ETE 211/EEE 211

Digital Logic Design

Synchronous Sequential Logic - Analysis

Analysis of Clocked Sequential Circuits

•The analysis of a sequential circuit consists of obtaining a table or a diagram for the time sequence of inputs, outputs, and internal states.

•It is also possible to write Boolean expressions that describe the behavior of the sequential circuit.

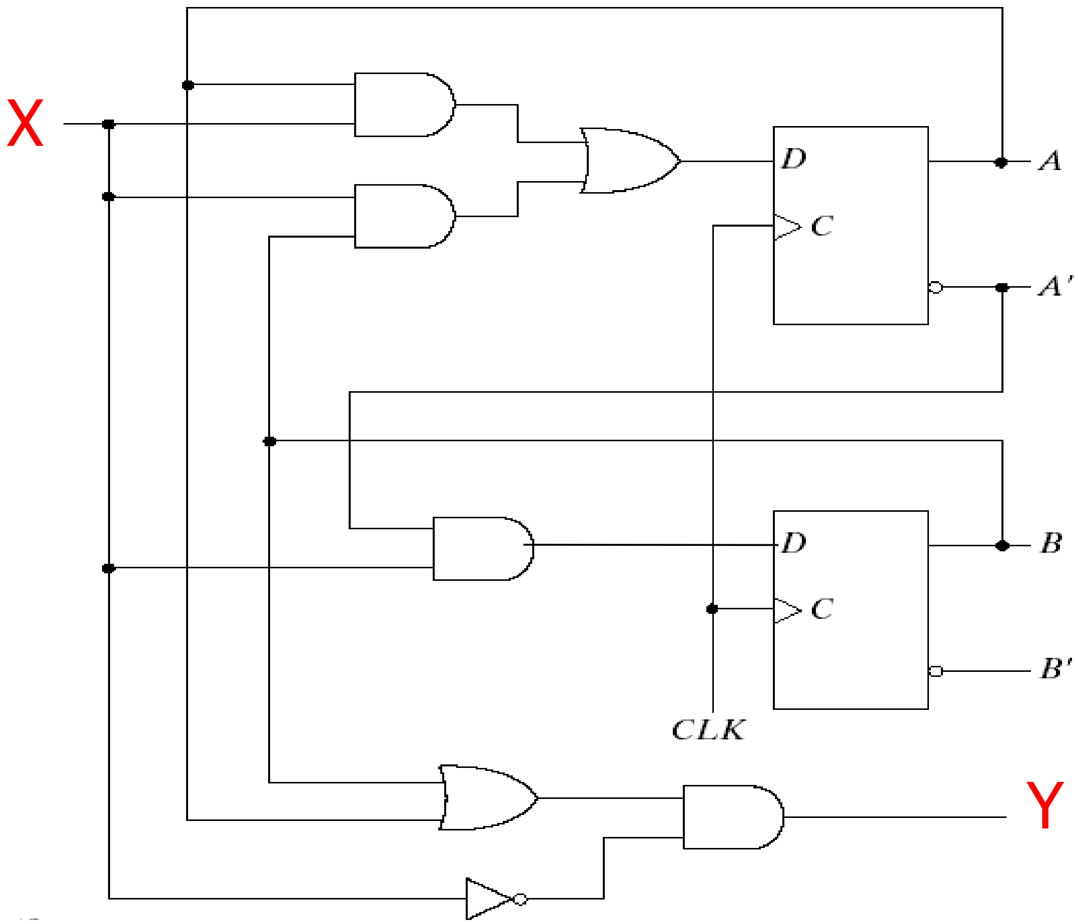
•These expressions must include the necessary time sequence, either directly or indirectly.

•The behavior of a clocked sequential circuit can be described algebraically by means of state equations.

• **State equation** specifies the next state as a function of the present state and inputs.

- ✓A state equation is an algebraic expression that specifies the condition for a flip-flop state transition.
- ✓The left side of the equation with (t+1) denotes the next state of the flip-flop one clock edge later.
- ✓The right side of the equation is Boolean expression that specifies the present state and input conditions that make the next state TRUE.

Example of Sequential Circuit



State Equation

$A(t+1) = A(t)\,x(t) + B(t)\,x(t)$

$B(t+1) = A`(t)\,x(t)$

$Y(t) = (A(t) + B(t))\,x(t)`$

•This sequential circuit consists of two D flip-flops A and B, an input X and an output Y.

