PRESENTATION

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- **Introducing and ensembling several models in the reich lab website:**
- Umass - MechBayes
- UCLA – SuEIR
- Imperial College (model1, model2, model3)
- Ensemble Model

UMASS - MECHBAYES

Comparing with traditional SEIRD model, there is one more status- H, which is hospitalized-and-will-die, between I and D.

$$\frac{dS}{dt} = -\beta(t)\frac{SI}{N}$$
$$\frac{dE}{dt} = \beta(t)\cdot\frac{SI}{N} - \sigma E$$
$$\frac{dI}{dt} = \sigma E - \gamma I$$
$$\frac{dR}{dt} = (1-\rho)\gamma I$$
$$\frac{dH}{dt} = \rho\gamma I - \lambda H$$
$$\frac{dD}{dt} = \lambda H$$

PARAMETER MEANING

Model uses a time-dependent beta

$$\frac{dS}{dt} = -\beta(t)\frac{SI}{N}$$
$$\frac{dE}{dt} = \beta(t)\cdot\frac{SI}{N} - \sigma E$$
$$\frac{dI}{dt} = \sigma E - \gamma I$$
$$\frac{dR}{dt} = (1-\rho)\gamma I$$
$$\frac{dH}{dt} = \rho\gamma I - \lambda H$$
$$\frac{dD}{dt} = \lambda H$$

The parameters are:

- $\beta(t)$: (time-varying) contact rate
- σ: rate of transition from E to I
- γ: rate of transition from I to (R/H)
- ρ: fatality rate (i.e., probability of transitioning from I to H instead of I to R)
- λ: rate of transition from H to D (inverse of expected number of days in H compartment before death)

ONE MORE VARIABLE

$$\frac{dS}{dt} = -\beta(t)\frac{SI}{N}$$
$$\frac{dE}{dt} = \beta(t) \cdot \frac{SI}{N} - \sigma E$$
$$\frac{dI}{dt} = \sigma E - \gamma I$$
$$\frac{dR}{dt} = (1-\rho)\gamma I$$
$$\frac{dH}{dt} = \rho\gamma I - \lambda H$$
$$\frac{dD}{dt} = \lambda H$$

One additional variable $C(t)$ is added to track cumulative number of infections for the purposes of the observation model:

$$\frac{dC}{dt} = \sigma E$$

INITIAL VALUE

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta(t)\frac{SI}{N} \\
\frac{dE}{dt} &= \beta(t)\cdot\frac{SI}{N} - \sigma E \\
\frac{dI}{dt} &= \sigma E - \gamma I \\
\frac{dR}{dt} &= (1-\rho)\gamma I \\
\frac{dH}{dt} &= \rho\gamma I - \lambda H \\
\frac{dD}{dt} &= \lambda H
\end{aligned}
$$

Parameters and Intial State Variables

$$
\begin{aligned}
I_0 &\sim \text{Unif}(0, 0.02N) \\
E_0 &\sim \text{Unif}(0, 0.02N) \\
H_0 &\sim \text{Unif}(0, 0.02N) \\
D_0 &\sim \text{Unif}(0, 0.02N) \\
\sigma &\sim \text{Gamma}(5, 5\hat{d}_E) \\
\gamma &\sim \text{Gamma}(7, 7\hat{d}_I) \\
\beta_0 &\sim \text{Gamma}(1, \hat{d}_I/\hat{R}) \\
\lambda &\sim \text{Gamma}(10, 100) \\
\rho &\sim \text{Beta}(10, 90) \\
p &\sim \text{Beta}(15, 35) \\
p_d &\sim \text{Beta}(90, 10)
\end{aligned}
$$

EXPLANATIONS OF THESE DISTRIBUTIONS

$$\frac{dS}{dt} = -\beta(t)\frac{SI}{N}$$
$$\frac{dE}{dt} = \beta(t)\cdot\frac{SI}{N} - \sigma E$$
$$\frac{dI}{dt} = \sigma E - \gamma I$$
$$\frac{dR}{dt} = (1-\rho)\gamma I$$
$$\frac{dH}{dt} = \rho\gamma I - \lambda H$$
$$\frac{dD}{dt} = \lambda H$$

