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# Digital Logic Design I

## Chapter 1

### Digital Systems and Binary Numbers

# Outline of Chapter 1

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- ▣ 1.1 Digital Systems
- ▣ 1.2 Binary Numbers
- ▣ 1.3 Number-base Conversions
- ▣ 1.4 Octal and Hexadecimal Numbers
- ▣ 1.5 Complements
- ▣ 1.6 Signed Binary Numbers
- ▣ 1.7 Binary Codes
- ▣ 1.8 Binary Storage and Registers (Optional)
- ▣ 1.9 Binary Logic (Optional)

# Digital Systems and Binary Numbers

- ▣ Digital age and information age
- ▣ Digital computers
  - ◆ General purposes
  - ◆ Many scientific, industrial and commercial applications
- ▣ Digital systems
  - ◆ Telephone switching exchanges
  - ◆ Digital camera
  - ◆ Electronic calculators, PDA's
  - ◆ Digital TV
- ▣ Discrete information-processing systems
  - ◆ Manipulate discrete elements of information
  - ◆ For example,  $\{1, 2, 3, \dots\}$  and  $\{A, B, C, \dots\}$ ...

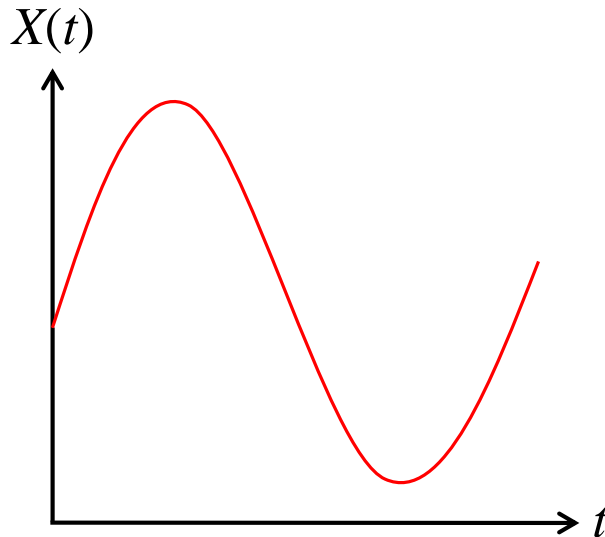
# Analog and Digital Signal

## ■ Analog system

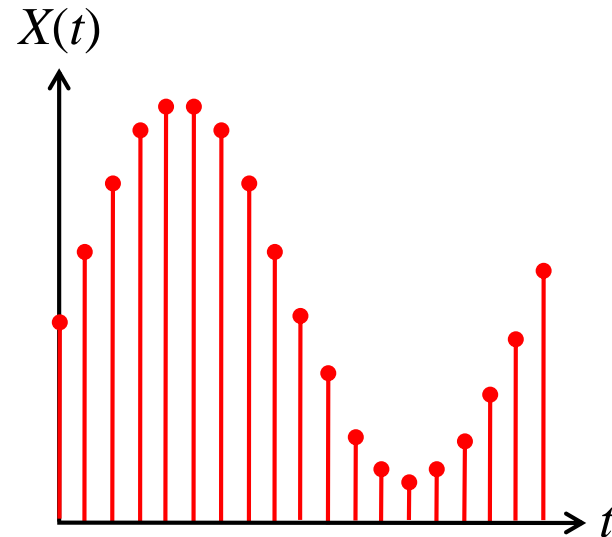
- ◆ The physical quantities or signals may vary continuously over a specified range.

