Explore or Exploit: A Revealed Preference Approach to Preference Discovery

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Abstract

This paper relaxes the standard revealed preference conditions to account for preference discovery, where consumers may be initially unaware of their preferences for certain goods and only discover them through consumption. We develop a set of necessary and sufficient conditions for rationalizability under this assumption by modifying Afriat's inequalities to include additional parameters that capture the process of preference learning. We further extend the model to allow for rationalization by weakly separable utility functions, offering a structured analysis of intra- and inter-group trade-offs between experienced and non-experienced goods. Our results provide a framework for empirical testing of consumer behavior where preferences are discovered endogenously through experience.

Keywords: Revealed Preference, Preference Discovery, Weakly-Separable Preferences

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1 Introduction

Explore-exploit decisions are ubiquitous in nature. Consider a diner choosing a meal at a restaurant: she can order her usual dish, whose taste and satisfaction are familiar (exploit), or try a completely new dish she has never tasted before (explore). While she may strictly prefer the new dish over her usual order she is uncertain about her own taste and has to discover or learn it through consumption. This situation illustrates the fundamental tension between exploring unfamiliar options to learn about one's preferences and exploiting known options to secure known rewards. In this paper we examine consumer behavior where preferences are discovered through consumption. We derive necessary and sufficient conditions under which such behavior can be rationalized, modifying the standard revealed preference conditions to account for endogenous preference learning.

Inconsistent choices are frequently observed in both empirical and experimental studies (Thaler (1980), Tversky and Kahneman (1981), Iyengar and Lepper (2000), Feldman and Rehbeck (2022), Costa-Gomes et al. (2022)). Do such inconsistencies necessarily imply irrational behavior? In this paper we argue that they do not. Rational preferences can lead to behavior that appears irrational (such as choice reversal) when individuals are uncertain of their preferences. According to the *discovered preference hypothesis* (Plott et al. (1996), Braga and Starmer (2005)) individuals may not have complete knowledge of their preferences until they discover them through consumption [1]. Without full discovery, choices may not reflect the true underlying preference ordering.

Continuing with the example of diner's choices - suppose she experiments and orders a dish she has never tried before. After tasting it, she realizes that it is not as satisfying as her usual order. On her next visit, she returns to her preferred dish. To an external observer, this choice reversal may appear irrational. Standard revealed preference analysis would interpret

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[1]The concept of preference discovery is closely related to experience goods Nelson (1970). Briefly stated experience goods are goods whose quality and attributes are hard to infer before consumption. The difference is that in preference discovery there is uncertainty about ones own taste while for experience goods there is uncertainty about the attributes of the good before consumption.

this pattern as a violation of the basic axiom of acyclic choices, leading to the conclusion that her behavior is not rationalizable. However, this line of reasoning is flawed. The observed choice reversal reflects not a reversal in preferences, but the process of discovering them. It captures the trade-off between exploration and exploitation. Experimenting with untried alternatives in order to learn one's preferences is a rational behavior by any reasonable normative standard. In this paper, we demonstrate how the standard revealed preference conditions must be relaxed to accommodate such discovery-based behavior.

Revealed preference analysis is a powerful tool to estimate welfare implications of policy interventions (Brown and Calsamiglia (2007), Deb et al. (2023), Chambers and Echenique (2024)). It is frequently used to measure behavioral response to counter-factual changes in prices and income. However, as argued above, failing to account for preference discovery in such analyses can lead to misguided conclusions about rationality and welfare. The goal of the paper is to address this problem.

We formalize preference discovery by introducing an augmented utility function defined over twice as many variables as a standard utility function. For each good i, the utility function takes two arguments: x^i, representing the experienced quantity, and y^i, the non-experienced quantity. We impose the constraint that exactly one of these two variables equals the actual quantity consumed, while the other is set to zero. Whether x^i or y^i equals the consumed quantity depends on whether the good is part of the experienced set at the time of consumption. A good enters the experienced set at time t once past cumulative consumption exceeds a fixed threshold. We refer to this mechanism, which governs the transition from non-experienced to experienced, as *threshold-based discovery.*

The advantage of an augmented utility function is that beside capturing the preferences between different consumption bundles it allows us to capture the evolving nature of consumer awareness. Also, it distinguishes between informed (experienced) and exploratory (non-experienced) choices. Using this framework we show how standard revealed preference conditions must be relaxed to accommodate preference discovery.

Additionally, when estimating the augmented utility function from a finite dataset of observed prices and quantities, it may be desirable to impose structure that reflects realistic behavioral assumptions. In particular, we consider weak separability of the augmented utility function. This allows us to test whether the observed data is consistent with utility maximization under the assumption that preferences over experienced goods are independent of the consumption of non-experienced goods. The assumption that the marginal rate of substitution between experienced goods is unaffected by non-experienced consumption is both natural and empirically plausible. We formalize rationalization under a weakly separable augmented utility function and derive testable implications for revealed preference analysis.

This paper contributes to the literature on preference discovery. Prior theoretical studies have examined the learning process by which decision makers sample new alternatives of unknown rewards (Piermont et al. (2016), Cooke (2017)) and characterized optimal experimentation strategy (Weitzman (1978), Aghion et al. (1991)). Delaney et al. (2020) conduct an experiment on preference discovery focusing on extensive margin: what is learned and what is not. They find that that participants often stop sampling after some point and fail to learn their preferences for all available alternatives. This incomplete learning leads to measurable welfare loss. We differ from the above mentioned papers as we examine preference discovery from a revealed preference perspective. [2] Rather than focusing on subjective learning process or optimal experimentation, we ask under what conditions can observed behavior, potentially shaped by evolving awareness, be rationalized as utility-maximizing.

Our paper is conceptually similar to papers that take a revealed preference approach to search (Caplin and Dean (2011), Caplin et al. (2011), Masatlioglu et al. (2012), Masatlioglu and Nakajima (2013)) and costly information acquisition (Caplin and Dean (2015), Chambers et al. (2020)). All the papers mentioned here use a choice-theoretic framework where

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[2] Ferreira and Gravel (2025) provide revealed preference characterization for various behavioral models of preference discovery. However they have a choice-theoretic framework whereas our focus is on consumer behavior.

decision maker has to choose from a menu of finite alternatives. The focus of this paper is to examine consumer behavior where the decision maker chooses a consumption bundle from a continuous budget set. We derive testable implications for the rationalization of consumer behavior similar to Richter (1966), Afriat (1967), Varian (1982, 1983) with the added component of preference discovery. [3]

On the technical side, our study is most closely related to two papers. Demuynck and Seel (2018) derive revealed preference conditions for consumer behavior where individuals use consideration sets to simplify their maximization problem. Goods outside the consideration set are constrained to be zero. Similar to their setting we partition the available goods in two subsets (experienced and non-experienced) but we do not impose the constraint that the quantity of one subset of goods has to be zero. Instead we impose the zero constraint restriction on the non-experienced quantity of experienced goods and experienced quantity of non-experienced goods.