- σ is the rate for leaving the exposed compartment; i.e., $1/\sigma$ is the expected duration in the exposed compartment. The prior satisfies $\mathbb{E}[\sigma] = 1/\hat{d}_E$, where $\hat{d}_E$ is an initial guess of the duration in the exposed compartment. Currently $\hat{d}_E = 4.0$ based on published estimates (shortened slightly to account for possible infectiousness prior to developing symptoms)
- γ is the rate for leaving the infectious compartment; i.e., $1/\gamma$ is the expected duration in the infectious compartment. The prior satisfies $\mathbb{E}[\gamma] = 1/\hat{d}_I$, where $\hat{d}_I$ is an initial guess for the duration in the infectious compartment. The current setting is $\hat{d}_I = 2.0$ to model the likely isolation of individuals after symptom onset.
- β_0 is the initial contact rate. In the SEIR model, it is known that $R_0 = \beta/\gamma$, so we set our prior to have mean $\mathbb{E}[\beta_0] = \hat{R}/\hat{d}_I$ where $\hat{R} = 3.0$ is an initial guess for R_0 and $\hat{d}_I = 2.0$, as described above.

ABOUT TIME-DEPENDENT BETA

$$\beta_{k+1} = \beta_k \times \exp(\epsilon_k), \qquad \epsilon_k \sim \mathcal{N}(0, 0.2)$$
$$X(k+1) = \texttt{odesolve}\big(X(k),\, dX/dt,\, \beta_k\big)$$

The contact rate β_k undergoes an exponentiated Gaussian random walk starting from β_0 (defined above) with scale τ. The function `odesolve` finds the state vector at time $k+1$ by simulating the ODE for one time step with (constant) contact rate β_k.

PARAMETER ESTIMATION-SQUARE LOSS

- Here we use the square loss between the true number of total deaths and the predicted number of total death as the objective function, to minimize it.

```
ssr<-sum((predictions$D-data$D)^2)
```

NOTE:

Since the fitted value and prediction value are far away from the observed value, we give up aggregating UMass model into our ensemble model.

UCLA - SUEIR

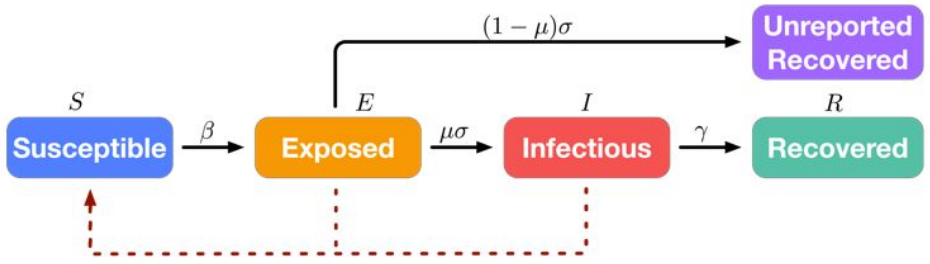
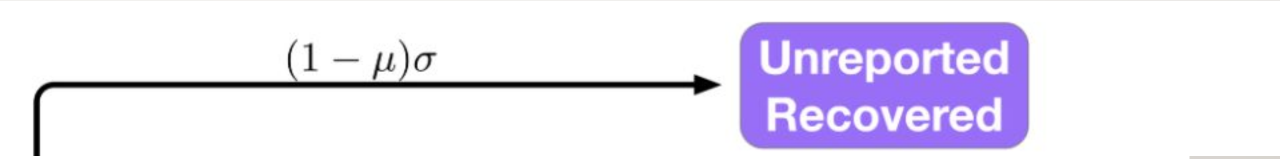
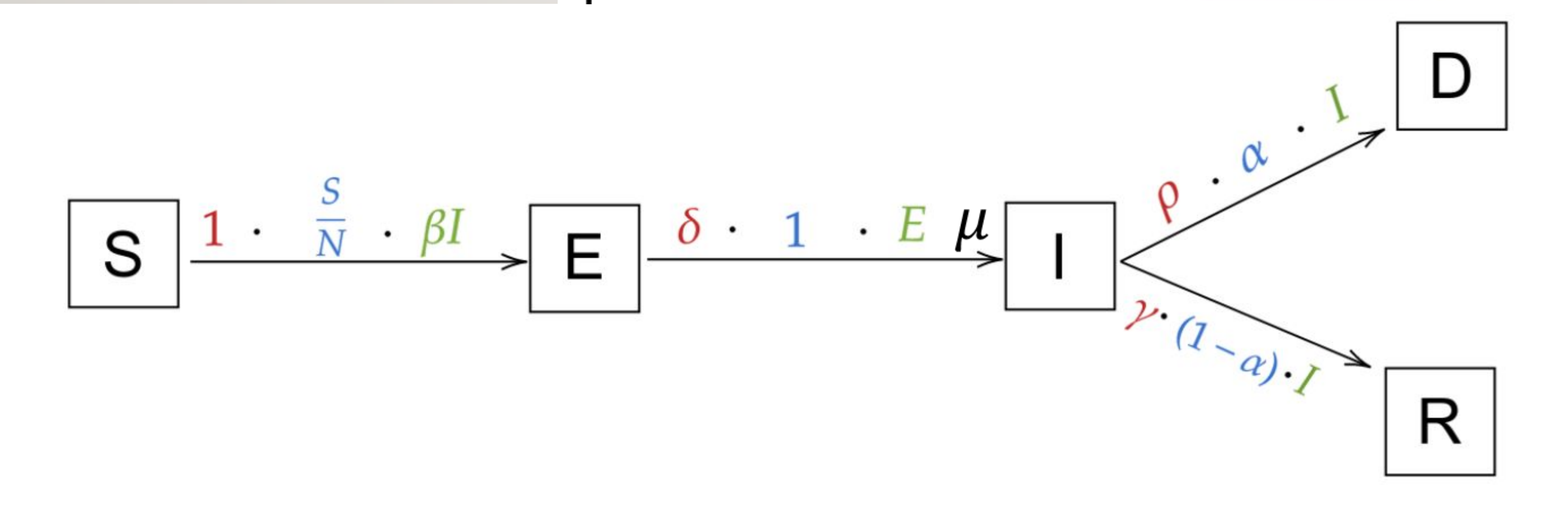


