



Optimization



Agenda

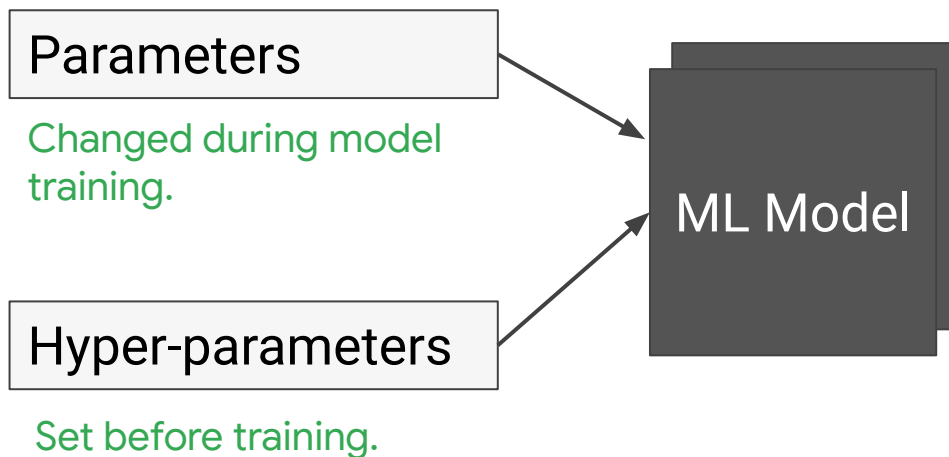
Defining ML Models

Introducing Loss Functions

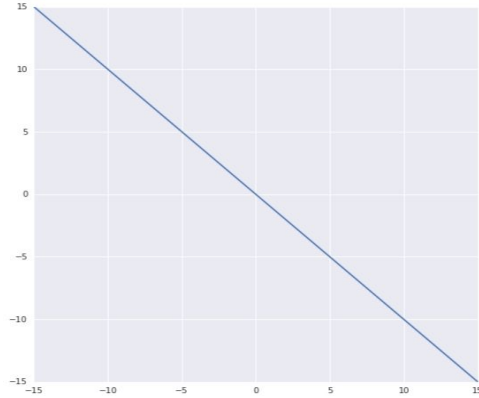
TensorFlow Playground



ML models are mathematical functions with parameters and hyper-parameters



Linear models have two types of parameters: Bias and weight



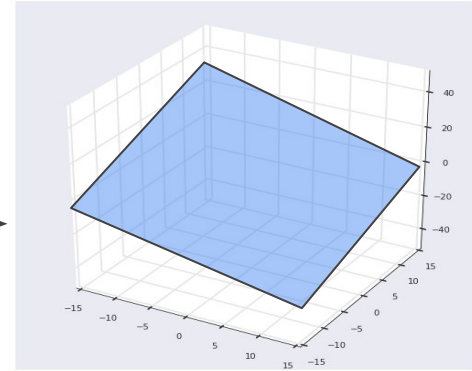
Linear

Output Bias Input Weight
Term

$$\blacktriangleleft y = \boxed{b} + x \times \boxed{m}$$

$$y = \boxed{b} + X \times \boxed{w} \blacktriangleright$$

Model
Parameters

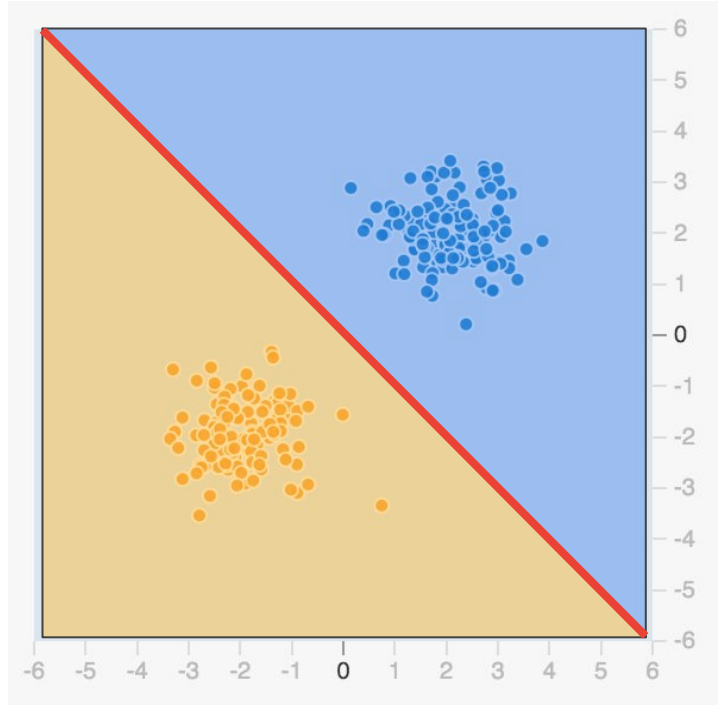


Hyperplane



How can linear models classify data?

Classification explained
graphically.



How do we predict a baby's health *before* they are born?

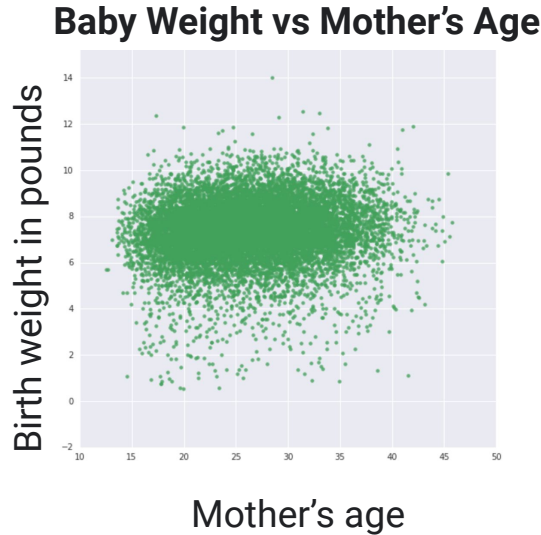
Which of these could be a *feature* in your model?

- A. Mother's Age
- B. Birth Time
- C. Baby Weight

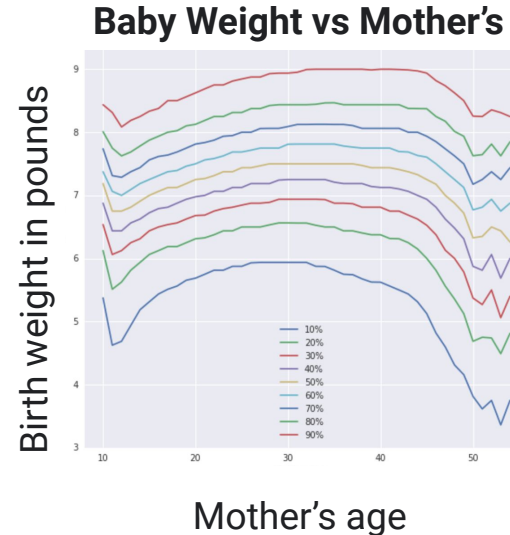
Which could be a *label*?