Crawford and Polisson (2015) analyze a situation where prices for goods that are consumed with zero quantities are missing from a consumer's dataset. They derive a set of necessary and sufficient conditions for theoretical consistency in the presence of partially observed prices. They focus on solving the problem of welfare analysis in case of missing prices, which is very different from our paper. Unlike their analysis, our model does not prescribe a solution for missing prices.

The rest of the paper is organized as follows. Section II presents the testable implication for the standard model of utility maximization. In Section III we describe how the standard revealed preference conditions of Section II need to be relaxed to allow for preference discovery. In Section IV we derive the conditions for rationalization by a weakly separable augmented utility function and Section V concludes.

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[3]See Crawford and De Rock (2014) and Dziewulski et al. (2024) for excellent surveys of revealed preference analysis of consumer theory. For a textbook treatment of revealed preference theory see Chambers and Echenique (2016).

2　Revealed Preference

Revealed preference theory for consumer behavior takes as primitive a finite dataset of observed prices and quantities. A dataset is denoted by $D = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ where $\boldsymbol{p}_t \in \mathbb{R}_{++}^n$ is a strictly positive vector of prices at time period t and $\boldsymbol{q}_t \in \mathbb{R}_+^n$ is the associated vector (non-negative) of quantities purchased for each of the n different goods. A dataset records the decision-maker's chosen bundle $\boldsymbol{q}_t$ when faced with prices $\boldsymbol{p}_t$.

Definition 1 : A dataset $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ is rationalizable if there exists a locally non-satiated utility function $u : \mathbb{R}_+^n \to \mathbb{R}$ such that $\forall t \in T$,

$$\boldsymbol{q}_t \in \arg\max_{\boldsymbol{q}} u(\boldsymbol{q}) \quad subject \ to \ \ \boldsymbol{p}_t \boldsymbol{q} \leq \boldsymbol{p}_t \ \boldsymbol{q}_t$$

In other words, the decision maker's bundle of choice in each period t maximizes the utility function u over the set of all affordable consumption bundles.

Given any dataset S we can construct the directly revealed preference relation R over the set of observed consumption bundles $\{\boldsymbol{q}_t\}_{t \in T}$. The bundle $\boldsymbol{q}_t$ is directly revealed preferred to $\boldsymbol{q}_v$ (written as $\boldsymbol{q}_t R \boldsymbol{q}_v$) if $\boldsymbol{p}_t \boldsymbol{q}_t \geq \boldsymbol{p}_t \boldsymbol{q}_v$ i.e. the bundle $\boldsymbol{q}_v$ was affordable when $\boldsymbol{q}_t$ was bought. We denote the transitive closure of R as the indirectly revealed preference relation R^T i.e. $\boldsymbol{q}_t R^T \boldsymbol{q}_v$ if there exists a sequence $r, s, \ldots, w$ of observations in T such that $\boldsymbol{q}_t R \boldsymbol{q}_r$, $\boldsymbol{q}_r R \boldsymbol{q}_s$, $\ldots$, $\boldsymbol{q}_w R \boldsymbol{q}_v$. A dataset $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ satisfies the *Generalized Axiom of Revealed Preference (GARP)* if for all $\boldsymbol{q}_t R^T \boldsymbol{q}_v$ it is not the case that $\boldsymbol{p}_v \boldsymbol{q}_v > \boldsymbol{p}_v \boldsymbol{q}_t$ i.e if $\boldsymbol{q}_t$ is indirectly revealed preferred to $\boldsymbol{q}_v$, it is not the case that $\boldsymbol{q}_v$ was more expensive than $\boldsymbol{q}_t$ when $\boldsymbol{q}_v$ was bought.

The following theorem shows that GARP is both necessary and sufficient for a given dataset to be rationalizable.

Theorem 1 : *(Afriat, 1967; Varian, 1982). Let $S = \{\boldsymbol{p}^t, \boldsymbol{q}^t\}_{t \in T}$ be a finite dataset. The following statements are equivalent:*

(i) S is rationalizable by a locally non-satiated utility function.

(ii) S satisfies GARP.

(iii) For all observations $t \in T$, there exist numbers U_t and $\lambda_t > 0$ such that for all observations $t, v \in T$,

$$U_t - U_v \leq \lambda_v \, \boldsymbol{p}_v \, (\boldsymbol{q}_t - \boldsymbol{q}_v)$$

(iv) S is rationalizable by a strictly monotone, continuous and concave utility function.

Equivalence between statements (i) and (iv) shows that the hypothesis of rationalization under a concave, strictly monotonic and continuous utility function has the same testable implications as the hypothesis of rationalization under a locally non-satiated utility function.

The linear inequalities in (iii) are called the Afriat inequalities. They allow us to explicitly construct the utility levels U_t and the marginal utility of income λ_t for each time period t. Intuitively, these inequalities correspond to the first-order conditions for the maximization of a utility function subject to the budget constraint. Suppose that the dataset is rationalizable by a concave, differentiable and strictly monotone utility function. The first order condition for the constrained maximization problem is as follows:

$$\nabla u(\boldsymbol{q}_v) \leq \lambda_v \, \boldsymbol{p}_t$$

where λ_t is the Lagrangian multiplier for the budget constraint.

By concavity of the utility function we have:

$$u(\boldsymbol{q}_t) - u(\boldsymbol{q}_v) \leq \nabla u(\boldsymbol{q}_v)(\boldsymbol{q}_t - \boldsymbol{q}_v)$$

Substituting the first order conditions in the concavity restriction and letting $U_t = u(\boldsymbol{q}_t)$ we obtain the Afriat inequalities:

$$U_t - U_v = u(\boldsymbol{q}_t) - u(\boldsymbol{q}_v) \leq \nabla u(\boldsymbol{q}_v)(\boldsymbol{q}_t - \boldsymbol{q}_v) \leq \lambda_v \, \boldsymbol{p}_v(\boldsymbol{q}_t - \boldsymbol{q}_v)$$

3 Preference Discovery

In this section we model a situation where the decision maker discovers her preference through consumption. The decision maker has a 'true' underlying rational preference (complete and transitive) for all bundles of goods. However, she is uncertain about her taste and discovers it over time through consumption.

