UNIT 2

Lecture 30

Normalization

Attribute Closure

Closure of a set of FDs (Attribute Closure)

1. The closure of a set of FDs is the set of all FDs implied by a given set of FDs.
2. It can be calculated using Armstrong Axioms.
3. It can be used to check if a FD follows from a given set.
4. Can check if a set of attributes is a candidate key.

Armstrong Axiom to find Closure

$X^+ := X$;

Repeat

Old $X^+ := X^+$;

For each functional dependency Y → Z in F do

If $X^+ \supseteq Y$ then $X^+ := X^+ \cup Z$;

Until (X^+ = Old X^+);

Q.1 Suppose we are given a relation schema R = (A, B, C, G, H, I) and the set of FDs. A → B, A → C, CG → H, CG → I, B → H. Find $(AG)^+$.

Sol : Given relation R (A, B, C, G, H, I) and set of functional dependencies are

A → B, A → C, CG → H, CG → I, B → H, so to find $(AG)^+$.

Iteration	Using	Result $(AG)^+$	Old $(AG)^+$
1		AG	AG
2	A → B	ABG	AG
3	A → C	ABCG	AG
4	CG → H	ABCGH	AG
5	CG → I	ABCGHI	AG
6	B → H	ABCGHI	AG
7		ABCGHI	ABCGHI

So $(AG)^+$ = ABCGHI.

Since $(AG)^+$ contains all the attributes of relation R so AG is the key of R.

Q.2 What is closure of attributes, give the algorithm for it with given relation R (A, B, C, D, E, F) and following set of functional dependencies.

A → BC, E → CF, B → E, CD → EF

(i) Computer $(AB)^+$.

(ii) Is AC → CF implied by above FD's?

Sol : Given relation R (A, B, C, D, E, F) and set of functional dependencies are

A → BC, E → CF, B → E, CD → EF, so to find $(AB)^+$.

Iteration	Using	Result $(AB)^+$	Old $(AB)^+$
1		AB	AB
2	A → BC	ABC	AB
3	B → E	ABCE	AB
4	E → CF	ABCEF	AB
5		ABCEF	ABCEF

So $(AB)^+$ = ABCEF.

Q.2 What is closure of attributes, give the algorithm for it with given relation R (A, B, C, D, E, F) and following set of functional dependencies.

A → BC, E → CF, B → E, CD → EF

(i) Computer $(AB)^+$.

(ii) Is AC → CF implied by above FD's?

Sol : Now to check whether AC → CF is implied by FDs we need to find $(AC)^+$.

Iteration	Using	Result $(AC)^+$	Old $(AC)^+$
1		AC	AC
2	A → BC	ABC	AC
3	B → E	ABCE	AC
4	E → CF	ABCEF	AC
5		ABCEF	ABCEF

So $(AC)^+$ = ABCEF. Since $(AC)^+$ contains CF, so we can say AC → CF.

Q.3 Consider a relation R (A, B, C, D, E) with the following functional dependencies

AB → C, CD →E, DE → B.

Is AB a candidate key of this relation? If not, is ABD? Explain your answer.

Sol : For AB to be a candidate key of this relation it can determine all the attributes of this relation that is for AB to be a candidate key if and if only if AB → ABCDE, so we need to find $(AB)^+$.

Iteration	Using	Result $(AB)^+$	Old $(AB)^+$
1		AB	AB
2	AB → C	ABC	AB
3		ABC	ABC

So $(AB)^+$ = ABC. Since $(AB)^+$ do not contain all the attribute of this relation, AB is not a candidate key of this relation.

Q.3 Consider a relation R (A, B, C, D, E) with the following functional dependencies

AB → C, CD →E, DE → B.

Is AB a candidate key of this relation? If not, is ABD? Explain your answer.

Sol : For ABD to be a candidate key of this relation it can determine all the attributes of this relation that is for ABD to be a candidate key if and if only if ABD → ABCDE, so we need to find $(ABD)^+$.

Iteration	Using	Result $(ABD)^+$	Old $(ABD)^+$
1		ABD	ABD
2	AB → C	ABCD	ABD
3	CD → E	ABCDE	ABCD
4	DE → B	ABCDE	ABCD
5		ABCDE	ABCDE

So $(ABD)^+$ = ABCDE. Since $(ABD)^+$ contains all the attribute of this relation, ABD is a candidate key of this relation.

Q.4 Let the relation :

R = {A, B, C, D, E, F, G} satisfies following FD's

Let F = {A → B, BC → DE, AEF → G, B → F}

Find A^+, BC^+, AEF^+.

Q.5 Let the relation

R (A, B, C, D, E, F, G)

Satisfies the following FD's

A → B,

BC → DE,

AEF → G

Compute the closure $\{A, C\}^+$.

Is the FD ACF → DG implied by this set.

GATE Question

The following functional dependencies are given :

AB →CD, AF →D, DE → F, C → G, F → E, G →A

Which one of the following option is false?