## ■ Digital system

- ◆ The physical quantities or signals can assume only discrete values.
- ◆ Greater accuracy



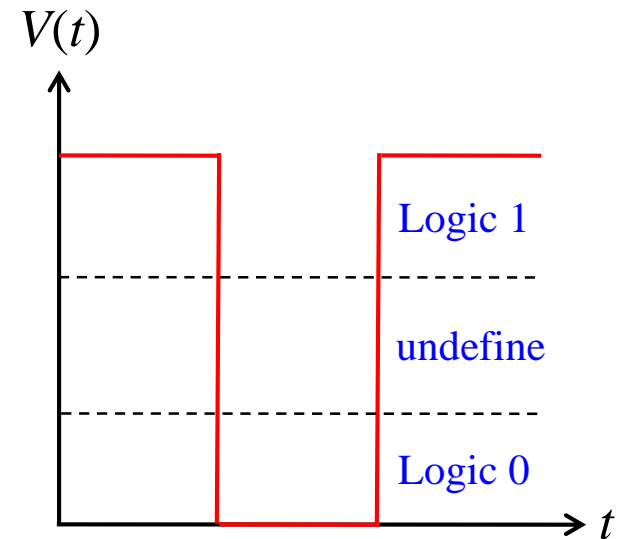
Analog signal



Digital signal

# Binary Digital Signal

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
  - ◆ Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
  - ◆ Digits 0 and 1
  - ◆ Words (symbols) False (F) and True (T)
  - ◆ Words (symbols) Low (L) and High (H)
  - ◆ And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.



Binary digital signal

# Decimal Number System

Base (also called radix) = 10

10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Digit Position

Integer & fraction

Digit Weight

Weight =  $(Base)^{Position}$

Magnitude

Sum of “Digit x Weight”

Formal Notation



	$P$	2	1	0		-1	-2
		5	1	2	.	7	4
	$W$	100	10	1		0.1	0.01
					.		
	$M$	500	10	2		0.7	0.04
		$d_2 * B^2 + d_1 * B^1 + d_0 * B^0 + d_{-1} * B^{-1} + d_{-2} * B^{-2}$					
		$(512.74)_{10}$					

# Octal Number System

□ Base = 8

◆ 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }

□ Weights

◆ Weight =  $(Base)^{Position}$

□ Magnitude

◆ Sum of “*Digit x Weight*”

□ Formal Notation

64	8	1		1/8	1/64
<b>5</b>	<b>1</b>	<b>2</b>	•	<b>7</b>	<b>4</b>
2	1	0		-1	-2

$$\textcolor{blue}{5} * 8^2 + \textcolor{red}{1} * 8^1 + \textcolor{red}{2} * 8^0 + \textcolor{red}{7} * 8^{-1} + \textcolor{red}{4} * 8^{-2}$$

$$=(330.9375)_{10}$$
$$(\textcolor{red}{512.74})_8$$

# Binary Number System

□ Base = 2

◆ 2 digits { 0, 1 }, called *binary digits* or “*bits*”

□ Weights

◆ Weight =  $(Base)^{Position}$

□ Magnitude

◆ Sum of “*Bit x Weight*”

□ Formal Notation

□ Groups of bits

8 bits = *Byte*

4	2	1		1/2	1/4
<b>1</b>	<b>0</b>	<b>1</b>	•	<b>0</b>	<b>1</b>
2	1	0		-1	-2

$$1 * 2^2 + 0 * 2^1 + 1 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2}$$

$$=(5.25)_{10}$$
$$(101.01)_2$$

**1 0 1 1**

**1 1 0 0 0 1 0 1**



# Hexadecimal Number System

□ Base = 16

◆ 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }

□ Weights

◆ Weight =  $(Base)^{Position}$

□ Magnitude

◆ Sum of “*Digit x Weight*”

□ Formal Notation

256	16	1		1/16	1/256
<b>1</b>	<b>E</b>	<b>5</b>	•	<b>7</b>	<b>A</b>
2	1	0		-1	-2

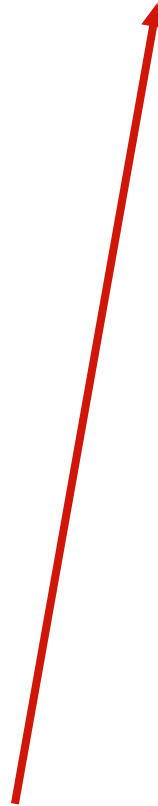
$$1 * 16^2 + 14 * 16^1 + 5 * 16^0 + 7 * 16^{-1} + 10 * 16^{-2}$$

$$=(485.4765625)_{10}$$

$$(1E5.7A)_{16}$$

# The Power of 2

n	$2^n$
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$



n	$2^n$
8	$2^8=256$
9	$2^9=512$
10	$2^{10}=1024$
11	$2^{11}=2048$
12	$2^{12}=4096$
20	$2^{20}=1M$
30	$2^{30}=1G$
40	$2^{40}=1T$