Exploring the data visually



Scatterplots are made from samples of large datasets rather than from the whole dataset.



Graph representing groups of data, specifically, quantiles.



Equation for a linear model tying mother's age and baby weight

The slope of the line is given by w_1 .

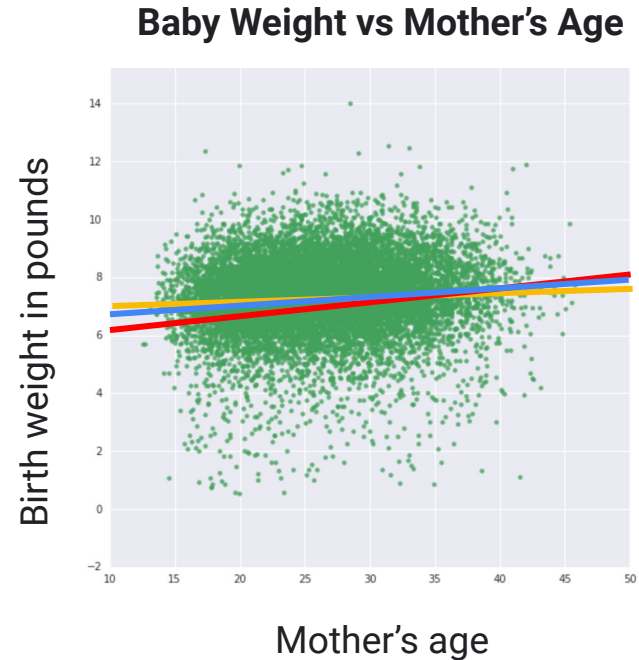
$$y = w_1 x_1 + b$$

- x_1 is the **feature** (e.g. mother's age)
- w_1 is the **weight** for x_1

Line: $y = .02x + 6.83$

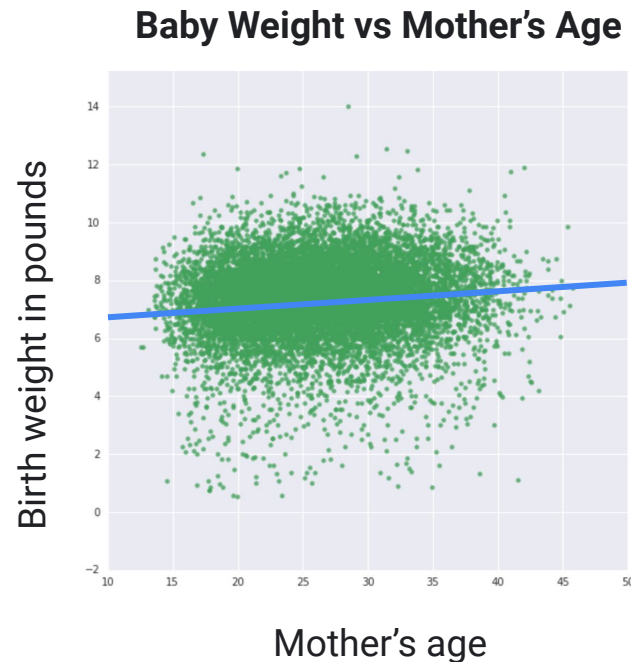
Line: $y = .03x + 6.49$

Line: $y = .01x + 7.14$

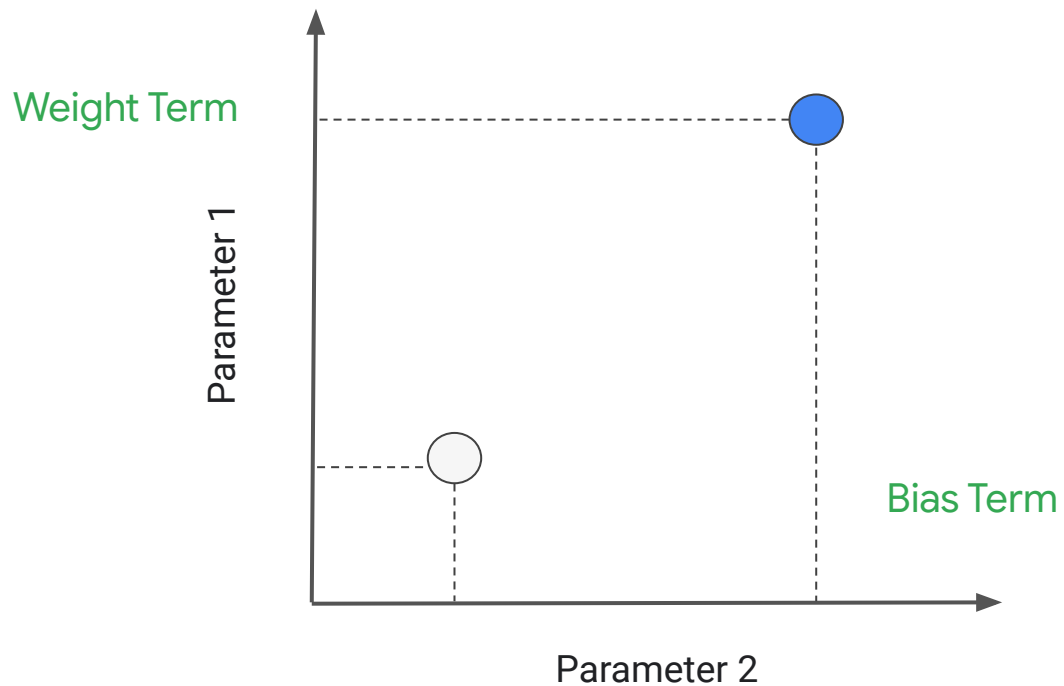


Can't we just solve the equation using all the data?

When an analytical solution is no longer an option, you use gradient descent.



Searching in parameter-space



Agenda

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Compose a loss function by calculating errors

Error = actual (true) - predicted value

Compute the errors:

+0.70

+1.10

+0.65

-1.20

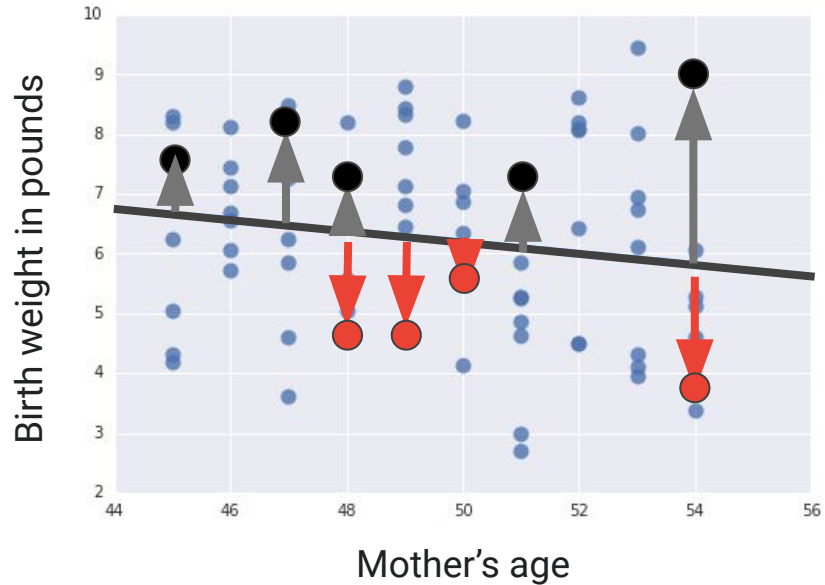
-1.15

+1.10

+3.09

-2.10

Each error makes
sense. How about all
the errors added
together?



