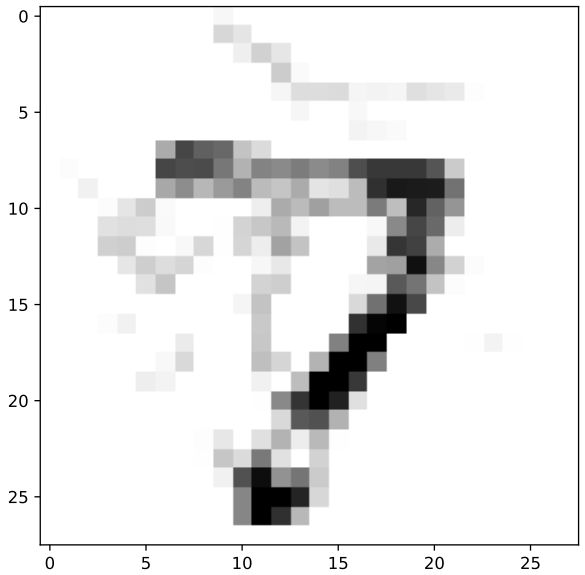
28×28 input image



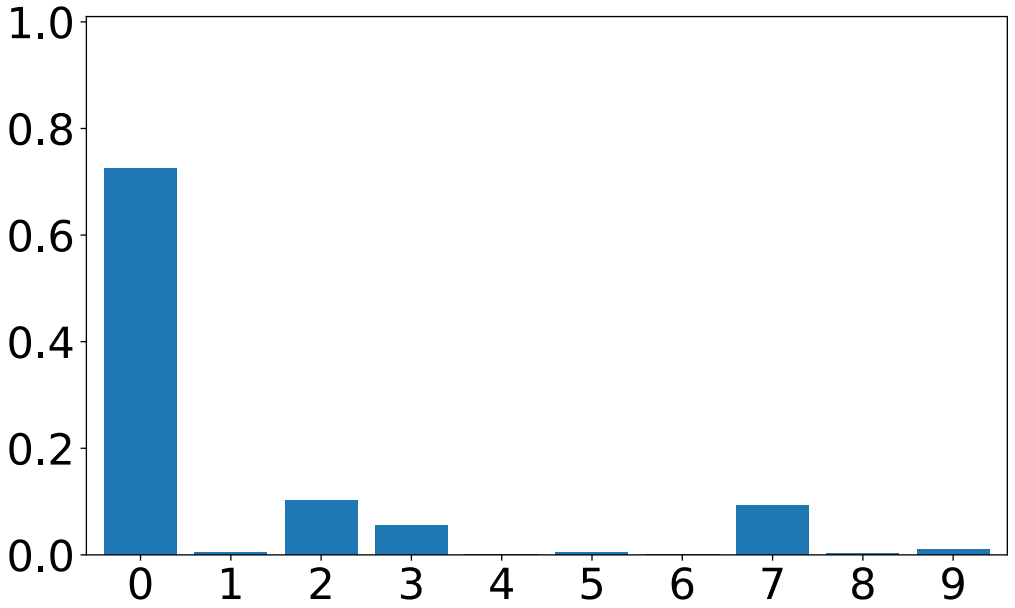
10-dim prediction vector

Targeted Adversarial Example

Question: How is the fake image generated?



28×28 input image



10-dim prediction vector

Targeted Attack

Question: How is the fake image generated?

1. Given a real image $\mathbf{x}^\star$ which we want to attack.
2. Set a fake target, e.g., $\tilde{\boldsymbol{y}} = [0, 0, 0, \cdots, 0, 1]$.
3. Fix the network parameters to $\mathbf{W}^\star$.
4. Set up the loss function

$$\text{Loss}(\tilde{\mathbf{x}}, \mathbf{x}^\star, \tilde{\mathbf{y}}) = \text{Dist}(\tilde{\mathbf{y}}, \mathbf{f}(\tilde{\mathbf{x}}; \mathbf{W}^\star)) + \lambda \left|\left|\tilde{\mathbf{x}} - \mathbf{x}^\star\right|\right|_2^2$$

make the neural net think the generated image $\tilde{\mathbf{x}}$ is digit "0".

the generated image $\tilde{\mathbf{x}}$ should look similar to the real one $\mathbf{x}^\star$.

Targeted Attack

Question: How is the fake image generated?

1. Given a real image $\mathbf{x}^\star$ which we want to attack.
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4. Set up the loss function

$$\text{Loss}(\tilde{\mathbf{x}}, \mathbf{x}^\star, \tilde{\mathbf{y}}) = \text{Dist}(\tilde{\mathbf{y}}, \mathbf{f}(\tilde{\mathbf{x}}; \mathbf{W}^\star)) + \lambda \left|\left|\tilde{\mathbf{x}} - \mathbf{x}^\star\right|\right|_2^2$$

make the neural net think the generated image $\tilde{\mathbf{x}}$ is digit "0".

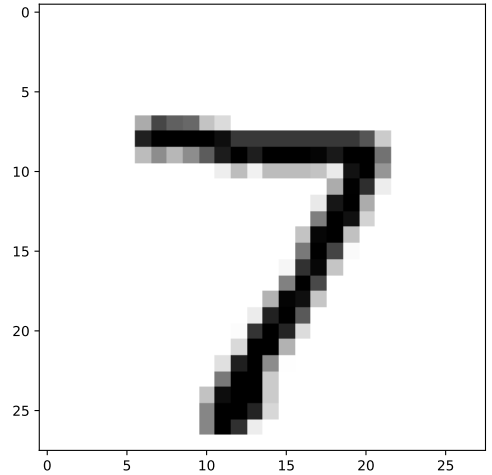
the generated image $\tilde{\mathbf{x}}$ should look similar to the real one $\mathbf{x}^\star$.

5. Generate a fake image $\tilde{\mathbf{x}}$ by $\tilde{\mathbf{x}} = \underset{\mathbf{x}}{\text{argmin}}\, \text{Loss}(\mathbf{x}, \mathbf{x}^\star, \tilde{\mathbf{y}})$.

Targeted Attack

Step 1: Given a real image which we want to attack.

```
from keras.layers import Input
from keras import backend as K

i = 0
digit = x_test[i].reshape((1, 28, 28, 1))
print('The true label is ' + str(y_test[i]))

true_img = Input(tensor=K.constant(digit))
```




```
The true label is 7
```

Targeted Attack

Step 2: Set a fake target to be digit "**9**".

```
import numpy as np
from keras.layers import Input
from keras import backend as K

j = 9 # the fake lable
y_tilde = np.zeros((1, 10))
y_tilde[0, j] = 1
print(y_tilde)

fake_target = Input(tensor=K.constant(y_tilde))
```

```
[[0. 0. 0. 0. 0. 0. 0. 0. 0. 1.]]
```

Targeted Attack

Step 3: Freeze the parameters. Define input image and prediction.

```
from keras.layers import Input

model.trainable = False
fake_img = Input(shape=(28, 28, 1))
pred = model(fake_img)
```

Targeted Attack

Step 4: Set the loss function.

- $\text{Loss}(\tilde{\mathbf{x}}, \mathbf{x}^\star, \tilde{\mathbf{y}}) = \text{CrossEntropy}\left(\tilde{\mathbf{y}}, \mathbf{f}\left(\tilde{\mathbf{x}}; \mathbf{W}^\star\right)\right) + \lambda\left|\left|\tilde{\mathbf{x}} - \mathbf{x}^\star\right|\right|_1$

or $\text{Loss}(\tilde{\mathbf{x}}, \mathbf{x}^\star, \tilde{\mathbf{y}}) = \text{CrossEntropy}\left(\tilde{\mathbf{y}}, \mathbf{f}\left(\tilde{\mathbf{x}}; \mathbf{W}^\star\right)\right) + \lambda\left|\left|\tilde{\mathbf{x}} - \mathbf{x}^\star\right|\right|_2^2$

make the neural net think the generated image $\tilde{\mathbf{x}}$ is digit "0".

the generated image $\tilde{\mathbf{x}}$ should look similar to the real one $\mathbf{x}^\star$.