Consider a set of goods $G = \{1, 2, \ldots, n\}$. At each time period t we partition G in two subsets: experiencet set (E_t) and non-experienced set (NE_t) where;

(i) $E_t \cap NE_t = \emptyset$

(ii) $E_t \cup NE_t = G$

(Threshold-based-discovery) We define the threshold quantity for each good $\bar{q}^i$ as the total consumption required to consider it an experienced good. Specifically,

$$i \in E_t \ \text{ if } \ \sum_{v=1}^{t-1} q_v^i \geq \bar{q}^i$$

where q_v^i is the quantity consumed of good i at time period v.

We assume that agents have perfect recall and therefore if any good is in the experienced set in period t then it remains in the experienced set for all subsequent periods.

(iii) $E_t \subseteq E_v$ and $NE_t \supseteq NE_v$ $\forall v, t \in T$ such that $v \geq t$

Given a finite dataset $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ and the vector of threshold quantities $\bar{\boldsymbol{q}}$ we construct a new dataset $S_{PD} = \{(\boldsymbol{p}_t, \boldsymbol{p}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$, where $\boldsymbol{x}_t \in \mathbb{R}_+^n$ and $x_t^i = q_t^i$ if $i \in E_t$ and $x_t^i = 0$ if $i \notin E_t$. Similarly, $\boldsymbol{y}_t \in \mathbb{R}_+^n$ and $y_t^i = q_t^i$ if $i \notin E_t$ and $y_t^i = 0$ if $i \in E_t$. In words, for each observed consumption bundle $\boldsymbol{q}_t$ we construct two n-dimensional vectors, *a)* $\boldsymbol{x}_t$: experienced quantity for each good, and *b)* $\boldsymbol{y}_t$: non-experienced quantity for each good. Note that $x_t^i + y_t^i = q_t^i$ $\forall i \in G$ and $t \in T$.

The new dataset reflects the observed purchase behavior over experienced and non-experienced set given prices $\boldsymbol{p}_t$ and experienced goods set E_t.

Definition 2 : *Given the threshold quantities* $\bar{\boldsymbol{q}} \in \mathbb{R}_+^n$, *a dataset* $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ *is rationalizable with preference discovery if there exists a continuous, concave and strictly monotonic augmented utility function* $u : \mathbb{R}_+^{2n} \to \mathbb{R}$ *such that* $\forall\, t \in T$:

$$(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \arg\max_{\boldsymbol{x}, \boldsymbol{y}} u(\boldsymbol{x}, \boldsymbol{y}) \quad \textit{subject to} \ \ \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y}) \leq \boldsymbol{p}_t \ \boldsymbol{q}_t$$

$$x^i = 0 \quad \forall\, i \notin E_t$$

$$y^i = 0 \quad \forall\, i \in E_t$$

At each time period t, the decision maker's chosen bundle of experienced goods and non-experienced goods are consistent with the model of utility maximization subject to the constraints. Along with the standard budget constraint we impose additional constraints that the non-experienced quantity of experienced goods and experienced quantity of non-experienced goods are zero.

Definition 2 is different from Definition 1 in two main aspects. *First*, the augmented utility function in Definition 2 is defined over $2n$ variables unlike the utility function in Definition 1 which is defined over n variables. The increase in dimensionality allows us to recover preferences between exploration and exploitation along with the preferred tradeoffs within experienced and non-experienced sets.

Secondly, there are n additional constraints in Definition 2 compared to Definition 1. Note that in the maximization problem the constrained set is restricted to lie in a n-dimensional linear subspace of the full budget set. Essentially, this keeps the dimension of maximization problem unchanged between the two models.

We want to emphasize here that constructing the new dataset $S_{DP} = \{(\boldsymbol{p}_t, \boldsymbol{p}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$ from $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ do not require a huge amount of additional data. Once we fix the threshold quantities $\bar{\boldsymbol{q}}$ and the set of experienced goods in time period 1 (E_1), it is straightforward to construct S_{DP}.

We have the following main result:

Theorem 2 *Let $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ be a finite dataset with the threshold quantities $\bar{\boldsymbol{q}}$. The following statements are equivalent:*

(i) *S is rationalizable with preference discovery.*

(ii) *For all observations $t \in T$ and $i \in G$, there exist numbers U_t, $\lambda_t > 0$ and $\phi_t^i \geq 0$ such that for all observations $t, v \in T$,*

$$U_t - U_v \leq \lambda_v \sum_{i \in E_v} p_v^i (x_t^i - x_v^i) + \lambda_v \sum_{i \notin E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(x_t^i - x_v^i) +$$
$$\lambda_v \sum_{i \notin E_v} p_v^i (y_t^i - y_v^i) + \lambda_v \sum_{i \in E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(y_t^i - y_v^i)$$

(iii) *For all observations $t \in T$, there exist numbers U_t, $\lambda_t > 0$ and vectors $\boldsymbol{P}_t \in \mathbb{R}_{++}^n$, $\boldsymbol{Q}_t \in \mathbb{R}_{++}^n$ such that for all observations $t, v \in T$,*

$$U_t - U_v \leq \lambda_v \boldsymbol{P}_v (\boldsymbol{x}_t - \boldsymbol{x}_v) + \lambda_v \boldsymbol{Q}_v (\boldsymbol{y}_t - \boldsymbol{y}_v)$$

where, for all $t \in T$ and $i \in G$: $P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.

(iv) *There exist vectors $\boldsymbol{P}_t \in \mathbb{R}_{++}^n$ and $\boldsymbol{Q}_t \in \mathbb{R}_{++}^n$ such that $\{(\boldsymbol{P}_t, \boldsymbol{Q}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$ satisfies GARP where, for all $t \in T$ and $i \in G$: $P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.*

(v) *There exist vectors $\boldsymbol{P}_t \in \mathbb{R}_{++}^n$ and $\boldsymbol{Q}_t \in \mathbb{R}_{++}^n$ such that $\{(\boldsymbol{P}_t, \boldsymbol{Q}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$ is rationalizable by a continuous, concave and strictly monotonic utility function $u : \mathbb{R}_+^{2n} \to \mathbb{R}$ where, for all $t \in T$ and $i \in G$: $P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.*

Theorem 2 establishes a set of necessary and sufficient conditions for maximizing behavior in the presence of preference discovery. In the remaining part of the section we provide a brief explanation of the theorem. Proof is deferred to Appendix A.

The inequalities in statement (ii) are similar to the Afriat inequalities. The difference between these inequalities and the ones in standard model is that with preference discovery

we have n additional unknowns. Intuitively, these can be interpreted as the Lagrangian multiplier associated with the n zero constraints. Note that the inequalities are still linear in the unknowns $(U_t, \lambda_t, \phi_t^i)$.