(A) $\{CF\}^+ = \{ACDEFG\}$

(B) $\{BG\}^+ = \{ABCDG\}$

(C) $\{AF\}^+ = \{ACDEFG\}$

(D) $\{AB\}^+ = \{ABCDFG\}$

[**GATE 2006**]

GATE Question

In a scheme with attribute A, B, C, D and E following set of functional dependencies are given

A → B

A → C

CD → E

B → D

E → A

Which of the following functional dependencies is NOT implied by the above set?

(a) CD → AC

(b) BD → CD

(c) BC → CD

(d) AC → BC

[GATE 2005]

GATE Question

Prime Attribute – An attribute of relation schema R is called a prime attribute of R if it is a member of some candidate key of R.

Nonprime Attribute – An attribute is called nonprime if it is not a prime attribute, i.e. if it is not a member of any candidate key.

A prime attribute of a relation scheme R is an attribute that appears

(A) in all candidate keys of R.

(B) in some candidate key of R.

(C) in a foreign keys of R.

(D) only in the primary key of R.

[GATE 2014]

Finding candidate key of Relation

Q.1 Consider a relation R (A, B, C, D, E) with a set of FDs

F = {A → BC, C → D, D → E}.

Find candidate key of R.

Sol : Super key of this relation is = ABCDE

Essential attribute for candidate key = A

Find A^+ = ABCDE

Since A contains all the attributes of this relation, A is an candidate key of this relation.

So prime attribute is = A

Non prime attributes are = B, C, D, E

Finding candidate key of Relation

Q.2 Consider a relation R (A, B, C, D, E) with a set of FDs

F = {AB → C, C → D, D → E, E → B}.

Find candidate key of R.

Sol : Super key of this relation is = ABCDE

Essential attribute for candidate key = A

Find A^+ = A

Since A do not contain all the attributes of this relation, A is not an candidate key of this relation. That means relation R has multiple candidate keys and every candidate key must contain attribute A. So we try all the combinations of A.

So, AB^+ = ABCDE, implies that AB is an candidate key.

AC^+ = ABCDE, implies that AC is an candidate key.

AD^+ = ABCDE, implies that AD is an candidate key.

AE^+ = ABCDE, implies that AE is an candidate key.

So we have 4 candidate keys.

Prime attributes are = A, B, C, D, E

Non prime attributes are = ø

Finding candidate key of Relation

Q.3 Consider a relation R (A, B, C, D, E) with a set of FDs

F = {A → BC, CD → E, B → D, E → A}.

Find candidate key of R.

Sol : Super key of this relation is = ABCDE

Essential attribute for candidate key = ø

That means relation R has multiple candidate keys and no attribute is common in any candidate key. So we need to find the closure of all the determinants.

So, A^+ = ABCDE, implies that A is an candidate key.

CD^+ = ABCDE, implies that CD is an candidate key.

B^+ = BD, implies that B is not an candidate key.

E^+ = ABCDE, implies that E is an candidate key.

So we try the combination of B with C and D and check whether BC or CD is an candidate key.

So, BC^+ = ABCDE

BD^+ = BD

So, the candidate keys of this relation are A, BC, CD, E.

Prime attributes are = A, B, C, D, E

Non prime attributes are = ø

GATE Question

Consider the relation schema R = (E, F, G, H, I, J, K, L, M, N) and the set of functional dependencies {{E, F} → {G}, {F} → {I, J}, {E, H} → {K, L}, {K} → {M}, {L} → {N}} on R. What is the key for R?

(a) {E, F}

(b) {E, F, H}

(c) {E, F, H, K, L}

(d) {E}

[GATE 2014]

GATE Question

Which of the following is NOT a super key in a relational schema with attributes V, W, X, Y, Z and primary key VY?

(A) VXYZ

(B) VWXZ

(C) VWXY

(D) VWXYZ

[GATE 2016]

GATE Question

The maximum number of super keys for the relation schema R (E, F, G, H) with E as the key is ___.

[GATE 2014]

GATE Question

Relation R has eight attributes ABCDEFGH. Fields of R contain only atomic values. F = {CH → G, A →BC, B →CFH, E →A, F →EG} is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

Q. How many candidate keys does the relation R have?

(A) 3

(B) 4

(C) 5

(D) 6

[GATE 2013]

GATE Question

Consider a relation scheme R = {A, B, C, D, E, H}on which the following functional dependencies hold : {A →B, BC →D, E →C, D →A}. Which are the candidate keys of R?

(A) AE, BE

(B) AE, BE, DE

(C) AEH, BEH, BCH

(D) AEH, BEH, DEH

[GATE 2005]

GATE Question

Let R = (A, B, C, D, E, F) be a relation scheme with the following dependencies C →F, E →A, EC →D, A →B. which of the following is the key for R?

(A) CD

(B) EC

(C) AE

(D) AC

[GATE 1999]

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The link for my youtube channel is

https://www.youtube.com/channel/UCRWGtE76JlTp1iim6aOTRuw?sub_confirmation=1