UNIT 2

Lecture 31

Normalization

FD Closure and Minimal Cover

Closure of a set of FDs (Attribute Closure)

1. The closure of a set of FDs is the set of all FDs implied by a given set of FDs.
2. It can be calculated using Armstrong Axioms.
3. It can be used to check if a FD follows from a given set.
4. Can check if a set of attributes is a candidate key.

Armstrong Axiom to find Closure

$X^+ := X;$

Repeat

Old $X^+ := X^+$;

For each functional dependency $Y \rightarrow Z$ in F do

If $X^+ \supseteq Y$ then $X^+ := X^+ \cup Z$;

Until ($X^+ =$ Old X^+);

Inference Rules for functional dependencies

1. Reflexive Rule

   If $X \supset Y$ then $X \rightarrow Y$

2. Augmentation Rule

   If $X \rightarrow Y$ then $XZ \rightarrow YZ$

3. Transitive Rule

   If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

4. Decomposition or Projective Rule

   If $X \rightarrow YZ$ then $X \rightarrow Y$, and $X \rightarrow Z$

5. Union or Additive Rule

   If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$

6. Pseudo transitive Rule

   If $X \rightarrow Y$ and $YZ \rightarrow W$ then $XZ \rightarrow W$

Q.1 Consider the closure of following set F of functional dependencies for the relation scheme. R = {A, B, C, D, E, F}, F = {A → B, A → C, CD → E, CD → F, B → F}. Find F^+.

Sol : The closure of a set of functional dependencies (F^+) is the set of all functional dependencies implied by the set F.

So, F^+ = {A →B, A → C, CD → E, CD → F, B → F}

Since, A → B and B → F exists we have A → F by transitivity.

So, F^+ = {A →B, A → C, CD → E, CD → F, B → F, A → F}

Since, A → C and CD → E exists we have AD → E by pseudo transitivity

So, F^+ = {A →B, A → C, CD → E, CD → F, B → F, A → F, AD → E}

Since, A → C and CD → F exists we have AD → F by pseudo transitivity

So, F^+ = {A →B, A → C, CD → E, CD → F, B → F, A → F, AD → E, AD→F}

Covering Constraint

- A set of functional dependencies F is said to **cover** another set of functional dependencies E if every FD in E is also in F^+; i.e. if any dependency in E can be inferred from F; we can say that E is **covered by** F.
- E.g. Consider the relation R (A, B, C, D) with 2 sets of FD's F = {A $\rightarrow$ B, B $\rightarrow$ C, C $\rightarrow$ D} and G = {A $\rightarrow$ B, C $\rightarrow$ D}.
- So, we can say F is said to cover G because all FDs that is implied by G^+ is already in F^+.

Equivalence of Sets of Functional Dependencies

- Two sets of Functional dependencies E and F are **equivalent** if $E^+ = F^+$.
- Hence, equivalence means that every FD in E can be inferred from F, and every FD in F can be inferred from E; that is, E is equivalent to F if both the conditions E covers F and F covers E hold.
- We can determine whether F covers E by calculating X^+ with respect to F for each FD $X \rightarrow Y$ in E, and then checking this X^+ includes the attributes in Y. If this is the case for every FD in E, then F covers E.
- We determine whether E and F are equivalent by checking that E covers F and F covers E.

Q.2 Consider the following set of FDs F = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ AD, E $\rightarrow$ H}, and G = {A $\rightarrow$ CD, E $\rightarrow$ AH}. Check whether they are equivalent.

Sol : Two sets of FDs F and G are equivalent set of FDs if and only if $F^+ = G^+$.

So, F = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ AD, E $\rightarrow$ H}

From E $\rightarrow$ AD we have E $\rightarrow$ A and E $\rightarrow$ D [By decomposition Rule]

So, F^+ = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ A, E$\rightarrow$D, E$\rightarrow$ H}

From A $\rightarrow$ C and AC $\rightarrow$ D we have, A $\rightarrow$ D [By pseudo transitive Rule]

So, F^+ = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ A, E$\rightarrow$D, E$\rightarrow$ H, A $\rightarrow$D}

From E $\rightarrow$A and A $\rightarrow$ C we have E $\rightarrow$ C [By transitive Rule]

So, F^+ = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ A, E$\rightarrow$D, E$\rightarrow$ H, A $\rightarrow$D, E$\rightarrow$H}

From AC $\rightarrow$ D we have AC $\rightarrow$ CD [By augmenting C in both sides]

So, F^+ = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ A, E$\rightarrow$D, E$\rightarrow$ H, A $\rightarrow$D, E$\rightarrow$H, AC $\rightarrow$ CD}

From AC $\rightarrow$ CD we have AC $\rightarrow$ C and AC $\rightarrow$ D [By decomposition Rule]

So, F^+ = {A $\rightarrow$ C, AC $\rightarrow$ D, E $\rightarrow$ A, E$\rightarrow$D, E$\rightarrow$ H, A $\rightarrow$D, E$\rightarrow$H, AC $\rightarrow$C} .... (i)

Q.2 Consider the following set of FDs F = {$A \rightarrow C$, $AC \rightarrow D$, $E \rightarrow AD$, $E \rightarrow H$}, and G = {$A \rightarrow CD$, $E \rightarrow AH$}. Check whether they are equivalent.

