

Maximum differentiation with partial managerial delegation

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Abstract

This paper derives the robustness of maximum differentiation principle in a Hotelling model of duopoly under endogenous relative performance based managerial delegation. Specifically, we analyze a partial delegation setting where owners determine location choices while delegating pricing decisions to the managers. Additionally, we demonstrate that equilibrium location choices remain robust to the sequence of owners' decisions, whether they first decide on locations and then contracts or vice versa.

Keywords: Differentiated Duopoly, Relative Performance evaluation, Maximum Differentiation.

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1 Introduction

Linear city model of Hotelling (1929) is a widely used framework for analyzing differentiated duopolies. In this setup, firms strategically choose product/location differentiation to soften price competition. In their seminal paper, d’Aspremont et al. (1979) established the *maximum differentiation* principle, wherein firms locate at opposite ends of the product differentiation spectrum. This paper derives the robustness of the maximum differentiation principle to relative performance-based partial managerial delegation in a Hotelling model.

Advantages of delegating firm’s strategic choices to a manager is well established in the theory of firms (Kopel and Pezzino (2018)). Altering manager’s objective away from pure profit maximization by committing to a publicly observed incentive contract may earn higher profits for the owner. Broadly, two types of incentive schemes have been examined in the managerial delegation literature: (1) *sales-based delegation*, where owners offer a sales based incentive contract to managers, in which owners measure manager’s performance based on a combination of sales and profit (Vickers (1985), Fershtman and Judd (1987), Sklivas (1987)), and (2) *relative performance-based delegation*, where the owners evaluate managers based on a combination of own profit and that of competitor’s (Miller and Pazgal (2001), Miller and Pazgal (2002)).¹ This paper contributes to the latter.

While most studies on delegation in differentiated duopolies focus on sales-based delegation (Bárcena-Ruiz and Casado-Izaga (2005), Matsumura and Matsushima (2012), Wang and Buccella (2020)) a smaller body examines relative performance based delegation (Liang et al. (2011), Kou and Zhou (2015)). For instance, Kou and Zhou (2015) investigate the impact of relative performance evaluation on firms’ equilibrium location. They find that equilibrium outcome ranges from minimum to maximum differentiation depending on the weight placed on relative performance in managers’ contract.

Building on Kou and Zhou (2015), this paper introduces *endogenous* contract design,

¹There is a third type where the incentive contracts are based on profit and market share (Jansen et al. (2007)).

where owners choose the strength of the relative performance parameter in managerial contracts. This allows for asymmetric contracts across firms, unlike the exogenous and uniform contracts assumed by Kou and Zhou (2015). Additionally, we shift focus from *full delegation* (managers decide both price and location) to *partial delegation*, where owners retain control over location decisions, delegating only pricing to managers. We further explore whether the sequence of owner's decision —*location-then-contract* or *contract-then-location*—affects equilibrium outcomes.

Our findings demonstrate that the principle of maximum differentiation holds in both decision sequences. In *location-then-contract* game the unique equilibrium features a pure profit maximization contract. Whereas in the *contract-then-location* game, two equilibria emerge, where the optimal contract can either be one of pure profit maximization or pure relative performance.

2 Model

There are two firms (Firm A & Firm B) competing in a horizontally differentiated market. Consumers are uniformly distributed with unit density along the linear city $[0,1]$. Demand is inelastic and each consumer demands at most one unit of the good. Firms can locate themselves anywhere within the linear city (product space) but not outside the boundary points. Let x_A denote the location of Firm A and $1 - x_B$ be the location of Firm B. Without loss of generality we assume that Firm A is located to the left of Firm B i.e. $(x_A + x_B \leq 1)$. Both owners want to maximize their respective firm's profit, $p_i D_i$, where p_i is the price charged by firm i and D_i is the demand for firm i 's product ($i, j = A, B, i \neq j$). For simplicity we assume that marginal cost for both firms is zero.