Figure 3. Illustration of the SuEIR model.

We assume that there are much amount of unreported recovered population, thus we add a U status like the picture on the top. Then, to predict the number of total deaths, we also include Deaths status like the following pict





ODE

- $$\frac{dS}{dt} = -\beta * (\mathrm{I} + \mathrm{E}) * \frac{S}{N}$$
$$\frac{dE}{dt} = \beta * (\mathrm{I} + \mathrm{E}) * \frac{S}{N} - \delta * E$$
$$\frac{dI}{dt} = \mu * \delta * E - (1 - \alpha) * \gamma * I - \alpha * \rho * I$$
$$\frac{dR}{dt} = (1 - \alpha) * \gamma * I$$
$$\frac{dD}{dt} = \alpha * \rho * I$$

PARAMETER ESTIMATION

- Here we use the square loss between the number of observations and predictions in deaths as the objective function.

```
ssr<-sum((predictions$D-data$D)^2)
```

810429.5
[1] 900.2386

PREVIOUS WEEK PREDICTIONS

DATE	predictions	observations
7/25	148988.2	146965
7/26	149861.1	147442
7/27	150732.0	148567
7/28	151600.7	149927
7/29	152467.4	151360
7/30	153332.1	152573
7/31	154194.7	153814

MSE= 2768843
RMSE=1663.984

FORECASTING IN DEATHS 8/1-8/7

DATE	predictions
08/01	155055.2
08/02	155913.7
08/03	156770.2
08/04	157624.6
08/05	158477.0
08/06	159327.4
08/07	160175.8

IMPERIAL COLLEGE (MODEL 1): TIME-WINDOW 7 DAYS

I_t is daily incidence (here is the daily death). R_t is the instantaneous reproduction number and w_s is the serial interval distribution. Here we can assume w_s is a uniform distribution, that is the weight for every I_{t-s} are same. And R_t remains constant in the time-window.

The core here is to calculate R_t.

$$I_t \sim Pois\left(R_t \sum_{s=0}^{t} I_{t-s} w_s\right)$$

How we choose Rt

We assume the prior of Rt is a Gamma distribution with mean = the mean of transmission rate of the past 7 days, and sd = the sd of the transmission rate of the past 7 days (in our code, we define the
today's transmission rate = today's number of existing infectious people / yesterday's number of existing infectious people).
And we can calculate the likelihood of It.

$$P\left(I_t \mid I_0, \ldots, I_{t-1}, w, R_t\right) = \frac{\left(R_t \Lambda_t\right)^{I_t} e^{-R_t \Lambda_t}}{I_t\,!}$$

$$\Lambda_t = \sum_{s=1}^{t} I_{t-s} w_s$$

ESTIMATE RT AND DAILY INCIDENCE IN THE TIME-WINDOW

We can quickly get that the joint posterior distribution of $R_{t,k}(here\ k = 7\ days)$ and $I_{t-k+1}, \dots, I_t$ has the following form.

