Week 4. Presentation.

I. UMASS model.

- add a H status for "hospitalized-and-will-die" in SEIRD model.
- equations:

$$
\begin{cases}
\frac{dS}{dt} = -\beta(t)\frac{SI}{N} \\
\frac{dE}{dt} = \beta(t)\cdot\frac{SI}{N} - \alpha E \\
\frac{dI}{dt} = \alpha E - \gamma I \\
\frac{dR}{dt} = (1-\rho)\gamma I \\
\frac{dH}{dt} = \rho\gamma I - \lambda H \\
\frac{dD}{dt} = \lambda H
\end{cases}
$$

parameters:

① α: rate of transition from E to I
prior $\text{Gamma}(5, 5\hat{d}_E) = \text{Gamma}(5, 5\times 4)$

② γ: rate of transition from I to (R/H)
prior: $\text{Gamma}(7, 7\hat{d}_I) = \text{Gamma}(7, 7\times 2)$

③ ρ: fatality rate
prior: $\text{Beta}(10, 90)$

④. λ: rate of transition from H to D.

prior: Gamma(10, 100)

⑤ β_0: initial contact rate.

prior: $\text{Gamma}(1, \hat{\alpha}_I / \hat{R}) = \text{Gamma}(1, \frac{2}{3})$

$\beta(t)$:

$$\beta_{K+1} = \beta_K \cdot \exp(\varepsilon_K) \qquad \varepsilon_K \sim N(0, 0.2)$$

- objective function in the codes:

sum(predict$D − observation$D)^2 + sum(predict.daily$D − observe.daily$D))^2

↑ minimize

⇒ α.best, γ.best, ρ.best, lamba.best

```
> best$par
[1] 0.33973528 0.37704547 0.03049904 0.04532215
>
```

- MSE: between predict$D and observation$D in the previous week (6/19 − 6/25)

⇒ to decide the weight of this model in the ensemble model.

```
> w.exam
[1] 117261.1 117629.9 117982.6 118319.9 118642.4 118950.7 119245.5
>
```

MSE = 7218301

- forcast of the following week.
(6/26 - 7/2)

```
> w.fore
[1] 119527.3 119796.7 120054.3 120300.5 120535.8 120760.8 120975.8
>
```

II. UCLA Model - SuEIR

Unreported Recovery: The SuEIR Model

It is observed that COVID-19 has an incubation period ranging from 2 to 14 days[3]. However, during this period, individuals who have been exposed to the virus can also infect the susceptible group. In practice, the common situation is that the number of reported cases (including confirmed cases and recovered cases) are not equal to their real numbers as many infectious cases have not been tested, which will not pass to the next compartment. Therefore, we use the similar idea of SEIR and proposed a new epidemic model that takes the untested/unreported cases into consideration, which are illustrated in Figure 3.

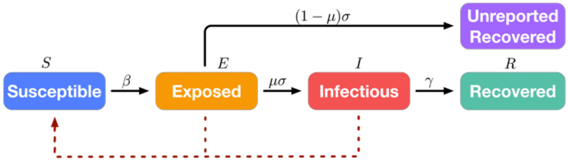

Figure 3. Illustration of the SuEIR model.

In particular, the compartment **Exposed** in our model is considered as the cases that have already been infected and have not been tested. Therefore, they also have the capability to infect the susceptible individuals. Moreover, some of such cases can receive a test and be passed to the **Infectious** compartment (as well as reported to the public), while the rest of them will recover/die but not appear in the publicly reported cases. Therefore, we introduce a new parameter $\mu < 1$ in the evolution dynamics of I_t to control the ratio of the exposed cases that are confirmed and reported to the public.

$$\frac{dS_t}{dt} = -\frac{\beta(I_t + E_t)S_t}{N}$$

$$\frac{dE_t}{dt} = \frac{\beta(I_t + E_t)S_t}{N} - \sigma E_t$$

$$\frac{dI_t}{dt} = \mu\sigma E_t - \gamma I_t$$

$$\frac{dR_t}{dt} = \gamma I_t$$

However, in our code, to predict the number of total deaths, we add Deaths status into this model. So the equations in our model is:

$$\frac{dS}{dt} = -\beta * (\mathrm{I} + \mathrm{E}) * \frac{S}{N}$$

$$\frac{dE}{dt} = \beta * (\mathrm{I} + \mathrm{E}) * \frac{S}{N} - \delta * E$$

$$\frac{dI}{dt} = \mu * \delta * E - (1-\alpha) * \gamma * I - \alpha * \rho * I$$

$$\frac{dR}{dt} = (1-\alpha) * \gamma * I$$

$$\frac{dD}{dt} = \alpha * \rho * I$$

other than μ, the other parameters are just like before.