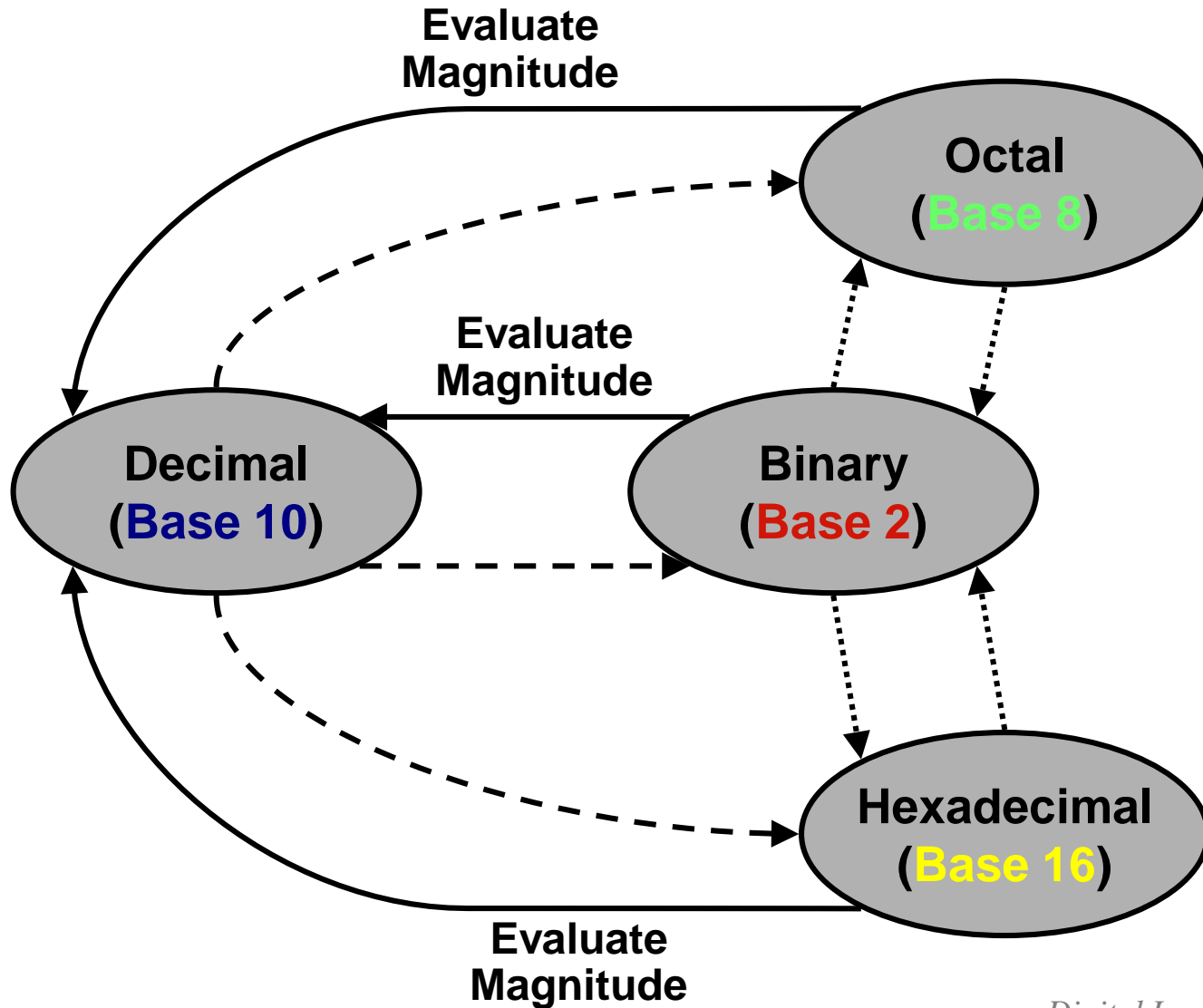
**Kilo**

**Mega**

**Giga**

**Tera**

# Number Base Conversions





# Decimal (*Integer*) to Binary Conversion

- Divide the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

**Example: (13)<sub>10</sub>**

	Quotient	Remainder	Coefficient
<b>13</b> / 2 =	<b>6</b>	<b>1</b>	$a_0 = \mathbf{1}$
<b>6</b> / 2 =	<b>3</b>	<b>0</b>	$a_1 = \mathbf{0}$
<b>3</b> / 2 =	<b>1</b>	<b>1</b>	$a_2 = \mathbf{1}$
<b>1</b> / 2 =	<b>0</b>	<b>1</b>	$a_3 = \mathbf{1}$

**Answer:**  $(\mathbf{13})_{10} = (a_3 a_2 a_1 a_0)_2 = (\mathbf{1101})_2$

**MSB**
**LSB**


# Decimal (*Fraction*) to Binary Conversion

- ▣ Multiply the number by the 'Base' (=2)
- ▣ Take the integer (either 0 or 1) as a coefficient
- ▣ Take the resultant fraction and repeat the division