One loss function metric is Root Mean Squared Error (RMSE)

1 Get the errors for the training examples.

+0.70
+1.10
+0.65
-1.20
-1.15
+1.10
+3.09
-2.10

2 Compute the squares of the error values.

0.49
1.21
0.42
1.44
1.32
1.21
9.55
4.41

3 Compute the mean of the squared error values.

2.51

$$\sqrt{\frac{1}{n} \times \sum_{i=1}^n (\hat{Y}_i - Y_i)^2}$$

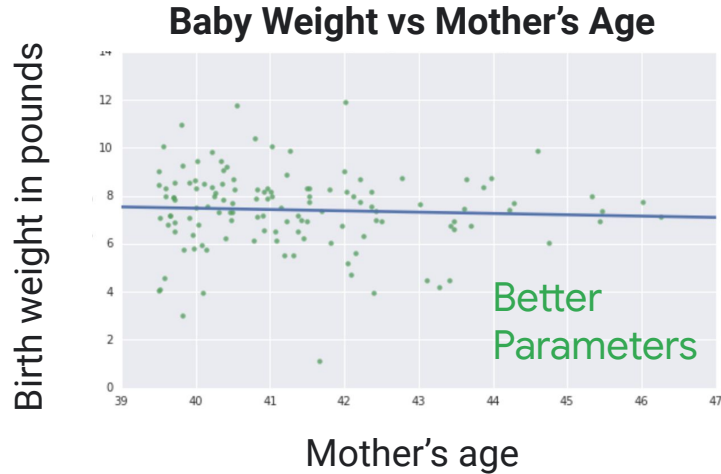
\hat{Y}_i predicted value

Y_i labeled value

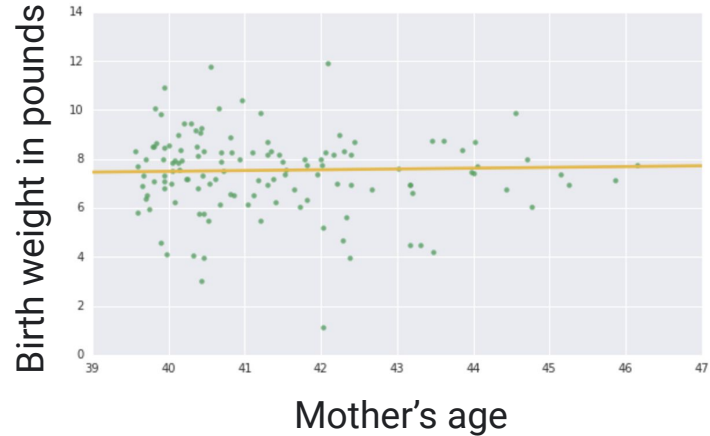
4 Take a square root of the mean. 1.58



Lower RMSE indicates a better performing model



RMSE=.145



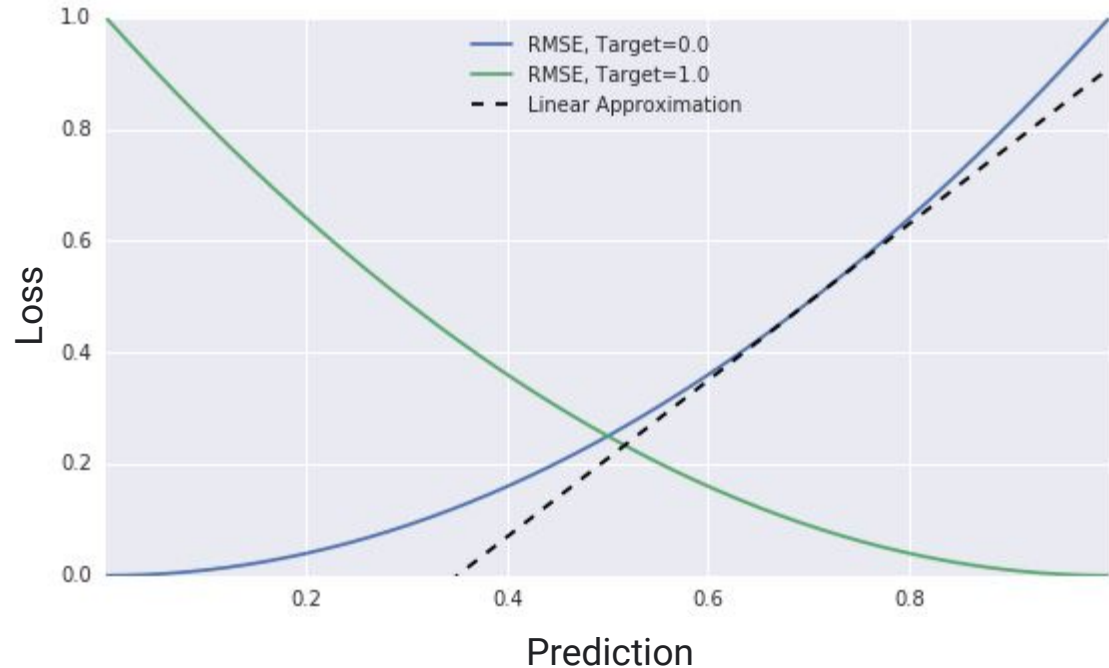
RMSE=.149

Need a way to find the best values for weight and bias.



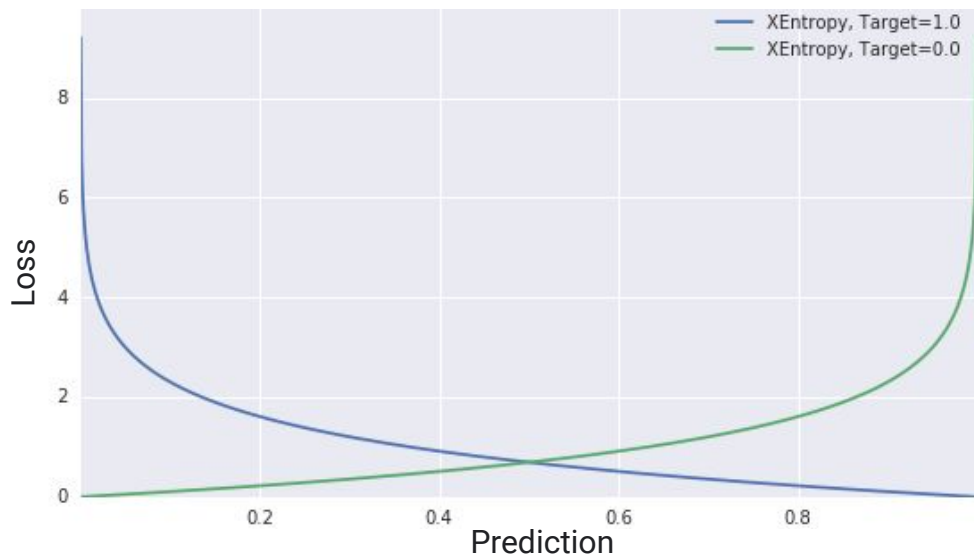
Problem: RMSE doesn't work as well for classification

RMSE doesn't penalize
bad classifications
appropriately.



