Data Structure and Algorithms-I

Arrays: Memory Mapping, Linear and Binary Search, Linear Time Sorting (Counting Sort)

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Arrays

- An array is an indexed sequence of components
  - The components of an array are all of the same type
- Typically, the array occupies sequential storage locations
- Array is a static data structure, that is, the length of the array is determined when the array is created, and cannot be changed
- Each component of the array has a fixed, unique index
  - Indices range from a lower bound to an upper bound
- Any component of the array can be inspected or updated by using its index
  - This is an efficient operation: $O(1)$ = constant time

a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]
7	6	11	17	3	15	5	19	30	14

Representation of Arrays in Memory

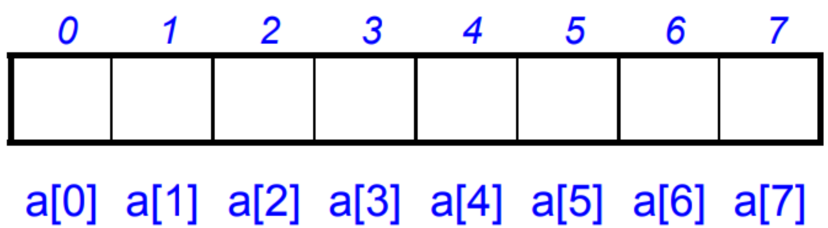
- **Linear (1 D) Arrays:**

  A 1-dimensional array a is declared as:

  int a[8];

  The elements of the array a may be shown as

  a[0]   a[1]   a[2]   a[3]   a[4]   a[5]   a[6]   a[7]



Representation of Arrays in Memory

- **2 D Arrays:**

A 2-dimensional array a is declared as:

```
int a[3][4];
```

The elements of the array a may be shown as a table

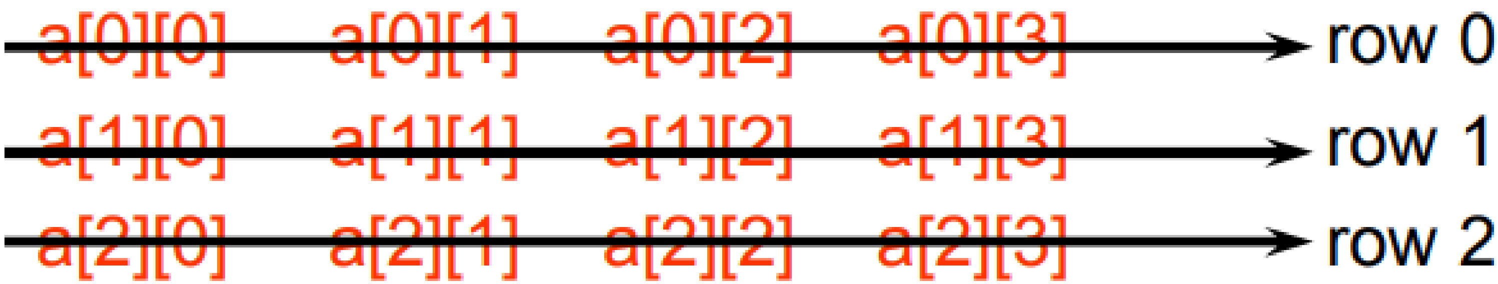
a[0][0]	a[0][1]	a[0][2]	a[0][3]
a[1][0]	a[1][1]	a[1][2]	a[1][3]
a[2][0]	a[2][1]	a[2][2]	a[2][3]

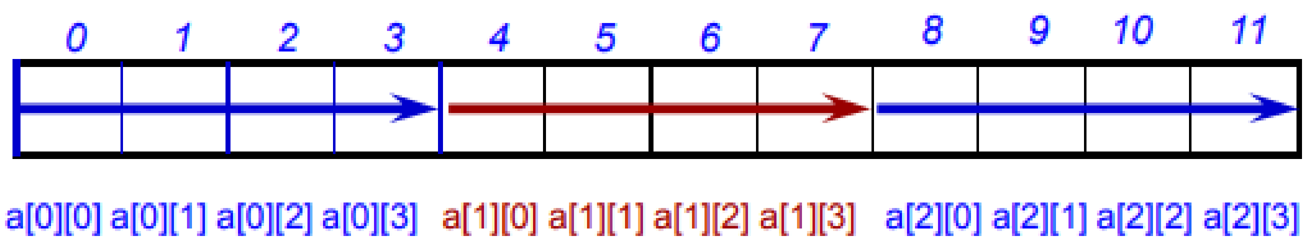
In which order are the elements stored?

