Dependent

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# Final Term:- Lecture 1.

Assignment:- (i) Example 9.5 and 9.6.
(ii) What do you mean by scattering and decribe rutherford scattering theory.

## WKB- Approximation:-

→ Wentzel - Kramer - Brillion approximation this approximation. is used for the treatments of systems of slowly varying potential'
→ Potential is nearly constant.
→ This approximation is semi-classical Approximation.
→ Motion of particle in time-independent Potential.

$$\frac{-\hbar^2}{2m}\nabla^2\psi(r) + V(r)\psi(r) = E\psi(r)$$

Multiple $\frac{-2m}{\hbar^2}$ on both side.

$$\nabla^2\psi + \frac{1}{\hbar^2}P^2(r)\,\psi(r) = 0$$

$$\because \frac{2m}{\hbar^2}E - \frac{2mV(r)}{\hbar^2}.$$

$$\because \frac{2m(E-V)}{\hbar^2} \leftarrow P^2(r).$$

For such potential solution is

$$\boxed{\psi(r) = Ae^{\pm iP.\vec{r}/\hbar}}$$

In WKB method
$\psi(r)$ ... $e^{iS(\vec{r})/\hbar}$

→ Where $A(\vec{r})$ and $S(\vec{r})$ needed to be determined → But Both $A(\vec{r})$ and $S(\vec{r})$ are real.

→ Put $\psi(\vec{r})$ into

$$\nabla^2 \psi(r) + \frac{1}{\hbar^2} P^2(r)\, \psi(r) = 0$$

$$\underbrace{\nabla^2 \left[ A(\vec{r})\, e^{iS(\vec{r})/\hbar} \right]}_{\text{Product.}} + \frac{1}{\hbar^2} P^2(r) \left[ A(\vec{r})\, e^{iS(\vec{r})/\hbar} \right] = 0$$

$$A\left[\frac{\hbar^2}{A} \nabla^2 A - (\nabla S)^2 + P^2(r)\right] + i\hbar\left[2(\vec{\nabla} A)(\vec{\nabla} S) + A\nabla^2 S\right] = 0$$

→ <u>Two parts</u>

<u>Real part</u>.

$$\nabla^2 A - (\vec{\nabla} S)^2 + P^2(r) = 0.$$

$$(\vec{\nabla} S)^2 = P^2(r)$$

$$(\vec{\nabla} S)^2 = 2m(E - V(r)) \rightarrow (A)$$

$\frac{\hbar^2}{A} = \nabla^2 A$ ↳ neglected since its very small as compared to $(\nabla S)^2$ and $P^2(r)$.

<u>Imaginary Parts</u>- $2(\nabla A)(\nabla \vec{S}) + A\nabla^2 S = 0$

$$\Rightarrow \vec{\nabla} S = \pm P(r) = \pm\sqrt{2m(E-V)}$$

$$\frac{dS}{dx} = \pm P(x) = \pm\sqrt{2m(E-V)}$$

→ 2nd Term:- $2(\nabla A)(\nabla S) + A\nabla^2 S = 0$

Classical $\rightarrow$ $E > $ Potential

Quantum $\rightarrow$ $P > E$.

$$\boxed{\vec{\nabla} S x = \pm P(x)}.$$

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$$2\left(\frac{d}{dx}\ln A\right) P(x) + \frac{d}{dx} P(x) = 0$$

$$\frac{d}{dx}\ln A = \frac{1}{A}\frac{dA}{dx}$$

$$\nabla^2 S = \nabla \nabla S$$
$$= \nabla P(x)$$
$$= \frac{d}{dx}(P(x)).$$

Integrate $\frac{dS}{dx}$

$$\int dS = \pm \int \sqrt{2m(E-V)}\, dx$$

$$S(x) = \pm \int dx \sqrt{2m(E-V)} = \pm \int P(x)\, dx.$$

$$-\frac{d}{dx}\left[2\ln A + \ln P(x)\right] = 0$$

$$A(x) = ?$$

$$2\ln A + \ln P(x) = C$$

$$\boxed{A(x) = \frac{C}{\sqrt{P(x)}}}$$

$$\psi_{\pm}(x) = \frac{C_{\pm}}{\sqrt{P(x)}} \exp\left[\pm\frac{i}{\hbar}\int^{x} P(x)'\, dx'\right].$$

Amplitude is proportional to $\frac{1}{\sqrt{P(x)}}$

$\rightarrow$ Amplitude give us probability.

