Lecture – 4

Boolean Algebra and Logic Simplifications

Lesson Outcomes

After completing this lecture, students will be able to

- ❑ Apply the basic laws and rules of Boolean algebra
- ❑ Apply DeMorgan's theorems to Boolean expressions
- ❑ Describe gate combinations with Boolean expressions and evaluate Boolean expressions
- ❑ Simplify expressions by using the laws and rules of Boolean algebra
- ❑ Convert any Boolean expression into a sum-of-products (SOP) and product-of-sums (POS) form
- ❑ Relate a Boolean expression to a truth table
- ❑ Use a Karnaugh map to simplify Boolean expressions and truth table functions
- ❑ Apply Boolean algebra and the Karnaugh map method in an application

Key Terms

- **Complement** The inverse or opposite of a number. In Boolean algebra, the inverse function, expressed with a bar over a variable.
- **“Don’t care”** A combination of input literals that cannot occur and can be used as a 1 or a 0 on a Karnaugh map for simplification.
- **Karnaugh map** An arrangement of cells representing the combinations of literals in a Boolean expression and used for a systematic simplification of the expression.
- **Product-of-sums (POS)** A form of Boolean expression that is basically the ANDing of ORed terms.
- **Product term** The Boolean product of two or more literals equivalent to an AND operation.
- **Sum-of-products (SOP)** A form of Boolean expression that is basically the ORing of ANDed terms.
- **Minimization** The process that results in an SOP or POS Boolean expression that contains the fewest possible literals per term.
- **Sum term** The Boolean sum of two or more literals equivalent to an OR operation.

Boolean theorems

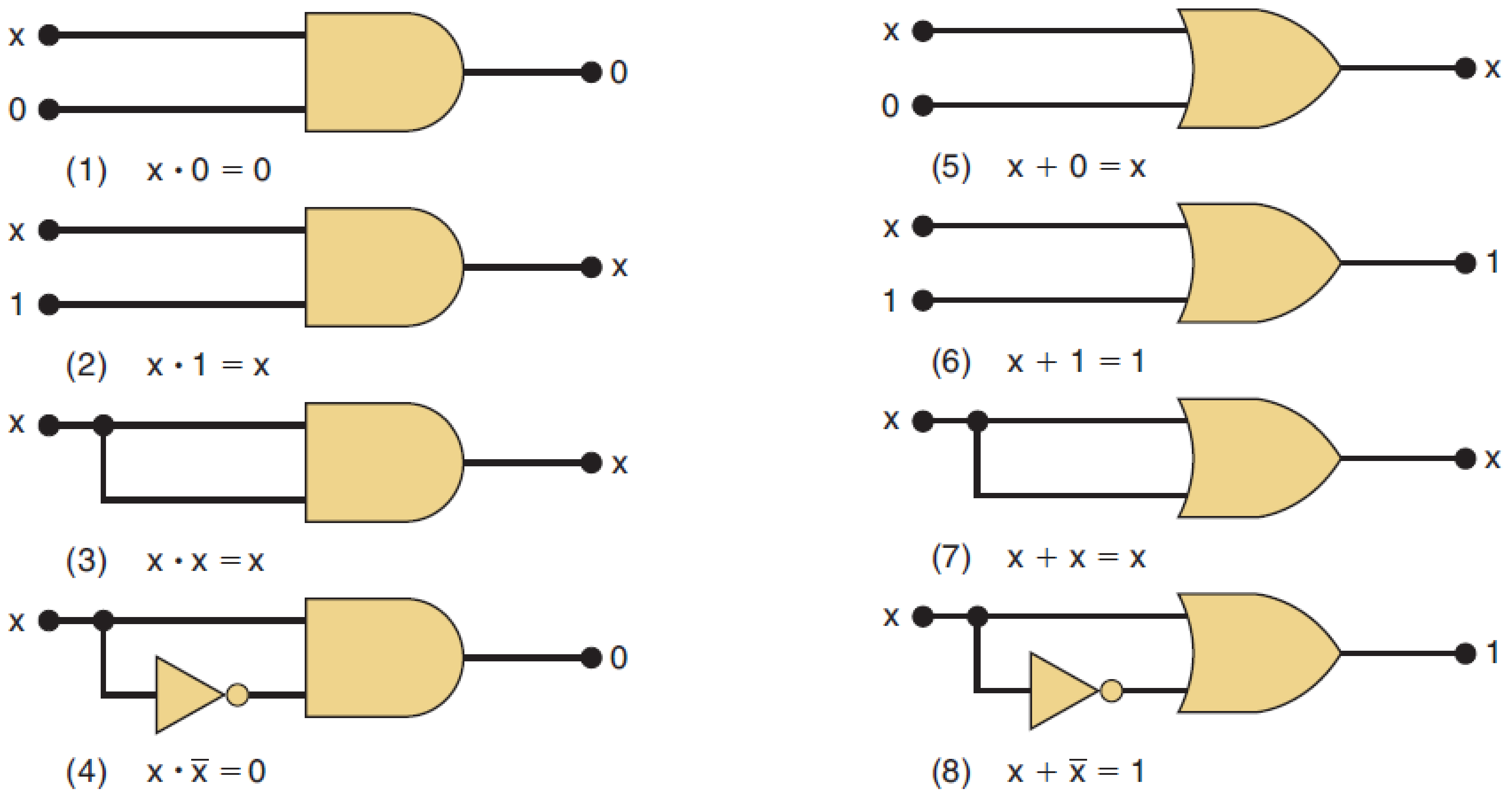


FIGURE 3-25 Single-variable theorems.