Targeted Attack

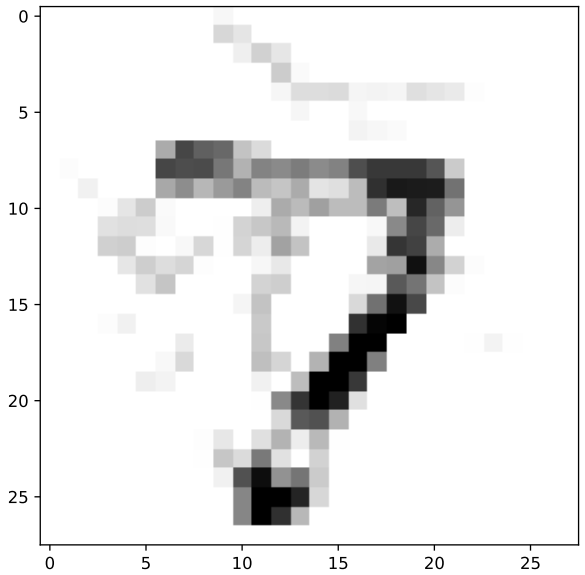
Step 4: Set the loss function.

- $\text{Loss}(\tilde{\mathbf{x}}, \mathbf{x}^{\star}, \tilde{\mathbf{y}}) = \text{CrossEntropy}\left(\tilde{\mathbf{y}}, \mathbf{f}\left(\tilde{\mathbf{x}};\ \mathbf{W}^{\star}\right)\right) + \lambda \left|\left|\tilde{\mathbf{x}} - \mathbf{x}^{\star}\right|\right|_1$

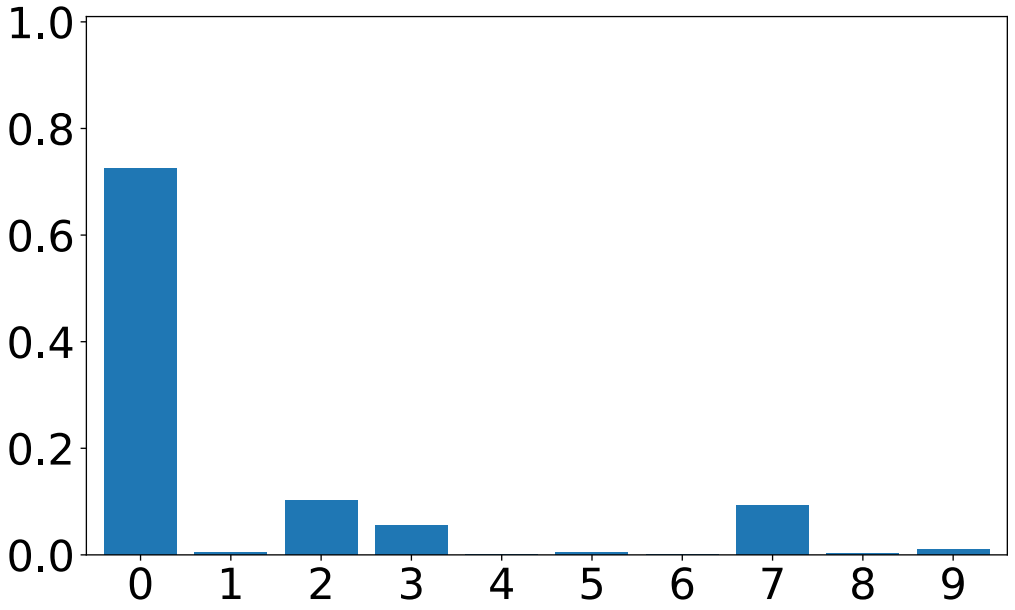
```
import keras
from keras import backend as K

param = 30

loss1 = keras.metrics.binary_crossentropy(pred, fake_target)
#loss2 = K.mean(K.square(true_img - fake_img))
loss2 = K.mean(K.abs(true_img - fake_img))

loss = loss1 + param * loss2

grads = K.gradients(loss, [fake_img])[0]
fetch_loss_and_grads = K.function([fake_img], [loss, grads])
```

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "0".



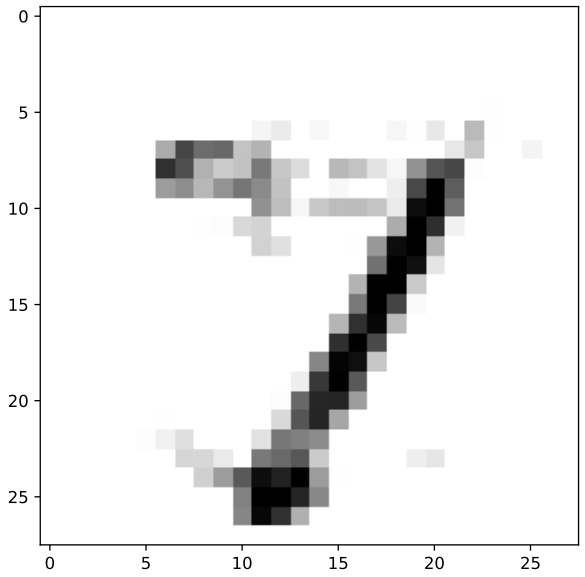
28×28 generated image



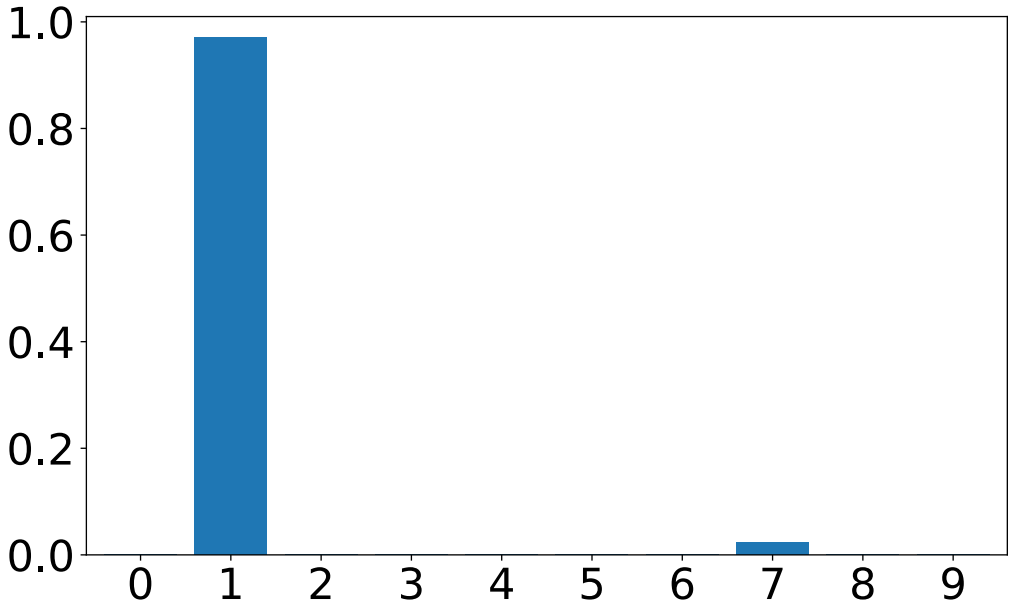
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "1".