State Table

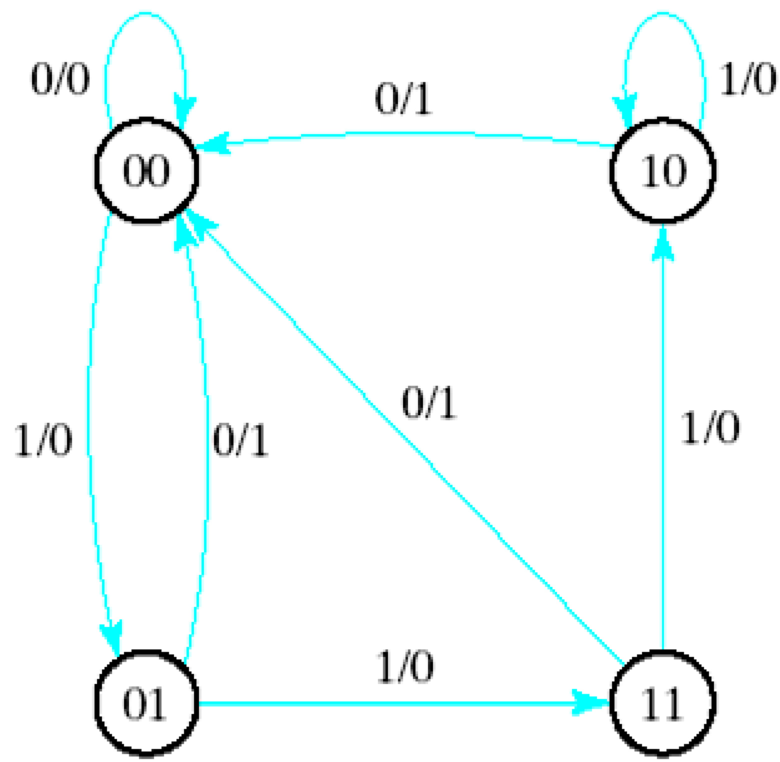
The time sequence of inputs, outputs, and flip-flop states can be enumerated in a state table (sometimes called transition table).

Present State A	Present State B	Input X	Next State A	Next State B	Output Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

State Diagram

- The information available in a state table can be represented graphically in the form of a state diagram.

- A state is represented by a circle, and the transitions between states are indicated by directed lines connecting the circles.



1/0 : means input =1
output=0

Flip-Flop Input Equations

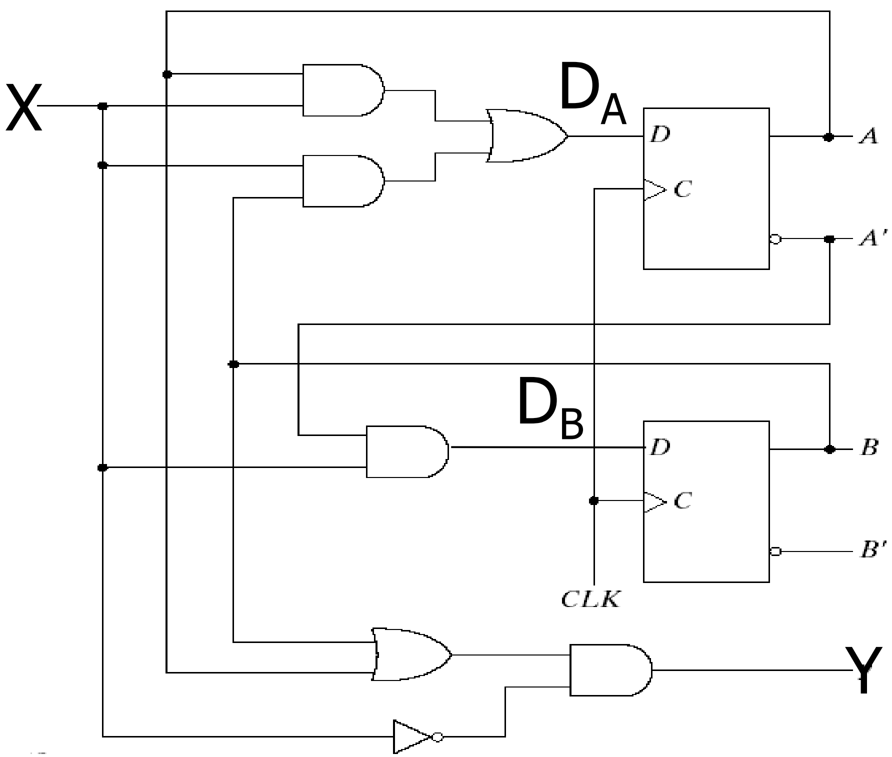
- The part of the combinational circuit that generates external outputs is described algebraically by a set of Boolean functions called output equations.
- The part of the circuit that generates the inputs to flip-flops is described algebraically by a set of Boolean functions called flip-flop input equations.