Statement (iii) inequalities are equivalent to Afriat inequalities with prices for zero constrained quantities replaced with 'shadow' prices, $\boldsymbol{P_t}$ and $\boldsymbol{Q}_t$. The intuition behind it is that since certain quantities are constrained to be zero, we can construct shadow prices (greater than actual prices) such that the optimal quantity without the constraint is zero. It is straightforward to see that under the assumption of monotonic utility function, the shadow prices are not unique.

Statement (iv) establishes GARP conditions for rationalizability with preference discovery. The difference from the standard model is that instead of imposing GARP on the original dataset S, rationalizability is equivalent to GARP on the dataset $\{(\boldsymbol{P}_t, \boldsymbol{Q}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$, where prices for zero- constrained quantities are replaced with shadow prices.

Finally, statement (v) provides an equivalent condition for rationalizability with preference discovery using the standard model of utility maximization

4 Weakly Separable Preferences

A natural restriction that one may wish to impose on the form of augmented utility function with preference discovery is weak separability. In this section we derive the conditions needed on the observed behavior for it to be rationalizable by a weakly separable utility function.

A utility function is weakly separable if there is a group of goods such that the marginal rate of substitution between any two goods in that group is independent of the quantity consumed of any good outside the group. This group of goods is referred to as the weakly separable group.

We extend the standard revealed preference analysis for weakly separable utility function to our model of utility maximization with preference discovery. In this model separability is

imposed on the experienced quantity of goods, $\boldsymbol{x}$. The restriction implies that the marginal rate of substitution between any two experienced goods is independent of the quantities consumed of the non-experienced goods.

Intuitively what this means is that the within group tradeoff/preferences for the experienced goods can be described independently of the preference between explore and exploit as well as the tradeoff between non-experienced goods. Consequently we can define a sub-utility function over the experienced quantities of goods. To determine the demanded quantities it suffices to know the prices of the experienced goods and the total outlay for exploitation by the decision maker.

We first restate the known revealed preference conditions for the standard model that imposes weak separability on the utlity function. Then we state the conditions for weakly separable utility maximization with preference discovery.

Consider the set of goods $G = \{1, 2, \ldots, n\}$ and a finite dataset $S = \{(\boldsymbol{p}_t, \boldsymbol{\pi}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$. For all $t \in T$, $\boldsymbol{x}_t \in \mathbb{R}_+^{n_1}$ is the bundle of goods in the weakly separable group and $\boldsymbol{y}_t \in \mathbb{R}_+^{n_2}$ is the bundle of remaining goods bought in period t, where $n_1 + n_2 = n$. Similarly, we partition the price vector into a price vector for goods in the separable group $\boldsymbol{p}_t$ and a price vector for the remaining goods $\boldsymbol{\pi}_t$. The dataset is rationalized by a utility function separable in $\boldsymbol{x}-$ goods, u, if there exist sub-utility function v and macro-utility function $\bar{u}$ such that:

$$u(\boldsymbol{x}, \boldsymbol{y}) = \bar{u}(v(\boldsymbol{x}), \boldsymbol{y})$$

Formally, we have the following definition for rationalizability:

Definition 3 *A datsaet $S = \{(\boldsymbol{p}_t, \boldsymbol{\pi}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$ is rationalizable by a weakly -separable utility function if there exists a continuous, concave and monotonic sub-utility function $v :$ $\mathbb{R}_+^{n_1} \to \mathbb{R}$ and a continuous, concave, and monotonic 'macro' utility function $\bar{u} : \mathbb{R}_+^{n_2+1} \to \mathbb{R}$ such that;*

$$\boldsymbol{x}_t \in \arg\max_{\boldsymbol{x}} v(\boldsymbol{x}) \quad \text{subject to} \ \ \boldsymbol{p}_t \boldsymbol{x} \leq \boldsymbol{p}_t \ \boldsymbol{x}_t \, ; \ \ and$$

$$(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \arg\max_{\boldsymbol{x}, \boldsymbol{y}} \bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \quad \textit{subject to} \quad \boldsymbol{p}_t \boldsymbol{x} + \boldsymbol{\pi}_t \boldsymbol{y} \leq \boldsymbol{p}_t \boldsymbol{x}_t + \boldsymbol{\pi}_t \ \boldsymbol{y}_t$$

The following theorem by Varian (1983) provide restrictions on the observed behavior required for rationalizability by a weakly separable utility function.

Theorem 3 *(Varian, 1983). Let $S = \{(\boldsymbol{p}_t, \boldsymbol{\pi}_t), (\boldsymbol{x}_t, \boldsymbol{y}_t)\}_{t \in T}$ be a finite dataset. The following statements are equivalent:*

(i) *S is rationalizable by a weakly separable utility function.*

(ii) *For all observations $t \in T$, there exist numbers U_t, V_t, $\lambda_t > 0$ and $\mu_t > 0$ such that for all observations $t, v \in T$:*

$$U_t - U_v \leq \lambda_v \, \boldsymbol{\pi}_v \, (\boldsymbol{y}_t - \boldsymbol{y}_v) + \frac{\lambda_v}{\mu_v}(V_t - V_v)$$

$$V_t - V_v \leq \mu_v \boldsymbol{p}_v (\boldsymbol{x}_t - \boldsymbol{x}_v)$$

(iii) *The dataset $\{\boldsymbol{p}_t, \boldsymbol{x}_t\}_{t \in T}$ and $\{(1/\mu_t, \boldsymbol{\pi}_t), (V_t, \boldsymbol{y}_t)\}_{t \in T}$ satisfy GARP for some choice of (V_t, μ_t) that satisfy the Afriat inequalities*

Now we state our definition of rationalization by a weakly separable augmneted utility under preference discovery and a theorem characterizing the rationalizable behavior.

Definition 4 *: Given the threshold quantities $\bar{\boldsymbol{q}} \in \mathbb{R}^n_+$, a dataset $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ is rationalizable by a weakly separable augmented utility function under preference discovery if:*

(i) *There exists a continuous, concave and strictly monotonic sub-utility function $v : \mathbb{R}^n_+ \to \mathbb{R}$ such that $\forall t \in T$:*

$$\boldsymbol{x}_t \in \arg\max_{\boldsymbol{x}} v(\boldsymbol{x}) \quad \textit{subject to} \quad \boldsymbol{p}_t \boldsymbol{x} \leq \boldsymbol{p}_t \boldsymbol{x}_t$$

$$x^i = 0 \quad \forall i \notin E_t$$

(ii) *There exists a continuous, concave and strictly monotonic macro utility function $\bar{u}$:* $\mathbb{R}^{n+1}_+ \to \mathbb{R}$ *such that* $\forall\, t \in T$:

$$(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \arg\max_{\boldsymbol{x}, \boldsymbol{y}} \bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \quad \text{subject to} \quad \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y}) \le \boldsymbol{p}_t\, \boldsymbol{q}_t$$

$$x^i = 0 \quad \forall\, i \notin E_t$$

$$y^i = 0 \quad \forall\, i \in E_t$$

We re-emphasize that once we fix $\bar{\boldsymbol{q}}$, it is straightforward to construct $\{(\boldsymbol{x}_t, \boldsymbol{y_t})\}_{t \in T}$ from $\{\boldsymbol{q}_t\}_{t \in T}$. For all $t \in T$, $x^i_t = q^i_t$ if $i \in E_t$ otherwise $x^i_t = 0$. Similarly, $y^i_t = q^i_t$ if $i \notin E_t$ otherwise $y^i_t = 0$.