- Row major order (C, C++, Java support it)
- Column major order (Fortran supports it)

Representation of Arrays in Memory

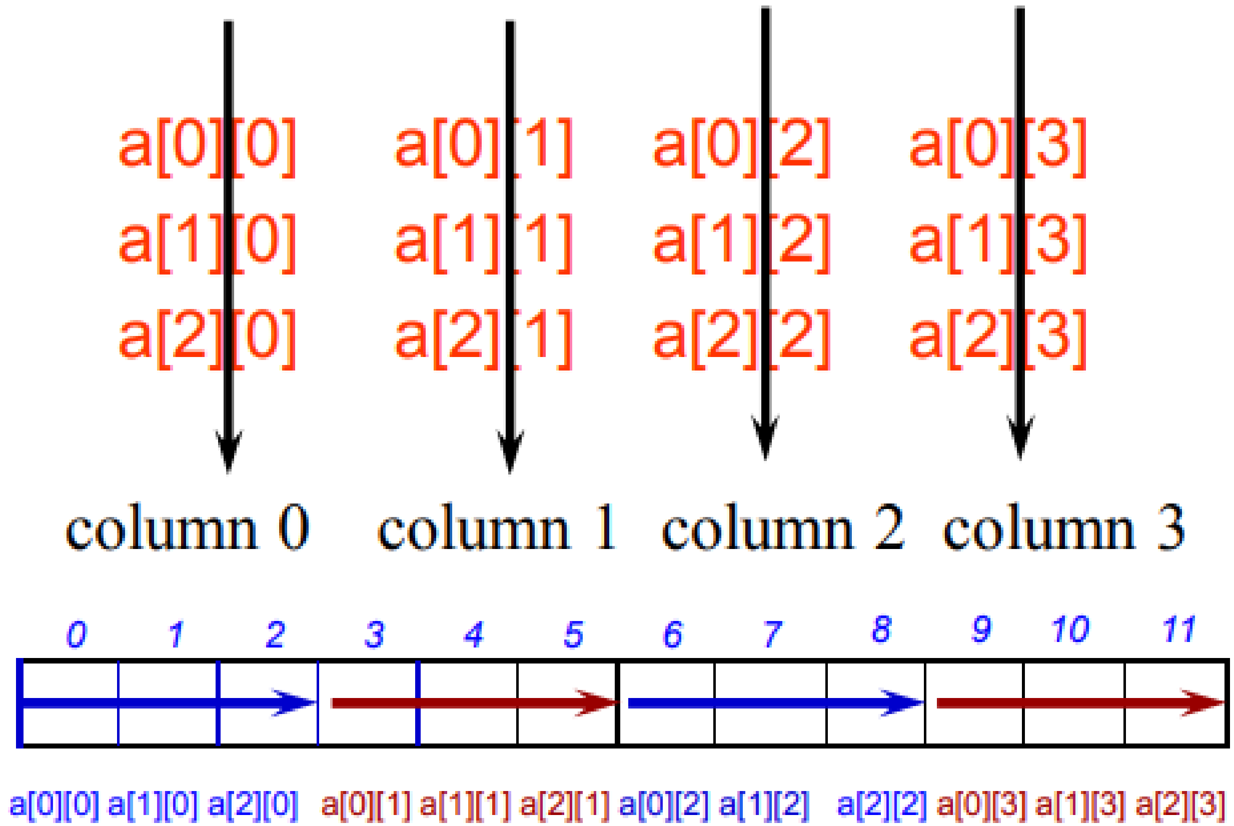
Row Major Order: the array is stored as a sequence of 1-D arrays consisting of rows