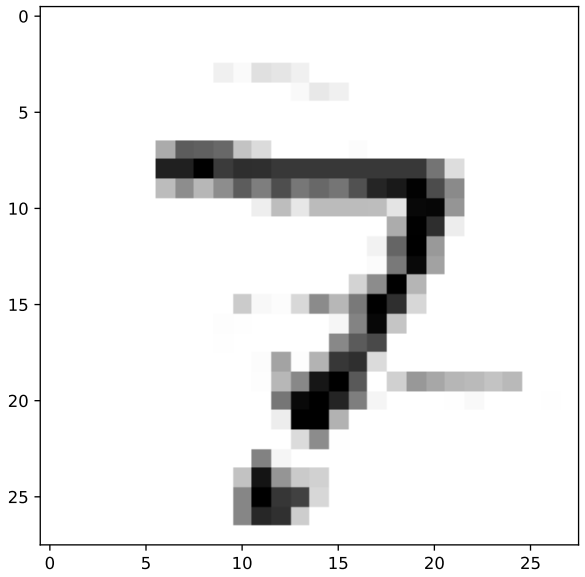
28×28 generated image



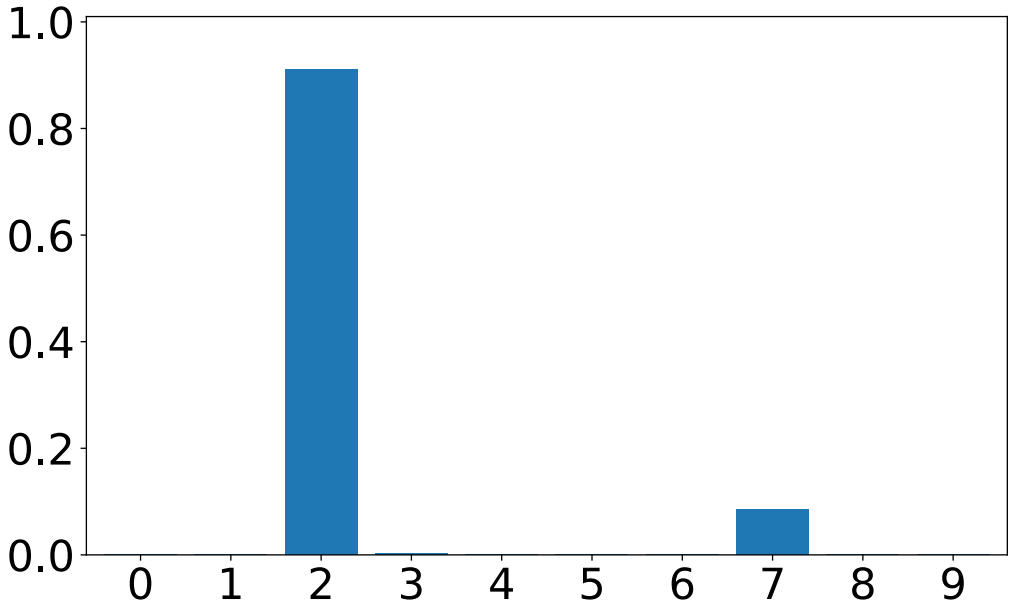
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "2".



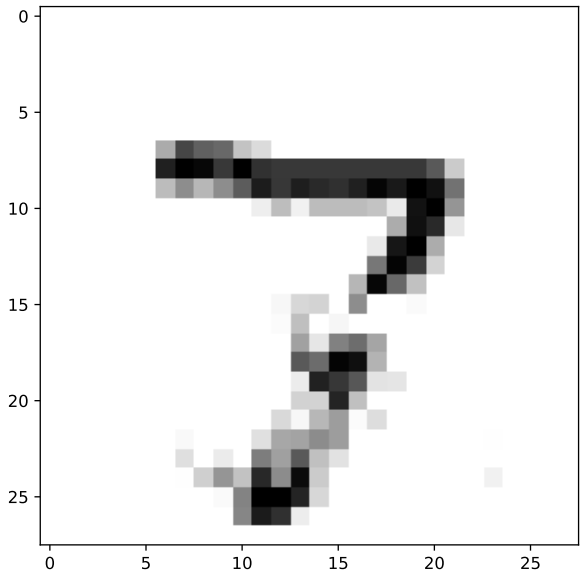
28×28 generated image



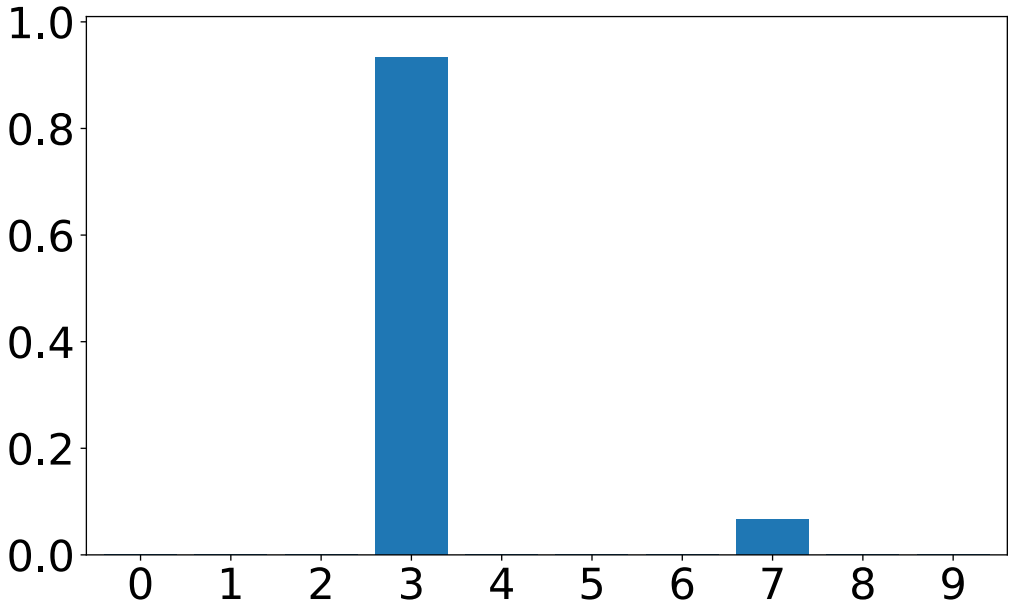
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit “3”.