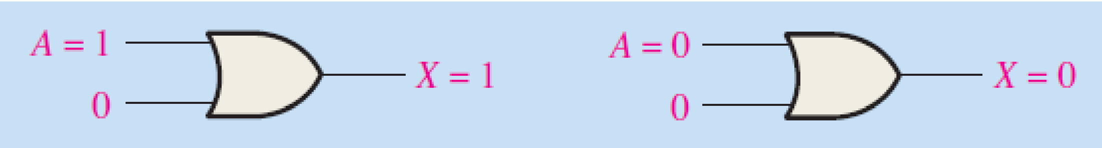
Boolean theorems

TABLE 4–1

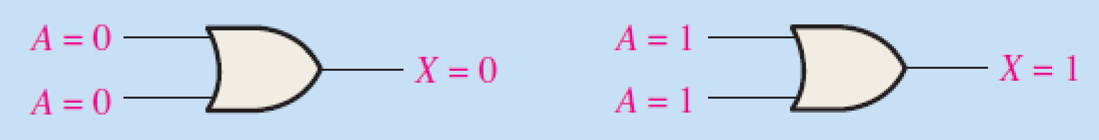
Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

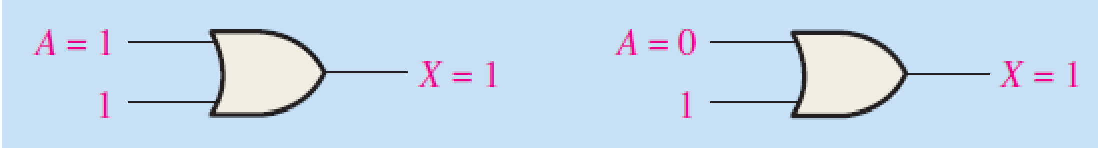
A, B, or C can represent a single variable or a combination of variables.



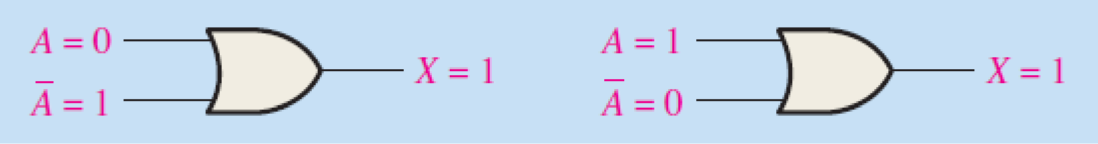
$X = A + 0 = A$



$X = A + A = A$

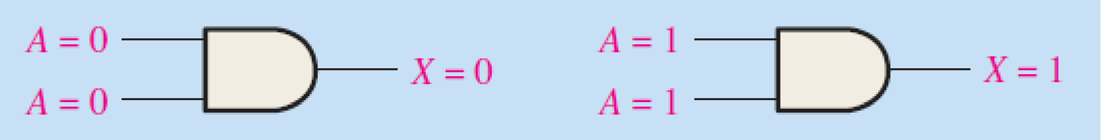


$X = A + 1 = 1$

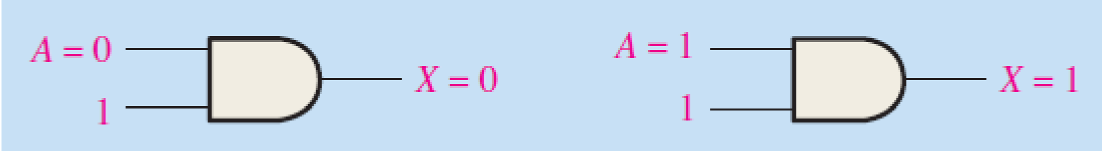


$X = A + \overline{A} = 1$

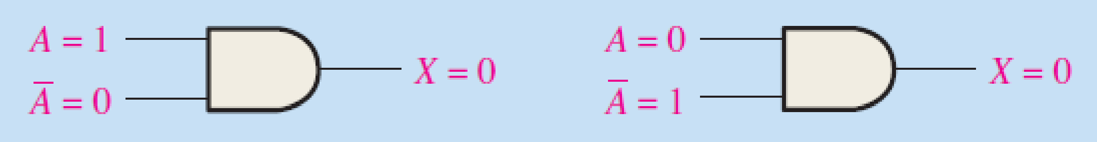
$X = A \cdot 0 = 0$



$X = A \cdot A = A$



$X = A \cdot 1 = A$



$X = A \cdot \overline{A} = 0$

Boolean theorem (contd..)

$$\overline{\overline{A}} = A$$

$$\begin{aligned} A + AB &= A \cdot 1 + AB = A(1 + B) \\ &= A \cdot 1 \\ &= A \end{aligned}$$

$$\begin{aligned} A + \overline{A}B &= (A + AB) + \overline{A}B \\ &= (AA + AB) + \overline{A}B \\ &= AA + AB + A\overline{A} + \overline{A}B \\ &= (A + \overline{A})(A + B) \\ &= 1 \cdot (A + B) \\ &= A + B \end{aligned}$$

$$\begin{aligned} (A + B)(A + C) &= AA + AC + AB + BC \\ &= A + AC + AB + BC \\ &= A(1 + C) + AB + BC \\ &= A \cdot 1 + AB + BC \\ &= A(1 + B) + BC \\ &= A \cdot 1 + BC \\ &= A + BC \end{aligned}$$

DeMorgan's theorem

- **DeMorgan's 1st Theorem:** The complement of a product of variables is equal to the sum of the complements of the variables.

$$\boxed{\overline{XY} = \overline{X} + \overline{Y}}$$

- **DeMorgan's 2nd Theorem:** The complement of a sum of variables is equal to the product of the complements of the variables.

$$\boxed{\overline{X + Y} = \overline{X}\,\overline{Y}}$$

Simplification of expression using DeMorgan's theorem

EXAMPLE 4–3

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

EXAMPLE 4–4

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

EXAMPLE 4–5

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

Solution

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan’s theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan’s theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

EXAMPLE 4–6

Apply DeMorgan’s theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{\overline{(A + B)} + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = \overline{((A + B)\overline{C}\,\overline{D})}(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

EXAMPLE 4–7

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

Solution

Start by complementing the exclusive-OR expression and then applying DeMorgan's theorems as follows:

$$\overline{A\overline{B} + \overline{A}B} = (\overline{A\overline{B}})(\overline{\overline{A}B}) = (\overline{A} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{B}) = (\overline{A} + B)(A + \overline{B})$$