$\tau(also\ k\ in\ our\ code)$ here is the time window which means how long time you think I will influence the R. We can simply assume it to be a week- 7days, and we will discuss how to optimize it.

$$P\left(I_{t-\tau+1},\dots,I_t,R_{t,\tau} \mid I_0,\dots,I_{t-\tau},w\right) = P\left(I_{t-\tau+1},\dots,I_t \mid I_0,\dots,I_{t-\tau},w,R_{t,\tau}\right)P\left(R_{t,\tau}\right)$$

$$= \left(\prod_{s=t-\tau+1}^{t} \frac{\left(R_{t,\tau}\Lambda_s\right)^{I_s} e^{-R_{t,\tau}\Lambda_s}}{I_s\,!}\right)\left(\frac{R_{t,\tau}^{a-1} e^{-R_{t,\tau}/b}}{\Gamma(a)b^a}\right)$$

$$= R_{t,\tau}^{a+\sum_{s=t-\tau+1}^{t} I_s - 1} e^{-R_{t,\tau}\left(\sum_{s=t-\tau+1}^{t}\Lambda_s + \frac{1}{b}\right)} \prod_{s=t-\tau+1}^{t} \frac{\Lambda_s^{I_s}}{I_s\,!} \frac{1}{\Gamma(a)b^a}$$

With the joint posterior distribution of $R_{t,k}(here\ k = 7\ days)$ and $I_{t-k+1}, \dots, I_t$, and given $R_{t,k}$, these daily incidence follow the Poisson distribution mentioned before, and given $I_{t-k+1}, \dots, I_t$, $R_{t,k}$ has the Gamma distribution with parameter

$$\left(a + \sum_{s=t-\tau+1}^{t} I_s, \frac{1}{\frac{1}{b} + \sum_{s=t-\tau+1}^{t} \Lambda_s} \right)$$

$$\Lambda_t = \sum_{s=1}^{t} I_{t-s} w_s$$

In our training data, the last date is 7/31, so, t here = the date 7/31, the time window 7/25-7/31. So we use Gibbs sampling to estimate $R_{t,k}$ and $I_{t-k+1}, \dots, I_t$.

We have $R_{t,k.post} = 1.014501, (I_{t-k+1}, \dots, I_t).post =$

```
[1] 1008.166 1007.557 1008.073 1009.282 1008.600 1008.664 1008.289
```

PREVIOUS WEEK PREDICTIONS

DATE	predictions	observations
7/25	147083.9	146965
7/26	148093.8	147442
7/27	149104.2	148567
7/28	150113.9	149927
7/29	151123.1	151360
7/30	152134.6	152573
7/31	153146.2	153814

MSE=208120.2
RMSE=456.2019

(SHORT TERM FORECAST)

- After estimating R_t, we can calculate the prediction of I_t

$$I_t \sim Pois\left(R_t \sum_{s=0}^{t} I_{t-s} w_s\right)$$

WHEN CHOOSING THE LENGTH OF TIME-WINDOW=7 DAYS FORECASTING IN DEATHS 8/1-8/7

DATE	predictions
08/01	154828
08/02	155843
08/03	156856
08/04	157871
08/05	158886
08/06	159903
08/07	160917

IC(MODEL2):
CHOICE OF TIME WINDOW - K

- We'll use accumulated predictive error (APE) to choose best time window, the formula is like this:

$$\mathrm{APE}_k = \sum_{t=0}^{T-1} -\log P\left(I_{t+1} = I_{t+1}^{*} \mid I_{t-k+1}^{t}\right)$$

- The optimal window length k^* is then $k^* = argmin_k APE_k$. Here T is the last time point in the existing incidence curve.

WHAT IS I'S DISTRIBUTION?

$$x \mid I_{s-k+1}^{s} \sim \mathrm{NB}\left(\alpha_{\tau(s)}, \frac{\Lambda_{s+1}\beta_{\tau(s)}}{1+\Lambda_{s+1}\beta_{\tau(s)}}\right)$$

$$\Lambda_t = \sum_{s=1}^{t} I_{t-s} w_s$$

α and β is the parameter of the posterior distribution of R_s

$$\left(a + \sum_{s=t-\tau+1}^{t} I_s, \frac{1}{\frac{1}{b} + \sum_{s=t-\tau+1}^{t} \Lambda_s} \right)$$