Problem: RMSE doesn't work as well for classification

Bad classifications
are penalized
appropriately.





$$\frac{-1}{N} \times \sum_1^N y_i \times \log(\hat{y}_i) + (1 - y_i) \times \log(1 - \hat{y}_i)$$



Computing cross-entropy loss

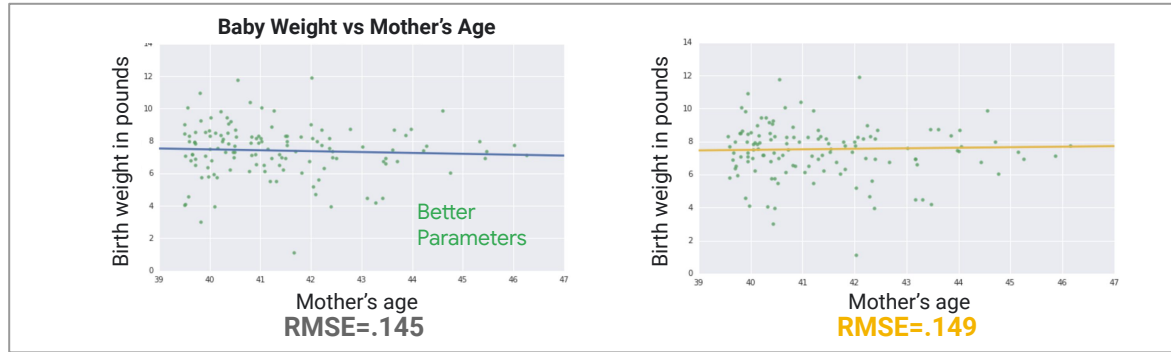
$$\frac{-1}{N} \times \sum_1^N \overbrace{y_i \times \log(\hat{y}_i)}^{\text{Positive term}} + \overbrace{(1 - y_i) \times \log(1 - \hat{y}_i)}^{\text{Negative term}}$$



X	y_i	\hat{y}_i
	1	.7
	0	.2

$$\left((1.0 * \log(.7)) + \cancel{(1 - 1.0) * \log(1 - .7)} + (0.0 * \log(.2)) + (1 - 0.0) * \log(1 - .2) \right) * (-\frac{1}{2}) = .13$$



From loss functions to gradient descent

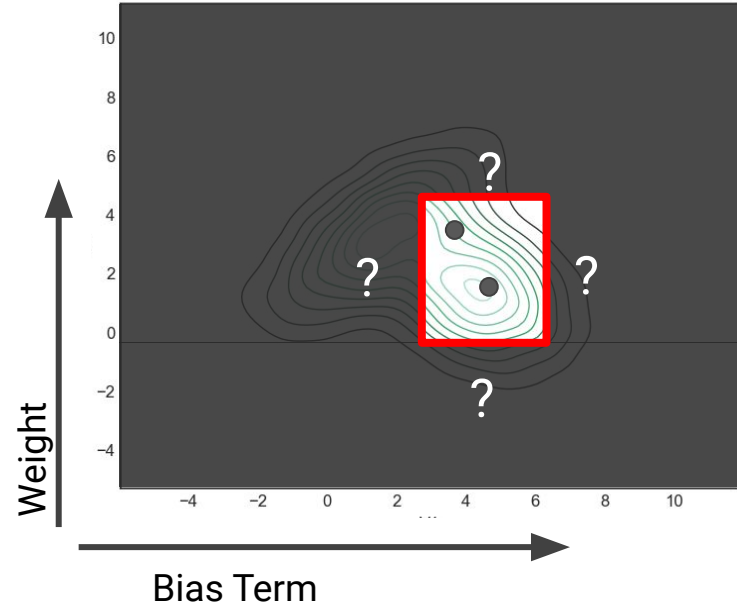
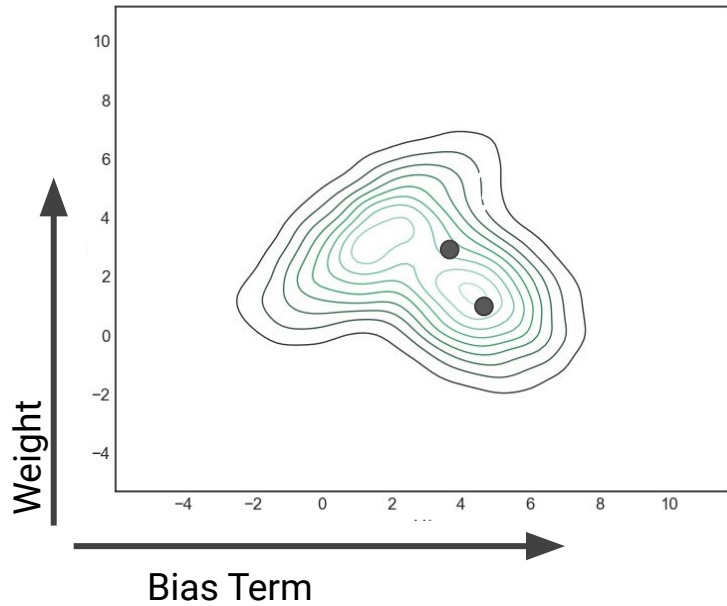


X	Y_i	\hat{Y}_i	$\frac{-1}{N} \times \sum_1^N$ <div style="display: inline-block; vertical-align: middle; text-align: left;"> <div style="display: inline-block; vertical-align: middle; text-align: center;"> <div style="margin-bottom: 5px;">Positive term</div> $y_i \times \log(\hat{y}_i)$ </div> <div style="display: inline-block; vertical-align: middle; text-align: center;"> <div style="margin-bottom: 5px;">Negative term</div> $+ (1 - y_i) \times \log(1 - \hat{y}_i)$ </div> </div>
	1	.7	$\left(1.0 * \log(.7) + \cancel{(1-1.0) * \log(1-.7)} \right)$
	0	.2	$\left(\cancel{0.0 * \log(.2)} + (1-0.0) * \log(1-.2) \right)$

$\left. \begin{array}{l} \text{Positive term} \\ \text{Negative term} \end{array} \right\} * (-\frac{1}{2}) = .13$



Loss functions lead to loss surfaces



Finding the bottom

Which direction should I head?