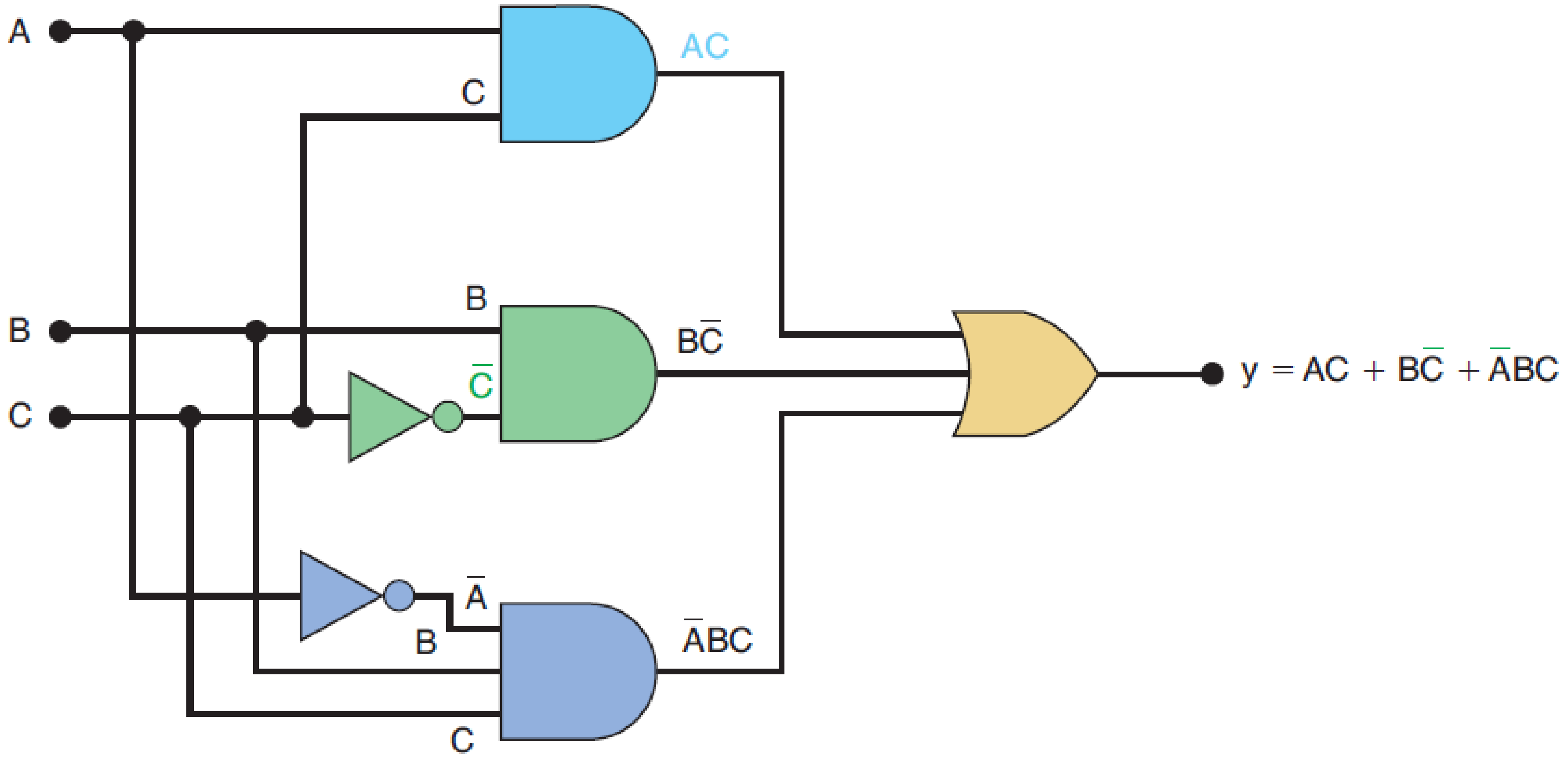
Next, apply the distributive law and rule 8 ($A \cdot \overline{A} = 0$).

$$(\overline{A} + B)(A + \overline{B}) = \overline{A}A + \overline{A}\,\overline{B} + AB + B\overline{B} = \overline{A}\,\overline{B} + AB$$

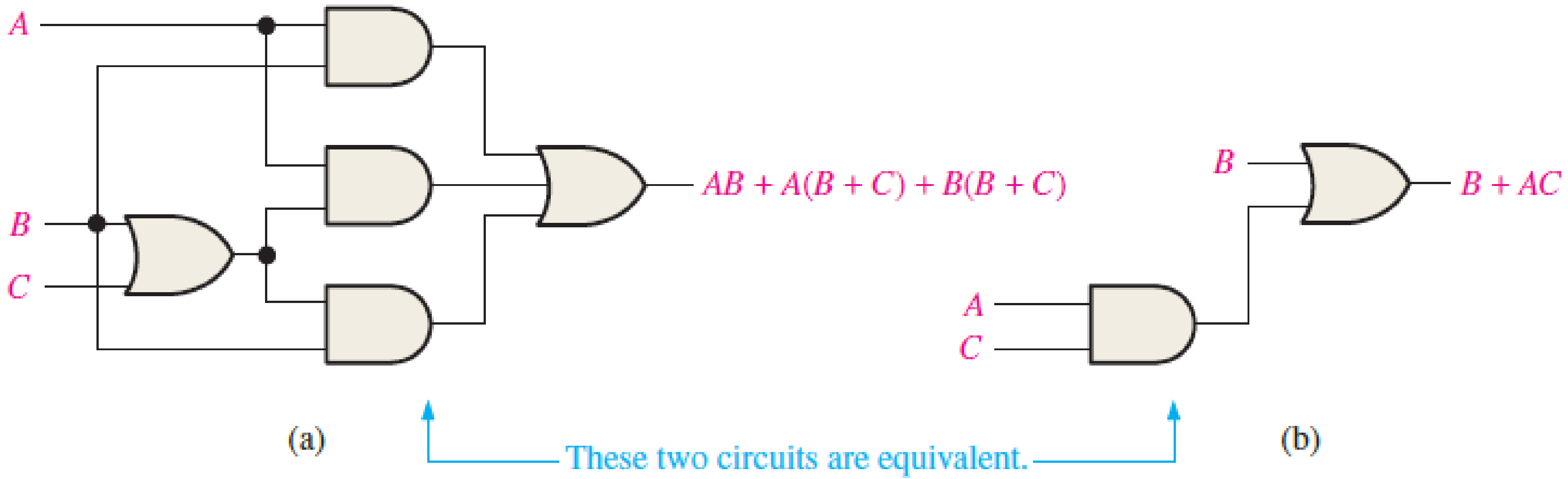
The final expression for the XNOR is $\overline{A}\,\overline{B} + AB$. Note that this expression equals 1 any time both variables are 0s or both variables are 1s.