The utility derived from consuming the good is V , a constant sufficiently large to ensure full market coverage. Transportation costs are quadratic in the distance between the consumer and the firm, with a normalized coefficient of one. The median consumer located at

x who is indifferent between buying from either firm, satisfies:

$$V - p_A - (x - x_A)^2 = V - p_B - (1 - x_B - x)^2 \quad (1)$$

Consumers to the left of the median consumer purchase from Firm A, while those to the right purchase from Firm B. Solving for the median consumer, the demand from Firm i is:

$$D_i = \frac{p_j - p_i}{2(1 - x_i - x_j)} + \frac{1 + x_i - x_j}{2} \quad (2)$$

Following the literature on relative performance based managerial delegation, firm owners employ managers to set prices and offer them linear incentive contract. Once the contracts have been decided they are publicly observable. Managers are risk-neutral and receive a payoff $A_i + B_i\Pi_i$, where A_i and B_i are constants, $B_i > 0$, and Π_i is a convex combination of firm's own profit and relative performance. Owners choose A_i equal to the manager's opportunity cost which is normalized to zero. Manager's objective is to maximize a weighted function of its own profit (π_i) and its relative performance ($\pi_i - \pi_j$) i.e. $\Pi_i = (1 - \theta_i)\pi_i + \theta_i(\pi_i - \pi_j) = \pi_i - \theta_i\pi_j$, where $\theta_i \in [0, 1]$ measures the weight attached by a firm's owner on relative performance.

Under partial delegation, owners only delegate pricing decisions to the managers. Location choices are made by the owners themselves. We analyze two three-stage games, where the sequence of owners' decisions varies. *First Stage*: Owners choose firm's location — x_A, x_B (relative performance weights — θ_A, θ_B); *Second Stage*: Owners choose relative performance weights — θ_A, θ_B (firm's location — x_A, x_B); *Third Stage*: Managers set prices (p_A, p_B). All decisions at a particular stage are taken independently and simultaneously.

In both games the third stage is the same. After observing the location choices and incentive contract choices from the previous two stages, managers make pricing decisions. Managers maximize their objective functions, $\Pi_i = \pi_i - \theta_i\pi_j = p_iD_i - \theta_ip_jD_j$, with respect to their own prices. Using standard optimization techniques we get the following first order

conditions:

$$2p_i = (1 - \theta_i)p_j + (1 - x_i - x_j)(1 + x_i - x_j) \quad (3)$$

Solving the first order conditions we get equilibrium prices:

$$p_i^* = \frac{(1 - x_i - x_j)[4 - (1 + \theta_i)(1 + x_j - x_i)]}{4 - (1 - \theta_i)(1 - \theta_j)} \quad (4)$$

Substituting equilibrium prices into demand function (2) yields the equilibrium demand D_i^* .

3 First stage: Location, Second stage: Incentive Contract

We first consider the *location-then-contract* game. We use backward induction to find the subgame perfect equilibrium.

In the second stage each owner independently chooses θ_i to maximize their firm's own profit, $\pi_i^* = p_i^* D_i^*$. As the incentive parameter $\theta_i \in [0, 1]$, we use Kuhn-Tucker theorem to get the first order conditions for the constrained optimization problem:

$$\frac{\partial \pi_i^*}{\partial \theta_i} = p_i^* \frac{\partial D_i^*}{\partial \theta_i} + D_i^* \frac{\partial p_i^*}{\partial \theta_i} \leq 0 \quad (5)$$

$$\frac{\partial \pi_i^*}{\partial \theta_i} \theta_i = 0 \quad (6)$$

with $\frac{\partial \pi_i^*}{\partial \theta_i} < 0$ when $\theta_i = 0$ and $\frac{\partial \pi_i^*}{\partial \theta_i} = 0$ when $\theta_i > 0$.

Solving equation (5) we get the following critical points:

$$\theta_i = \frac{(3 + x_i - x_j)(1 - \theta_j)}{-5 + x_i - x_j + \theta_j(1 + 3x_i - 3x_j)} \quad (7)$$

For all feasible location choices $\{(x_i, x_j) : 0 \leq x_i \leq 1, 0 \leq x_j \leq 1, x_i + x_j \leq 1\}$ and rival

firm's incentive contract ($0 \leq \theta_j \leq 1$) the critical point is negative. Hence by Kuhn-Tucker theorem, we get the corner solution, $\theta_A^* = 0$ and $\theta_B^* = 0$. In equilibrium, owners choose the incentive parameter in such a way that the managers' objective is same as the owners' i.e. to maximize the firms' profit only.

Substituting the equilibrium values of the incentive parameters ($\theta_A^* = 0$ and $\theta_B^* = 0$) in third stage equilibrium price p_i^* (equation (4)) and in the demand functions D_i^* we eliminate θ_A and θ_B from the profit functions, $\pi_i^{**} = p_i^{**} D_i^{**}$.

$$\pi_i^{**} = \frac{(1 - x_i - x_j)(3 + x_i - x_j)^2}{18} \quad (8)$$

In the first stage, firm owners will maximize the profit function with respect to location choices, x_A and x_B . Restricting the location choices of the firm ($0 \leq x_A \leq 1$ and $0 \leq 1 - x_B \leq 1$) and using standard optimization techniques we get the equilibrium location choices as, $x_A^* = x_B^* = 0$.