How large or small a step?



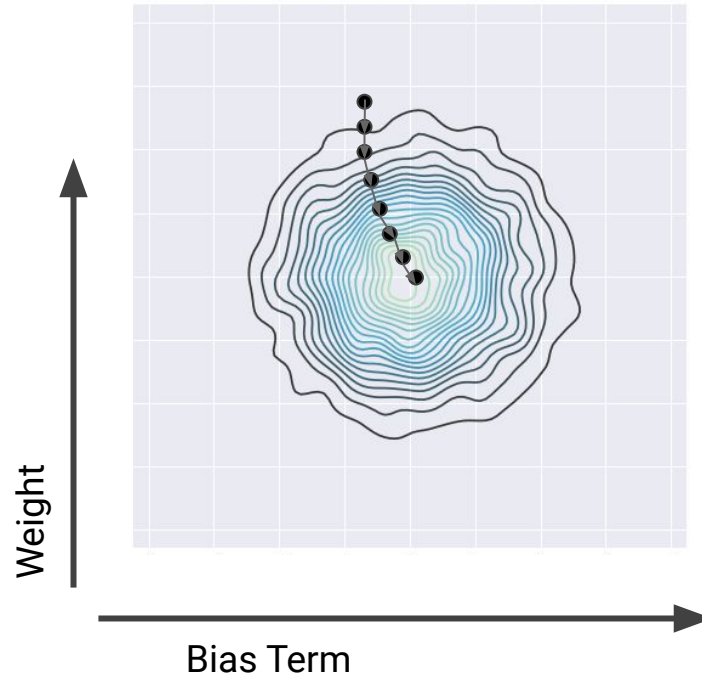
A simple algorithm to find the minimum

```
while loss is > Epsilon:  
    direction = computeDirection()  
    for i in range(weights.size):  
        weights[i] = weights[i] + stepSize * direction[i]  
    loss = computeLoss()
```

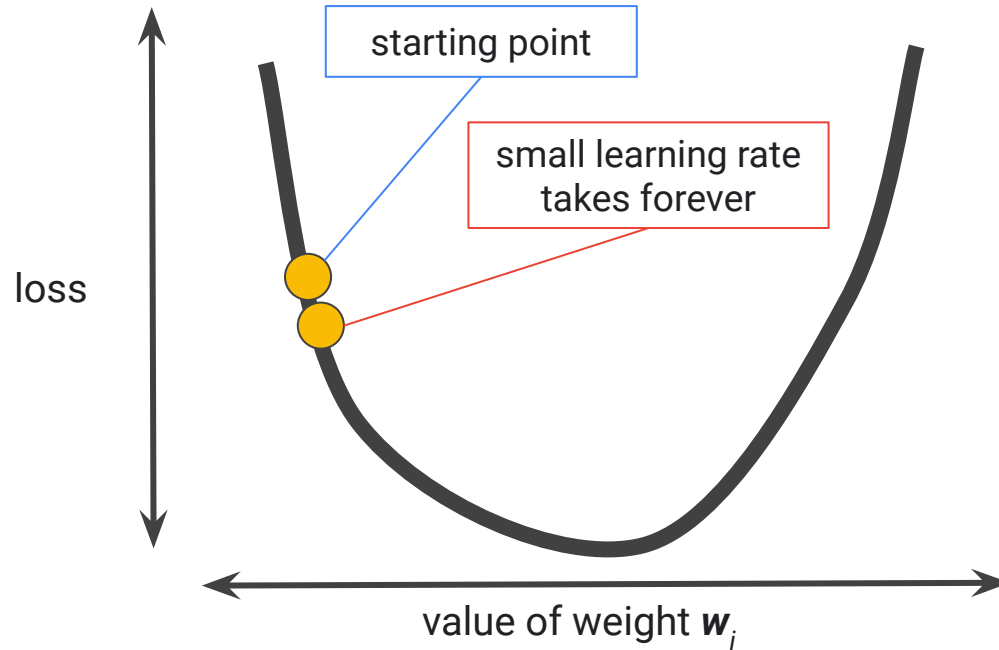
Epsilon = A tiny Constant



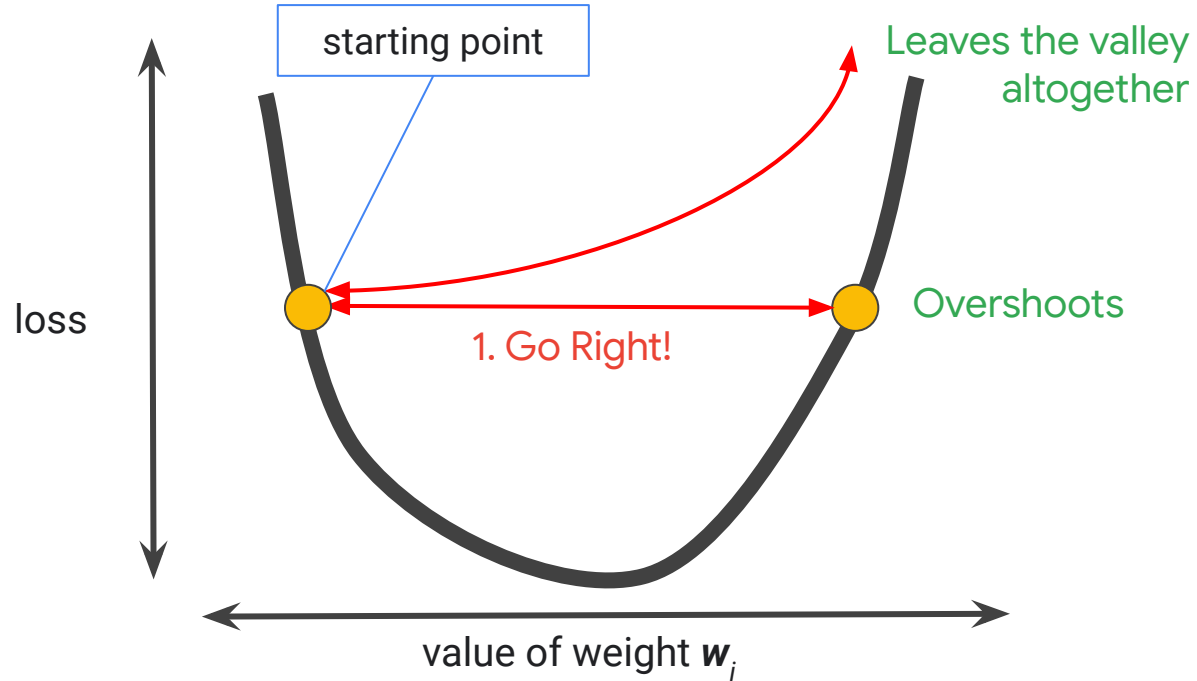
Search for a minima by descending the gradient



