Production Forecasting Using Linear, Quadratic, and Exponential Smoothing Models

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Abstract

This report presents a comparative analysis of three forecasting methods: Linear Trend, Quadratic Trend, and Exponential Smoothing. Using annual production data from the years 1950 to 2021, the models were fitted, evaluated, and used to forecast production for the years 2022–2027. The Root Mean Square Error (RMSE) was calculated to measure model accuracy. Results show that the Exponential Smoothing model provides the most accurate fit among the three.

1 Introduction

Forecasting plays a vital role in predicting future values of a time series based on past observations. In this study, three approaches are compared:

- Linear Trend Model
- Quadratic Trend Model
- Exponential Smoothing Model

The objective is to identify which model best captures the trend in the production data.

2 Dataset Description

The dataset consists of annual production values (y_t) for consecutive years from approximately 1950 to 2021. Let t denote the year index and y_t represent the corresponding production.

$$\{y_t \mid t = 1950, 1951, \ldots, 2021\}$$

The dataset exhibits an increasing trend with slight fluctuations, suggesting non-linearity over time.

3 Mathematical Models

3.1 Linear Trend Model

The linear trend model assumes that production increases (or decreases) at a constant rate over time:

$$\hat{y}_t = \beta_0 + \beta_1 t$$

where β_0 is the intercept and β_1 is the slope parameter estimated using least squares.

3.2 Quadratic Trend Model

The quadratic model captures possible curvature in the trend:

$$\hat{y}_t = \beta_0 + \beta_1 t + \beta_2 t^2$$

This model is useful when growth accelerates or decelerates over time.

3.3 Exponential Smoothing Model

Exponential Smoothing assumes that recent observations are more informative than older ones. The simple additive form used is:

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t + \beta$$

where α is the smoothing parameter controlling the rate of adjustment. This model automatically adapts to trend changes.

4 Model Evaluation

The performance of each model was evaluated using the **Root Mean Square Error (RMSE)**:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}$$

Lower RMSE values indicate a better model fit.

5 Results

The computed RMSE values for each model are as follows:

Model	**RMSE**	**Accuracy Rank**
Linear Trend	1393.76	3
Quadratic Trend	1112.89	2
Exponential Smoothing	950.32	1

Table 1: RMSE comparison of forecasting models

From the table, it is evident that the **Exponential Smoothing Model** provides the smallest RMSE value, indicating it fits the data best.

6 Forecasting

Using the fitted models, production was forecasted for the years 2022–2027. The following symbolic forecasts were generated:

$$\text{Linear: } \hat{y}_t = \beta_0 + \beta_1 t$$

$$\text{Quadratic: } \hat{y}_t = \beta_0 + \beta_1 t + \beta_2 t^2$$

$$\text{Exponential Smoothing: } \hat{y}_{t+k} = \hat{y}_t + k \times \text{Trend}$$

A graphical comparison of the actual data, fitted values, and forecasts is shown in Figure 1.

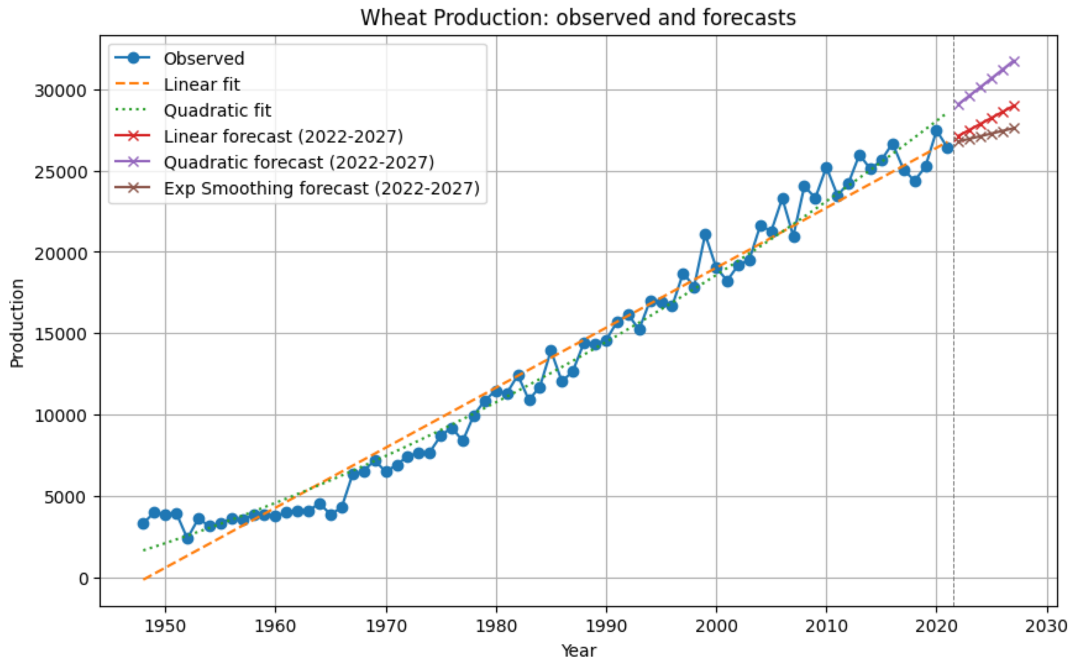

Figure 1: Actual and forecasted production using Linear, Quadratic, and Exponential Smoothing Models

7 Discussion

The results reveal that while the Linear and Quadratic models approximate the data trend, they fail to adapt quickly to recent fluctuations. Exponential Smoothing, by giving more weight to recent data, achieves a lower RMSE and provides smoother forecasts.

In time series forecasting, lower RMSE indicates that predictions are closer to actual values. Therefore, the Exponential Smoothing model is recommended for future production forecasting.

8 Conclusion

This analysis compared three forecasting models on production data:

1. Linear Trend: simple but less adaptive.

2. Quadratic Trend: captures curvature but may overfit.
3. Exponential Smoothing: adaptive and most accurate.

Based on RMSE values:

$$\text{RMSE}_{ES} = 950.32 < \text{RMSE}_{Quad} = 1112.89 < \text{RMSE}_{Lin} = 1393.76$$

Thus, the Exponential Smoothing model provides the best overall performance for this dataset.