**Example:**  $(0.625)_{10}$

		Integer	Fraction	Coefficient
$0.625$	$* 2 =$	$1$	$. 25$	$a_{-1} = 1$
$0.25$	$* 2 =$	$0$	$. 5$	$a_{-2} = 0$
$0.5$	$* 2 =$	$1$	$. 0$	$a_{-3} = 1$

**Answer:**  $(0.625)_{10} = (0.a_{-1} a_{-2} a_{-3})_2 = (0.101)_2$



# Decimal to Octal Conversion

Example:  $(175)_{10}$

	Quotient	Remainder	Coefficient
$175 / 8 =$	<b>21</b>	<b>7</b>	$a_0 = 7$
$21 / 8 =$	<b>2</b>	<b>5</b>	$a_1 = 5$
$2 / 8 =$	<b>0</b>	<b>2</b>	$a_2 = 2$

Answer:  $(175)_{10} = (a_2 a_1 a_0)_8 = (257)_8$

Example:  $(0.3125)_{10}$

	Integer	Fraction	Coefficient
$0.3125 * 8 =$	<b>2</b>	<b>5</b>	$a_{-1} = 2$
$0.5 * 8 =$	<b>4</b>	<b>0</b>	$a_{-2} = 4$

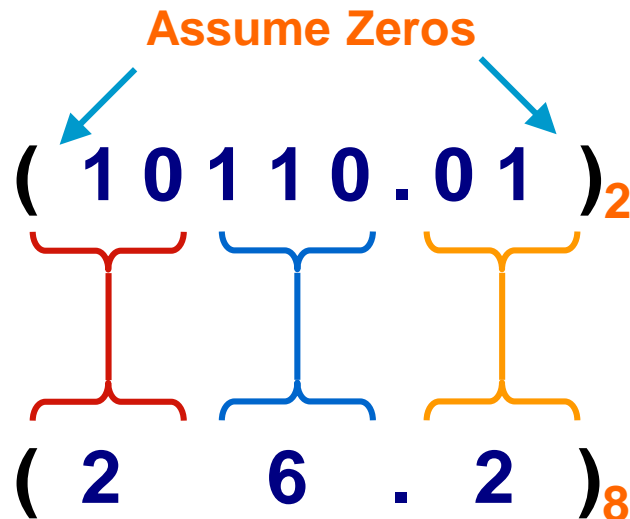
Answer:  $(0.3125)_{10} = (0.a_{-1} a_{-2} a_{-3})_8 = (0.24)_8$

