Assignment 4

* DeMorgan's Theorem :-
  - $Y = \overline{AB} = \overline{A} + \overline{B}$
  - $Y = \overline{A+B} = \overline{A} \cdot \overline{B}$

Example :- $F = \overline{x} y \overline{z} + x \overline{y} \overline{z}$

$\overline{F} = (x + \overline{y} + z)(\overline{x} + y + z)$

* Minterms and Maxterms for 2 variables :-

x	y	Index	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}\overline{y}$	$M_0 = x+y$
0	1	1	$m_1 = \overline{x} y$	$M_1 = x+\overline{y}$
1	0	2	$m_2 = x\overline{y}$	$M_2 = \overline{x}+y$
1	1	3	$m_3 = xy$	$M_3 = \overline{x}+\overline{y}$

- Minterm evaluates with 1 for each combination of x and y.
- Maxterm evaluates with 0.
- Maxterm is the complement of the minterm.

$M_i = \overline{m_i}$ and $m_i = \overline{M_i}$

* Minterms and Maxterms for 3 variables:-

x	y	z	Index	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}\bar{y}\bar{z}$	$M_0 = x+y+z$
0	0	1	1	$m_1 = \bar{x}\bar{y}z$	$M_1 = x+y+\bar{z}$
0	1	0	2	$m_2 = \bar{x}y\bar{z}$	$M_2 = x+\bar{y}+z$
0	1	1	3	$m_3 = \bar{x}yz$	$M_3 = x+\bar{y}+\bar{z}$
1	0	0	4	$m_4 = x\bar{y}\bar{z}$	$M_4 = \bar{x}+y+z$
1	0	1	5	$m_5 = x\bar{y}z$	$M_5 = \bar{x}+y+\bar{z}$
1	1	0	6	$m_6 = xy\bar{z}$	$M_6 = \bar{x}+\bar{y}+z$
1	1	1	7	$m_7 = xyz$	$M_7 = \bar{x}+\bar{y}+\bar{z}$

* For minterms, '1' means the variable is "Not complemented" and '0' means the variable is "complemented".

* For Maxterms, '0' means variable is "not complemented", and '1' means the variable is complemented.

* Sum-Of-Minterm (SOM):-

- Sum-Of-Minterm(SOM) canonical form:
  Sum of minterms of entries that evaluate to '1'.

x	y	z	F	Minterm
0	0	0	0	
0	0	1	1	$m_1 = \bar{x}\bar{y}z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$m_6 = xy\bar{z}$
1	1	1	1	$m_7 = xyz$

$$F = m_1 + m_6 + m_7 = \Sigma(1,6,7) = \bar{x}\bar{y}z + xy\bar{z} + xyz$$

* Product-of-Maxterm (POM) :-

POM of entries that evaluate to '0'.

x	y	z	F	Maxterm
0	0	0	1	
0	0	1	1	
0	1	0	0	$M_2 = (x+\bar{y}+z)$
0	1	1	1	
1	0	0	0	$M_4 = (\bar{x}+y+z)$
1	0	1	1	
1	1	0	0	$M_6 = (\bar{x}+\bar{y}+z)$
1	1	1	1	

$F = M_2 \cdot M_4 \cdot M_6 = \Pi(2,4,6) = (x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$

* Converting to Sum-of-Minterms Form:

A Function that is not in the SOM form can be converted to that form by means of a truth table.

- Consider $F = \bar{y} + \bar{x}\bar{z}$

x	y	z	F	Minterm
0	0	0	1	$m_0 = \bar{x}\bar{y}\bar{z}$
0	0	1	1	$m_1 = \bar{x}\bar{y}z$
0	1	0	1	$m_2 = \bar{x}y\bar{z}$
0	1	1	0	
1	0	0	1	$m_4 = x\bar{y}\bar{z}$
1	0	1	1	$m_5 = x\bar{y}z$
1	1	0	0	
1	1	1	0	

$F = \Sigma(0,1,2,4,5)$

$= m_0 + m_1 + m_2 + m_4 + m_5$

$= \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$

* Converting to Product-of-Maxterms Form:

A function that is not in the POM form can be converted to that form by means of a truth table.