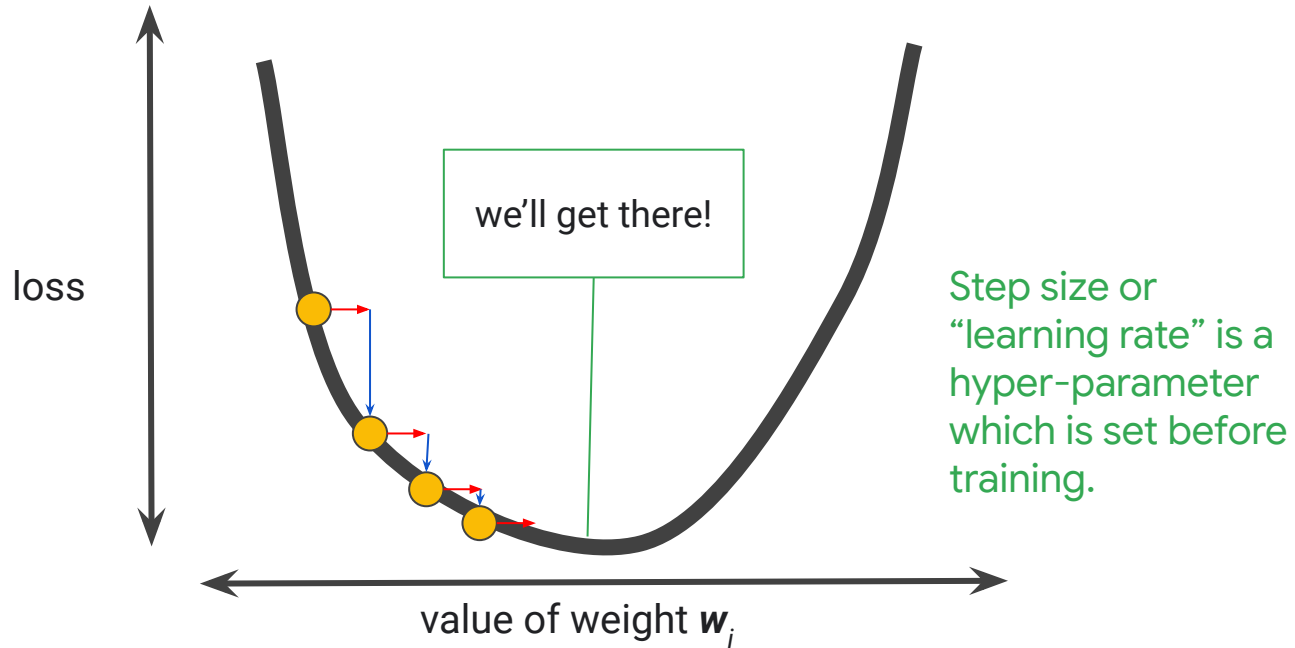
Small step sizes can take a very long time to converge



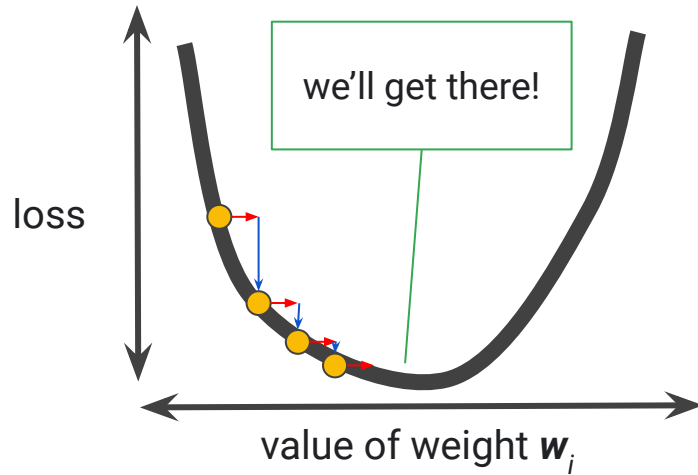
Large step sizes may never converge to the true minimum



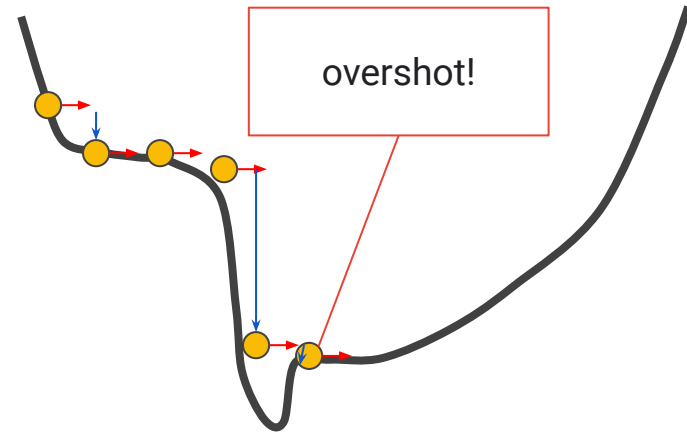
A correct and constant step size can be difficult to find



A correct and constant step size can be difficult to find



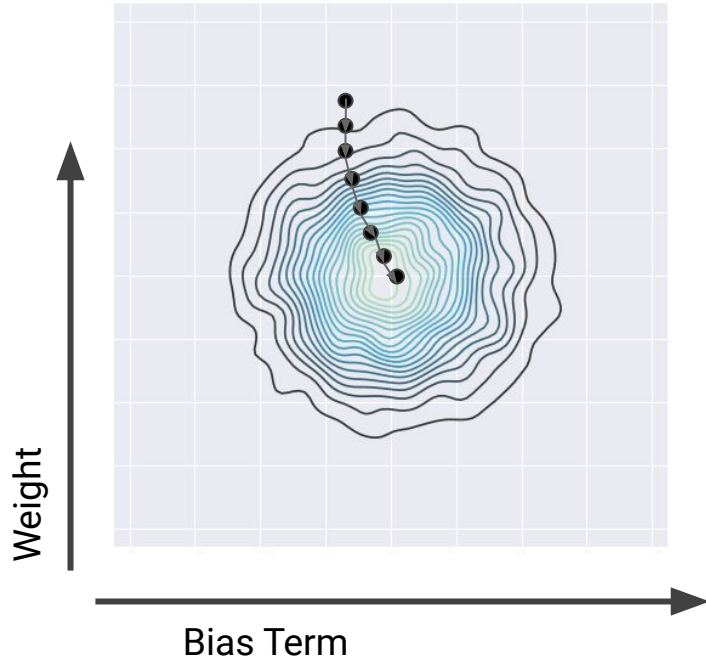
Step size or “learning rate” is a hyper-parameter which is set before training.



One size does not fit all models.