# Binary – Octal Conversion

■  $8 = 2^3$

- Each group of 3 bits represents an octal digit

**Example:**



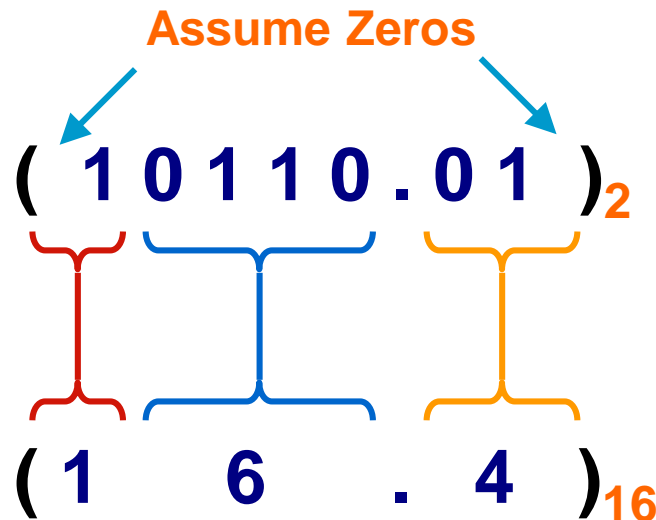
Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works **both** ways (*Binary to Octal & Octal to Binary*)

# Binary – Hexadecimal Conversion

- 16 = 2<sup>4</sup>
- Each group of 4 bits represents a hexadecimal digit

**Example:**



Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

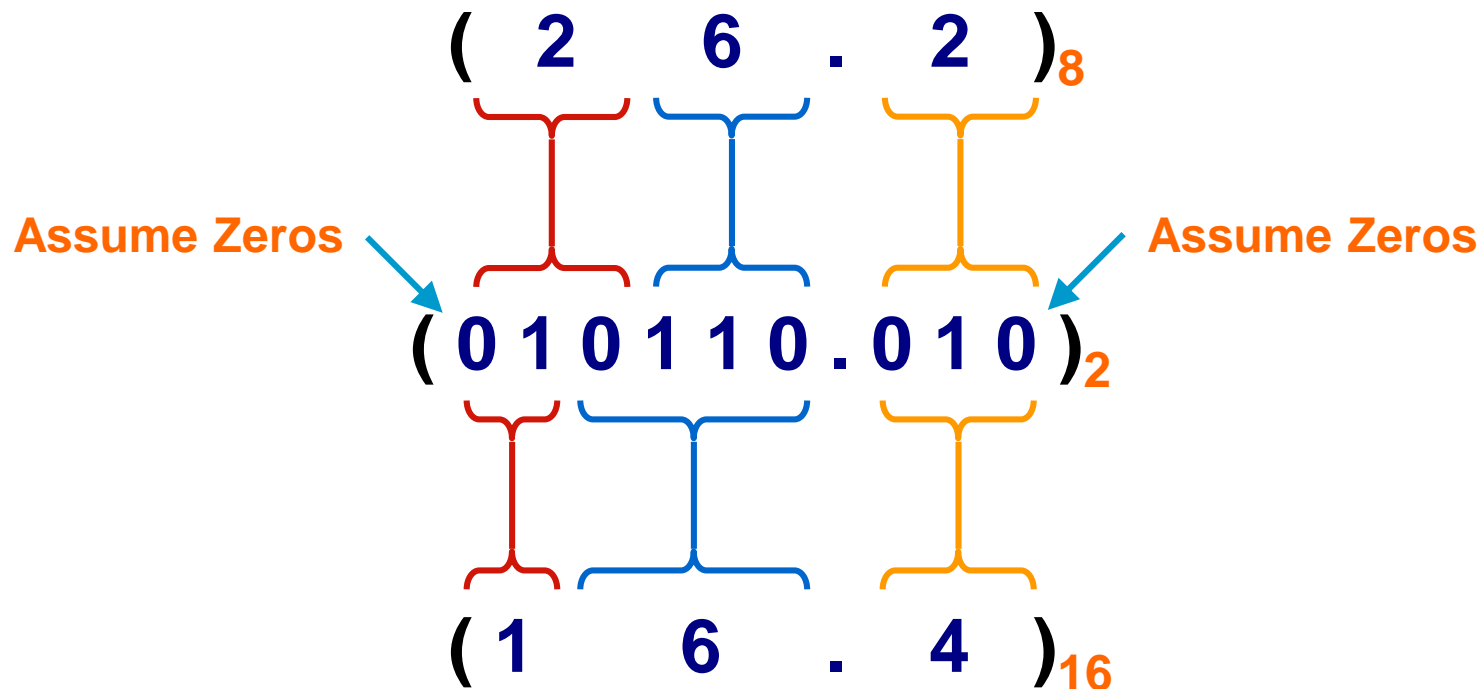
Works **both** ways (*Binary to Hex & Hex to Binary*)



