

UNIT 2

Lecture 35

Normalization

Properties of Decomposition

Properties of Decomposition

- **Decomposition** - Decomposition means breaking the relation schema into smaller schema. We may also call this schema refinement.
- Suppose a relation schema $R (A, B, C, D, E, F, G)$ is split into $R1 (A, B, C, D)$ and $R2 (E, F, G)$. We can say that $R1$ and $R2$ are decompositions of R .
- We can say a relation R is decomposed into a set of relations $R1$ and $R2$ if and only if it satisfies the following properties of decomposition.
 1. Attribute Preservation
 2. Lossless Join Decomposition
 3. Dependency Preservation

Attribute Preservation

- If R_1 and R_2 are projections of some relation R and R_1 and R_2 between them include all the attributes of R , then we say that R is decomposed into R_1 and R_2 .
- For e.g., Consider the relation $R (A, B, C, D, E, F, G)$ and the decomposition of this relation are $R_1 (A, B, C, D)$ and $R_2 (E, F, G)$ are the attribute preserving decompositions.

Loss-Less Join Decomposition

- We say that the decomposition of R into R1, with attribute set X1 and R2 with attribute set X2, is loss-less join decomposition if by joining R1 and R2 we get back the original relation R.
- The word loss-less refers to loss of information and not loss of tuples.
- Let R be a relation with attribute set X and F is the set of FDs that hold over R. The decomposition of R into R1 and R2 with attributes X1 and X2 respectively is loss-less join if and only if F^+ contains either

$$X1 \cap X2 \rightarrow X1$$

Or,
$$X1 \cap X2 \rightarrow X2$$

- In other words, the attributes common to R1 and R2 must contain a key for either R1 or R2.
- If an FD, $X \rightarrow Y$ holds over a relation schema R and $X \cap Y$ is empty, the decomposition of R into $R - Y$ and XY is loss-less.
- For E.g. Consider the relation schema R(A, B, C) with FDs $AB \rightarrow C$ and $C \rightarrow B$. The relation is not in BCNF since $C \rightarrow B$ holds in R. We decompose this relation into R1(A, C) (i.e. $R - Y$) and R2(B, C) (i.e. XY) is a loss-join decomposition because $B \cap C$ is empty.

Dependency Preservation

- If R is decomposed into X , Y and Z , and we enforce the FDs that hold on X , on Y and on Z , then all FDs that were given to hold on R must also hold.
- The decomposition of relation R with FDs F into R_1 and R_2 is said to be dependency preserving iff $(F_{R_1} \cup F_{R_2})^+ = F^+$.
- In other words, if we take the dependencies of F_{R_1} and F_{R_2} and compute the closure of their union, we get back the original FDs in F .
- For e.g., Consider the relation $R(A, B, C, D)$ with FDs $A \rightarrow B$ and $C \rightarrow D$ is decomposed into $R_1(A, B)$ and $R_2(C, D)$ is a dependency preserving decomposition.
- It is always possible to find a dependency preserving decomposition with respect to a F such that the resulting relations are in 3NF.
- In general, there may not be a dependency preserving decomposition that also decomposes relations in BCNF.
- For e.g., Consider a relation $R(A, B, C)$ with FDs, $AB \rightarrow C$ and $C \rightarrow B$ is a relation that cannot be decomposed to satisfy both dependency preserving and BCNF.

Properties of Decomposition

Q. 1 Suppose that we decompose the schema $R = (A, B, C, D, E)$ into

(A, B, C)

(A, D, E)

Show that this decomposition is a loss-less join decomposition if the following set F of functional dependencies holds :

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

Sol: A decomposition $\{R_1, R_2\}$ is loss-less join decomposition if $R_1 \cap R_2 \rightarrow R_1$ or $R_1 \cap R_2 \rightarrow R_2$.

Let $R_1 = (A, B, C)$, and $R_2 = (A, D, E)$, So $R_1 \cap R_2 = A$. Since A is a candidate key of both R_1 and R_2 therefore $R_1 \cap R_2 \rightarrow R_1$ and $R_1 \cap R_2 \rightarrow R_2$ (i.e. A is a candidate key of both relations R_1 and R_2). So the decomposition is loss-less join decomposition.

Properties of Decomposition

Q. 2 Suppose that we decompose the schema $R = (A, B, C, D, E)$ into

(A, B, C)

(C, D, E)

Show that this decomposition is not a loss-less join decomposition if the following set F of functional dependencies holds :

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

Sol: A decomposition $\{R_1, R_2\}$ is loss-less join decomposition if $R_1 \cap R_2 \rightarrow R_1$ or $R_1 \cap R_2 \rightarrow R_2$.

Let $R_1 = (A, B, C)$, and $R_2 = (C, D, E)$, So $R_1 \cap R_2 = C$. Since C is not a candidate key of any relations R_1 and R_2 , therefore $R_1 \cap R_2$ cannot determine either R_1 or R_2 (i.e. C is not a candidate key of any of the relations R_1 and R_2). So the decomposition is not a loss-less join decomposition, it is lossy decomposition.

Properties of Decomposition

Q. 3 Give a loss-less join decomposition into BCNF of schema

$R = (A, B, C, D, E)$ with a set of FDs

$F = \{A \rightarrow BC,$

$CD \rightarrow E,$

$B \rightarrow D,$

$E \rightarrow A\}$

Sol : The candidate keys of this relation are A, BC, CD and E. The relation schema $R = (A, B, C, D, E)$ with a set of FDs is not in BCNF because the nontrivial Functional dependency $B \rightarrow D$ holds in R and B is not a candidate key of R. So the decomposition of $R_1(A, B, C, E)$ and $R_2(B, D)$ is loss-less join decomposition and in BCNF.