Result 1: *In a constrained Hotelling's model with quadratic transportation costs and Relative Performance based partial managerial delegation, where the owner decides location in the first stage and contract design in the second stage, the optimal locations (degree of product differentiation) are $x_A^* = 0$ and $1 - x_B^* = 1$. Consequently, the equilibrium outcomes are such that, $\theta_A^* = \theta_B^* = 0$; $p_A^* = p_B^* = 1$; $D_A^* = D_B^* = 1/2$; $\pi_A^* = \pi_B^* = 1/2$.*

4 First stage: Incentive Contract, Second stage: Location

In *incentive-then-location* game the order of owners' decisions is reversed. That is, in the first stage they design the incentive contract and then in the second stage they choose the location of their firms.

4.1 Second stage: Location

We start our analysis from the second stage of the game. Owners will independently choose x_i to maximize their own profit, $\pi_i^* = p_i^* D_i^*$. We again use the Kuhn-Tucker theorem to get the first order conditions:

$$\frac{\partial \pi_i^*}{\partial x_i} = p_i^* \frac{\partial D_i^*}{\partial x_i} + D_i^* \frac{\partial p_i^*}{\partial x_i} \leq 0 \quad (9)$$

$$\frac{\partial \pi_i^*}{\partial x_i} x_i = 0 \quad (10)$$

with $\frac{\partial \pi_i^*}{\partial x_i} < 0$ when $x_i = 0$ and $\frac{\partial \pi_i^*}{\partial x_i} = 0$ when $x_i > 0$.

Solving equation (9) we get the critical point equations $x_i^*(\theta_i, \theta_j, x_j)$.² Analyzing the critical point equation we get the following set of inequalities ($\frac{\partial x_i^*}{\partial x_j} \leq 0$, $\frac{\partial x_i^*}{\partial \theta_i} \geq 0$, $\frac{\partial x_i^*}{\partial \theta_j} \leq 0$). The reaction function or the best response (BR) of firm i is given by:

$$x_i^{\text{BR}}(\theta_i, \theta_j, x_j) = \max\{0, x_i^*(\theta_i, \theta_j, x_j)\} \quad (11)$$

The reaction functions give the best response of the firms as a function of incentive contracts and the location choice of the rival firm. Next we solve for the equilibrium location choices which is given by the intersection of the two best response functions.

Consider the case where $\theta_i \geq \theta_j$. Analytical analysis confirm that $x_j^{\text{BR}}(\theta_i, \theta_j, x_i) = 0$. When $x_j = 0$, $x_i^{\text{BR}}(\theta_i, \theta_j, 0) = \max\{0, x_i^*(\theta_i, \theta_j, 0)\}$ where

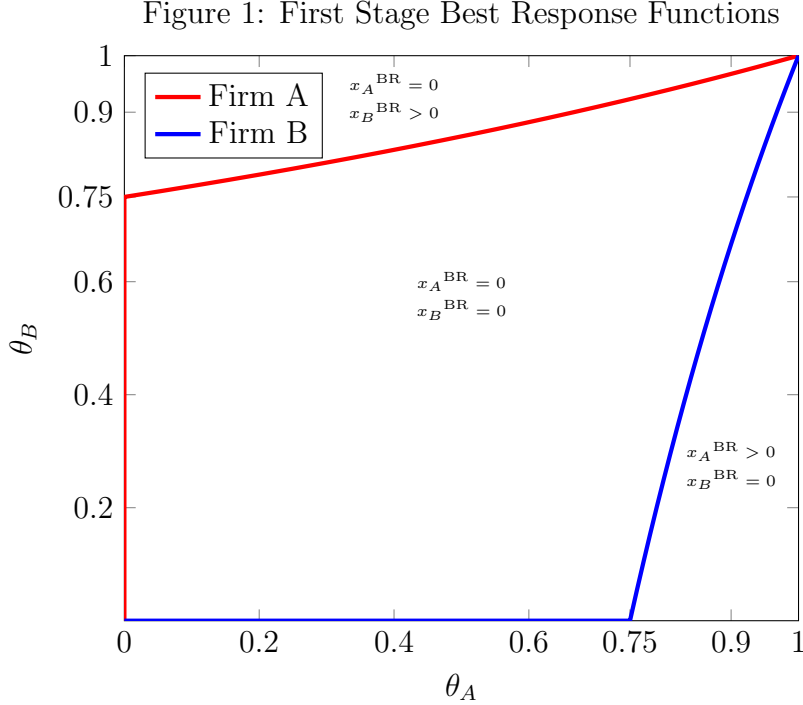
$$x_i^*(\theta_i, \theta_j, 0) = \frac{\theta_i^2 \theta_j + 2\theta_i^2 - 3\theta_i \theta_j + 3\theta_i + 5 - 2\sqrt{\theta_i^4 \theta_j^2 - 2\theta_i^4 \theta_j + \theta_i^4 - 6\theta_i^3 \theta_j + 6\theta_i^3 + 3\theta_i^2 \theta_j^2 - 2\theta_i^2 \theta_j + 11\theta_i^2 - 6\theta_i \theta_j + 6\theta_i + 4}}{3(\theta_i^2 \theta_j + \theta_i \theta_j + \theta_i - 1)} \quad (12)$$