# Octal – Hexadecimal Conversion

- Convert to **Binary** as an intermediate step

**Example:**



Works **both** ways (*Octal to Hex & Hex to Octal*)

# Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

# Addition

## ▣ Decimal Addition


$$\begin{array}{r} \text{1} \quad \text{1} \quad \quad \leftarrow \text{Carry} \\ \quad \text{5} \quad \text{5} \\ + \quad \text{5} \quad \text{5} \\ \hline \text{1} \quad \text{1} \quad \text{0} \end{array}$$

$\searrow = \text{Ten} \geq \text{Base}$   
 $\rightarrow$  Subtract a Base

# Binary Addition

## ▣ Column Addition

$$\begin{array}{rcccccc} & 1 & 1 & 1 & 1 & 1 & 1 & & \\ & & 1 & 1 & 1 & 1 & 0 & 1 & = 61 \\ + & & & 1 & 0 & 1 & 1 & 1 & = 23 \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 & & = 84 \end{array}$$

  $\geq (2)_{10}$

# Binary Subtraction

- Borrow a “Base” when needed

$$\begin{array}{r}
 100 \\
 - 46 \\
 \hline
 54
 \end{array}$$

# Binary Multiplication

▣ Bit by bit

$$\begin{array}{r} \phantom{10111}10111 \\ \phantom{10111}1010 \\ \hline \phantom{10111}00000 \\ \phantom{10111}10111 \\ \phantom{10111}00000 \\ 10111 \\ \hline 11100110 \end{array}$$

# 1.5 Complements

- There are two types of complements for each base- $r$  system: the radix complement and diminished radix complement.

- Diminished Radix Complement -  $(r-1)$ 's Complement**

- Given a number  $N$  in base  $r$  having  $n$  digits, the  $(r-1)$ 's complement of  $N$  is defined as:

$$(r^n - 1) - N$$

- Example for 6-digit decimal numbers:**

- 9's complement is  $(r^n - 1) - N = (10^6 - 1) - N = 999999 - N$
- 9's complement of 546700 is  $999999 - 546700 = 453299$

- Example for 7-digit binary numbers:**

- 1's complement is  $(r^n - 1) - N = (2^7 - 1) - N = 1111111 - N$
- 1's complement of 1011000 is  $1111111 - 1011000 = 0100111$

- Observation:**

- Subtraction from  $(r^n - 1)$  will never require a borrow
- Diminished radix complement can be computed digit-by-digit
- For binary:  $1 - 0 = 1$  and  $1 - 1 = 0$

# Complements

## □ 1's Complement (*Diminished Radix* Complement)

◆ All '0's become '1's

◆ All '1's become '0's

Example  $(10110000)_2$

$\Rightarrow (01001111)_2$

If you add a number and its 1's complement ...

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0 \\ +\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \end{array}$$



# Complements

## ■ Radix Complement

The  $r$ 's complement of an  $n$ -digit number  $N$  in base  $r$  is defined as  $r^n - N$  for  $N \neq 0$  and as 0 for  $N = 0$ . Comparing with the  $(r - 1)$ 's complement, we note that the  $r$ 's complement is obtained by adding 1 to the  $(r - 1)$ 's complement, since  $r^n - N = [(r^n - 1) - N] + 1$ .