Representation of Arrays in Memory

Column Major Order: The array is stored as a sequence of arrays consisting of columns instead of rows



Representation of Arrays in Memory: Parameters

- **Base Address (*b*):** The memory address of the first byte of the first array component.
- **Component Length (*L*):** The memory required to store one component of an array.
- **Upper and Lower Bounds (l_i, u_i):** Each index type has a smallest value and a largest value.
- **Dimension**

Representation of Arrays in Memory

- **Array Mapping Function (AMF)**
  - AMF converts index value to component address

- **Linear (1D) Arrays:**

  a : array $[l_1 \mathinner{..} u_1]$ of element_type

  $$\text{Then addr}(a[i]) = b + (i - l_1) \times L$$
  $$= c_0 + c_1 \times i$$

  Therefore, the time for calculating the address of an element is same for any value of i.

Representation of Arrays in Memory

- **Array Mapping Function (AMF): 2D Arrays**

  Row Major Order:

  a : array $[l_1 \,..\, u_1, l_2 \,..\, u_2]$ of element_type

  Then addr$(a[i, j]) = b + (i - l_1)\times(u_2 - l_2 + 1)\times L + (j - l_2)\times L$

  $= c_0 + c_1\times i + c_2\times j$

  Therefore, the time for calculating the address of an element is same for any value of (i, j).

Representation of Arrays in Memory

- **Array Mapping Function (AMF): 2D Arrays**

  Column Major Order:

  a : array $[l_1 \,..\, u_1, l_2 \,..\, u_2]$ of element_type

$$\text{Then addr}(a[i,j]) = b + (j - l_2)\times(u_1 - l_1 + 1)\times L + (i - l_1)\times L$$
$$= c_0 + c_1 \times i + c_2 \times j$$

Therefore, the time for calculating the address of an element is same for any value of (i, j).

Representation of Arrays in Memory

- **Array Mapping Function (AMF): 3D Arrays :**

  a : array $[l_1 \,..\, u_1, l_2 \,..\, u_2\,, l_3 \,..\, u_3]$ of element_type

$$\text{Then addr}(a[i, j, k]) = b + (i - l_1)\times(u_2 - l_2 + 1)\times(u_3 - l_3 + 1)\times L + (j - l_2)\times(u_3 - l_3 + 1)\times L + (k - l_3)\times L$$

$$= c_0 + c_1\times i + c_2\times j + c_3\times k$$

Therefore, the time for calculating the address of an element is same for any value of $(i, j\,, k)$.

Summary on Arrays

- **Advantages:**
  - Array is a random access data structure.
  - Accessing an element by its index is very fast (constant time)
- **Disadvantages:**
  - Array is a static data structure, that is, the array size is fixed and can never be changed.
  - Insertion into arrays and deletion from arrays are very slow.
- An array is a suitable structure when
  - a lot of searching and retrieval are required.
  - a small number of insertions and deletions are required.

Searching Algorithms:
Linear Search, Binary Search

The Searching Problem

- The process of finding a particular element in an array is called searching. There two popular searching techniques:
  - *Linear search*, and
  - *Binary search*.

- The ***linear search*** compares each element in an unsorted array with the ***search key***.
  - Running time: $O(n)$

- Given a sorted array, ***Binary Search*** algorithm can be used to perform fast searching of a search key on the sorted array.
  - Running time: $O(\log n)$

Linear Search

- Each member of the array is visited until the search key is found.

- **Example:**

  Write a program to search for the search key entered by the user in the following array:

  (9, 4, 5, 1, 7, 78, 22, 15, 96, 45)

  You can use the linear search in this example.