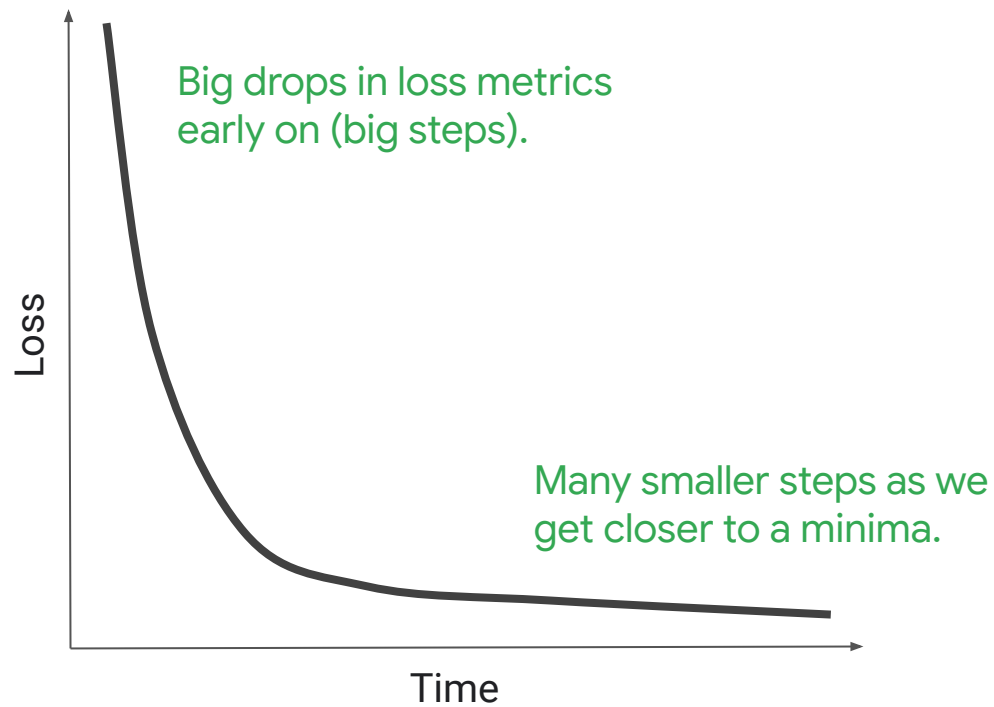
Are you done yet?



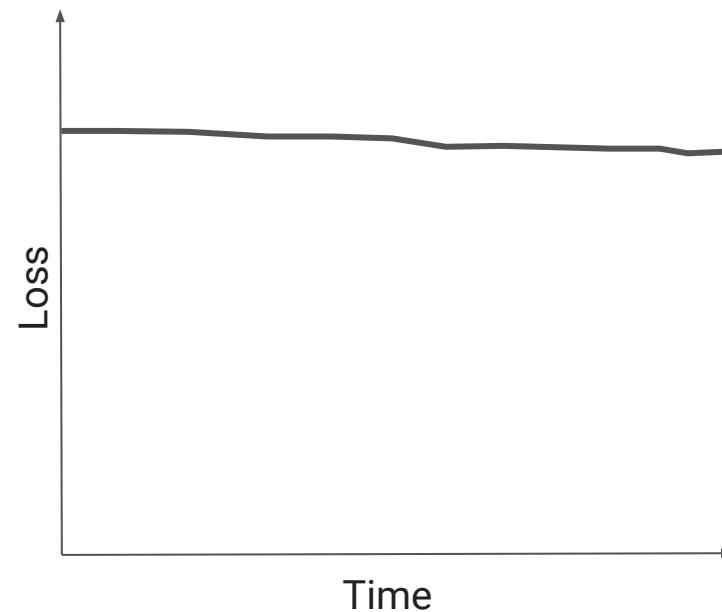
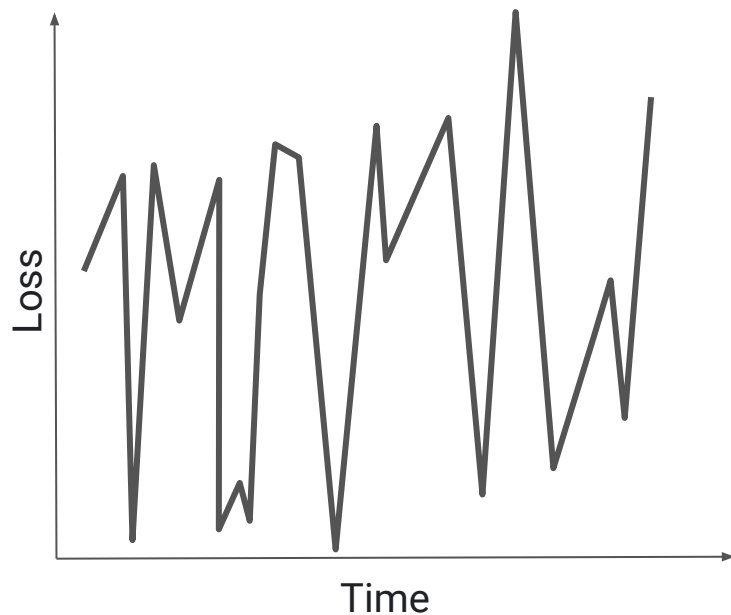
```
while loss is > Epsilon:  
    derivative = computeDerivative()  
    for i in range(weights.size):  
        weights[i] = weights[i] - derivative[i]  
    loss = computeLoss()
```



A typical loss curve



Troubleshooting a Loss Curve



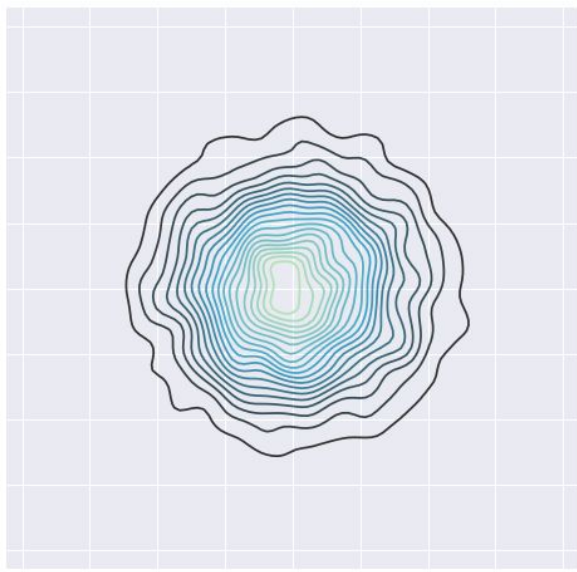
Adding a scaling hyperparameter

```
while loss is > Epsilon:
    derivative = computeDerivative()
    for i in range(weights.size):
        weights[i] = weights[i] - learning_rate * derivative[i]
    loss = computeLoss()
```