$x$ and $x + dx \rightarrow \frac{1}{\sqrt{P(x)}}$

$\rightarrow$ That is what expected classically

is proportional inverse speed

Gradient $\xrightarrow[\text{decribe}]{}$ slope.

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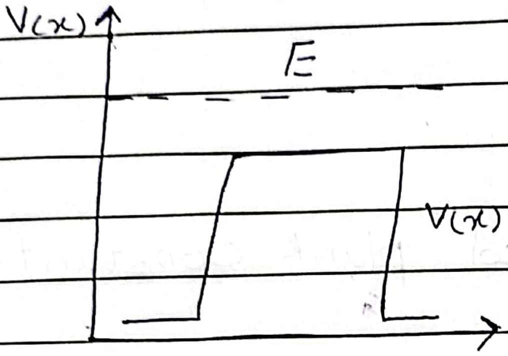
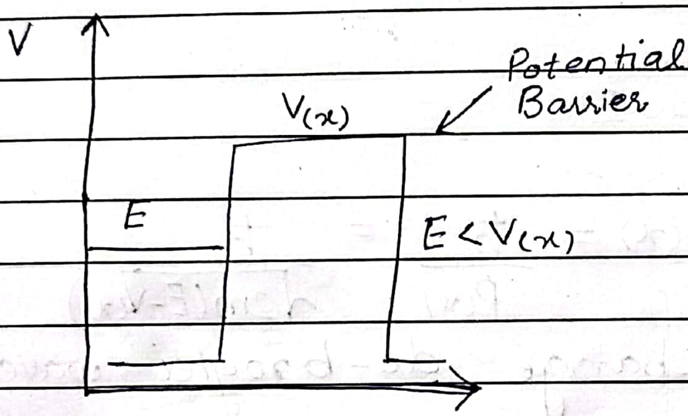
# Lecture 2:-
## WKB approximation:-

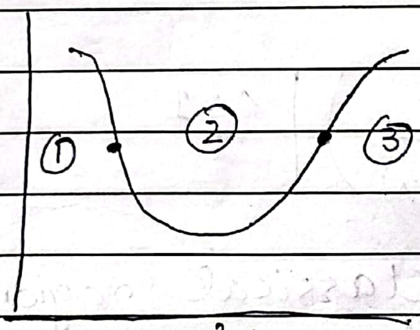
$E > V(x) \longleftarrow$ Classically allowed.
$E < V(x) \longleftarrow$ classically Forbidden.




$E > V(x).$




$E < V(x)$




$\dfrac{\hbar \nabla^2 S}{(\nabla S)^2}$

$\Psi(r) = A(r)\, e^{iS(r)/\hbar}$

$S(r) =$ Phase

$A(\vec{r}) =$ Amplitude.

$|\hbar \nabla^2 S| << (\nabla S)^2$

$\hbar S'' << S'^2$

$\left| \dfrac{\hbar S''}{S'^2} \right| << 1$

$\left| \hbar \dfrac{d}{dx}\left(\dfrac{1}{S'}\right) \right| << 1$

$S'(x) = \pm P(x)$ $\qquad \lambda = \frac{h}{P}$

$\left| \hbar \frac{d}{dx}\left(\frac{1}{\hbar / \lambda(x)}\right)\right| << 1$ $\qquad P = h/\lambda \Rightarrow P(x) = h/\lambda(x)$

$\left|\frac{\hbar}{2\pi}\frac{d}{dx}\left(\frac{1}{\hbar}\lambda(x)\right)\right| << 1$

$\frac{d}{dx}\left(\frac{\lambda(x)}{2\pi}\right) << 1$

$\hbar = \frac{h}{2\pi}$ reduced plank constant.