Linear Search

```
/* This program is an example of the Linear Search*/
#include <stdio.h>
#define SIZE 10
int LinearSearch(int [], int);
int main() {
	int a[SIZE]= {9, 4, 5, 1, 7, 78, 22, 15, 96, 45};
	int key, pos;
	printf(“Enter the Search Key\n”);
	scanf(“%d”, &key);
	pos = LinearSearch(a, key);
	if(pos == –1)
		printf(“The search key is not in the array\n”);
	else
		printf(“The search key %d is at location %d\n”, key, pos);
	return 0;
}
```

Linear Search

```
int LinearSearch (int b[ ], int skey) {
    int i;
    for (i=0; i < SIZE; i++)
        if(b[i] == skey)
            return i;
    return –1;
}
```

Binary Search

- Given a sorted array, ***Binary Search*** algorithm can be used to perform fast searching of a search key on the sorted array.

- The following program implements the binary search algorithm for the search key entered by the user in the following array:

  (3, 5, 9, 11, 15, 17, 22, 25, 37, 68)

Binary Search

```
#include <stdio.h>
#define SIZE 10
int BinarySearch(int [ ], int);
int main(){
      int a[SIZE]= {3, 5, 9, 11, 15, 17, 22, 25, 37, 68};
      int key, pos;
      printf("Enter the Search Key\n");
      scanf("%d",&key);
      pos = BinarySearch(a, key);
      if(pos == –1)
           printf("The search key is not in the array\n");
      else
           printf("The search key %d is at location %d\n", key, pos);
      return 0;
}
```

Binary Search

```
int BinarySearch (int A[], int skey){
    int low=0, high=SIZE–1, middle;
    while(low <= high){
        middle = (low+high)/2;
        if (skey == A[middle])
            return middle;
        else if(skey <A[middle])
            high = middle – 1;
        else
            low = middle + 1;
    }
    return –1;
}
```

Linear-Time Sorting Algorithm: Counting Sort

Sorting in Linear Time

- Counting sort
  - No comparisons between elements!
  - ***But***...depends on assumption about the numbers being sorted
    - We assume numbers are in the range 1, …, k
  - The algorithm:
    - Input: A[1..n], where A[i] $\in$ {1, 2, 3, …, k}
    - Output: B[1..n], sorted (notice: not sorting in place)
    - Also: Array C[1..k] for auxiliary storage

Counting Sort

```
COUNTING-SORT(A, B, k)
for i ← 0 to k
        do C[i] ← 0
for j ← 1 to length[A]
        do C[A[j]] ← C[A[j]] + 1
▷ C[i] now contains the number of elements equal to i.
for i ← 1 to k
        do C[i] ← C[i] + C[i − 1]
▷ C[i] now contains the number of elements less than or equal to i.
for j ← length[A] downto 1
        do B[C[A[j]]] ← A[j]
           C[A[j]] ← C[A[j]] − 1
```

COUNTING-SORT assumes that each of the input elements is an integer in the range 0 to k, inclusive.

$A\,[1..n]$ is the input array, $B\,[1..n]$ is the output array, $C\,[0..k]$ is a temporary working array.

Counting Sort

```
COUNTING-SORT(A, B, k)
for i ← 0 to k
    do C[i] ← 0
for j ← 1 to length[A]
    do C[A[j]] ← C[A[j]] + 1
▷ C[i] now contains the number of elements equal to i.
for i ← 1 to k
    do C[i] ← C[i] + C[i − 1]
▷ C[i] now contains the number of elements less than or equal to i.
for j ← length[A] downto 1
    do B[C[A[j]]] ← A[j]
       C[A[j]] ← C[A[j]] − 1
```

Takes time O(k)

Takes time O(n)

What will be the running time?

- Total time: $O(n + k)$
  - Usually, $k = O(n)$
  - Thus counting sort runs in $O(n)$ time

Counting Sort

```
for i ← 0 to k
    do C[i] ← 0
for j ← 1 to length[A]
    do C[A[j]] ← C[A[j]] + 1
```