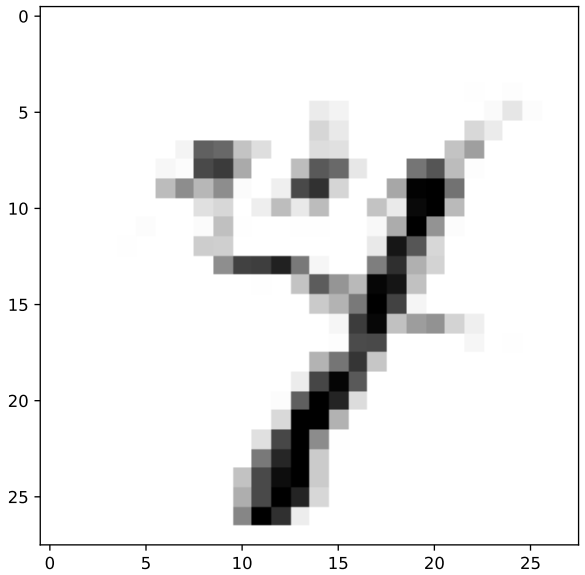
28×28 generated image



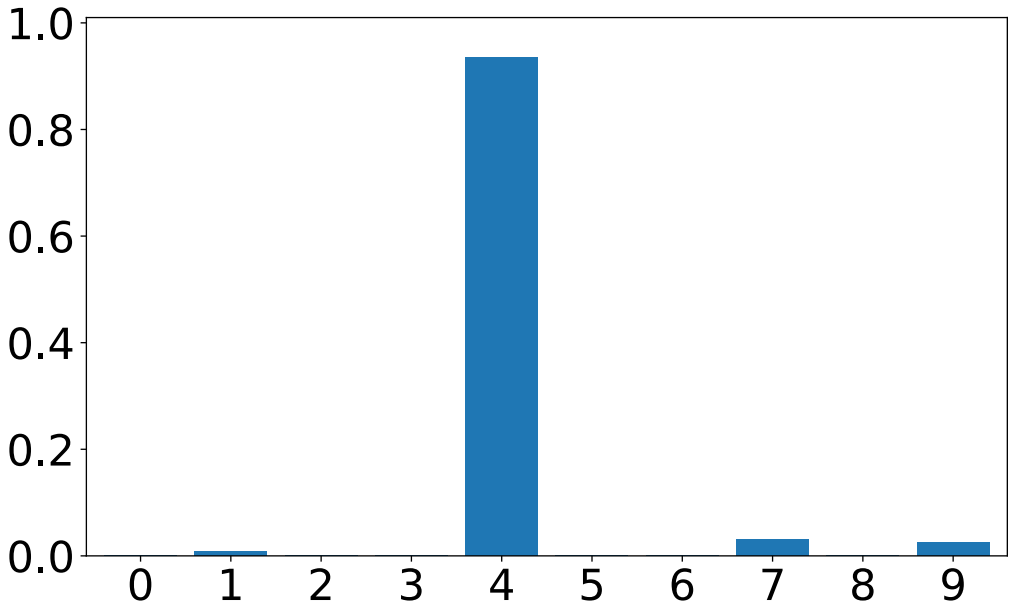
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "**4**".



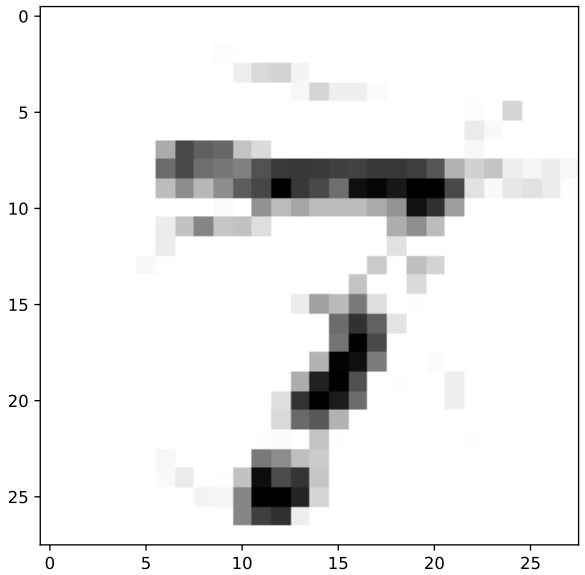
28×28 generated image



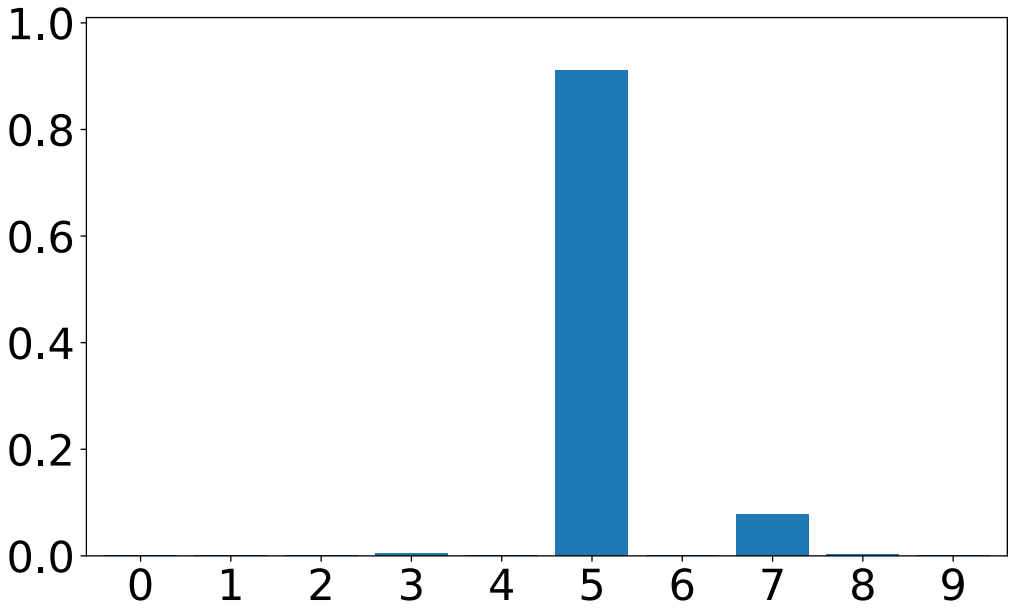
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "5".



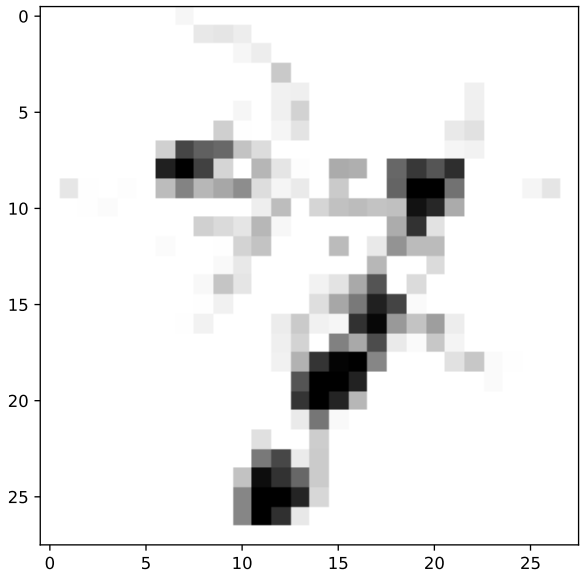
28×28 generated image



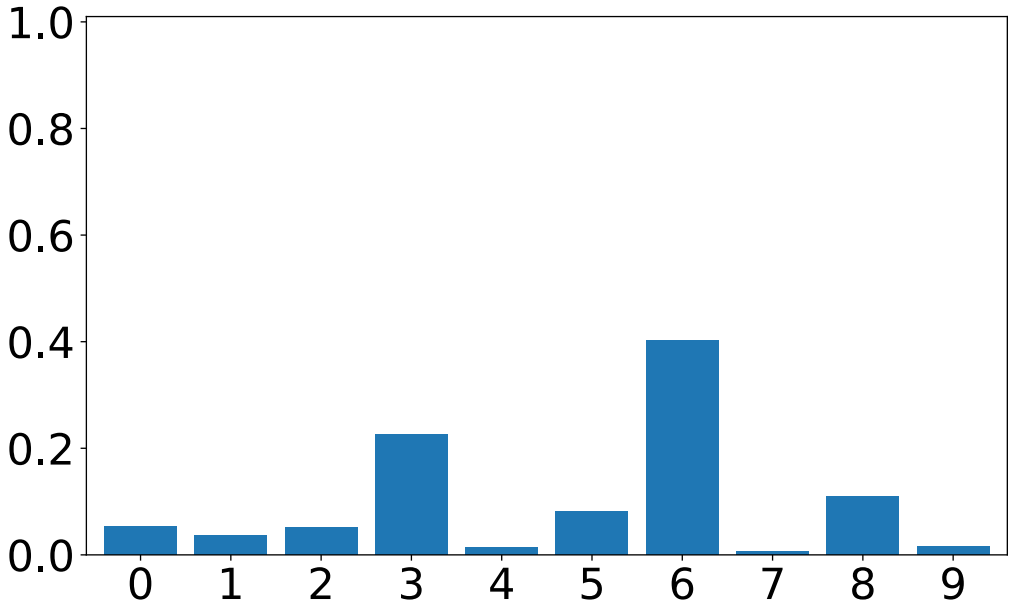
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "6".