We have the following main result:

Theorem 4 *Let* $S = \{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t \in T}$ *be a finite dataset with the threshold quantities* $\bar{\boldsymbol{q}}$. *The following statements are equivalent:*

(i) *S is rationalizable by a weakly separable augmented utility function under preference discovery.*

(ii) *For all observations $t \in T$, there exist numbers U_t, V_t, $\lambda_t > 0$, $\mu_t > 0$, $\phi^i_t \ge 0$ $(i \in E_t)$ and $\gamma^i_t \ge 0$ $(i \notin E_t)$ such that for all observations $t, v \in T$:*

$$U_t - U_v \le \lambda_v \sum_{i \notin E_v} p^i_v(y^i_t - y^i_v) + \lambda_v \sum_{i \in E_v}\left(p^i_v + \frac{\phi^i_v}{\lambda_v}\right)(y^i_t - y^i_v) + \frac{\lambda_v}{\mu_v}(V_t - V_v)$$

$$V_t - V_v \le \mu_v \sum_{i \in E_v} p^i_v(x^i_t - x^i_v) + \mu_v \sum_{i \notin E_v}\left(p^i_v + \frac{\gamma^i_v}{\lambda_v}\right)(x^i_t - x^i_v)$$

(iii) *For all observations $t \in T$ there exist numbers U_t, V_t, $\lambda_t > 0$, $\mu_t > 0$ and vectors $\boldsymbol{P}_t \in \mathbb{R}^n_{++}$, $\boldsymbol{Q}_t \in \mathbb{R}^n_{++}$ such that for all observations $t, v \in T$:*

$$U_t - U_v \le \lambda_v \boldsymbol{Q}_v(\boldsymbol{y}_t - \boldsymbol{y}_v) + \frac{\lambda_v}{\mu_v}(V_t - V_v)$$

$$V_t - V_v \leq \mu_v \boldsymbol{P}_v(\boldsymbol{x}_t - \boldsymbol{x}_v)$$

where, for all $t \in T$ and $i \in G : P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.

(iv) For all observations $t \in T$ there exist vectors $\boldsymbol{P}_t \in \mathbb{R}_{++}^n$ and $\boldsymbol{Q}_t \in \mathbb{R}_{++}^n$ such that the dataset $\{\boldsymbol{P}_t, \boldsymbol{x}_t\}_{t \in T}$ and $\{(1/\mu_t, \boldsymbol{Q}_t), (V_t, \boldsymbol{y}_t)\}_{t \in T}$ satisfy GARP for some choice of (V_t, μ_t) that satisfy the inequalities in (iii) where for all $t \in T$ and $i \in G : P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.

Proof is given in Appendix B.

One thing to note here is that in contrast to the inequalities in Theorem 3, the inequalities in Theorem 4 are non-linear in the unknowns. Hence finding the solution is a non-linear programming problem and therefore computationally burdensome [4].

5   Conclusion

This paper develops a theoretical framework for analyzing consumer choice under preference discovery, extending the standard model of utility maximization to accommodate situations where agents have taste uncertainty. The key innovation lies in incorporating shadow prices and threshold constraints into the revealed preference framework, leading to a set of necessary and sufficient conditions for rationalizability.

In Theorem 2 we have shown that observed behavior can be rationalized under preference discovery if the dataset satisfies a modified version of Afriat's inequalities, incorporating additional parameters to represent the endogenous discovery of preferences. In one of the equivalent formulations, these parameters appear as shadow prices. These shadow prices play a critical role in adapting standard revealed preference techniques (such as GARP) to characterize rationalizability in this extended setting.

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[4]Cherchye et al. (2015) provide an integer programming approach to the test of weak separability. We do not show it formally but a similar approach can be used to derive revealed preference conditions under preference discovery.

Further, in Theorem 4, we extended the analysis to encompass rationalizability by weakly separable augmented utility functions. This extension allows for a more structured interpretation of the consumer's decision process, distinguishing between goods in the experienced set and those in the non-experienced set. Our characterization captures both intra-group trade-offs and overall utility maximization illustrating how the realistic assumption of separability affects the testable implications.

Our framework bridges dynamic learning models with revealed preference theory and contributes to a richer understanding of consumer behavior by accounting for a more realistic assumption: that consumers have uncertain preferences and discover it through experience. By transforming observed consumption into exploration and exploitation components, our characterization allows us to distinguish between preference uncertainty and bounded rationality due to limited attention.

An important avenue for future research is the application of our results to experimental data. Learning and gradual stability of choices are common observations in experimental studies. The framework developed here provides testable conditions for rationalizability under preference discovery, which can be directly implemented in experimental settings where information is revealed through consumption. Experiments that study how agents explore and update their choices over time can provide critical empirical validation of the model, as well as insights into how individuals discover preferences in unfamiliar environments. Our results offer a framework for designing such experiments and a rationale for observed deviation from rationality.

Another direction for future research is to explore alternative preference discovery mechanisms beyond the *threshold-based-discovery* used here. In our model goods become experienced once cumulative consumption exceeds a fixed threshold. This is just one way to model how preferences may be discovered over time. For example, discovery could depend on recent consumption patterns, probabilistic rules, or time-varying exposure requirements. These alternatives can better capture how preferences are learned through consumption, and

deriving their testable implications for rationalizability would extend the model's relevance.

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Appendix A - Proof of Theorem 2

Proof. We will first show (i) $\Leftrightarrow$ (ii). Then we will prove the equivalence bewteen statements (ii) and (iii). Finally, we show that statements (iii), (iv) and (v) are equivalent.

