

Assignment 7

Use a K map to minimize the following :-

POS expression :

$$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

0 0 0 0 0 1 0 1 0 0 1 1 1 1 0

AB \ C	0	1
00	0	0
01	0	0
11	0	1
10	1	1

Simplified POS expression :-

$$\bar{F} = \bar{A} + B\bar{C}$$

$$F = \overline{\bar{A} + B\bar{C}}$$

$$= \bar{A}(\overline{B\bar{C}})$$

$$= A(\bar{B} + C)$$

AB \ C	0	1
00	0	0
01	0	0
11	0	1
10	1	1

Simplified SOP expression :-

$$F = A\bar{B} + AC \quad [\text{SOP form}]$$

$$= \cancel{A}(\bar{B} + C)$$

* Simplify the function expressed in product of maxterm form. $F(x, y, z) = \Pi(0, 2, 5, 7)$

no. of maxterm or sum = 4

$xy \backslash z$	0	1
00	0	1
01	2	3
11	6	7
10	4	5

\Rightarrow POS expression:-

$$\bar{F} = \bar{x}\bar{z} + xz$$

$$\bar{\bar{F}} = \overline{\bar{x}\bar{z} + xz}$$

$$F = \overline{\bar{x}\bar{z} + xz}$$

$$= \overline{\bar{x}\bar{z}} \cdot \overline{xz}$$

$$= (\overline{\bar{x}\bar{z}})(\overline{xz})$$

$$= (\bar{x} + \bar{z})(x + z)$$

$$= (x + z)(\bar{x} + \bar{z})$$

\Rightarrow ~~POS~~ ^{SOP} expression:-

$xy \backslash z$	0	1
00	0	1
01	0	1
11	1	0
10	1	0

$$F = \bar{x}z + x\bar{z} \text{ [SOP]}$$

* Use K-map to minimize the following expression :-

$$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABC\bar{D} + ABCD$$

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	0	0
10	0	1	1	0

simplified SOP expression :-

$$F = \bar{A}\bar{C} + \bar{A}B + A\bar{B}D$$

In POS expression :-

$$F = \bar{A}(\bar{B} + \bar{C})$$

* simplify the function expressed in sum of minterms :-

$$F(W, X, Y, Z) = \sum (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

no. of minterms = 11

WX \ YZ	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$$\therefore F = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}$$

*Don't care conditions!

Inputs				Output
A	B	C	D	Y
0	0	0	0	X
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

AB \ CD				
	00	01	11	10
00	X			
01			1	
11	X	X	X	X
10	1	1	X	X

Without don't cares,

$$Y = A\bar{C}\bar{B} + \bar{A}BCD$$

With don't cares,

$$Y = A + BCD$$

*Simplify the boolean function in

(a) Sum of product and

(b) Product of Sum form

(a) $F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10)$

(a)

AB \ CD	00	01	11	10
00	0 1	1 1	3	2 1
01	4	5 1	7	6
11	12	13	15	14
10	8 1	9 1	11	10 1

$$\begin{aligned}
 F &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} \\
 &= \bar{B}\bar{C}(\bar{A} + A) + \bar{B}\bar{D}(\bar{A} + A) + \bar{A}\bar{C}\bar{D} \\
 &= \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}
 \end{aligned}$$

In product of maxterms, F can be expressed as
 $F(A, B, C, D) = \prod(3, 4, 6, 7, 11, 12, 13, 14, 15)$

(b)

AB \ CD	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

$$\bar{F} = \bar{C}\bar{D} + AB + B\bar{D}$$

$$F = \overline{\bar{C}\bar{D} + AB + B\bar{D}}$$

$$= (\bar{C}\bar{D})(\overline{AB})(\overline{B\bar{D}})$$

$$= (\bar{C} + \bar{D})(\bar{A} + \bar{B})(\bar{B} + D)$$

* Exercise :-

$$F(A, B, C, D) = \sum (5, 7, 8, 10, 11, 14, 15)$$

$$d = \sum (0, 12, 13)$$

SOP expression :-

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$F = BD + AB + AC + AD + \bar{B} \bar{C} \bar{D}$$

POS expression :-

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$\bar{F} = \bar{A} \bar{B} + \bar{A} \bar{D} + B \bar{C} \bar{D} + A \bar{C} D + \bar{B} \bar{C} D$$

$$\begin{aligned} F &= \overline{\bar{A} \bar{B} + \bar{A} \bar{D} + B \bar{C} \bar{D} + A \bar{C} D + \bar{B} \bar{C} D} \\ &= (\overline{\bar{A} \bar{B}})(\overline{\bar{A} \bar{D}})(\overline{B \bar{C} \bar{D}})(\overline{A \bar{C} D})(\overline{\bar{B} \bar{C} D}) \\ &= (\bar{A} + \bar{B})(\bar{A} + \bar{D})(\bar{B} + \bar{C} + \bar{D})(\bar{A} + \bar{C} + \bar{D})(\bar{B} + \bar{C} + \bar{D}) \\ &= (A + B)(A + D)(B + C + D)(\bar{A} + \bar{C} + \bar{D})(B + C + \bar{D}) \end{aligned}$$