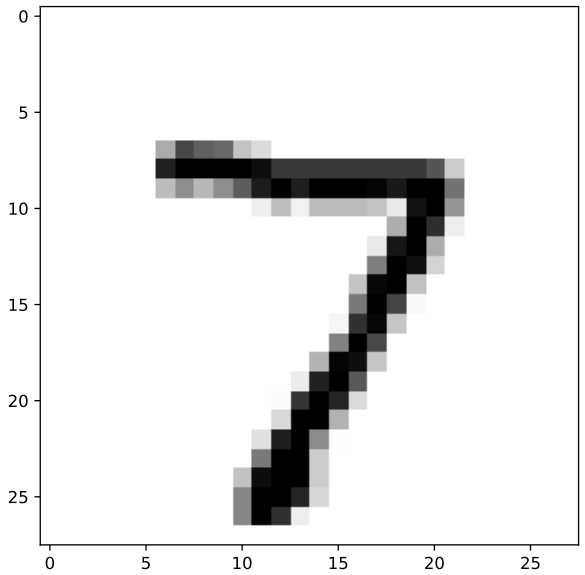
28×28 generated image



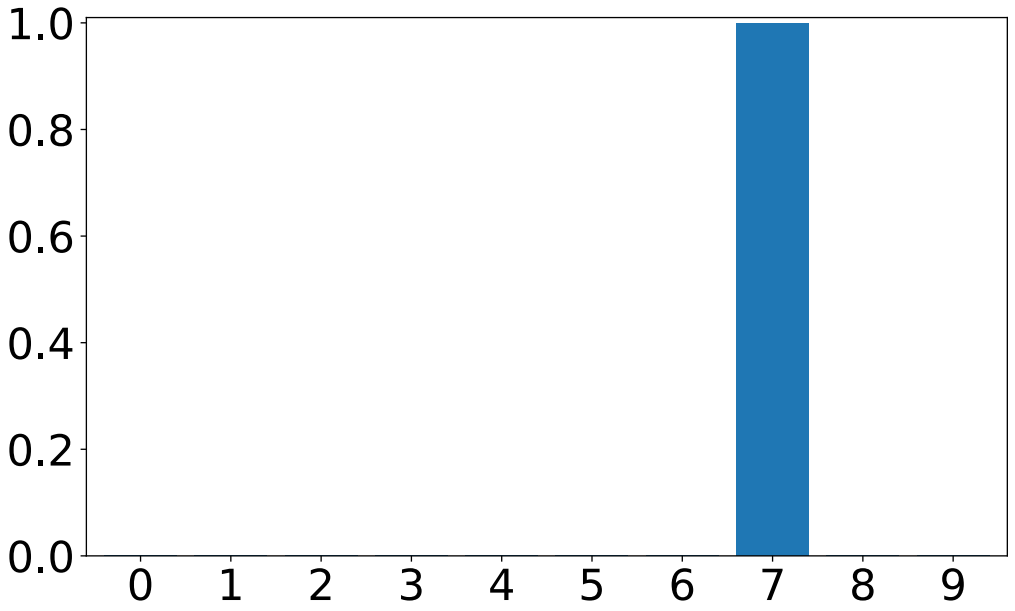
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit "7".



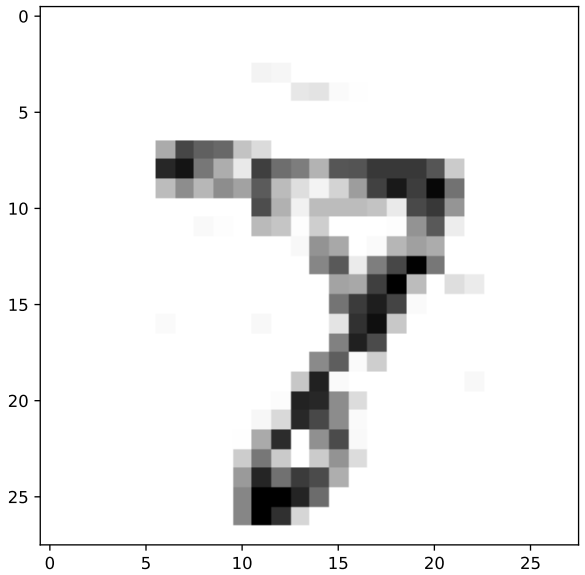
28×28 generated image



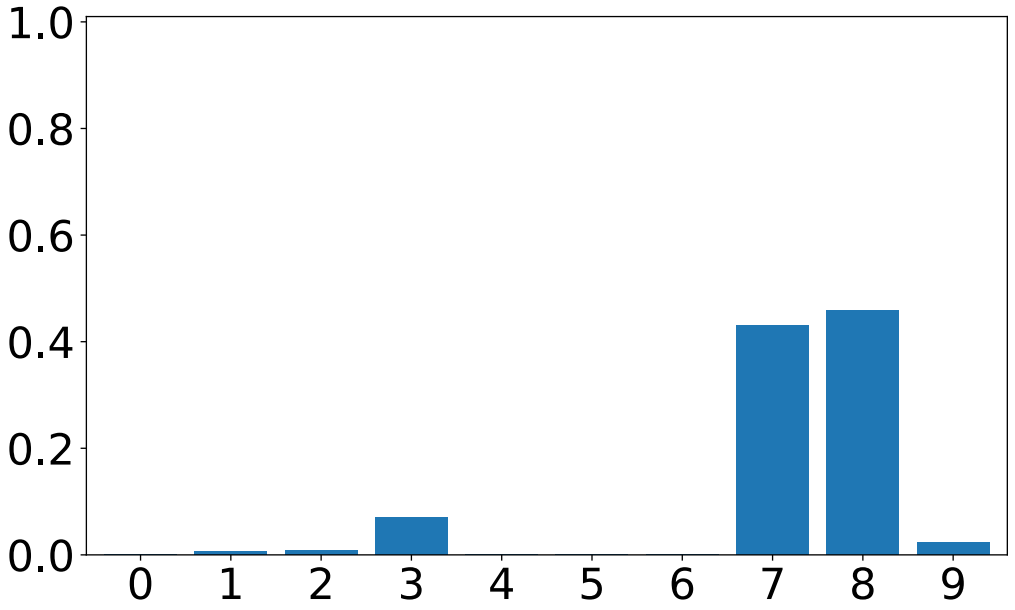
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit “8”.