Boolean expression into logic circuit

PROBLEM. Construct a logic circuit for the expression $y = AC + B\overline{C} + \overline{A}BC$



PROBLEM. Show that the following two circuits are equivalent.



SOLUTION:

$AB + A(B + C) + B(B + C) = AB + AB + AC + BB + BC$

Apply rule $(BB = B)$ to the fourth term. $AB + AB + AC + B + BC$

Apply rule $(AB + AB = AB)$ to the first two terms. $AB + AC + B + BC$

Apply rule $(B + BC = B)$ to the last two terms. $AB + AC + B$

Apply rule $(AB + B = B)$ to the first and third terms. $B + AC$

Simplifying the Boolean expression

EXAMPLE 4–11

Simplify the following Boolean expression: $\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$

Solution Factor BC out of the first and last terms. $BC(\overline{A} + A) + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$

Apply rule $(\overline{A} + A = 1)$ $\quad = BC \cdot 1 + A\overline{B}(\overline{C} + C) + \overline{A}\,\overline{B}\,\overline{C}$

Apply rule $(\overline{C} + C = 1)$ $\quad = BC + A\overline{B} \cdot 1 + \overline{A}\,\overline{B}\,\overline{C}$

$= BC + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C} \; = \; BC + \overline{B}(A + \overline{A}\,\overline{C})$

Apply rule $(A + \overline{A}\,\overline{C} = A + \overline{C})$ $\quad = BC + \overline{B}(A + \overline{C}) \; = \; BC + A\overline{B} + \overline{B}\,\overline{C}$

Simplifying the Boolean expression

EXAMPLE 4–12

Simplify the following Boolean expression: $\overline{AB + AC} + \overline{A}\,\overline{B}C$

Solution Apply DeMorgan's theorem to the first term. $(\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$

Apply DeMorgan's theorem to each term in parentheses.

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C = \overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

$$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} \qquad \text{Apply rule } (\overline{A}\,\overline{A} = \overline{A})$$

Apply rule $[\overline{A}\,\overline{B} + \overline{A}\,\overline{B}C = \overline{A}\,\overline{B}(1 + C) = \overline{A}\,\overline{B}]$

$$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} = \overline{A} + \overline{B}\,\overline{C}$$

Apply rule $[\overline{A} + \overline{A}\,\overline{B} = \overline{A}(1 + \overline{B}) = \overline{A}]$

Sum of product (SOP)

EXAMPLE 4–14

Convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD + EF)$ (b) $(A + B)(B + C + D)$ (c) $\overline{(\overline{A + B}) + C}$

Solution

(a) $AB + B(CD + EF) = AB + BCD + BEF$

(b) $(A + B)(B + C + D) = AB + AC + AD + BB + BC + BD$

(c) $\overline{(\overline{A + B}) + C} = (\overline{\overline{A + B}})\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$

Standard sum of product (SSOP)

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Solution

The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D + \overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D + \overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result.

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C + \overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D + \overline{D}$ as follows:

$$\begin{aligned}\overline{A}\,\overline{B} &= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D}) \\ &= \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\overline{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Product of sum (POS)

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Solution

The domain of this POS expression is A, B, C, D. Take one term at a time. The first term, $A + \overline{B} + C$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12 as follows:

$$A + \overline{B} + C = A + \overline{B} + C + D\overline{D} = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})$$

The second term, $\overline{B} + C + \overline{D}$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply rule 12 as follows:

$$\overline{B} + C + \overline{D} = \overline{B} + C + \overline{D} + A\overline{A} = (A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

The third term, $A + \overline{B} + \overline{C} + D$, is already in standard form. The standard POS form of the original expression is as follows:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Converting standard SOP to truth table

EXAMPLE 4–20

Develop a truth table for the standard SOP expression $\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$.

Solution: There are three variables in the domain, so there are eight possible combinations of binary values of the variables as listed in the left three columns of Table 4–6. The binary values that make the product terms in the expressions equal to 1 are $\underline{AB}C$: 001; $A\underline{BC}$: 100; and ABC: 111. For each of these binary values, place a 1 in the output column as shown in the table. For each of the remaining binary combinations, place a 0 in the output column.

TABLE 4–6

Inputs			Output	
A	B	C	X	Product Term
0	0	0	0	
0	0	1	1	$\overline{A}\,\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Converting standard POS to truth table

EXAMPLE 4–21

Determine the truth table for the following standard POS expression:

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

Solution: Out of 8 combinations, the binary values that make the sum terms in the expression equal to 0 are $A + B + C$: 000; $A + \underline{B} + C$: 010; $A + \underline{B} + \underline{C}$: 011; $\underline{A} + B + \underline{C}$: 101; and $\underline{A} + \underline{B} + C$: 110. For each of the remaining binary combinations, place a 1 in the output column.