(i) $\implies$ (ii) — Assume that the dataset is rationalizable. Let $u : \mathbb{R}^{2n}_+ \to \mathbb{R}$ be the differentiable, concave, and monotonic utility function that rationalizes the dataset. By the hypothesis of utility maximization the following first order conditions (FOCs) must be satisfied for all observations $t \in T$:

$$\frac{\partial u}{\partial x^i_t} \leq \lambda_t p^i_t \quad \text{for } i \in E_t \,; \quad \frac{\partial u}{\partial x^i_t} = \lambda_t p^i_t + \phi^i_t \quad \text{for } i \notin E_t$$

$$\frac{\partial u}{\partial y^i_t} \leq \lambda_t p^i_t \quad \text{for } i \notin E_t \,; \quad \frac{\partial u}{\partial y^i_t} = \lambda_t p^i_t + \phi^i_t \quad \text{for } i \in E_t$$

where, $\lambda_t > 0$ is Lagrangian multiplier for the budget constraint. It is strictly positive because of the assumption of monotonicity. $\phi^i_t \geq 0$ is the associated Lagrangian multiplier for the zero constrained quantities.

By concavity of the utility function we have the following condition:

$$u(\boldsymbol{x}_t, \boldsymbol{y}_t) \leq u(\boldsymbol{x}_v, \boldsymbol{y}_v) + \sum_{i \in E_v} \frac{\partial u}{\partial x^i_v}(x^i_t - x^i_v) + \sum_{i \notin E_v} \frac{\partial u}{\partial x^i_v}(x^i_t - x^i_v) +$$
$$\sum_{i \notin E_v} \frac{\partial u}{\partial y^i_v}(y^i_t - y^i_v) + \sum_{i \in E_v} \frac{\partial u}{\partial y^i_v}(y^i_t - y^i_v)$$

Substituting the FOCs in the concavity condition and letting $U_t = u(\boldsymbol{x}_t, \boldsymbol{y}_t)$ we get:

$$U_t \leq U_v + \lambda_v \sum_{i \in E_v} p^i_v(x^i_t - x^i_v) + \lambda_v \sum_{i \notin E_v} (p^i_v + \frac{\phi^i_v}{\lambda_v})(x^i_t - x^i_v) +$$
$$\lambda_v \sum_{i \notin E_v} p^i_v(y^i_t - y^i_v) + \lambda_v \sum_{i \in E_v} (p^i_v + \frac{\phi^i_v}{\lambda_v})(y^i_t - y^i_v)$$

We note here that differentiability is not a necessary requirement for the proof. As long as the utility function has subgradient at all points the above inequalities will hold.

(ii) $\implies$ (i) — Consider the utility function:

$$u(\boldsymbol{x}, \boldsymbol{y}) = \min_t \{U_t + \lambda_t \sum_{i \in E_t} p_t^i(x^i - x_t^i) + \lambda_t \sum_{i \notin E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(x^i - x_t^i) +$$

$$\lambda_t \sum_{i \notin E_t} p_t^i(y^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y^i - y_t^i)\}$$

Note that it is strictly monotone, continuous and concave. First we show that $u(\boldsymbol{x}_t, \boldsymbol{y}_t) = U_t$. For some $v \in T$ we have:

$$u(\boldsymbol{x}_t, \boldsymbol{y}_t) = U_v + \lambda_v \sum_{i \in E_v} p_v^i(x_t^i - x_v^i) + \lambda_v \sum_{i \notin E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(x_t^i - x_v^i) +$$

$$\lambda_v \sum_{i \notin E_v} p_v^i(y_t^i - y_v^i) + \lambda_v \sum_{i \in E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(y_t^i - y_v^i)$$

$$\implies u(\boldsymbol{x}_t, \boldsymbol{y}_t) \le U_t + \lambda_t \sum_{i \in E_t} p_t^i(x_t^i - x_t^i) + \lambda_t \sum_{i \notin E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(x_t^i - x_t^i) +$$

$$\lambda_t \sum_{i \notin E_t} p_t^i(y_t^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y_t^i - y_t^i)$$

$$\implies u(\boldsymbol{x}_t, \boldsymbol{y}_t) \le U_t$$

If the inequality was strict we would one of the inequalities in (ii). Hence we have $u(\boldsymbol{x}_t, \boldsymbol{y}_t) = U_t$. Now we must show that given any bundle $(\boldsymbol{x}, \boldsymbol{y})$ where,

$$\boldsymbol{p}_t(\boldsymbol{x}_t + \boldsymbol{y}_t) \ge \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y})$$
$$x^i = 0 \ \forall \ i \notin E_t$$
$$y^i = 0 \ \forall \ i \in E_t$$

it must be the case that $u(\boldsymbol{x}_t, \boldsymbol{y}_t) \ge u(\boldsymbol{x}, \boldsymbol{y})$.

By construction we have:

$$u(\boldsymbol{x}, \boldsymbol{y}) = \min_v \{ U_v + \lambda_v \sum_{i \in E_v} p_v^i(x^i - x_v^i) + \lambda_v \sum_{i \notin E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(x^i - x_v^i) +$$

$$\lambda_v \sum_{i \notin E_v} p_v^i(y^i - y_v^i) + \lambda_v \sum_{i \in E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(y^i - y_v^i) \}$$

$$\implies u(\boldsymbol{x}, \boldsymbol{y}) \leq U_t + \lambda_t \sum_{i \in E_t} p_t^i(x^i - x_t^i) + \lambda_t \sum_{i \notin E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(x^i - x_t^i) +$$

$$\lambda_t \sum_{i \notin E_t} p_t^i(y^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y^i - y_t^i)$$

$$\implies u(\boldsymbol{x}, \boldsymbol{y}) \leq U_t + \lambda_t \sum_{i \in E_t} p_t^i(x^i - x_t^i) + \lambda_t \sum_{i \notin E_t} p_t^i(y^i - y_t^i)$$

$$\implies u(\boldsymbol{x}, \boldsymbol{y}) \leq U_t$$

The second implication is due to the fact that $x^i = 0$ & $x_t^i = 0$ $\forall\, i \notin E_t$ and $y^i = 0$ & $y_t^i = 0$ $\forall\, i \in E_t$. We get the third implication from the way we defined $(\boldsymbol{x}, \boldsymbol{y})$: $\boldsymbol{p}_t(\boldsymbol{x}_t + \boldsymbol{y}_t) \geq \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y})$.

This completes our proof that (i) $\Leftrightarrow$ (ii). Now we we show that (ii) $\Leftrightarrow$ (iii).

