Assignment 5

De Morgan's theorem:-

* $\overline{AB} = \bar{A} + \bar{B}$
* $\overline{A+B} = \bar{A}\bar{B}$

Applying De Morgan's theorems to the following expression:-

* $\overline{A+B+C+D} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$
* $\overline{ABCD} = \bar{A}+\bar{B}+\bar{C}+\bar{D}$
* $\overline{\bar{A}\bar{B}\bar{C}\bar{D}} = \bar{\bar{A}}+\bar{\bar{B}}+\bar{\bar{C}}+\bar{\bar{D}} = A+B+C+D$
* $\overline{(AB+C)(A+BC)} = \overline{(AB+C)} + \overline{(A+BC)}$

$= (\overline{AB})\,\bar{C} + \bar{A}(\overline{BC})$

$= (\bar{A}+\bar{B})\bar{C} + \bar{A}(\bar{B}+\bar{C})$

* $\overline{(A+B+C)D} = \overline{A+B+C} + \bar{D}$

$= \bar{A}\,\bar{B}\,\bar{C} + \bar{D}$

* $\overline{A\bar{B}+\bar{C}D+EF} = (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF})$

$= (\bar{A}+\bar{\bar{B}})(\bar{\bar{C}}+\bar{D})(\bar{E}+\bar{F})$

$= (\bar{A}+B)(C+\bar{D})(\bar{E}+\bar{F})$

* $\overline{\overline{(A+B)} + \overline{C}} = \overline{\overline{(A+B)}}\,(\overline{\overline{C}}) = (A+B)C$

* $\overline{(A+B)\overline{C}\,\overline{D} + E + \overline{F}} = \left(\overline{(A+B)\overline{C}\,\overline{D}}\right)\overline{E}\,\overline{\overline{F}}$

$= \left((\overline{A+B}) + \overline{\overline{C}} + \overline{\overline{D}}\right)\overline{E}F$

$= (\overline{A}\cdot\overline{B} + C + D)\overline{E}F$

* $\overline{A+B\overline{C}+D(\overline{E+\overline{F}})} = (\overline{\overline{A+B\overline{C}}})\left(\overline{D(\overline{E+\overline{F}})}\right)$

$= (A+B\overline{C})(\overline{D} + \overline{\overline{E+\overline{F}}})$

$= (A+B\overline{C})(\overline{D} + E + \overline{F})$

◫ Simplification using Boolean Algebra :-

(a) $AB + A(B+C) + B(B+C) = AB + AB + AC + BB + BC$

$= AB + AC + B + BC$

$= AB + AC + B(1+C)$

$= AB + AC + B$

$= B(A+1) + AC$

$= B + AC$

(b) $\overline{AB}+\overline{AC}+\bar{A}\bar{B}\bar{C} = \bar{A}+\bar{B}+\bar{A}+\bar{C}+\bar{A}\bar{B}\bar{C}$

$= \bar{A}+\bar{B}+\bar{C}+\bar{A}\bar{B}\bar{C}$ $\quad [\bar{A}+\bar{A}=\bar{A}]$

$= \bar{A}+\bar{B}+\bar{C}(1+\bar{A}\bar{B})$

$= \bar{A}+\bar{B}+\bar{C} = \overline{ABC}$

(c) $[A\bar{B}(C+BD)+\cancel{B}\bar{A}\bar{B}]C = (A\bar{B}C+A\bar{B}BD+\bar{A}\bar{B})C$

$= (A\bar{B}C+A\cdot 0\cdot D+\bar{A}\bar{B})C$ $\quad [\cancel{A}\ B\cdot\bar{B}=0]$

$= (A\bar{B}C+\bar{A}\bar{B})C$

$= A\bar{B}CC+\bar{A}\bar{B}C$

$= A\bar{B}C+\bar{A}\bar{B}C$ $\quad [C\cdot C=C]$

$= \bar{B}C(A+\bar{A})$

$= \bar{B}C$ $\quad [A+\bar{A}=1]$

Convert following expression into SOP form:-

(a) $AB+B(CD+EF) = AB+BCD+BEF$

(b) $\overline{A+B+C} = \overline{\overline{(A+B)}}\,\bar{C} = (A+B)\bar{C} = A\bar{C}+B\bar{C}$

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* Convert the following Boolean expression into standard SOP form:-

$A\bar{B}\bar{C} + \bar{A}\bar{B} + AB\bar{C}D$

$\Rightarrow A\bar{B}\bar{C} = A\bar{B}\bar{C}(D+\bar{D}) = A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

$\bar{A}\bar{B} = \bar{A}\bar{B}(C+\bar{C}) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}\bar{C}(D+\bar{D})$

$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

$\therefore A\bar{B}\bar{C} + \bar{A}\bar{B} + AB\bar{C}D = A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

Simplifying:-

* $xy + x\bar{y} = x(y+\bar{y}) = x \cdot 1 = x$ $[A+\bar{A} = 1]$

* $xyz + \bar{x}y + xy\bar{z} = y(xz + \bar{x} + x\bar{z})$

$= y(\bar{x} + x(z+\bar{z}))$

$= y(\bar{x}+x)$ $[z+\bar{z} = 1]$

$= y$ $[x+\bar{x} = 1]$

* $ABC + \bar{A}B + AB\bar{C} = B(AC + \bar{A} + A\bar{C})$

$$= B(\bar{A} + A(C+\bar{C}))$$

$$= B(A+\bar{A}) = B$$

* $\overline{(A+B)}\,(\overline{\bar{A}+\bar{B}}) = \bar{A}\bar{B}\,\bar{\bar{A}}\bar{\bar{B}} = \bar{A}\bar{B}AB = A\bar{A}B\bar{B}$

$$= 0 \quad [A \cdot \bar{A} = 0]$$

* $(B\bar{C} + \bar{A}D)(A\bar{B} + C\bar{D})$

$= B\bar{C}A\bar{B} + B\bar{C}C\bar{D} + \bar{A}DA\bar{B} + \bar{A}DC\bar{D}$

$= A\bar{C}B\bar{B} + B\bar{D}C\bar{C} + A\bar{A}\bar{B}D + \bar{A}CD\bar{D}$

$= 0+0+0+0 = 0 \quad [A \cdot \bar{A} = 0]$