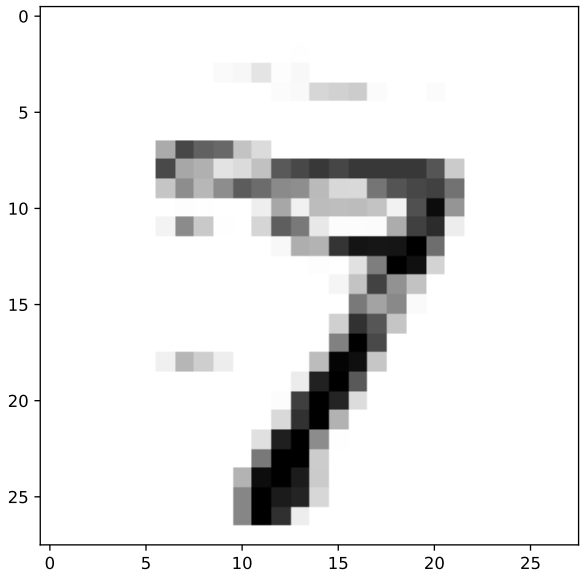
28×28 generated image



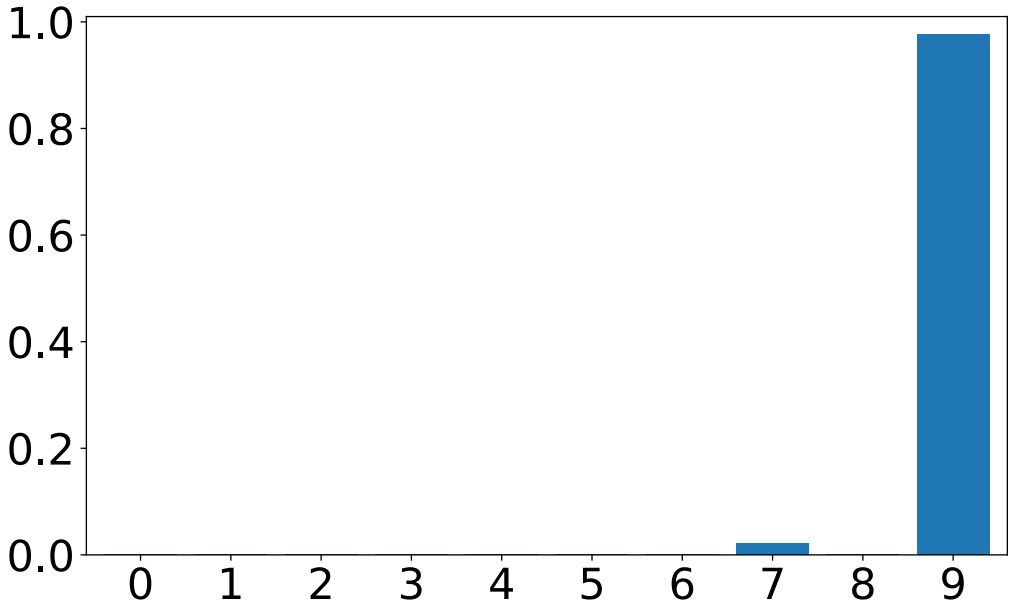
10-dim prediction vector

Targeted Attack

- Our trained CNN thinks the 28×28 input image is digit “9”.



28×28 generated image



10-dim prediction vector

Adversarial Training: Min-Max Model

Read the articles and slides for details: https://adversarial-ml-tutorial.org/

Adversarial Robust Model

- Standard model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right).$$

Adversarial Robust Model

- Standard model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j\ ;\ \mathbf{W})\big).$$

- Min-max model (robust to adversary):

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \left\{ \max_{||\boldsymbol{\delta}||<\sigma} \ \mathrm{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W})\big) \right\}.$$

Min-Max Model: Optimization

Min-max model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \left\{ \max_{||\boldsymbol{\delta}||<\sigma} \ \text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W})\big) \right\}.$$

Equivalent to

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \ \text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\tilde{\mathbf{x}}_j;\ \mathbf{W})\big).$$

- Here, $\tilde{\mathbf{x}}_j = \mathbf{x}_j + \boldsymbol{\delta}^\star$ is an adversarial example, where

$$\boldsymbol{\delta}^\star = \underset{||\boldsymbol{\delta}||<\sigma}{\text{argmax}} \ \text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W})\big).$$

Min-Max Model: Optimization

Min-max model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \left\{ \max_{||\boldsymbol{\delta}||<\sigma} \ \text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W})\big) \right\}.$$

Repeat:

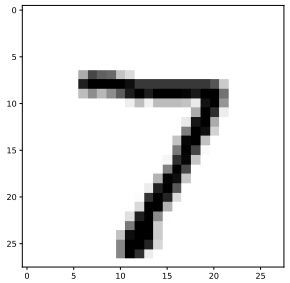
1. Randomly sample an index j from $\{1, \cdots, n\}$.
2. Adversary: $\boldsymbol{\delta}^{\star} = \underset{||\boldsymbol{\delta}||<\sigma}{\text{argmax}} \ \text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W})\big)$.
3. Update $\mathbf{W}$ using the derivative of $\text{Loss}\big(\mathbf{y}_j,\ \mathbf{f}\ (\mathbf{x}_j + \boldsymbol{\delta}^{\star}\ ;\ \mathbf{W})\big)$ w.r.t. $\mathbf{W}$.

Min-Max Model: Optimization

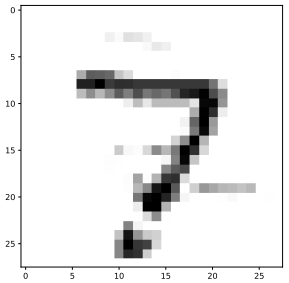
Another way to understand adversarial training.

Repeat:

1. Randomly select a real sample; generate an adversarial sample.
2. Replace real samples by adversarial sample. (Use the true label, e.g., “7”.)
3. Update the model parameters using the adversarial sample and true label.



Real sample



Adversarial sample

Adversarial Training: Gradient Regularization

Read the articles and slides for details: https://adversarial-ml-tutorial.org/

Gradient Regularization

- Standard model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right).$$

- Gradient regularization model:

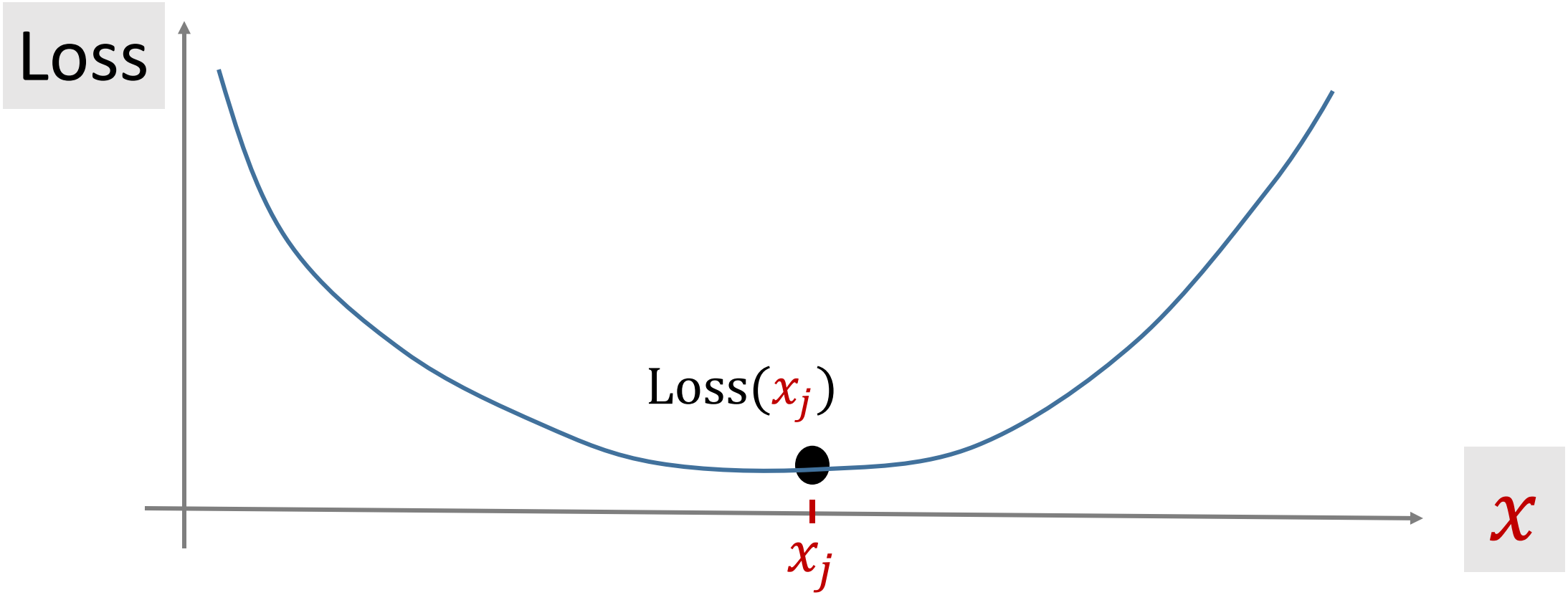
$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right) + \lambda \left|\left|g\left(\mathbf{x}_j\right)\right|\right|_2^2.$$

Here $g(\mathbf{x})$ is the gradient of Loss at $\mathbf{x}$.