The equilibrium location choices are $(x_A^{\text{BR}}(\theta_A, \theta_B, 0), 0)$ if $\theta_A \geq \theta_B$ and $(0, x_B^{\text{BR}}(\theta_A, \theta_B, 0))$ if $\theta_B \geq \theta_A$.

²There are two critical points for each firm. We select the one such that $\frac{\partial^2 \pi_i^*}{\partial x_i^2} \leq 0$.

4.2 First stage: Incentive Contract

Equilibrium analysis in the first stage proceeds as follows. The set $\{(\theta_A, \theta_B) : 0 \leq \theta_A \leq 1; 0 \leq \theta_B \leq 1\}$ can be partitioned into 3 subsets depending on the second stage equilibrium location choices: $(x_A^{\text{BR}} = 0, x_B^{\text{BR}} = 0)$, $(x_A^{\text{BR}} > 0 \text{ and } x_B^{\text{BR}} = 0)$ and $(x_A^{\text{BR}} = 0 \text{ and } x_B^{\text{BR}} > 0)$. Figure 1 provides a graphical representation of the set.



First consider the case where $(x_A^{\text{BR}} > 0 \text{ and } x_B^{\text{BR}} = 0)$. Substituting the equilibrium location choices in equilibrium price and demand equations we get the profit equation as a function of incentive parameters θ_A and θ_B :

$$\pi_A^{**}(\theta_A, \theta_B) = \frac{(1-x_A^{\text{BR}})(4-(1+\theta_A)(1-x_A^{\text{BR}}))}{4-(1-\theta_A)(1-\theta_B)} \left\{ \frac{(1+\theta_A)(1-x_A^{\text{BR}})-(1+\theta_B)(1+x_A^{\text{BR}})}{8-2(1-\theta_A)(1-\theta_B)} + \frac{1+x_A^{\text{BR}}}{2} \right\} \quad (13)$$

Using standard optimization technique we find that in this region the optimal solution is at the lower boundary point ($\frac{\partial \pi_A^{**}}{\partial \theta_A} < 0$). For each θ_B the optimal θ_A in this region is given by the equation:

$$x_A^*(\theta_A(\theta_B), \theta_B, 0) = 0 \quad (14)$$

Now consider the case where ($x_A^{\text{BR}} = 0$ and $x_B^{\text{BR}} = 0$). Substituting the equilibrium location choices in equilibrium price and demand equations we get the profit equation as a function of incentive parameters θ_A and θ_B :

$$\pi_A^{**}(\theta_A, \theta_B) = \frac{3 - \theta_A}{4 - (1 - \theta_A)(1 - \theta_B)} \left\{ \frac{\theta_A - \theta_B}{2(4 - (1 - \theta_A)(1 - \theta_B))} + \frac{1}{2} \right\} \quad (15)$$

Using standard optimization technique we find that in this region the optimal solution is at the lower boundary point ($\frac{\partial \pi_A^{**}}{\partial \theta_A} < 0$). For each θ_B the optimal θ_A in this region is given by the equation:

$$\begin{aligned} \theta_A(\theta_B) &= 0 & \text{if } \theta_B \leq 0.75 \\ x_B^*(\theta_A(\theta_B), \theta_B, 0) &= 0 & \text{if } \theta_B > 0.75 \end{aligned} \quad (16)$$