	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	2	0	2	3	0	1

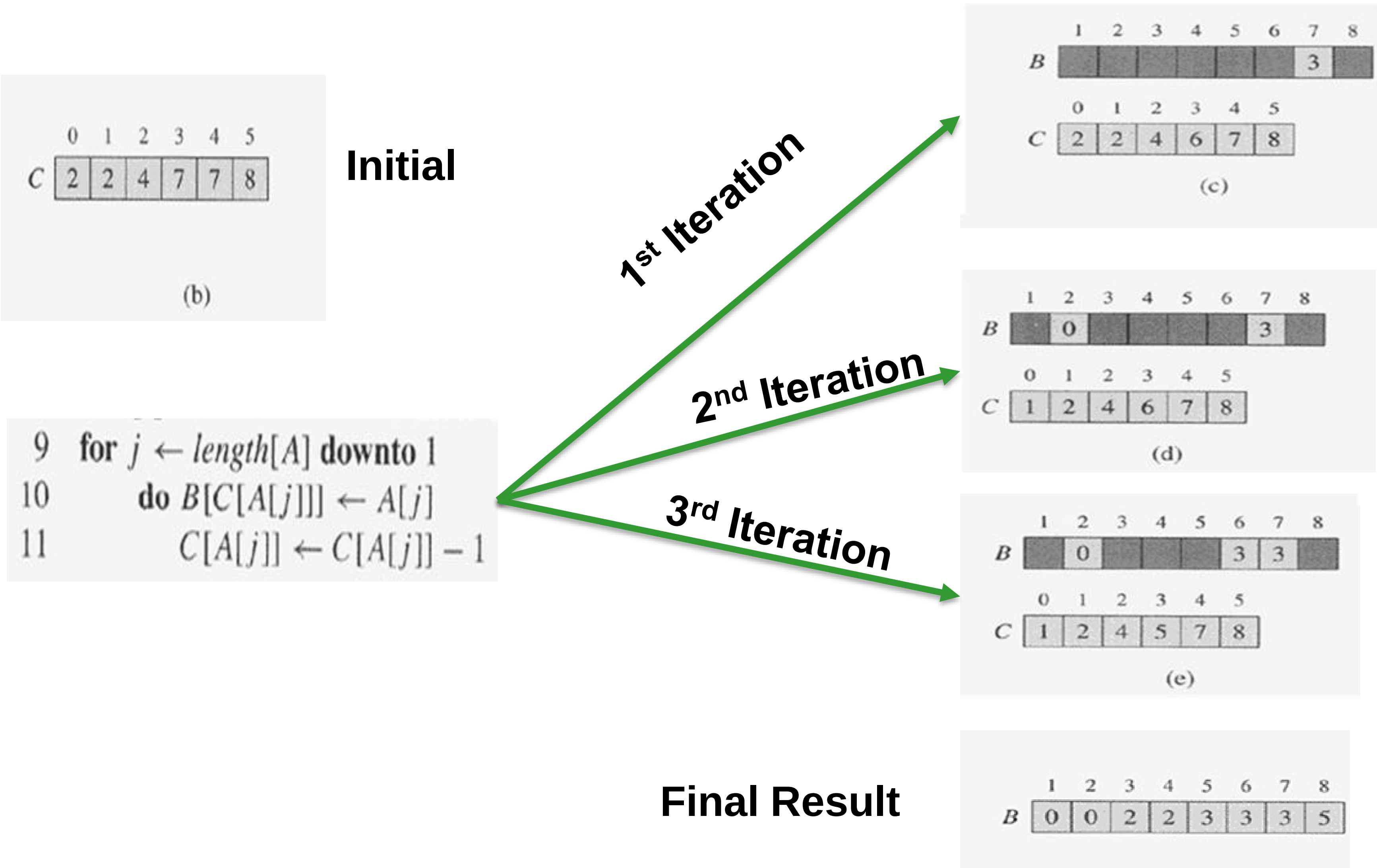
(a)

```
for i ← 1 to k
    do C[i] ← C[i] + C[i – 1]
```

	0	1	2	3	4	5
C	2	2	4	7	7	8

(b)

Counting Sort



Counting Sort

- *Why don't we always use counting sort?*
- Because it depends on range k of elements

- *Could we use counting sort to sort 32 bit integers? Why or why not?*
- Answer: No, k is too large ($2^{32} = 4,294,967,296$)