K.BEST=2
PREVIOUS WEEK PREDICTIONS

DATE	predictions	observations
7/25	146965.9	146965
7/26	147961.6	147442
7/27	148959.4	148567
7/28	149561.7	149927
7/29	150557.8	151360
7/30	152363	152573
7/31	153367	153814

MSE= 722497
RMSE=849.9982

WHEN CHOOSING THE LENGTH OF TIME-WINDOW=2 DAYS FORECASTING IN DEATHS 8/1-8/7

DATE	predictions
08/01	154824.8
08/02	155833.8
08/03	156844.0
08/04	157857.0
08/05	158864.3
08/06	159875.1
08/07	160887.6

IC (MODEL3):

- First, we have a known distribution for delay from report to death (gamma distribution with mean 10 days and standard deviation 2 days)

$$\delta \sim \Gamma(\mu, \sigma).$$

- This is a kind of distribution to represent the probability of how long a Infectious people will switch to death state.

HOW TO PREDICT DEATH?

$$\delta \sim \Gamma(\mu, \sigma).$$

$$D_{i,t} \sim Binom\left(\int_0^\infty \Gamma(x \mid \mu, \sigma) I^r_{i,t-x} dx, r_{i,\mu}\right).$$

Based on distribution of delay before, if we have a estimated Infectious I, and a death rate r, we can get the Death prediction by a binomial distribution.

HOW CAN WE GET R AND I?

- we estimate new reporting cases in the coming week by sampling from a Gamma distribution with mean and standard deviation estimated from the number of observed cases in the last week.

- r is the estimated ratio of deaths to reported cases for the last week of data

TO PREDICT DEATHS

- We already have I, r from last week's data and delay distribution. We can calculate the first part parameter in binomial distribution by weighted summing I data based on delay distribution. And simulate a binomial distribution to get the predicted D.

$$D_{i,t} \sim Binom\left(\int_0^\infty \Gamma(x \mid \mu, \sigma) I^r_{i,t-x} dx, r_{i,\mu}\right).$$

- Since this process refers to two random sampling, we’ll do this process 5000 times and get the mean of those results.

PREVIOUS WEEK PREDICTIONS

DATE	predictions	observations
7/25	147019	146965
7/26	147968	147442
7/27	148924	148567
7/28	149773	149927
7/29	150686	151360
7/30	151572	152573
7/31	152449	153814

MSE=449100.7
RMSE=670.1498

SHORT TERM FORECASTING IN DEATHS

DATE	predictions
08/01	154962.5
08/02	156109.5
08/03	157235.5
08/04	158331.6
08/05	159412.1
08/06	160497.3
08/07	161601.7

4 MODEL ENSEMBLE

- Since we have coded 5 models(UMASS, UCLA and 3 IC models), we decide to set the weights based on their performance in the previous week(7/25-7/31)—RMSE.

MODEL	UMASS	UCLA	IC(7 DAYS)	IC(2 DAYS)	IC(model 3)
RMSE	N/A	1663.984	456.2019	849.9982	670.1498
WEIGHT	N/A	0.1100	0.4013	0.2154	0.2732

```
> weight
[1] 0.1100339 0.4013457 0.2154059 0.2732145
```

The result of ensemble model of the previous week

DATE	predictions	observations
7/25	147250.3	146965
7/26	148225.4	147442
7/27	149202.9	148567
7/28	150065.4	149927
7/29	151029.8	151360
7/30	152161.9	152573
7/31	153118.6	153814

MSE= 268601.9
RMSE= 518.2682

FINAL FORECASTING IN DEATHS 8/01-8/07 AND COMPARISON WITH REAL RESULT

- Based on the weights in the ensemble model, we have the final predictions of total deaths:

DATE	predictions	TRUE
08/01	154889.1	154919
08/02	155921.6	155320
08/03	156947.7	155859
08/04	157966.7	157230
08/05	158980.1	158593
08/06	159996.0	159830
08/07	161016.2	161066

Prediction result on 08/03 comparison with Reich Lab model

DATE	predictions	TRUE
08/03	156947.7	155859

Since the source of data is different, the prediction and actual value are not completely identical. (We can see our true value and actual value on the reich lab website are not same)

1 ahead	
Actual	154,448
COVIDhub-baseline	152,801
COVIDhub-ensemble	153,024
CU-select	154,024
Columbia_UNC-SurvCon	153,196
Covid19Sim-Simulator	147,074
CovidAnalytics-DELPHI	153,030
DDS-NBDS	154,485
GT-DeepCOVID	153,445
Geneva-DetGrowth	154,411

THANK YOU!