Sol : Now, G = {$A \rightarrow CD$, $E \rightarrow AH$}

From $A \rightarrow CD$ and $E \rightarrow AH$ we have $A \rightarrow C$, $A \rightarrow D$, $E \rightarrow A$ and $E \rightarrow D$ [By decomposition]

So, G^+ = {$A \rightarrow C$, $A \rightarrow D$, $E \rightarrow A$, $E \rightarrow H$}

From $E \rightarrow A$, $A \rightarrow C$ and $A \rightarrow D$ we have, $E \rightarrow C$ and $E \rightarrow D$ [By transitive Rule]

So, G^+ = {$A \rightarrow C$, $A \rightarrow D$, $E \rightarrow A$, $E \rightarrow H$, $E \rightarrow C$, $E \rightarrow D$}

From $A \rightarrow D$ we have $AC \rightarrow CD$ [By Augmenting C]

So, G^+ = {$A \rightarrow C$, $A \rightarrow D$, $E \rightarrow A$, $E \rightarrow H$, $E \rightarrow C$, $E \rightarrow D$, $AC \rightarrow CD$}

From $AC \rightarrow CD$ we have $AC \rightarrow C$ and $AC \rightarrow D$ [By decomposition]

So, G^+ = {$A \rightarrow C$, $A \rightarrow D$, $E \rightarrow A$, $E \rightarrow H$, $E \rightarrow C$, $E \rightarrow D$, $AC \rightarrow C$, $AC \rightarrow D$} ……….. (ii)

From (i) and (ii) we have $F^+ = G^+$, so they are equivalent.

Q.3 Consider the following set of FDs F = {A $\rightarrow$ B, AB $\rightarrow$ C, C $\rightarrow$ AC, D $\rightarrow$ E}, and G = {A $\rightarrow$ BC, D $\rightarrow$ AE}. Check whether they are equivalent.

Sol : Two sets of FDs F and G are equivalent set of FDs if and only if $F^+ = G^+$.

So, F = {A $\rightarrow$ B, AB $\rightarrow$ C, C $\rightarrow$ AC, D $\rightarrow$ E}

From A $\rightarrow$ B and AB $\rightarrow$ C we have A $\rightarrow$ C [By pseudo transitive Rule]

So, F^+ = {A $\rightarrow$ B, AB $\rightarrow$ C, C $\rightarrow$ AC, D $\rightarrow$ E, A $\rightarrow$ C}

From C $\rightarrow$ AC we have, C $\rightarrow$ A and C $\rightarrow$ C [By decomposition Rule]

So, F^+ = {A $\rightarrow$ B, AB $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ C, D $\rightarrow$ E, A $\rightarrow$ C} .... (i)

Now, We calculate G^+, So, G = {A $\rightarrow$ BC, D $\rightarrow$ AE}

From A$\rightarrow$BC and D $\rightarrow$ AE we have A$\rightarrow$B, A$\rightarrow$C, D$\rightarrow$A, D$\rightarrow$E [By decomposition Rule]

So, G^+ = {A $\rightarrow$ B, A $\rightarrow$ C, D $\rightarrow$ A, D$\rightarrow$E}

From A $\rightarrow$ C we have AB $\rightarrow$ BC and from AB $\rightarrow$ BC we have AB $\rightarrow$ B and AB $\rightarrow$ C

So, G^+ = {A $\rightarrow$ B, A $\rightarrow$ C, AB $\rightarrow$ B, AB$\rightarrow$C, D $\rightarrow$ A, D $\rightarrow$ E} .... (ii)

From (i) and (ii), we have $F^+ \neq G^+$,so they are not equivalent.

Irreducible sets of Dependencies (Minimal Cover)

- We define a set S of FDs to be irreducible if and only if it satisfies the following three properties:
  1. The right-hand side (the dependent) of every FD in S involves just one attribute (i.e, it is a singleton set).
  2. The left hand side (the determinant) of every FD in S is irreducible in turn meaning that no attribute can be discarded from the determinant without changing the closure S^+ (i.e., without converting S into some set not equivalent to S). We will say such an FD is **left-irreducible**.
  3. No FD in S can be discarded from S without changing the closure S^+ (i.e., without converting S into some set not equivalent to S).

Irreducible sets of Dependencies (Minimal Cover)

E.g. 1

Consider a relation R(A, B, C, D, E) with a set of FDs

F = { A → BC,
C → D,
D → E}

is not an irreducible set of FDs because RHS of A → BC is not an singleton set.

