Differential entropy examples: information theory lecture 7

COMSM0075 Information Processing and Brain

`comsm0075.github.io`

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Differential entropy

Differential entropy is the name given to Shannon's entropy for continuous probability distributions: distributions where the sample space is $\mathcal{X} \subseteq \mathbf{R}^d$.

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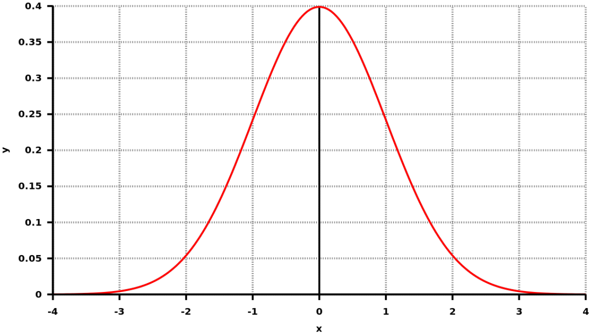

$p(x)$

Image from wikipedia.

Differential entropy

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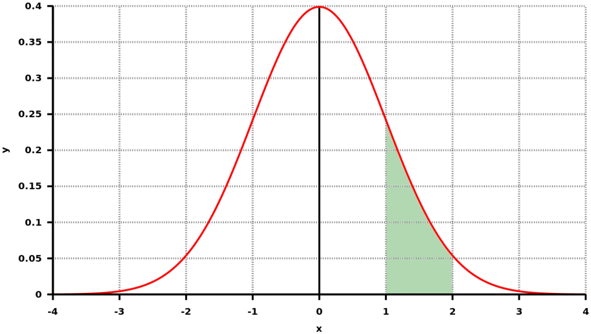


$$P(x_0 \leq x < x_1) = \int_{x_0}^{x_1} dx p(x)$$

is the probability $x_0 \leq x < x_1$.

Image from wikipedia.

Differential entropy

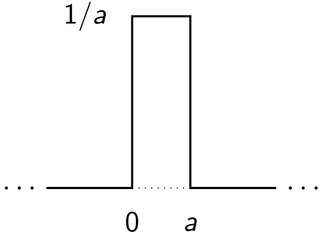
Differential entropy is the name given to Shannon's entropy for continuous probability distributions

$$h(X) = -\int dx p(x) \log_2 p(x)$$

Example - uniform

Consider a uniform distribution

$$p(x) = \begin{cases} 1/a & x \in [0, a] \\ 0 & \text{otherwise} \end{cases}$$



Example - uniform

Consider a uniform distribution

$$p(x) = \begin{cases} 1/a & x \in [0, a] \\ 0 & \text{otherwise} \end{cases}$$

so

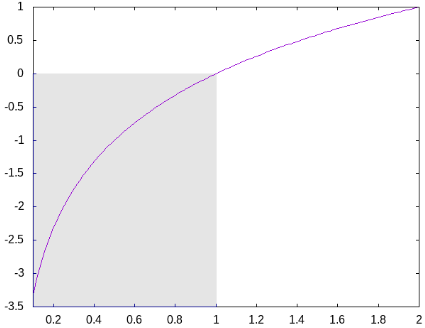
$$h(X) = -\int_{-\infty}^{\infty} dx\, p(x) \log_2 p(x) = -\frac{1}{a}\int_0^a dx\, \log_2 \frac{1}{a}$$

and so

$$h(X) = \log_2 a$$

Who ordered that!

$$h(X) = \log_2 a$$



The Gaußian distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

Substitute and integrate by parts to get

$$h(X) = \frac{1}{2} \log_2 2\pi e\sigma^2$$

where the e is just the exponential $\exp(1)$.

More of the negative values

As with the uniform distribution, this formula can give a positive or negative number depending on the size of σ*. Interestingly it can be proved that for fixed variance the Gaußian has the highest entropy.*

Densities are not probabilities

discrete case $p(x)$ is the probability of $X = x$

continuous case

$$\int_{x_0}^{x_1} dx p(x)$$

is the probability $x_0 \leq x < x_1$.

Densities are not probabilities

The usual sums and probabilities go to integrals and densities doesn't work because there is a p in the log:

$$h(X) = -\int dx p(x) \log_2 p(x)$$

Real numbers are like an infinity of numbers

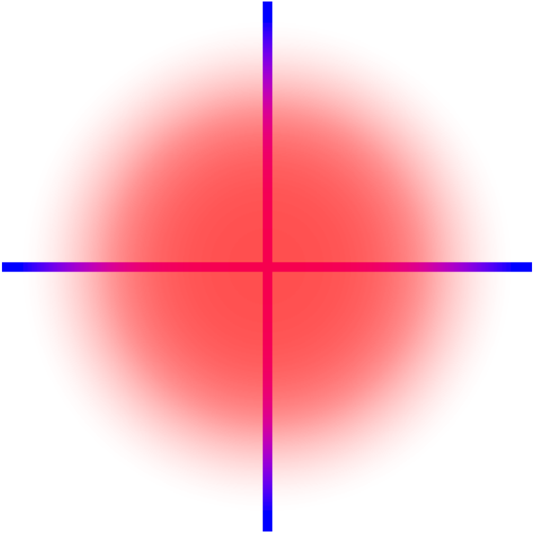
1.618033988749894848204586834365638117720309179805762862135
44862270526046281890244970720720418939113748475408807538689
17521266338622235369317931800607667263544333890865959395829
05638322661319928290267880675208766892501711696207032221043
21626954862629631361443814975870122034080588795445474924618
56953648644492410443207713449470495658467885098743394422125
44877066478091588460749988712400765217057517978834166256249
40758906970400028121042762177111777805315317141011704666599
14669798731761356006708748071013179523689427521948435305678
30022878569978297783478458782289110976250030269615617002504
64338243776486102838312683303724292675263116533924731671112
11588186385133162038400522216579128667529465490681131715993
43235973494985090409476213222981017261070596116456299098162
90555208524790352406020172799747175342777592778625619432082
7505131218156285512224809394...

We can't really see all the numbers

1.618033988749894848204586834365638117720309179805762862135
44862270526046281890244970720720418939113748475408807538689
17521266338622235369317931800607667263544333890865959395829
05638322661319928290267880675208766892501711696207032221043
21626954862629631361443814975870122034080588795445474924618
56953648644492410443207713449470495658467885098743394422125
44877066478091588460749988712400765217057517978834166256249
40758906970400028121042762177111777805315317141011704666599
14669798731761356006708748071013179523689427521948435305678
30022878569978297783478458782289110976250030269615617002504
64338243776486102838312683303724292675263116533924731671112
11588186385133162038400522216579128667529465490681131715993
43235973494985090409476213222981017261070596116456299098162
90555208524790352406020172799747175342777592778625619432082
75051312181562855122248093947...

Sensitivity

We need to model the sensitivity of the receiver as well the behaviour of the source.



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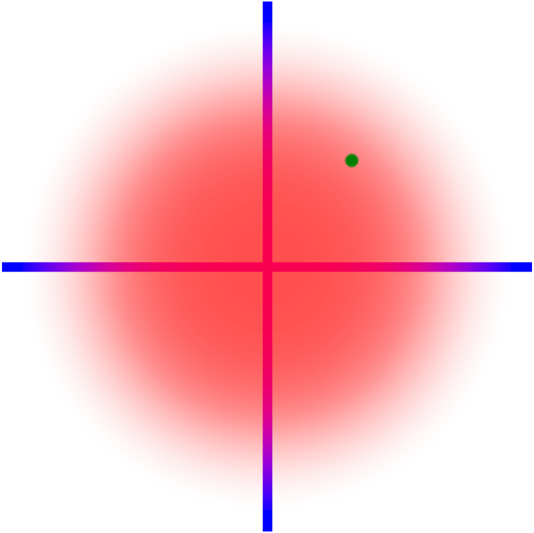

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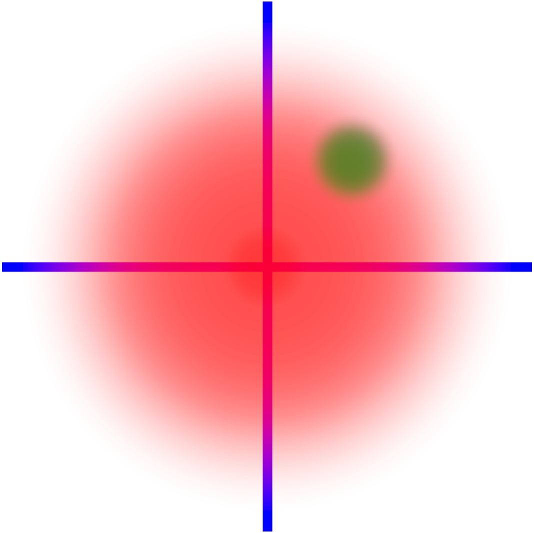

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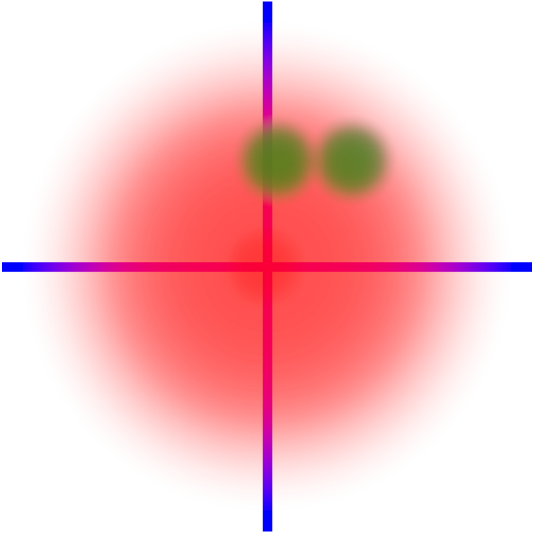

Sensitivity

Maybe $I(X, Y)$ is what we really want.