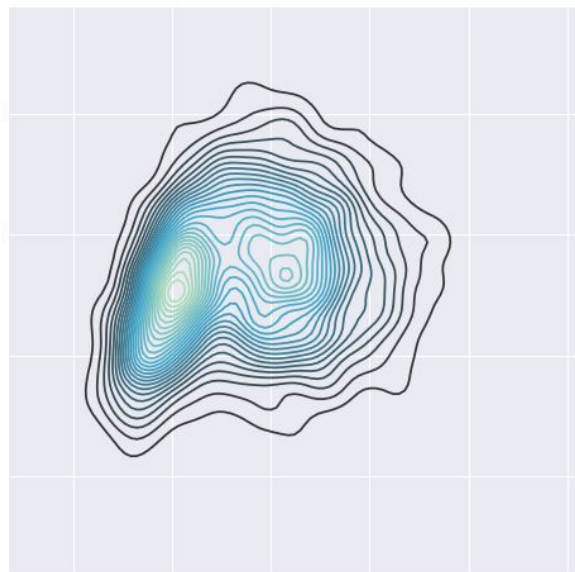


Problem: My model changes every time I retrain it

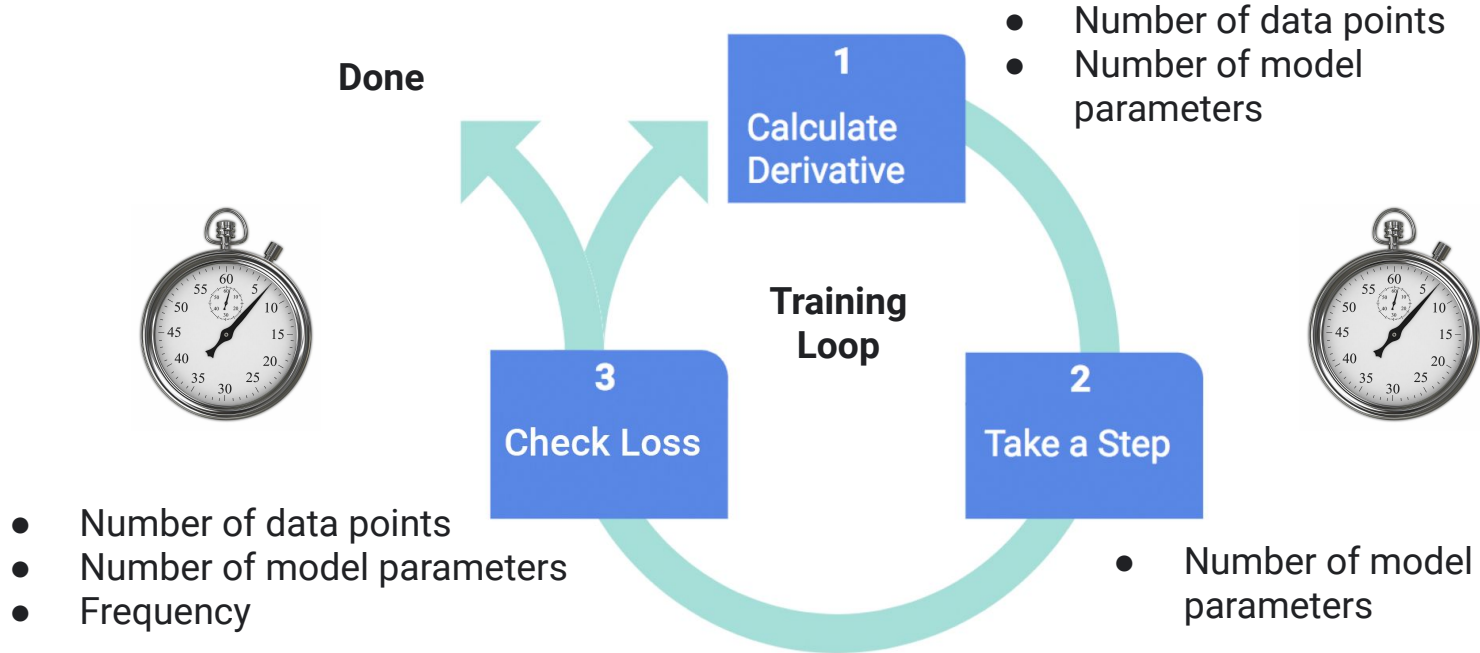
Loss Surface with a global minimum



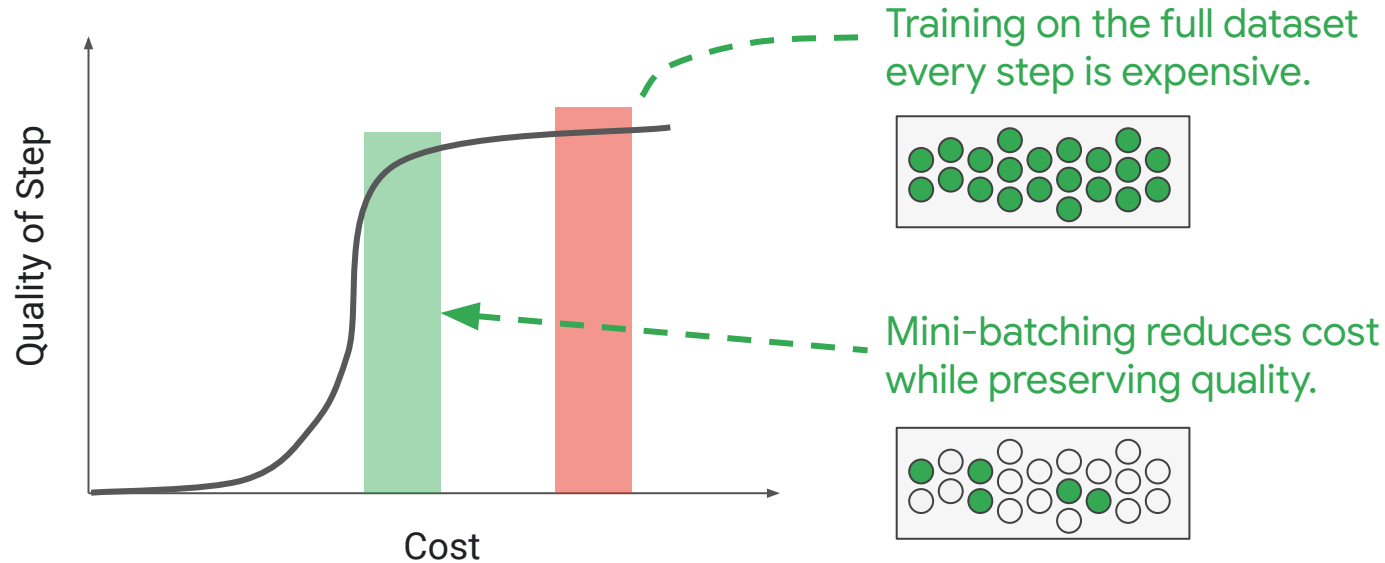
Loss Surface with more than one minima



Problem: Model training is still too slow



Calculating the derivative on fewer data points



Typical values for batch size: 10 - 1000 examples.



Checking loss with reduced frequency

```
while loss is > Epsilon:
    derivative = computeDerivative()
    for i in range(weights.size):
        weights[i] = weights[i] - learning_rate * derivative[i]
    if readyToSampleLoss():
        loss = sampleLoss()
```

Popular implementations for readyToSampleLoss():

- Time-based (e.g., every hour)
- Step-based (e.g., every 1000 steps)



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TensorFlow Playground



TensorFlow Playground Interface