To make an irreducible set of FDs we have

F = {A → B,
A → C,
C → D,
D → E}

Irreducible sets of Dependencies (Minimal Cover)

E.g. 2

Consider a relation R(A, B, C, D, E) with a set of FDs

F = { $A \rightarrow B$,
$AB \rightarrow D$,
$D \rightarrow E$}

is not an irreducible set of FDs because from $A \rightarrow B$ and $AB \rightarrow D$ we have $A \rightarrow D$ [From pseudo transitive rule] that means B in $AB \rightarrow D$ is an extraneous attribute.

To make an irreducible set of FDs we have

F = {$A \rightarrow B$,
$A \rightarrow D$,
$D \rightarrow E$}

Irreducible sets of Dependencies (Minimal Cover)

E.g. 3

Consider a relation R(A, B, C, D, E) with a set of FDs

F = { A → B,
B → C,
A → C,
D → E}

is not an irreducible set of FDs because from A → B and B → C we have A → C [From transitive rule] that means A → C is an redundant FD.

To make an irreducible set of FDs we have

F = {A → B,
B → C,
D → E}

Q.3 Given a relation schema R(A,B,C,D) and a set of FDs

$A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow B$

$AB \rightarrow C$

$AC \rightarrow D$

Compute an irreducible set of FDs that is equivalent to this set.

Sol : The first step is to rewrite the FDs such that each has a singleton right-hand side:

$A \rightarrow B$

$A \rightarrow C$ [By Decomposition]

$B \rightarrow C$

$A \rightarrow B$

$AB \rightarrow C$

$AC \rightarrow D$

We observe immediately that is FD $A \rightarrow B$ occurs twice, so one occurrence can be eliminated.

Next, attribute C can be eliminated from the left-hand side of the FD $AC \rightarrow D$, because we have $A \rightarrow C$, so $A \rightarrow AC$ by augmentation, and we are given $AC \rightarrow D$, so $A \rightarrow D$ by transitivity; thus the C on the left-hand side of $AC \rightarrow D$ is redundant.

So the remaining FDs are:

$A \rightarrow B$

$A \rightarrow C$

$B \rightarrow C$

$AB \rightarrow C$

$A \rightarrow D$

Next, we observe that the FD $AB \rightarrow C$ can be eliminated, because again we have $A \rightarrow C$, so $AB \rightarrow CB$ by augmentation, so $AB \rightarrow C$ by decomposition. So the remaining FDs are:

$A \rightarrow B$

$A \rightarrow C$

$B \rightarrow C$

$A \rightarrow D$

Finally, the FD $A \rightarrow C$ is implied by the FDs $A \rightarrow B$ and $B \rightarrow C$, so it can also be eliminated. We are left with

$A \rightarrow B$

$B \rightarrow C$

$A \rightarrow D$

This set is irreducible.

Q. Consider the Relation R(A,B,C,D,E,F,G,H,I,J) and a set of FDs

$ABD \rightarrow E$

$AB \rightarrow G$

$B \rightarrow F$

$C \rightarrow J$

$CJ \rightarrow I$

$G \rightarrow H$

Is this an irreducible set? What are the candidate keys?

Q. Consider the following set of FDs

$AB \rightarrow C$

$A \rightarrow DE$

$B \rightarrow F$

$F \rightarrow GH$

$D \rightarrow IJ$

Find

(i) Super Key

(ii) Irreducible set

(iii) Candidate Key

GATE Question

The following functional dependencies hold true for the relational schema R{V, W, X, Y, Z} :

$V \rightarrow W$,

$VW \rightarrow X$,

$Y \rightarrow VX$,

$Y \rightarrow Z$

Which of the following is irreducible equivalent for this set of functional dependencies?

(A)		(B)		(C)		(D)	
	$V \rightarrow W$		$V \rightarrow W$		$V \rightarrow W$		$V \rightarrow W$
	$V \rightarrow X$		$W \rightarrow X$		$V \rightarrow X$		$W \rightarrow X$
	$Y \rightarrow V$		$Y \rightarrow V$		$Y \rightarrow V$		$Y \rightarrow V$
	$Y \rightarrow Z$		$Y \rightarrow Z$		$Y \rightarrow X$		$Y \rightarrow X$
					$Y \rightarrow Z$		$Y \rightarrow Z$

[GATE 2017]

MSQ Question

The following functional dependencies hold true for the relational schema R{A, B, C} :

A → BC,

B → AC,

C → AB

Which of the following is irreducible equivalent for this set of functional dependencies?

(A)	**A → B**	**(B)**	**A → C**	**(C)**	**A → B**	**(D)**	**B → A**
	B → C		**B → A**		**B → C**		**C → B**
	C → A		**C → B**		**C → B**		**C → A**

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https://www.youtube.com/channel/UCRWGtE76JlTp1iim6aOTRuw?sub_confirmation=1