$\bar{\lambda}(x) = \frac{\lambda(x)}{2\pi}$

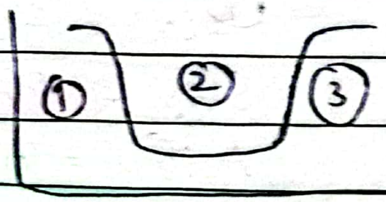
$\frac{d}{dx}\bar{\lambda}(x) << 1$

$\cancel{\hbar(x) = \hbar}$ $\qquad \bar{\lambda}(x) = \frac{\hbar}{P(x)} = \frac{\hbar}{\sqrt{2m(E-V(x))}}$

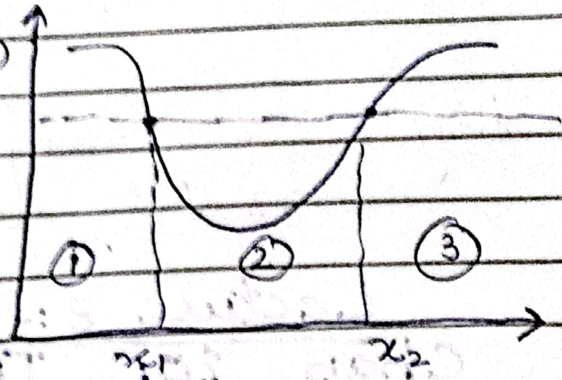
→ rate of change de-broglei wavelength is small.

$\left|\frac{d\bar{\lambda}(x)}{dx}\right| = \left|\frac{d}{dx}\left(\frac{\hbar}{P(x)}\right)\right| << 1$

Statisfied in classical region.



## Bound states for Potential Wells:




$x = x_1, \; x = x_2$

$E = V(x_1) = V(x_2)$.

$\psi_{1_{WKB}}(x), \; \psi_{2_{WKB}}(x),$

$\psi_{3_{WKB}}(x)$.

9.173

$$\psi_1 = \frac{c_1}{\sqrt{P(x)}} \exp\left[-\frac{1}{\hbar} \int P(x')\,dx'\right] \quad x < x_1$$

$E > V$
$E < V$

$$\psi_3 = \frac{c_3}{\sqrt{P(x)}} \exp\left[\frac{1}{\hbar} \int P(x')\,dx'\right] \quad x > x_2$$

$$\psi_2(x) = \frac{c_2}{\sqrt{P(x)}} \sin\left(\frac{1}{\hbar} \int_{x_1}^{x_2} P(x')\,dx' + \alpha\right) \quad x_1 < x < x_2$$

~~eqn 9.185~~

~~Oscillator Field:-~~

Oscillator field: $V(x)$ around $x_2$ Taylor series

$$\cancel{V(x)} \; V(x) \simeq V(x_2) + (x - x_2) \left.\frac{dV(x)}{dx}\right|_{x=x}$$

$$V(x) = E + (x - x_2) F_0$$

Schrondinger Equation $H\psi = E\psi$
$H = P + V$.

$$\frac{d^2\psi}{dx^2} - \frac{2mF_0}{\hbar^2}(x - x_2)\,\psi(x) = 0$$

$$y = \left(\frac{2mF_0}{\hbar^2}\right)^{1/3} (x - x_2)$$

$$\left(\frac{2mF_0}{\hbar^2}\right)^{2/3} \left[\frac{d^2\psi(y)}{dy^2} - y\,\psi(y)\right] = 0$$

— Functions.

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$$\psi(y) = A' Ai(y) = \frac{A'}{\pi}\int_0^\infty \cos\left(\frac{z^3}{3} + yz\right) dz.$$

$$Ai\,y \cong \frac{1}{\sqrt{\pi}\,|y|^{1/2}} \sin\left[\frac{2}{3}(y)^{2/3} + \frac{\pi}{4}\right] \quad y << 0$$

## Lecture 3:-

## Chap 10: Time dependent Perturbation

Methods

Imp

(i) Schrondinger picture
(ii) Heisenberg Picture
(iii) Interaction Picture
or
Dirac Picture.

**In schrondinger picture:**
State $|\psi\rangle$ is time Dependent while operator is time-independent.

**In Heisenberg picture:-**
state $|\psi\rangle$ is time-independent while Operator $\hat{O}$ is time-dependent.

**In interaction or Dirac Picture,** both state $|\psi\rangle$ and Operator $\hat{O}$ are time dependent.

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

Time-dependent SWE.