- The logic diagram of the circuit can be expressed algebraically with two flip-flop input equations and an output equation:

$D_A = AX + BX$

$D_B = A`X$

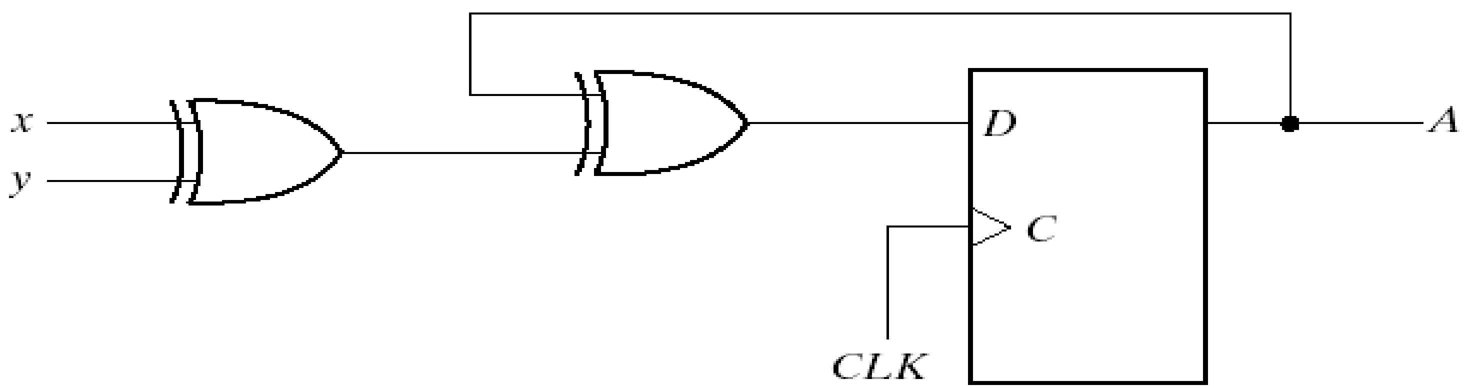
$y = (A + B) X'$



Analysis with D Flip-Flop

The circuit we want to analyze is described by the input equation $D_A = A \oplus x \oplus y$

- The D_A symbol implies a D flip-flop with output A.
- The x and y variables are the inputs to the circuit.
- No output equations are given, so the output is implied to come from the output of the flip-flop.



Analysis with D Flip-Flop

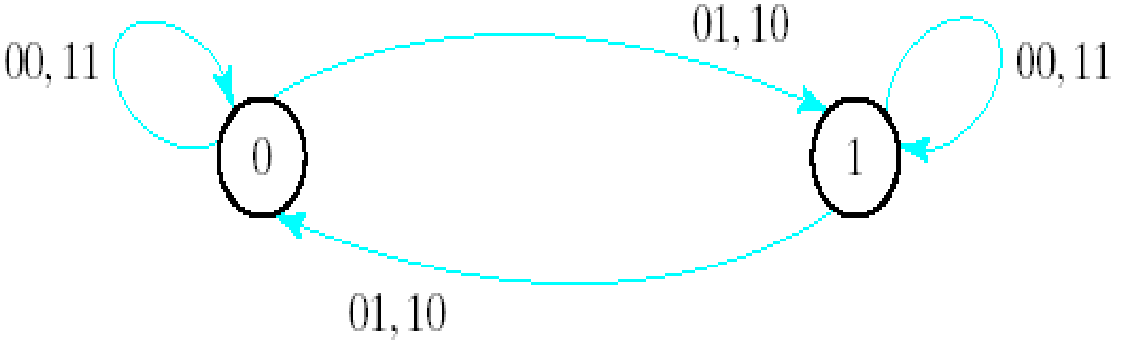
- The binary numbers under Axy are listed from 000 through 111
- The next state values are obtained from the state equation