- Consider $F = \bar{y} + \bar{x}\bar{z}$

x	y	z	F	Maxterm
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	$M_3 = (x+\bar{y}+\bar{z})$
1	0	0	1	
1	0	1	1	
1	1	0	0	$M_6 = (\bar{x}+\bar{y}+\bar{z})$
1	1	1	0	$M_7 = (\bar{x}+\bar{y}+\bar{z})$

$F = \Pi(3,6,7) = M_3 \cdot M_6 \cdot M_7$

$= (x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

* Conversion Between canonical Forms:-

x	y	z	F	Minterm	Maxterm
0	0	0	0		$M_0 = (x+y+z)$
0	0	1	1	$m_1 = \bar{x}\bar{y}z$	
0	1	0	1	$m_2 = \bar{x}y\bar{z}$	
0	1	1	1	$m_3 = \bar{x}yz$	
1	0	0	0		$M_4 = (\bar{x}+y+z)$
1	0	1	1	$m_5 = x\bar{y}z$	
1	1	0	0		$M_6 = (\bar{x}+\bar{y}+z)$
1	1	1	1	$m_7 = xyz$	

$F = m_1 + m_2 + m_3 + m_5 + m_7 = \Sigma(1,2,3,5,7) =$

$\bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xyz$

$F = M_0 \cdot M_4 \cdot M_6 = \Pi(0,4,6) = (x+y+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$

* Algebraic Conversion to Sum-Of-Minterms :-

- Expand all terms first to explicitly list all minterms
- And any term missing a variable v with $(v+\bar{v})$
- Example 1 : $f = x + \bar{x}\bar{y}$ (2 variables)

$$f = x(y+\bar{y}) + \bar{x}\bar{y}$$

$$f = xy + x\bar{y} + \bar{x}\bar{y}$$

$$f = m_3 + m_2 + m_0 = \Sigma(0,2,3)$$

- Example 2 : $g = a + \bar{b}c$ (3 variables)

$$g = a(b+\bar{b})(c+\bar{c}) + (a+\bar{a})\bar{b}c$$

$$g = abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + \bar{a}\bar{b}c$$

$$g = \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c} + abc$$

$$g = m_1 + m_4 + m_5 + m_6 + m_7 = \Sigma(1,4,5,6,7)$$

*Algebraic Conversion to Product-of-Maxterms :-

- Expand all terms first to explicitly list all maxterms.
- Or any term missing a variable v with $v.\bar{v}$.
- Example 1: $f = x + \bar{x}\bar{y}$ (2 variables)

Apply 2nd distributive law :-

$$f = (x+\bar{x})(x+\bar{y}) = 1 \cdot (x+\bar{y}) = (x+\bar{y}) = M_1$$

Example 2:- $g = a\bar{c} + bc + \bar{a}\bar{b}$ (3 variables)

$g = (a\bar{c} + bc + \bar{a})(a\bar{c} + bc + \bar{b})$ [distibutive]

$g = (\bar{c} + bc + \bar{a})(a\bar{c} + c + b)$ [$x + \bar{x}y = x + y$]

$g = (\bar{c} + b + \bar{a})(a + c + \bar{b})$ [$x + \bar{x}y = x + y$]

$g = (\bar{a} + b + \bar{c})(a + \bar{b} + c) = M_5 \cdot M_2 = \Pi(2,5)$

* Standard Sum-of-Products (SOP)

• A Simplification Example:-

$F(A,B,C) = \Sigma(1,4,5,6,7)$

Minterm Expression:-

$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

Simplifying:-

$F = \bar{A}\bar{B}C + A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC)$

$F = \bar{A}\bar{B}C + A(\bar{B}(\bar{C} + C) + B(\bar{C} + C))$

$F = \bar{A}\bar{B}C + A(\bar{B} + B)$

$F = \bar{A}\bar{B}C + A$

$F = \cancel{A}\bar{B}C + A$

Simplified F contains ~~3~~ 3 literals compared to 15.