## ■ Example: Base-10

The 10's complement of 012398 is 987602

The 10's complement of 246700 is 753300

$$\begin{array}{r} 753299 \\ + \phantom{000001} \\ \hline 753300 \end{array}$$

## ■ Example: Base-2

The 2's complement of 1101100 is 0010100

The 2's complement of 0110111 is 1001001

# Complements

## □ 2's Complement (*Radix* Complement)

◆ Take 1's complement then add 1

OR ◆ Toggle all bits to the left of the first '1' from the right

*Example:*

Number:

1's Comp.:

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0 \\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1 \\ + \qquad \qquad \qquad 1 \\ \hline 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0 \end{array}$$

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0 \\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1 \\ + \qquad \qquad \qquad 1 \\ \hline 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0 \end{array}$$

# Complements

## ■ Subtraction with Complements

- ◆ The subtraction of two  $n$ -digit unsigned numbers  $M - N$  in base  $r$  can be done as follows:

1. Add the minuend  $M$  to the  $r$ 's complement of the subtrahend  $N$ . Mathematically,  $M + (r^n - N) = M - N + r^n$ .
2. If  $M \geq N$ , the sum will produce an end carry  $r^n$ , which can be discarded; what is left is the result  $M - N$ .
3. If  $M < N$ , the sum does not produce an end carry and is equal to  $r^n - (N - M)$ , which is the  $r$ 's complement of  $(N - M)$ . To obtain the answer in a familiar form, take the  $r$ 's complement of the sum and place a negative sign in front.

# Complements

## ■ Example 1.7

84

67

- ◆ Given the two binary numbers  $X = 1010100$  and  $Y = 1000011$ , perform the subtraction (a)  $X - Y$ ; and (b)  $Y - X$ , by using 2's complement.

(a)	$X =$	1010100
	2's complement of $Y =$	<u>+0111101</u>
	Sum =	10010001
	Discard end carry $2^7 =$	<u>-10000000</u>
	Answer. $X - Y =$	0010001

(b)	$Y =$	1000011
	2's complement of $X =$	<u>+ 0101100</u>
	Sum =	1101111

There is no end carry.  
Therefore, the answer is  
 $Y - X = -$  (2's complement  
of 1101111)  $= - 0010001$ .

# Complements

- Subtraction of unsigned numbers can also be done by means of the  $(r - 1)$ 's complement. Remember that the  $(r - 1)$ 's complement is one less than the  $r$ 's complement.
- Example 1.8
  - Repeat Example 1.7, but this time using 1's complement.

(a)  $X - Y = 1010100 - 1000011$

$$\begin{array}{r} X = 1010100 \\ \text{1's complement of } Y = \pm 0111100 \\ \text{Sum} = 10010000 \\ \text{End-around carry} = \quad + \quad 1 \\ \hline \text{Answer. } X - Y = 0010001 \end{array}$$

(b)  $Y - X = 1000011 - 1010100$

$$\begin{array}{r} Y = 1000011 \\ \text{1's complement of } X = + 0101011 \\ \text{Sum} = 1101110 \end{array}$$



There is no end carry,  
Therefore, the answer is  $Y - X = - (1\text{'s complement of } 1101110) = - 0010001$ .

# 1.6 Signed Binary Numbers

- To represent negative integers, we need a notation for negative values.
- It is customary to represent the sign with a bit placed in the leftmost position of the number since binary digits.
- The convention is to make the **sign bit 0 for positive** and **1 for negative**.
- Example:

Signed-magnitude representation:	10001001
Signed-1's-complement representation:	11110110
Signed-2's-complement representation:	11110111

- **Table 1.3** lists all possible four-bit signed binary numbers in the three representations.