$A(t+1) = A \oplus x \oplus y$

- The state diagram consists of two circles-one for each state

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table



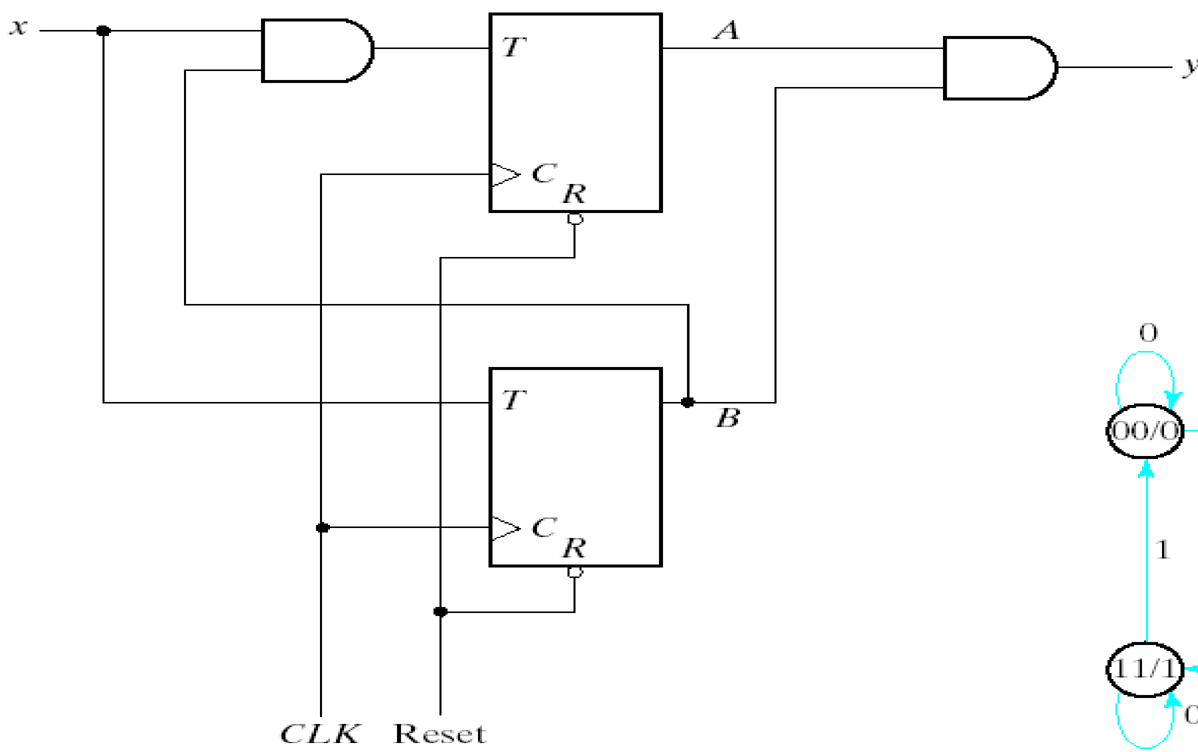
(c) State diagram

Analysis With T Flip-Flops

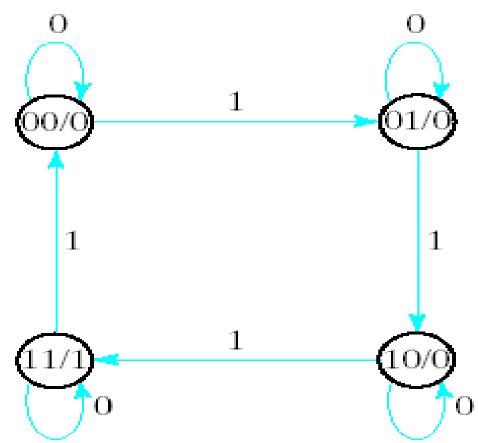
Characteristic equation

$$Q(t+1) = T \oplus Q = T'Q + TQ'$$

00/0 : means
state is 00
output is 0



(a) Circuit diagram



(b) State diagram

Analysis With T Flip-Flops

Two flip-flops A and B, one input x, and one output y.
Two input equations and an output equation:

$T_A = Bx$

$T_B = x$

$y = AB$

$A(t+1) = (Bx)'A + (Bx)A'$

$= AB' + Ax' + A'Bx$

$B(t+1) = x \oplus B$

Table 5-5
State Table for Sequential Circuit with T Flip-Flops

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1