TABLE 4–7

Inputs			Output	
A	B	C	X	Sum Term
0	0	0	0	$(A + B + C)$
0	0	1	1	
0	1	0	0	$(A + \overline{B} + C)$
0	1	1	0	$(A + \overline{B} + \overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A} + B + \overline{C})$
1	1	0	0	$(\overline{A} + \overline{B} + C)$
1	1	1	1	

Converting truth table to standard SOP and POS

EXAMPLE 4–22

From the truth table in Table 4–8, determine the standard SOP expression and the equivalent standard POS expression.

TABLE 4–8

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Converting truth table to standard SOP and POS

Solution

There are four 1s in the output column and the corresponding binary values are 011, 100, 110, and 111.

Convert these binary values to product terms as follows:

$$011 \longrightarrow \overline{A}BC \quad 100 \longrightarrow A\overline{B}\,\overline{C} \quad 110 \longrightarrow AB\overline{C} \quad 111 \longrightarrow ABC$$

The resulting standard SOP expression for the output X is $X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$

For the POS expression, the output is 0 for binary values 000, 001, 010, and 101.

Convert these binary values to sum terms as follows:

$$000 \longrightarrow A + B + C \quad 001 \longrightarrow A + B + \overline{C} \quad 010 \longrightarrow A + \overline{B} + C \quad 101 \longrightarrow \overline{A} + B + \overline{C}$$

The resulting standard POS expression for the output X is

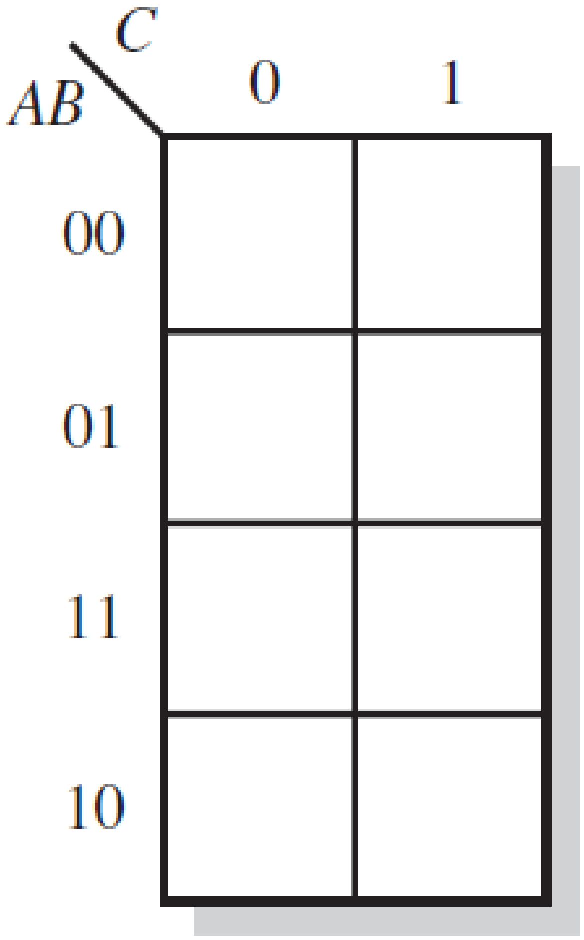
$$X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

The Karnaugh map

- A **Karnaugh map** is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value. Instead of being organized into columns and rows like a truth table, the Karnaugh map is an array of cells in which each cell represents a binary value of the input variables.

- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.

- The number of cells in a Karnaugh map, as well as the number of rows in a truth table, is equal to the total number of possible input variable combinations. For three variables, the number of cells is 23 = 8. For four variables, the number of cells is 24 = 16.



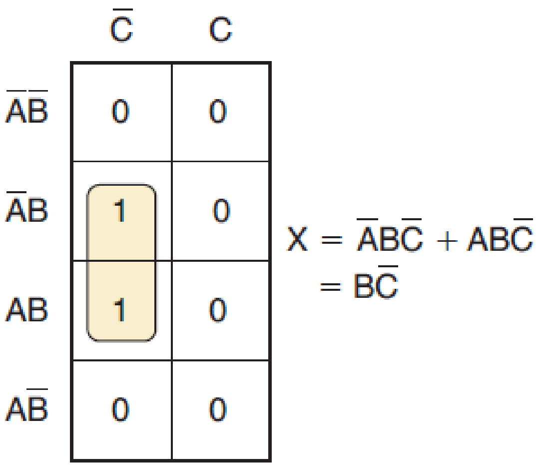
2-variable Karnaugh map

A 4-variable Karnaugh map

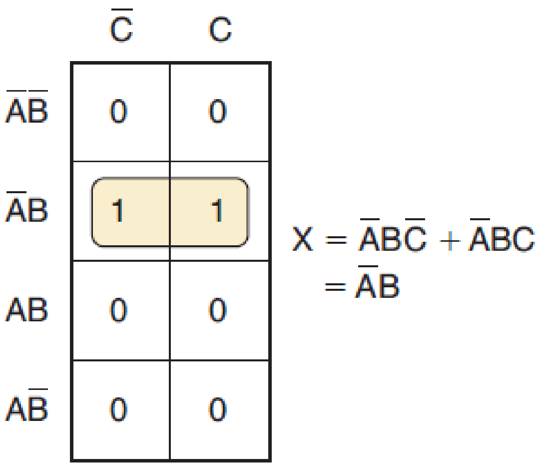
AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

Loop (group of 2)



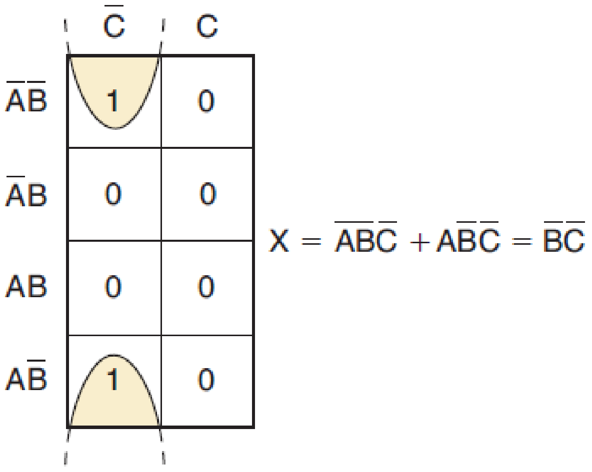
(a)