Gradient Regularization

- Gradient regularization model:

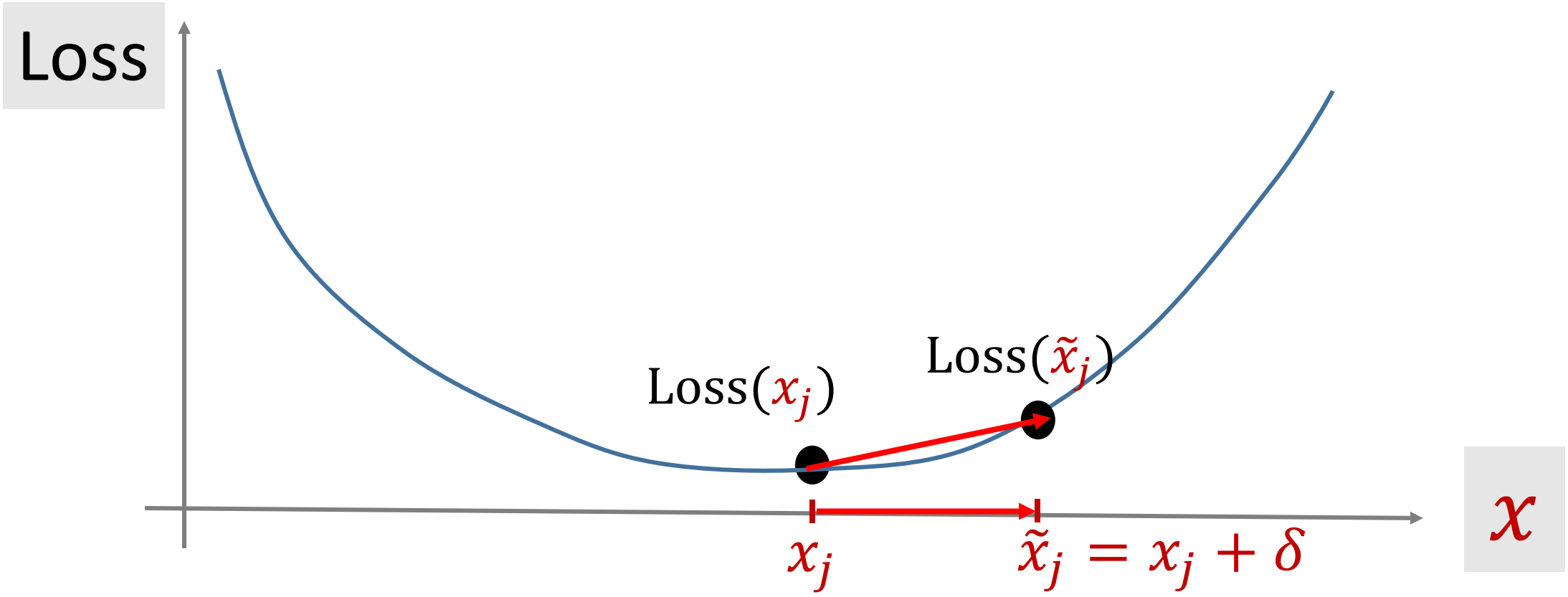
$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right)\ +\ \lambda \left|\left|g\left(\mathbf{x}_j\right)\right|\right|_2^2.$$



Gradient Regularization

- Gradient regularization model:

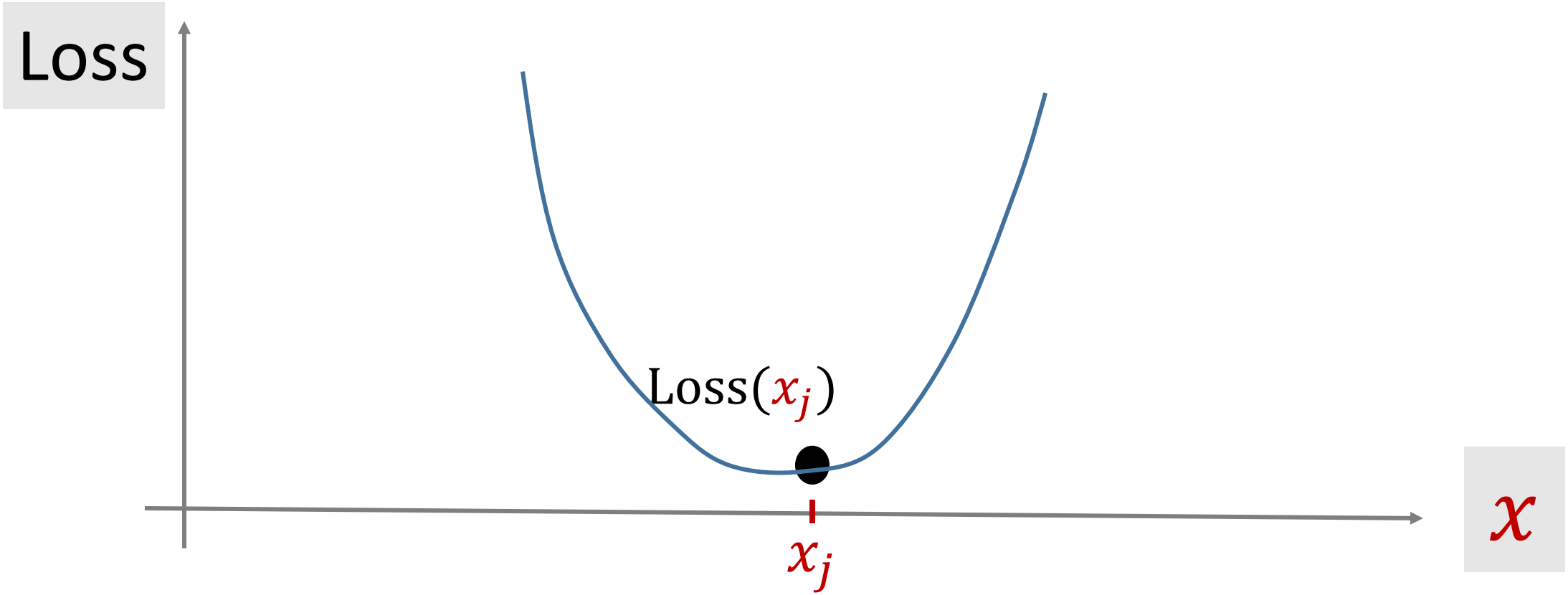
$$\min_{\mathbf{W}} \sum_{j=1}^{n} \mathrm{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right)\ +\ \lambda \left|\left|g\left(\mathbf{x}_j\right)\right|\right|_2^2.$$



Gradient Regularization

- Standard model:

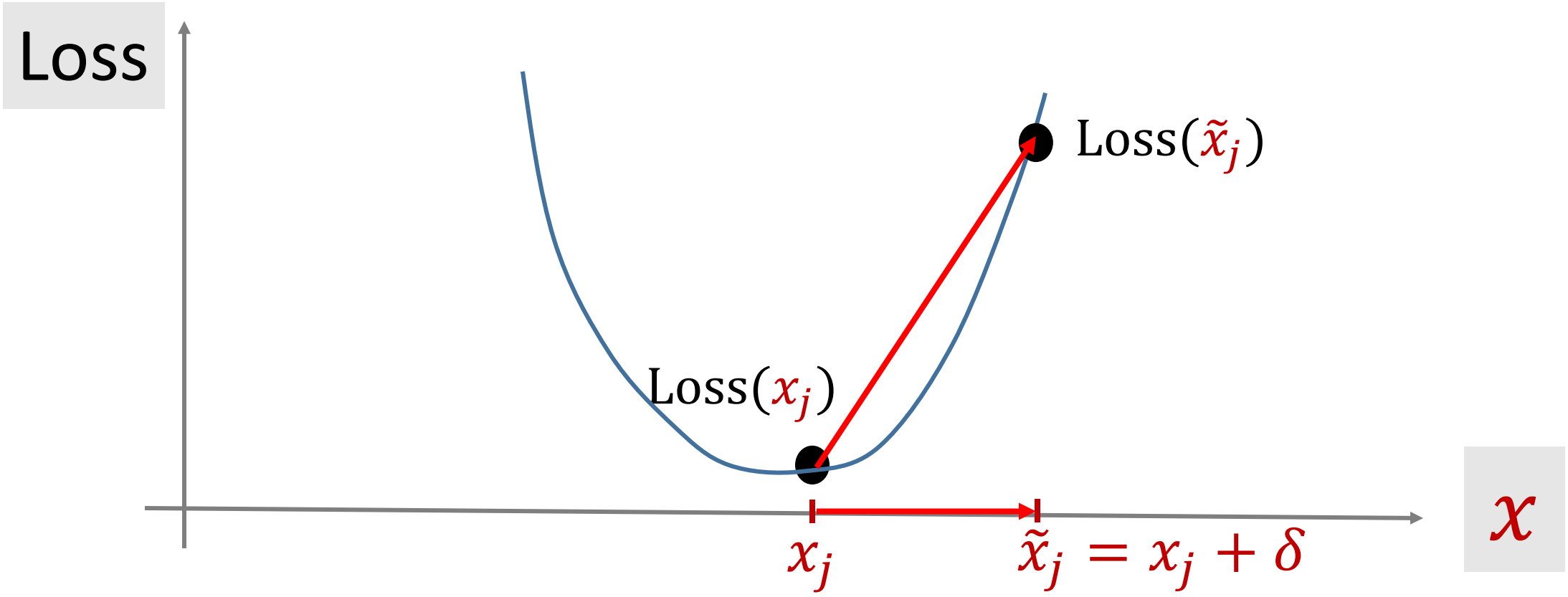
$$\min_{\mathbf{W}} \sum_{j=1}^{n} \text{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right).$$



Gradient Regularization

- Standard model:

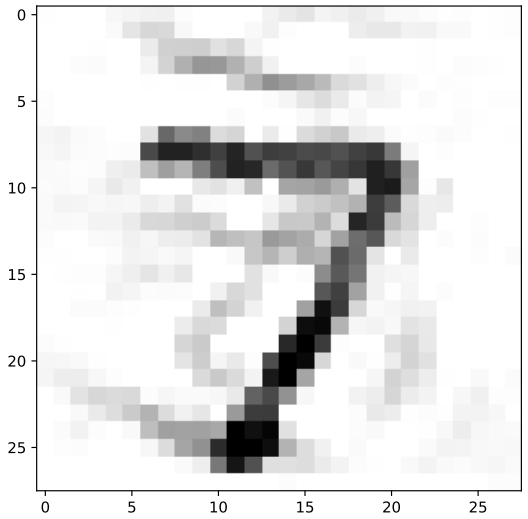
$$\min_{\mathbf{W}} \sum_{j=1}^{n} \text{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right).$$

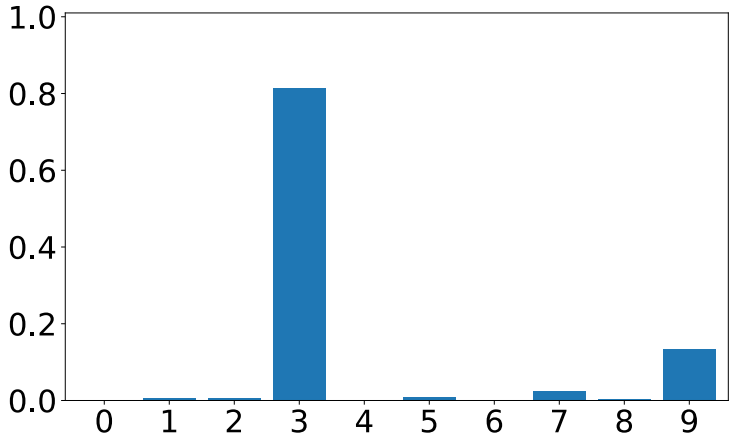


Summary

Attack Neural Networks

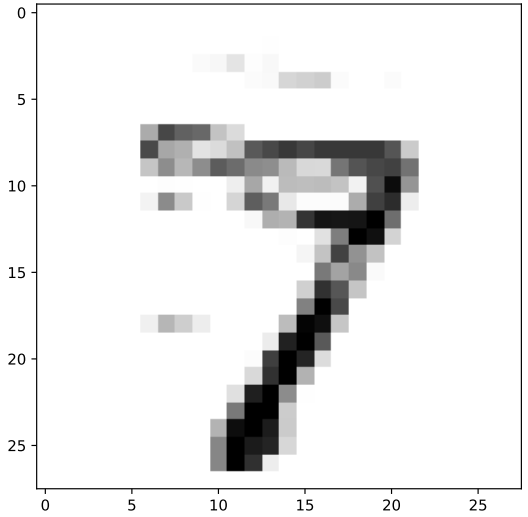
- Untargeted attack.
  - Perturb the image to increase **`Dist(true_target, prediction)`**.
  - Just let the neural network make wrong prediction.
  - Do not know what the outcome will be.

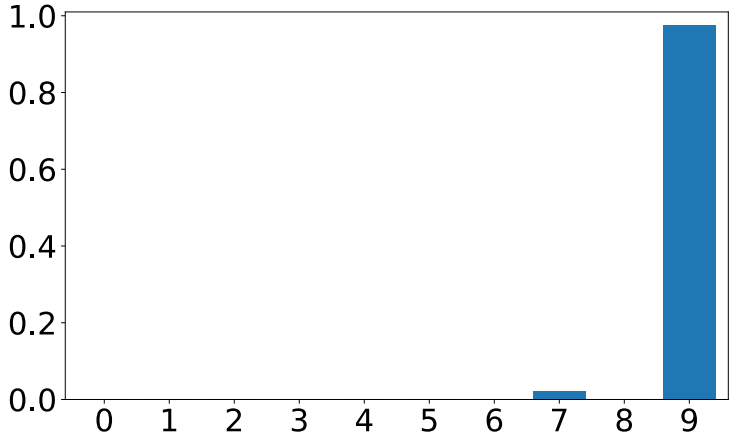




Attack Neural Networks

- Untargeted attack.
- Targeted attack.
  - Set a **`fake_label`** and minimize **`Dist(fake_target, prediction)`**.
  - Also keep it small: **`Dist(fake_image, true image)`**.
  - The neural network will make the wrong prediction as we expected.





Adversarial Training

- Defend from attacks.
  - Why?
  - Slightly change a stop sign.
  - A self-driving car can ignore the stop sign.

Adversarial Training

- Defend from attacks.
- Min-max model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \left\{ \max_{||\boldsymbol{\delta}||<\sigma} \text{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j + \boldsymbol{\delta}\ ;\ \mathbf{W}\right)\right) \right\}.$$

- Gradient regularization model:

$$\min_{\mathbf{W}} \sum_{j=1}^{n} \text{Loss}\left(\mathbf{y}_j,\ \mathbf{f}\left(\mathbf{x}_j\ ;\ \mathbf{W}\right)\right) + \lambda \left|\left| g\left(\mathbf{x}_j\right) \right|\right|_2^2,$$

where $g(\mathbf{x})$ is the gradient of Loss at $\mathbf{x}$.