Finally consider the case where ($x_A^{\text{BR}} = 0$ and $x_B^{\text{BR}} > 0$). Substituting the equilibrium location choices in equilibrium price and demand equations we get the profit equation as a function of incentive parameters θ_A and θ_B :

$$\pi_A^{**}(\theta_A, \theta_B) = \frac{(1 - x_B^{\text{BR}})(4 - (1 + \theta_A)(1 + x_B^{\text{BR}}))}{4 - (1 - \theta_A)(1 - \theta_B)} \left\{ \frac{(1 + \theta_A)(1 + x_B^{\text{BR}}) - (1 + \theta_B)(1 - x_B^{\text{BR}})}{8 - 2(1 - \theta_A)(1 - \theta_B)} + \frac{1 - x_B^{\text{BR}}}{2} \right\} \quad (17)$$

Using standard optimization technique we find that in this region the optimal solution is at the upper boundary point ($\frac{\partial \pi_A^{**}}{\partial \theta_A} > 0$). For each θ_B the optimal θ_A in this region is given by the equation:

$$x_B^*(\theta_A(\theta_B), \theta_B, 0) = 0 \quad (18)$$

Hence for each $\theta_B \in [0, 1]$ we have to compare Firm A's profit at two θ_A values to determine the best response θ_A^* . When $\theta_B \leq 0.75$ the two θ_A values are 0 and θ_A such that $x_A^*(\theta_A(\theta_B), \theta_B, 0) = 0$. When $\theta_B > 0.75$ the two θ_A values are θ_A such that $x_B^*(\theta_A(\theta_B), \theta_B, 0) = 0$ and θ_A such that $x_A^*(\theta_A(\theta_B), \theta_B, 0) = 0$.

Analytical analysis confirms that for each θ_B the smaller of the two candidate θ_A values gives a higher profit to Firm A. Hence the best response function of Firm A as a function of Firm B's incentive parameter is :

$$\begin{aligned}\theta_A^*(\theta_B) &= 0 & \text{if } \theta_B \leq 0.75 \\ x_B^*(\theta_A^*(\theta_B), \theta_B, 0) &= 0 & \text{if } \theta_B > 0.75\end{aligned}\tag{19}$$

Similar analysis for Firm B yields the reaction function for Firm B as a function of Firm A's incentive parameter:

$$\begin{aligned}\theta_B^*(\theta_A) &= 0 & \text{if } \theta_A \leq 0.75 \\ x_A^*(\theta_A, \theta_B^*(\theta_A), 0) &= 0 & \text{if } \theta_A > 0.75\end{aligned}\tag{20}$$

Best response function of the two firms is shown in Figure 1.

Result 2: *In a constrained Hotelling's model with quadratic transportation costs and Relative Performance based partial managerial delegation, where the owner decides contract design in the first stage and location in the second stage, Relative Performance parameters are strategic complements i.e. $\frac{\partial \theta_i}{\partial \theta_j} \geq 0$.*

The intuition behind the result is as follows. When the rival owner gives more weightage to relative performance, it incentivizes their manager to be more aggressive. The manager responds by decreasing the price of their product and capturing some of the market share of other firm. The optimal response by the owner is to increase the weight of relative performance in his contract.

Solving them simultaneously we get two equilibrium incentive contract choices : $\theta_A^* = \theta_B^* = 0$ and $\theta_A^* = \theta_B^* = 1$.

Result 3: *In a constrained Hotelling's model with quadratic transportation costs and Relative Performance based partial managerial delegation, where the owner decides contract design in the first stage and location in the second stage, the optimal locations (degree of*

product differentiation) are $x_A^* = 0$ and $1 - x_B^* = 1$. The two equilibrium outcomes are $\theta_A^* = \theta_B^* = 0$; $p_A^* = p_B^* = 1$; $D_A^* = D_B^* = 1/2$; $\pi_A^* = \pi_B^* = 1/2$ and $\theta_A^* = \theta_B^* = 1$; $p_A^* = p_B^* = 1/2$; $D_A^* = D_B^* = 1/2$; $\pi_A^* = \pi_B^* = 1/4$.

5 Conclusion

In this paper, we analyze the impact of partial managerial delegation and relative performance-based incentives on firm behavior in a Hotelling duopoly with quadratic transportation costs. We consider two sequence of owners' decision —*location-then-contract* or *contract-then-location*—and demonstrate that the maximum differentiation principle holds robustly across both settings. In the *location-then-contract* game, owners design contracts that align managerial incentives with pure profit maximization, leading to maximum differentiation. Conversely, in the *contract-then-location* game, the interplay of strategic complementarities in incentive design results in multiple equilibria, including one where owners fully adopt relative performance-based incentives and another where they focus solely on profit maximization. Despite the differences in contract choices, the equilibrium locations remain unchanged, underscoring the resilience of maximum differentiation. These results contribute to the literature on strategic delegation by exploring the interactions between managerial incentives and firm strategies in duopoly markets.

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