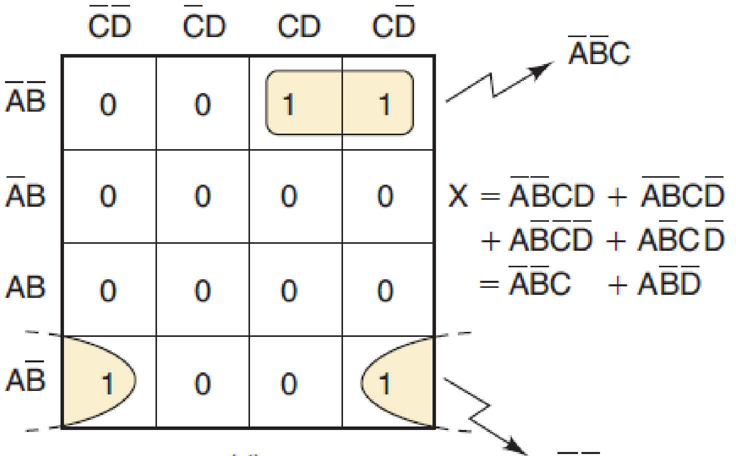
(b)

$$
\begin{aligned}
X &= \bar{A}\bar{B}C + \bar{A}BC + ABC + A\bar{B}C \\
&= \bar{A}C(\bar{B} + B) + AC(B + \bar{B}) \\
&= \bar{A}C + AC \\
&= C(\bar{A} + A) = C
\end{aligned}
$$

$$
\begin{aligned}
X &= \bar{A}B\bar{C} + AB\bar{C} \\
&= B\bar{C}(\bar{A} + A) \\
&= B\bar{C}(1) = B\bar{C}
\end{aligned}
$$

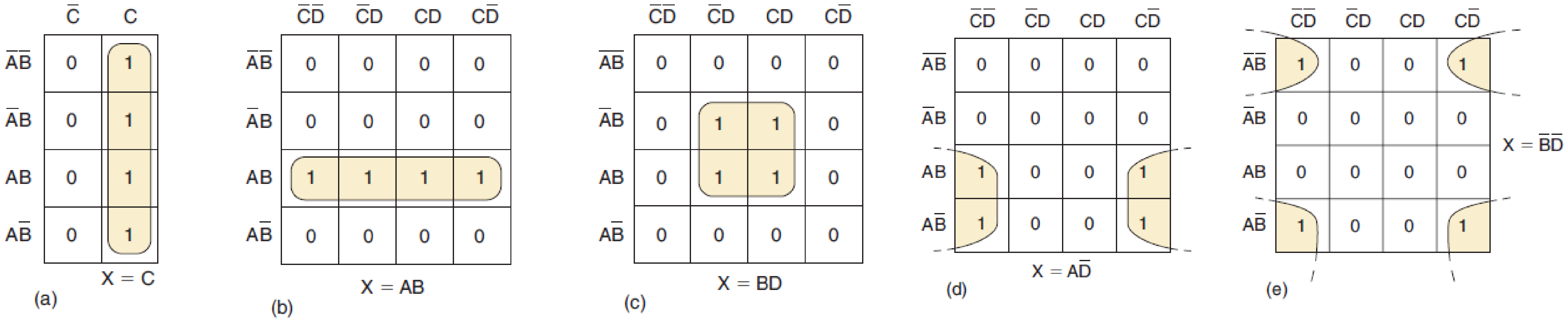


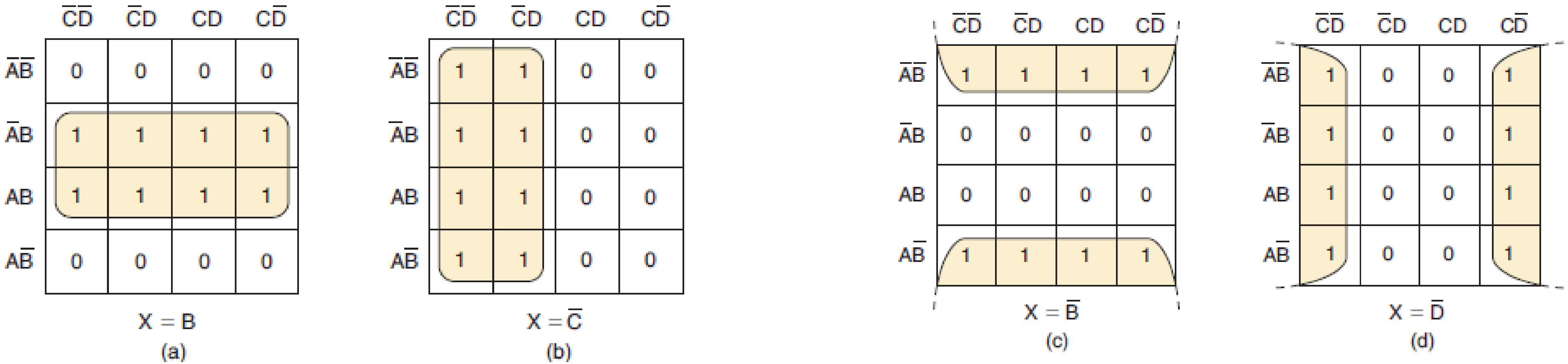
(c)



(d)

Loop (group of 4 and 8)