# Signed Binary Numbers

**Table 1.3**  
*Signed Binary Numbers*

<b>Decimal</b>	<b>Signed-2's Complement</b>	<b>Signed-1's Complement</b>	<b>Signed Magnitude</b>
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

# Signed Binary Numbers

## ■ Arithmetic addition

- ◆ The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. **If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.**
- ◆ The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits.
- ◆ A carry out of the sign-bit position is discarded.

## ■ Example:

+ 6	00000110	− 6	11111010
<u>+13</u>	<u>00001101</u>	<u>+13</u>	<u>00001101</u>
+ 19	00010011	+ 7	00000111
+ 6	00000110	− 6	11111010
<u>−13</u>	<u>11110011</u>	<u>−13</u>	<u>11110011</u>
− 7	11111001	− 19	11101101



# 1.7 Binary Codes

## ▣ BCD Code

- ◆ In this **code** each decimal digit is represented by a 4-bit binary number. **BCD** is a way to express each of the decimal digits with a binary **code**
- ◆ A number with k decimal digits will require 4k bits in BCD.
- ◆ Decimal 396 is represented in BCD with 12bits as 0011 1001 0110, with each group of 4 bits representing one decimal digit.
- ◆ The binary combinations 1010 through 1111 are not used and have no meaning in BCD.

**Table 1.4**  
*Binary-Coded Decimal (BCD)*

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

# Binary Code

## ■ Example:

- ◆ Consider decimal 185 and its corresponding value in BCD and binary:



$$(185)_{10} = (0001\ 1000\ 0101)_{\text{BCD}} = (10111001)_2$$

## ■ BCD addition

4	0100	4	0100	8	1000
<u>+5</u>	<u>+0101</u>	<u>+8</u>	<u>+1000</u>	<u>+9</u>	<u>+1001</u>
9	1001	12	1100	17	10001
			<u>+0110</u>		<u>+0110</u>
			10010		10111

# Binary Code

## ■ Example:

- ◆ Consider the addition of  $184 + 576 = 760$  in BCD:

BCD	1	1		
	0001	1000	0100	184
	<u>+ 0101</u>	<u>0111</u>	<u>0110</u>	<u>+576</u>
Binary sum	0111	10000	1010	
Add 6	<u>      </u>	<u>0110</u>	<u>0110</u>	<u>      </u>
BCD sum	0111	0110	0000	760

# Binary Codes

- Other Decimal Codes – weighted representation of decimal numbers in binary.

**Table 1.5**

*Four Different Binary Codes for the Decimal Digits*

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, -2, -1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
Unused bit combinations	1010	0101	0000	0001
	1011	0110	0001	0010
	1100	0111	0010	0011
	1101	1000	1101	1100
	1110	1001	1110	1101
	1111	1010	1111	1110

# Binary Codes)

## ▣ Gray Code

- ◆ The advantage is that only bit in the code group changes in going from one number to the next.
  - » Error detection.
  - » Representation of analog data.
  - » Low power design.

**Table 1.6**  
*Gray Code*

<b>Gray Code</b>	<b>Decimal Equivalent</b>
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

# Binary Codes

## ■ American Standard Code for Information Interchange (ASCII) Character Code

**Table 1.7**

*American Standard Code for Information Interchange (ASCII)*

$b_4b_3b_2b_1$	$b_7b_6b_5$							
	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	
1101	CR	GS	-	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	_	o	DEL

# Binary Codes

## ■ ASCII Character Code

### Control characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

# ASCII Character Codes

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- American Standard Code for Information Interchange (Refer to Table 1.7)
- A popular code used to represent information sent as character-based data.
- It uses 7-bits to represent:
  - ◆ 94 Graphic printing characters.
  - ◆ 34 Non-printing characters.
- Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return).
- Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).



# ASCII Properties

- ASCII has some interesting properties:
  - ◆ Digits 0 to 9 span Hexadecimal values  $30_{16}$  to  $39_{16}$
  - ◆ Upper case A-Z span  $41_{16}$  to  $5A_{16}$
  - ◆ Lower case a-z span  $61_{16}$  to  $7A_{16}$ 
    - » Lower to upper case translation (and vice versa) occurs by flipping bit 6.



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Thanks To  
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