(ii) $\implies$ (iii) — Suppose that for all observations $t \in T$ and $i \in G$, there exist numbers U_t, $\lambda_t > 0$ and $\phi_t^i \geq 0$ such that for all observations $t, v \in T$,

$$U_t - U_v \leq \lambda_v \sum_{i \in E_v} p_v^i(x_t^i - x_v^i) + \lambda_v \sum_{i \notin E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(x_t^i - x_v^i) +$$

$$\lambda_v \sum_{i \notin E_v} p_v^i(y_t^i - y_v^i) + \lambda_v \sum_{i \in E_v} (p_v^i + \frac{\phi_v^i}{\lambda_v})(y_t^i - y_v^i)$$

Let $P_v^i = p_v^i + \frac{\phi_v^i}{\lambda_v}$ $\forall\, i \notin E_v$ and $P_v^i = p_v^i$ $\forall\, i \in E_v$. Similarly, let $Q_v^i = p_v^i + \frac{\phi_v^i}{\lambda_v}$ $\forall\, i \in E_v$ and $Q_v^i = p_v^i$ $\forall\, i \notin E_v$. As $\lambda_t > 0$, P_v^i & Q_v^i always exist. Now we can rewrite the above inequality as

$$U_t - U_v \leq \lambda_v \, \boldsymbol{P}_v \, (\boldsymbol{x}_t - \boldsymbol{x}_v) + \lambda_v \, \boldsymbol{Q}_v \, (\boldsymbol{y}_t - \boldsymbol{y}_v)$$

where, for all $t \in T$ and $i \in G$: $P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.

(iii) $\implies$ (ii) — Suppose that for all observations $t \in T$, there exist numbers U_t, $\lambda_t > 0$

and vectors $\boldsymbol{P}_t \in \mathbb{R}^n_{++}$, $\boldsymbol{Q}_t \in \mathbb{R}^n_{++}$ such that for all observations $t, v \in T$,

$$U_t - U_v \leq \lambda_v\,\boldsymbol{P}_v\,(\boldsymbol{x}_t - \boldsymbol{x}_v)\,+\,\lambda_v\,\boldsymbol{Q}_v\,(\boldsymbol{y}_t - \boldsymbol{y}_v)$$

where, for all $t \in T$ and $i \in G$: $P_t^i = p_t^i$ if $i \in E_t$ and $Q_t^i = p_t^i$ if $i \notin E_t$.

We refer to it as Afriat type inequalities. Note that for all $i \in E_t$, the shadow prices P_t^i is not unique. Any $\hat{P}_t^i \geq P_t^i$ will also satisfy the inequality. Similarly for all $i \notin E_t$, any $\hat{Q}_t^i \geq Q_t^i$ will also satisfy the inequality.

For all $i \notin E_t$, let $\bar{P}_t^i$ be the shadow prices that satisfy the Afriat type inequalities with the added restriction that $\bar{P}_t^i \geq p_t^i$. Such a value will always exist by the reasoning given above. By the same argument, for all $i \in E_t$ there exist $\bar{Q}_t^i \geq p_t^i$ that satisfy the Afriat inequalities. Next, we define $\phi_t^i \geq 0$ as:

$$\phi_t^i = \begin{cases} \lambda_t(\bar{P}_t^i - p_t^i) & \forall i \notin E_t \\ \lambda_t(\bar{Q}_t^i - p_t^i) & \forall i \in E_t \end{cases}$$

Substituting it in the Afriat type inequalities we get:

$$U_t - U_v \leq \lambda_v \sum_{i \in E_v} p_v^i(x_t^i - x_v^i)\,+\,\lambda_v \sum_{i \notin E_v}(p_v^i + \frac{\phi_v^i}{\lambda_v})(x_t^i - x_v^i)\,+$$

$$\lambda_v \sum_{i \notin E_v} p_v^i(y_t^i - y_v^i)\,+\,\lambda_v \sum_{i \in E_v}(p_v^i + \frac{\phi_v^i}{\lambda_v})(y_t^i - y_v^i)$$

With this we have shown (ii) $\Leftrightarrow$ (iii).

(iii) $\Leftrightarrow$ (iv) $\Leftrightarrow$ (v) — This is evident from equivalence between statements (iii), (ii) and (iv) in Theorem 1.

This completes our proof for Theorem 2. ■

Appendix B - Proof of Theorem 4

Proof. We will first show (i) $\Leftrightarrow$ (ii). Then we will prove the equivalence bewteen statements (ii) and (iii). Finally, we will prove the equivalence between statements (iii) and (iv).

(i) $\implies$ (ii) — Assume that the dataset is rationalizable by a weakly separable utility function. Let $\bar{u}, u$ and v be the differentiable, concave and monotonic utility functions that

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rationalize the data, where

$$u(\boldsymbol{x}, \boldsymbol{y}) = \bar{u}(v(\boldsymbol{x}), \boldsymbol{y})$$

We will prove this in 4 steps.

Step 1: First, we state the first order conditions (FOCs) that must be satisfied under the hypothesis of utility maximization.

FOCs for utility function u :

$$\frac{\partial u}{\partial x_t^i} \leq \lambda_t p_t^i \quad \text{for } i \in E_t \,; \quad \frac{\partial u}{\partial x_t^i} = \lambda_t p_t^i + \phi_t^i \quad \text{for } i \notin E_t$$

$$\frac{\partial u}{\partial y_t^i} \leq \lambda_t p_t^i \quad \text{for } i \notin E_t \,; \quad \frac{\partial u}{\partial y_t^i} = \lambda_t p_t^i + \phi_t^i \quad \text{for } i \in E_t$$

where, for $i \in E_t$, $\frac{\partial u}{\partial x_t^i} = \lambda_t p_t^i$ if $x_t^i > 0$. The Lagrangian multiplier for the budget constraint $(\lambda_t > 0)$ is interpreted as the marginal utility of income and ϕ_t^i is the associated Lagrangian multiplier for the zero constrained quantities.

FOCs for sub-utility function v :

$$\frac{\partial v}{\partial x_t^i} \leq \mu_t p_t^i \quad \text{for } i \in E_t \,; \quad \frac{\partial v}{\partial x_t^i} = \mu_t p_t^i + \gamma_t^i \quad \text{for } i \notin E_t$$

where, for $i \in E_t$, $\frac{\partial v}{\partial x_t^i} = \mu_t p_t^i$ if $x_t^i > 0$. The Lagrangian multiplier $\mu_t > 0$ is interpreted as the marginal utility of income at $\boldsymbol{x}_t$ and γ_t^i is the associated Lagrangian multiplier for the zero constrained quantities $(i \notin E_t)$.