μ: ratio of reported cases

- objective function in my codes:

sum(predict $D - observation $D)^2 + sum(predict daily $D - observe.daily$D)^2

↑ minimize

⇒ beta.best, delta.best, mu.best, gamma.best, alpha.best, rho.best.

```
> best$par
[1] 0.24782938 0.03413010 0.79075559 0.06204655 0.03515935 0.10214740
```

- MSE: between predict$D and observation $D in the previous week (6/19 - 6/25)

⇒ to decide the weight of this model in the ensemble model.

```
> w.exam
[1] 118428.4 118936.2 119428.3 119905.2 120367.4 120815.3 121249.3
>
```

MSE = 1212767

- forcast of the following week (6/26 - 7/2)

```
> w.fore
[1] 121669.8 122077.1 122471.7 122853.9 123224.0 123582.6 123929.8
```

III Imperial College Model
— the length of time-window = 7 days

- Estimating current transimissibility.

  Estimating the current reproduction number R_t

  daily incidence $I_t \sim \text{Poisson}\left(R_t \overbrace{\sum_{s=1}^{t-1} I_{t-s} w_s}^{\Lambda_t}\right)$

$\boxed{\text{Note:}}$ • no data prior to the time window were used to estimate R_t.

  • The joint posterior distribution of R_t and the early epidemic curve (from which forcasts will be generated) were inferred using MCMC sampling.

- the last time point is 6/25/2020.

  in this part, assume the length of time-window is $\underbrace{7 \text{ days}}_{k=7}$.

  I_{t-k+1} ———————— I_t

  6/18 R_t constant   6/25

likelihood function:

$$f(I_{t-k+1}, \cdots, I_t \mid I_1, \cdots, I_{t-k}, R_t, w_s) = \prod_{s=t-k+1}^{t} \frac{(R_{t,k}\Lambda_s)^{I_s} \cdot e^{-R_{t,k}\Lambda_s}}{I_s !}$$

prior of $R_{t,k} \sim \text{Gamma}(\underset{\text{shape}}{a} \cdot \underset{\text{scale}}{b})$ (In code, mean = 5, sd = 5)

joint poterior of $R_{t,k}$ and $I_{t-k+1}, \cdots, I_t$.

$$P(I_{t-k+1}, \cdots, I_t, R_{t,k} \mid I_1, \cdots, I_{t-1}, w_s)$$

$$= P(I_{t-k+1}, \cdots, I_t \mid I_1, \cdots, I_{t-1}, w_s, R_{t,k}) \cdot P(R_{t,k} \mid I_1, \cdots, I_{t-1}, w_s)$$

$$\overset{R_{t,k} \perp I_1, \cdots, I_{t-1}}{=} P(I_{t-k+1}, \cdots, I_t \mid I_1, \cdots, I_{t-1}, w_s, R_{t,k}) \cdot P(R_{t,k})$$

$$= \text{likelihood} \cdot \text{prior}$$

$$= \prod_{s=t-k+1}^{t} \frac{(R_{t,k}\Lambda_s)^{I_s} \cdot e^{-R_{t,k}\Lambda_s}}{I_s!} \cdot \frac{1}{\Gamma(a) b^a} R_{t,k}^{a-1} e^{-\frac{1}{b} R_{t,k}}$$

$\Longrightarrow$ Given $R_{t,k}$, $I_{t-k+1}, \cdots, I_t \sim$ Poisson

Given $I_{t-k+1}, \cdots, I_t$ $R_{t,k} \sim \text{Gamma}(\alpha, \underset{\text{scale}}{\beta})$

$$\begin{cases} \alpha = a + \sum_{s=t-k+1}^{t} I_s \\ \beta = \dfrac{1}{\frac{1}{b} + \sum_{s=t-k+1}^{t} \Lambda_s} \end{cases}$$

$\Longrightarrow$ Using Gibbs sampling,

estimat $R_{t,k,post} = 0.7853442$

6/19 - 6/25. post =

```
> w.exam.post
[1] 838.8490 835.7520 834.9425 832.8555 831.4370 829.6710 828.8130
>
```

```
[1] 119236 120032 120901 121736 122543 123365 124225
```

- MSE = 914618.428571428

- Forcase 6/26 - 7/2

```
> tail(daily,7)
[1] 829.896 825.862 825.206 823.730 820.658 821.884 818.410
```

```
> w.fore
[1] 125239.9 126065.8 126891.0 127714.7 128535.4 129357.2 130175.6
```

- Then we will consider the choice of time-window k, since R_t is sensitive to the change of time-window

APE: accumulated predictive error to choose the best time window,

$$APE_k = \sum_{t=0}^{T-1} -\log \underbrace{P(I_{t+1} = I_{t+1}^{*} \mid I_{t-k+1}^{t})}$$

here, the probability mean the posterior predictive distribution of the number at time $t+1$.

- Ensemble model

Their performance in the previous week (6/18 - 6/25) decides their weights in the ensemble model.

☆ The more MSE, the less weight we should put on this model.

1st. calculate the reciprocal of each mse

⇒ standardized then to ~~SD~~ add up to 1