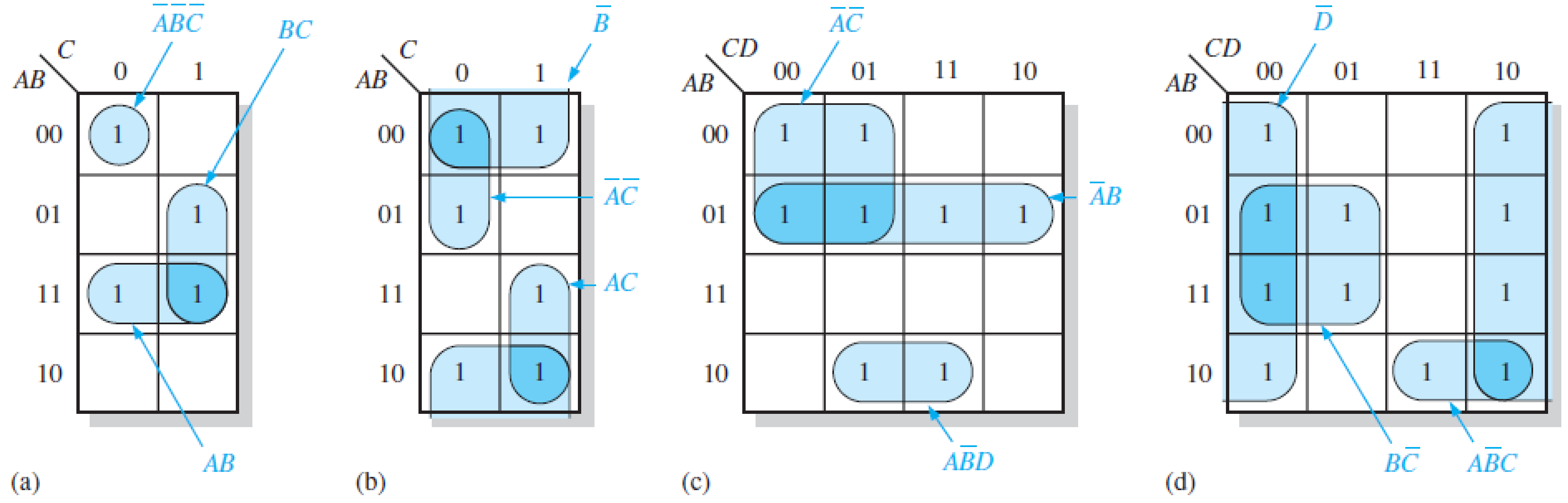




Logic simplification using Karnaugh map

EXAMPLE 4–29

Determine the product terms for each of the Karnaugh maps in Figure 4–36 and write the resulting minimum SOP expression.



Solution

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$

(b) $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$

(d) $\overline{D} + A\overline{B}C + B\overline{C}$

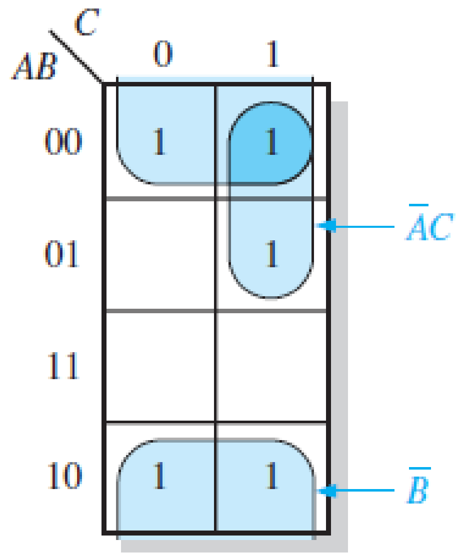
EXAMPLE 4–30

Use a Karnaugh map to minimize the following standard SOP expression:

(a) $A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$ (b) $\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$

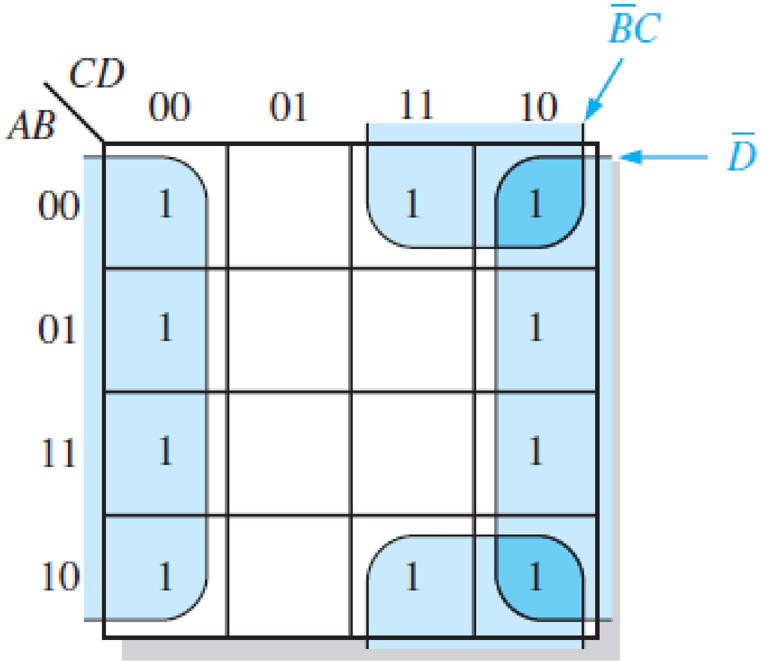
SOLUTION:

(a)



$$\boxed{\overline{B} + \overline{A}C}$$

(b) The first term $B\ C\ D$ must be expanded into $A\underline{B}\ \underline{C}\ \underline{D}$ and $\underline{A}\ \underline{B}\ \underline{C}\ \underline{D}$ to get the standard SOP expression, which is then mapped; the cells are grouped as shown in Figure.



