Week three task - the Kelly Criterion

The question

The **Kelly criterion** is an investment strategy which can be derived in its simplest form using the methods we have discussed. In a simple example you have the opportunity to bet on event, that two dice roll a seven for example. If your bet is a success you get r times your stake, if it fails you get nothing. The probability of success is p. What fraction of your float should you bet each time?

The answer

So there are two values for the reward, if the bet is successful the reward is

$$S_+ = (1 - f) + fr \tag{1}$$

times the stake, if it fails S is given by

$$S_- = (1 - f) \tag{2}$$

where f is the fraction of the float that is bet. Hence, the doubling rate is

$$R = p \log_2 [(1 - f) + fr] + (1 - p) \log_2(1 - f) \tag{3}$$

To minimize this we differentiate

$$\frac{dR}{df} = \frac{p(r - 1)}{1 + (r - 1)f} - \frac{1 - p}{1 - f} \tag{4}$$

and setting this equal to zero, and for convenient $\rho = r - 1$:

$$(1 - p)(1 + \rho f) = (1 - f)p\rho \tag{5}$$

or

$$f = p - \frac{1 - p}{\rho} \tag{6}$$

This is the Kelly criterion.

For $r = 1/p$ the bet is fair and so over time you won't profit. Substituting $r = 1/p$ in the Kelly criterion tells you you should bet nothing: $f = 0$. Only if $r > 1/p$ is $f > 0$ and in that case you should bet.