Step 2: Using the chain rule of differentiation on $u(\boldsymbol{x}, \boldsymbol{y}) = \bar{u}(v(\boldsymbol{x}), \boldsymbol{y})$:

$$\frac{\partial u}{\partial x_t^i} = \frac{\partial \bar{u}}{\partial x_t^i} = \frac{\partial \bar{u}}{\partial v_t} \cdot \frac{\partial v}{\partial x_t^i}$$

For some $i \in E_t$ where $x_t^i > 0$,[5] we substitute the FOCs from Step 1 in the equation above:

$$\frac{\partial \bar{u}}{\partial v_t} = \frac{\partial u / \partial x_t^i}{\partial v / \partial x_t^i} = \frac{\lambda_t p_t^i}{\mu_t p_t^i} = \frac{\lambda_t}{\mu_t}$$

---

[5]There is an implicit assumption that such an i will always exist. What this means is that at all time periods there is at least one good in the experienced set that is consumed in strictly positive quantity. Put differently, there is no period where the consumer spends all her income on exploration. If such an observation exists we can always delete it for computational purposes.

Step 3: By concavity of the macro-utility function $\bar{u}$:

$$\bar{u}(v(\boldsymbol{x}_t), \boldsymbol{y}_t) \leq \bar{u}(v(\boldsymbol{x}_m), \boldsymbol{y}_m) + \sum_{i \notin E_m} \frac{\partial \bar{u}}{\partial y_m^i}(y_t^i - y_m^i) + \sum_{i \in E_m} \frac{\partial \bar{u}}{\partial y_m^i}(y_t^i - y_m^i) + \frac{\partial \bar{u}}{\partial v_m}(v(\boldsymbol{x}_t) - v(\boldsymbol{x}_m))$$

Let $\bar{u}(v(\boldsymbol{x}_t), \boldsymbol{y}_t)$ be U_t and $v(\boldsymbol{x}_t) = V_t$. Using the fact that $\partial \bar{u}/\partial y_m^i = \partial u/\partial y_m^i$ and substituting the FOCs for u from Step 1 and substituting $\partial \bar{u}/\partial v_m$ from Step 2, we can re-write the concavity condition as:

$$U_t \leq U_m + \lambda_m \sum_{i \notin E_m} p_m^i(y_t^i - y_m^i) + \lambda_m \sum_{i \in E_m} (p_m^i + \frac{\phi_m^i}{\lambda_m})(y_t^i - y_m^i) + \frac{\lambda_m}{\mu_m}(V_t - V_m)$$

This is the first inequality in statement (ii) of Theorem 2.

Step 4: Using similar arguments as in Step 3 we can re-write the concavity condition for the sub-utility function v and get:

$$V_t - V_m \leq \mu_m \sum_{i \in E_m} p_m^i(x_t^i - x_m^i) + \mu_m \sum_{i \notin E_m} (p_m^i + \frac{\gamma_m^i}{\lambda_m})(x_t^i - x_m^i)$$

(ii) $\implies$ (i) — Consider the following sub-utility function v and the macro-utility function $\bar{u}$:

$$v(\boldsymbol{x}) = \min_t \{V_t + \mu_t \sum_{i \in E_t} p_t^i(x^i - x_t^i) + \mu_t \sum_{i \notin E_t} (p_t^i + \frac{\gamma_t^i}{\lambda_t})(x^i - x_t^i)\}$$

$$\bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) = \min_t \{U_t + \lambda_t \sum_{i \notin E_t} p_t^i(y^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y^i - y_t^i) + \frac{\lambda_t}{\mu_t}(v(\boldsymbol{x}) - V_t)\}$$

Note that v and $\bar{u}$ are strictly monotone, continuous and concave. Similar to the proof of Theorem 2 it is straightforward to show that $v(\boldsymbol{x}_t) = V_t$ and $\bar{u}(v(\boldsymbol{x}_t), \boldsymbol{y}_t) = U_t$. Now suppose that:

$$\boldsymbol{p}_t(\boldsymbol{x}_t + \boldsymbol{y}_t) \geq \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y})$$
$$x^i = 0 \ \forall \ i \notin E_t$$
$$y^i = 0 \ \forall \ i \in E_t$$

We need to show $\bar{u}(v(\boldsymbol{x_t}), \boldsymbol{y}_t) \geq \bar{u}(v(\boldsymbol{x}), \boldsymbol{y})$. By construction we have:

$$\bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) = \min_m \{ U_m + \lambda_m \sum_{i \notin E_m} p_m^i (y^i - y_m^i) + \lambda_m \sum_{i \in E_m} (p_m^i + \frac{\phi_m^i}{\lambda_m})(y^i - y_m^i)$$

$$+ \frac{\lambda_m}{\mu_m} (\min_j \{ V_j + \mu_j \sum_{i \in E_j} p_j^i (x^i - x_j^i) + \mu_j \sum_{i \notin E_j} (p_j^i + \frac{\gamma_j^i}{\mu_j}) \} - V_m) \}$$

$$\implies \bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \le U_t + \lambda_t \sum_{i \notin E_t} p_t^i (y^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y^i - y_t^i)$$

$$+ \frac{\lambda_t}{\mu_t} (V_t + \mu_t \sum_{i \in E_t} p_t^i (x^i - x_t^i) + \mu_t \sum_{i \notin E_t} (p_t^i + \frac{\gamma_t^i}{\lambda_t})(x^i - x_t^i) - V_t)$$

$$\implies \bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \le U_t + \lambda_t \sum_{i \notin E_t} p_t^i (y^i - y_t^i) + \lambda_t \sum_{i \in E_t} (p_t^i + \frac{\phi_t^i}{\lambda_t})(y^i - y_t^i)$$

$$+ \lambda_t \sum_{i \in E_t} p_t^i (x^i - x_t^i) + \lambda_t \sum_{i \notin E_t} (p_t^i + \frac{\gamma_t^i}{\mu_t})(x^i - x_t^i)$$

As $x^i = x_t^i = 0 \ \forall \, i \notin E_t$ and $y^i = y_t^i = 0 \ \forall \, i \in E_t$ we have:

$$\bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \le U_t + \lambda_t \sum_{i \notin E_t} p_t^i (y^i - y_t^i) + \lambda_t \sum_{i \in E_t} p_t^i (x^i - x_t^i)$$

$$\implies \bar{u}(v(\boldsymbol{x}), \boldsymbol{y}) \le U_t$$

where the last implication is due to the fact that $\boldsymbol{p}_t(\boldsymbol{x}_t + \boldsymbol{y}_t) \ge \boldsymbol{p}_t(\boldsymbol{x} + \boldsymbol{y})$. This completes our proof that (ii) $\implies$ (i)

(ii) $\Leftrightarrow$ (iii) — This is evident from the equivalence between statements (ii) and (iii) in Theorem 2.

(iii) $\Leftrightarrow$ (iv) — This follows from the equivalence between statements (iii) and (iv) in Theorem 2 by interpreting $1/\mu_t$ as the price and V_t as the quantity of a single aggregate commodity for the experienced goods.

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