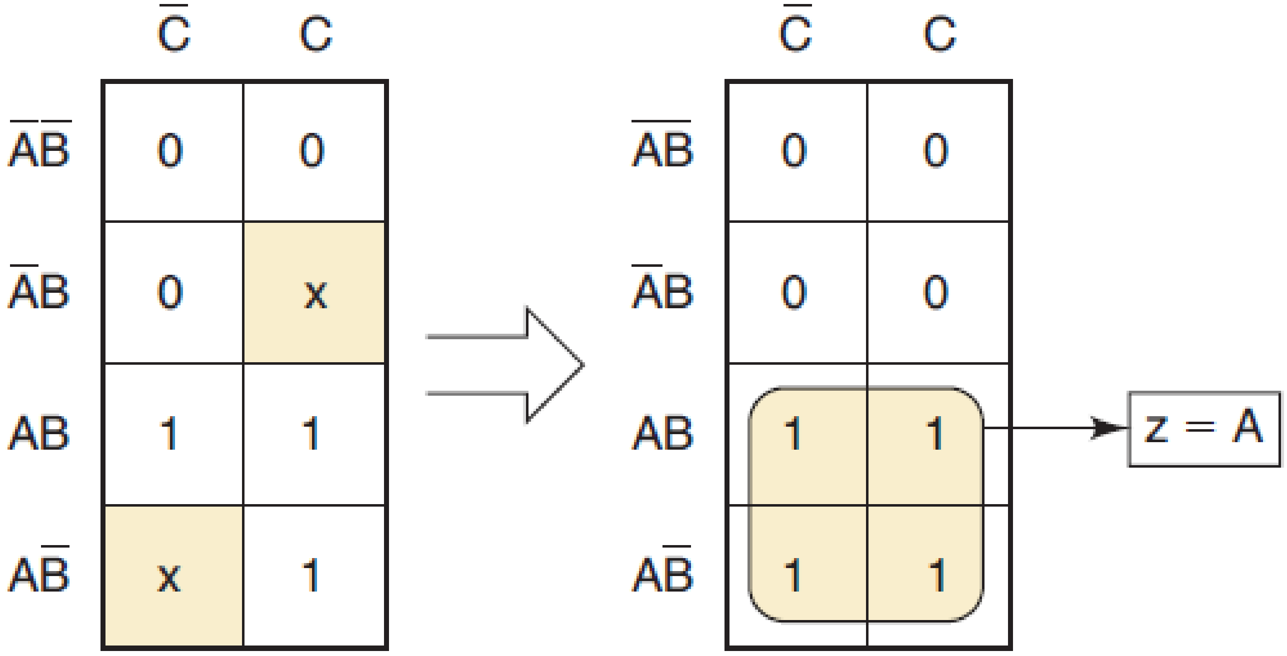
$$\boxed{\overline{D} + \overline{B}C}$$

“Don’t care” condition

- **DON’T CARE.** Some logic circuits can be designed so that there are certain input conditions for which there are no specified output levels, usually because these input conditions will never occur. In other words, there will be certain combinations of input levels where we “don’t care” whether the output is 1 or 0.

A	B	C	z	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	x	“don’t
1	0	0	x	care”
1	0	1	1	
1	1	0	1	
1	1	1	1	

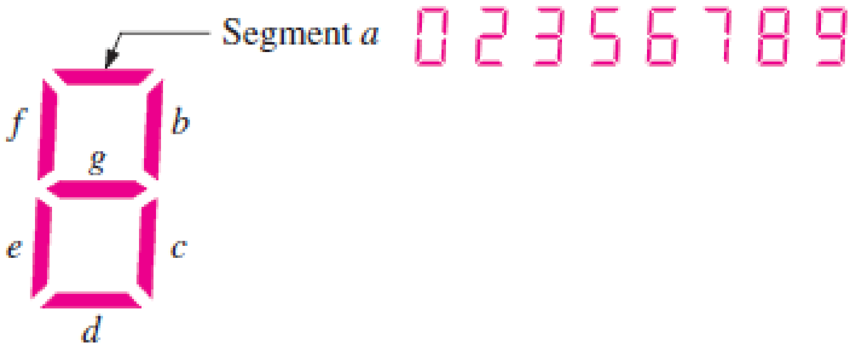
(a)



(b) (c)

DON'T CARE condition - application

PROBLEM. In a 7-segment display, each of the seven segments is activated for various digits. For example, segment *a* is activated for the digits 0, 2, 3, 5, 6, 7, 8, and 9, as illustrated in Figure. Since each digit can be represented by a BCD code, derive an SOP expression for segment *a* using the variables *ABCD* and then minimize the expression using a Karnaugh map.

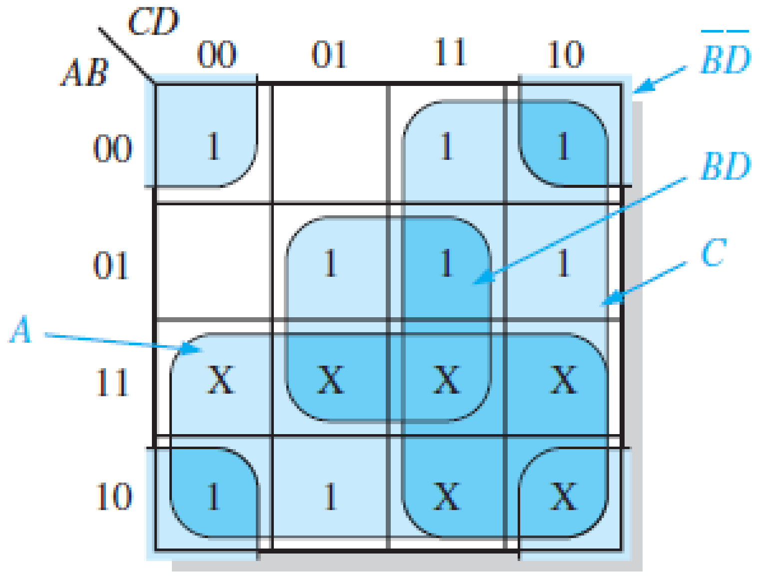


7-segment display.

Solution

The expression for segment a is

$$a = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D$$



From the Karnaugh map, the minimized expression for segment a is

$$a = A + C + BD + \overline{B}\,\overline{D}$$

References

1. ***Digital Fundamentals*** by Thomas Floyd, Pearson International Edition, 11th Edition, Chapter 4, Page 191-233.

Next class

Combinational Logic Analysis