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→ Time evolution state $|\psi\rangle$ is due to propagator or time-evolution - operator.

$$|\psi_0\rangle \xrightarrow[\text{operator}]{\text{Propagator}} |\psi_{(t)}\rangle$$

$$|\psi_{(t)}\rangle = \hat{U}_{(t,t_0)} \, |\psi_{(t_0)}\rangle$$

$$\hat{U}_{(t,t_0)} = e^{-i(t-t_0)\hat{H}/\hbar}$$

$$\hat{U}\hat{U}^{\dagger} = \hat{I}$$

Basically

$$\hat{U}\hat{U}^{-1} = \hat{I}$$

† → dagger operator which means transpose conjugate.

$$\hat{U}^{\dagger} = \hat{U}^{-1}$$

In unitary case

Heisenberg Picture

→ State is Frozen i.e not depending on time while operator $\hat{O}$ is evolutory.

$|\psi_{t}\rangle - |\psi_{(0)}\rangle$ at time $t=0$ state is same but operator evaluate.

## Lecture 4:-

### Time-dependent Pertubation:-

→ Three ~~particle~~ Picture of QM

$$|\psi_{new}\rangle = \underbrace{\hat{U}(t)}_{\text{Unitary Operator.}}|\psi_{(0)}\rangle$$

$$\langle\psi_{new}| = \langle\psi_{old}|\hat{U}^{\dagger}$$

† → represents transpose - conjugate.

$$\hat{A}_{new} = \hat{U}\hat{A}_{old}\hat{U}^{\dagger}$$

chap # 2

$$|\psi_{old}\rangle = \hat{U}^{\dagger}_{(t)}|\psi_{new}\rangle$$

$$\langle\psi_{old}| = \langle\psi_{new}|\hat{U}_{(t)}$$

$$\hat{A}_{old} = \hat{U}^{\dagger}\hat{A}_{new}\hat{U}.$$

→ Heisenberg picture is Obtained from Schrondinger picture.

→ In Herinberg picture state Vector is frozen.

$$|\psi_{(t)}\rangle_H = \hat{U}^{\dagger}_{(t)}|\psi_{new}(t)\rangle = |\psi_{(0)}\rangle$$

Schrondinger picture

$$|\psi_{(t)}\rangle = \hat{U}\underbrace{|\psi_{(0)}\rangle}_{|\psi_{old}\rangle}$$

$$\cancel{i(t-t_0)\hat{H}/\hbar}$$

$$\hat{U}(t,t_0) = e^{-i(t-t_0)\hat{H}/\hbar}$$

if initial time $t_0 = 0$

$$\Rightarrow \hat{U}(t) = e^{-it\hat{H}/\hbar}, \quad \hat{U}^{\dagger}(t) = e^{it\hat{H}/\hbar}$$

therefore:

$$|\psi_{(t)}\rangle_H = e^{it\hat{H}/\hbar}|\psi(t)\rangle$$

$$\frac{d|\psi(t)\rangle_H}{dt} = 0$$

$$\hat{A}_{old} = \hat{U}^{\dagger}\hat{A}_{new}\hat{U}$$

$$\langle\psi_{(t)}|\hat{A}|\psi_{(t)}\rangle = \langle\psi_{(t)}|\hat{U}^{\dagger}\hat{A}\hat{U}|\psi_{(t)}\rangle$$

$$\langle\psi_{(t)}|e^{it\hat{H}/\hbar}\hat{A}e^{-it\hat{H}/\hbar}|\psi_{(t)}\rangle$$

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$= \langle \Psi(t) | \hat{A}_H(t) | \Psi(t) \rangle.$

that $\hat{A}_H(t)$ is Heisenberg Operator.

$$\hat{A}_H(t) = e^{iHt/\hbar} \hat{A} e^{-itH/\hbar}$$

$$\rightarrow \frac{d}{dt}\hat{A}_H(t) = \frac{d}{dt}\left[\hat{U}^{\dagger}(t) \hat{A} U(t)\right] = \frac{1}{i\hbar}\left[\hat{A}_H, \hat{U}^{\dagger}\hat{H}U\right]$$

Since $\hat{U}$ and $\hat{H}$ commutes $[H, \hat{U}] = 0$

$$\frac{d}{dt}\hat{A}_H(t) = \frac{1}{i\hbar}\left[\hat{A}_H, \hat{U}^{\dagger}\hat{U}\hat{H}\right] \quad \hat{H}\hat{U} = \hat{U}\hat{H}$$

$$\hat{U}\hat{U}^{\dagger} = \hat{I} = 1$$

$$\boxed{\frac{d}{dt}\hat{A}_H(t) = \frac{1}{i\hbar}\left[\hat{A}_H, \hat{H}\right]}$$