Properties of Decomposition

Q. 4 Give a loss-less join, dependency preserving decomposition into 3NF of schema $R = (A, B, C, D, E)$ with a set of FDs

$F = \{A \rightarrow BC,$

$CD \rightarrow E,$

$B \rightarrow D,$

$E \rightarrow A\}$

Sol: The candidate keys of this relation are A, BC, CD and E.

The dependencies form the canonical cover.

So the decomposition of $R_1(A, B, C)$, $R_2(C, D, E)$, $R_3(B, D)$ and

$R_4(E, A)$ is a loss-less, dependency preserving decomposition into 3NF.

Properties of Decomposition

Q. 5 Suppose that we decompose the schema $R = (A, B, C, D, E)$ into

(A, B, C)

(A, D, E)

Show that this decomposition is not a dependency preserving decomposition if the following set F of functional dependencies holds :

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

Sol: A decomposition $\{R_1, R_2\}$ is dependency preserving decomposition iff $(F_{R_1} \cup F_{R_2})^+ = F^+$.

Properties of Decomposition

Q. 6 Suppose that we decompose the schema $R = (A, B, C, D, E)$ into

(A, B, C)

(C, D, E)

Show that this decomposition is not a dependency preserving decomposition if the following set F of functional dependencies holds :

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

Sol: A decomposition $\{R_1, R_2\}$ is dependency preserving decomposition iff $(F_{R_1} \cup F_{R_2})^+ = F^+$.

Properties of Decomposition

Q. 7 Suppose that we decompose the schema $R = (A, B, C, D, E)$ into
 (A, B) , (B, C) and (C, D)

Show that this decomposition is a dependency preserving decomposition if the following set F of functional dependencies holds :

$$F = \{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ A \rightarrow C \\ C \rightarrow D \end{array} \}$$

Sol: A decomposition $\{R_1, R_2, R_3\}$ is dependency preserving decomposition iff $(F_{R_1} \cup F_{R_2} \cup F_{R_3})^+ = F^+$.

Properties of Decomposition

Q. 8 Consider $R = (A, B, C, D, E)$ with

$$F = \{ \begin{array}{l} AD \rightarrow C, \\ CD \rightarrow A, \\ B \rightarrow D, \\ D \rightarrow BE \end{array} \}.$$

Determine whether $\{ABC, BCD, DE\}$ is loss-less. Find all keys for R .

Algorithm 15.3. Testing for Nonadditive Join Property

Input: A universal relation R , a decomposition $D = \{R_1, R_2, \dots, R_m\}$ of R , and a set F of functional dependencies.

Note: Explanatory comments are given at the end of some of the steps. They follow the format: (**comment**).

1. Create an initial matrix S with one row i for each relation R_i in D , and one column j for each attribute A_j in R .
2. Set $S(i, j) := b_{ij}$ for all matrix entries. (**Each b_{ij} is a distinct symbol associated with indices (i, j) **)
3. For each row i representing relation schema R_i
 {for each column j representing attribute A_j
 {if (relation R_i includes attribute A_j) then set $S(i, j) := a_j$ };}; (**Each a_j is a distinct symbol associated with index (j) **)
4. Repeat the following loop until a *complete loop execution* results in no changes to S
 {for each functional dependency $X \rightarrow Y$ in F
 {for all rows in S that have the same symbols in the columns corresponding to attributes in X
 {make the symbols in each column that correspond to an attribute in Y be the same in all these rows as follows: If any of the rows has an a symbol for the column, set the other rows to that *same* a symbol in the column. If no a symbol exists for the attribute in any of the rows, choose one of the b symbols that appears in one of the rows for the attribute and set the other rows to that same b symbol in the column ; } ; } ;}
5. If a row is made up entirely of a symbols, then the decomposition has the nonadditive join property; otherwise, it does not.

GATE Questions

Consider the schema $R(ABCD)$ and FDs $A \rightarrow B$ and $C \rightarrow D$. Then the decomposition of R into $R_1(AB)$ and $R_2(CD)$ is

- (A) Dependencies preserving and lossless join.
- (B) Lossless join but not dependency preserving.
- (C) Dependency preserving but not lossless join.
- (D) Not dependencies preserving and not lossless join.

[GATE 2001]

GATE Questions

Which one of the following statements about normal forms is FALSE?

- (A) BCNF is stricter than 3NF
- (B) Lossless, dependency-preserving decomposition into 3NF is always possible
- (C) Lossless, dependency-preserving decomposition into BCNF is always possible
- (D) Any relation with two attributes is in BCNF

[GATE 2005]

GATE Questions

Relation R is decomposed using a set of functional dependencies, F and relation S is decomposed using another set of functional dependencies G. One decomposition is definitely BCNF, the other is definitely 3NF, but it is not known which is which. To make a guaranteed identification, which one of the following tests should be used on the decompositions? (Assume that the closures of F and G are available).

- (A) Dependency-preservation
- (B) Lossless-join
- (C) BCNF definition
- (D) 3NF definition

[GATE 2002]

GATE Questions

Let the set of functional dependencies $F = \{QR \rightarrow S, R \rightarrow P, S \rightarrow Q\}$ hold on a relation schema $X = (PQRS)$. X is not in BCNF. Suppose X is decomposed into two schemas Y and Z , where $Y = (PR)$ and $Z = (QRS)$.

Consider the two statements given below.

I. Both Y and Z are in BCNF

II. Decomposition of X into Y and Z is dependency preserving and lossless

Which of the above statements is/are correct?

(A) Both I and II (B) I only (C) II only (D) Neither I nor II

[GATE 2019]

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