Heisenberg equation of motion

**Interaction picture:** its also called Dirac Picture.

→ Both state Vectors and operators evolves with time

$$|\Psi(t)\rangle_I = e^{iH_0t/\hbar} |\Psi(t)\rangle$$

$$\hat{H}(t) = \hat{H}_0 + V(t)$$

$i\hbar$

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle_I = \hat{V}_I(t) |\Psi(t)\rangle_I$$

Evolution of state vector is governed by

$$\hat{A}_I(t) = \hat{V}_I(t) = e^{itH_0/\hbar} \hat{V} e^{-itH_0}$$

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Interation Operator:

$$\boxed{\frac{d}{dt} A_I(t) = \frac{1}{i\hbar}\left[\hat{A}_I(t), H_0\right]}$$

Interaction Pic equation of motion.

Interaction Picture is intemediate ~~interaction~~ b/w Sch and Heisenberg

$$\hat{H}(t) = H_0 + V(t)$$

$$\hat{H}_0 |\psi_n\rangle = E_n |\psi_n\rangle .$$

$$|\psi_n(t)\rangle = e^{-it H_0/\hbar} |\psi_n\rangle$$

$$|\psi_n(t)\rangle = e^{-iE_n t/\hbar} |\psi_n\rangle .$$

→ $\hat{V}(t)$ is acting only

$$\hat{V}(t) = \begin{cases} \hat{V}(t) & 0 \le t \le \tau \\ 0 & t < 0, \quad t > \tau \end{cases}$$

acting $0 \le t \le \tau$.

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = (H_0 + V(t))|\psi(t)\rangle \rightarrow A$$

either absorbs or emits energy.

→ Main task of time dependent perturbation theroy

if system is in $|\psi_i\rangle$ of $\hat{H}_0$ what is the probability to be found in $|\psi(t)\rangle$

If A solution can be.

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Interation Operator :

$$\boxed{\frac{d}{dt} A_I(t) = \frac{1}{i\hbar}\left[\hat{A}_I(t), H_0\right]}$$

Interaction Pic equation of motion.

Interaction Picture is intemediate ~~interaction~~ b/w Sch and Heisenberg

$$\hat{H}(t) = H_0 + V(t)$$

$$\hat{H}_0 |\psi_n\rangle = E_n |\psi_n\rangle .$$

$$|\psi_n(t)\rangle = e^{-it H_0/\hbar} |\psi_n\rangle$$

$$|\psi_n(t)\rangle = e^{-iE_n t/\hbar} |\psi_n\rangle .$$

→ $\hat{V}(t)$ is acting only

$$\hat{V}(t) = \begin{cases} \hat{V}(t) & 0 \le t \le \tau \\ 0 & t < 0, \quad t > \tau \end{cases}$$

acting $0 \le t \le \tau$ .

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \left(H_0 + V(t)\right) |\psi(t)\rangle \rightarrow (A)$$

either absorbs or emits energy.

→ Main task of time dependent perturbation theroy

if system is in $|\psi_i\rangle$ of $\hat{H}_0$ what is the probability to be found in $|\psi(t)\rangle$

If (A) solution can be .

$$|\psi(t)\rangle = \sum_n c_n(t)\, e^{-iE_n t/\hbar}\, |\psi_n\rangle$$

or use interaction picture to calculate new state.

Reading Assignment:-

Do - transition probability for Harmonic perturbation.

Next Assignment :-

Lorentz transformation.

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$$P_{i\to f}(t) = \left| \langle \psi_f | U_I(t, t_i) | \psi_i \rangle \right|^2$$

$$U_I(t, t_i) = 1 - \frac{i}{\hbar} \int_{t_i}^{t} V_I(t')\, dt' \left(\frac{-i}{\hbar}\right)^2$$

$$\int_{t_i}^{t} V_I(t)\, dt_1$$

$$P_{if}(t) = \left| -\frac{i}{\hbar} \int_0^t \langle \psi_f | V(t') | \psi_i \rangle\, e^{i\omega_{fi} t'}\, dt' \right|^2$$