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Introduction to Algorithms Project Report

1)Shellsort is an in-place comparison sort. The idea is to arrange the list of elements so that, starting anywhere considering every hth element gives a sorted list.
For the given array A={9,8,7,6,5,4,3,2,1}. Value of the array after each iteration of for loop

9	8	7	6	5	4	3	2	1
2	8	7	6	5	4	3	**9**	1
2	**1**	7	6	5	4	3	9	**8**
2	1	7	6	5	4	3	9	8
2	1	7	6	5	4	3	9	8
2	1	**4**	6	5	**7**	3	9	8
2	1	4	**3**	5	7	**6**	9	8
2	1	4	3	5	7	6	9	8
2	1	4	3	5	7	6	9	8
1	2	4	3	5	7	6	9	8
1	2	4	3	5	7	6	9	8
1	2	**3**	**4**	5	7	6	9	8
1	2	3	4	5	7	6	9	8
1	2	3	4	5	7	6	9	8
1	2	3	4	5	**6**	**7**	9	8
1	2	3	4	5	6	7	9	8
1	2	3	4	5	6	7	**8**	**9**

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2) loop invariants:

a) first while loop produces h values. All h values are less than n i.e., Array size. All the hvalues produced are odd numbers. The i th iteration "h" value is equals to floor ((i+1 th iteration " h")/2) value.

b) Second while loop invariants: h is always greater than 0, h ordered sorted sets are formed at the end of each loop.

c) For loop: for every iteration A[j] is picked out of array. h separated sub array is produced.

d) Starting in position j-h, elements are successively moved h positions to the right until proper position for A[j] is found.

3)proof by contradiction .
Given condition B[i]<=C[i] for all 1<=i<=m before sorting ----------condition #1
Asked to prove B[i]<=C[i] condition holds true after sorting for all 1<=i<=m.
Lets assume that B[i]>C[i] for all 1<=i<=m after sorting
Then it means that B[i] value >= i elements of C for all 1<=i<=m---------2
But as per the 1^{st} condition it gets that any i elements of C >= any i elements of B
Here we gets that B[i]> i elements of B, which is not true for the sorted order
Therefore B[i]<=C[i] holds true after sorting

4)proof by contradiction
Given condition B[i]<=C[i+j] for 1<=i<=m-j before sorting ----------condition #1
Asked to prove B[i]<=C[i+j] for 1<=i<=m-j holds true after sorting
Lets assume that B[i]>C[i+j] for all 1<=i<=m-j after sorting,
From the 1^{st} condition we gets that any i C elements are greater than or equals to any i B elements
From that we gets that B[i]> i+j C elements> i B elements , it is a fallacy as B[i] cant be greater than i B elements after sorting.
There fore B[i]<=C[i+j] holds true after sorting

5)Given K>h>1. To show A remains K-ordered after A is h-sorted. Have to prove A[i]<=A[i+k] remains same after sorting h gap elements of Array A
we take a difference of h on both sides to show that A remains K ordered after h sorted.
B[]=….A[i-h],A[i],A[i+h]…..and C[]=…..A[i+k-h],A[i+k],A[i+k+h]…..
Have to prove The above sequences are independently sorted the condition still holds
i.e., B[i]<=C[i+k] for values 1….i-h<=i<=i+h…m--------condition 1
By contradiction : lets assume that B[i]>C[i+k] after sorting
It means that B[i] is greater than i+k elements of sorted array C.
As per 1^{st} condition we gets that any i C elements are greater than or equal to any i B elements.
From that we gets that B[i]>i+k C elements > I B elements , it is a not possible as B[i] can not be greater than I B elements after sorting.

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Therefore B[i]<=C[i+k] holds true

EMPIRICAL ANALYSIS

For Empirical analysis I used 4 different gap sequences

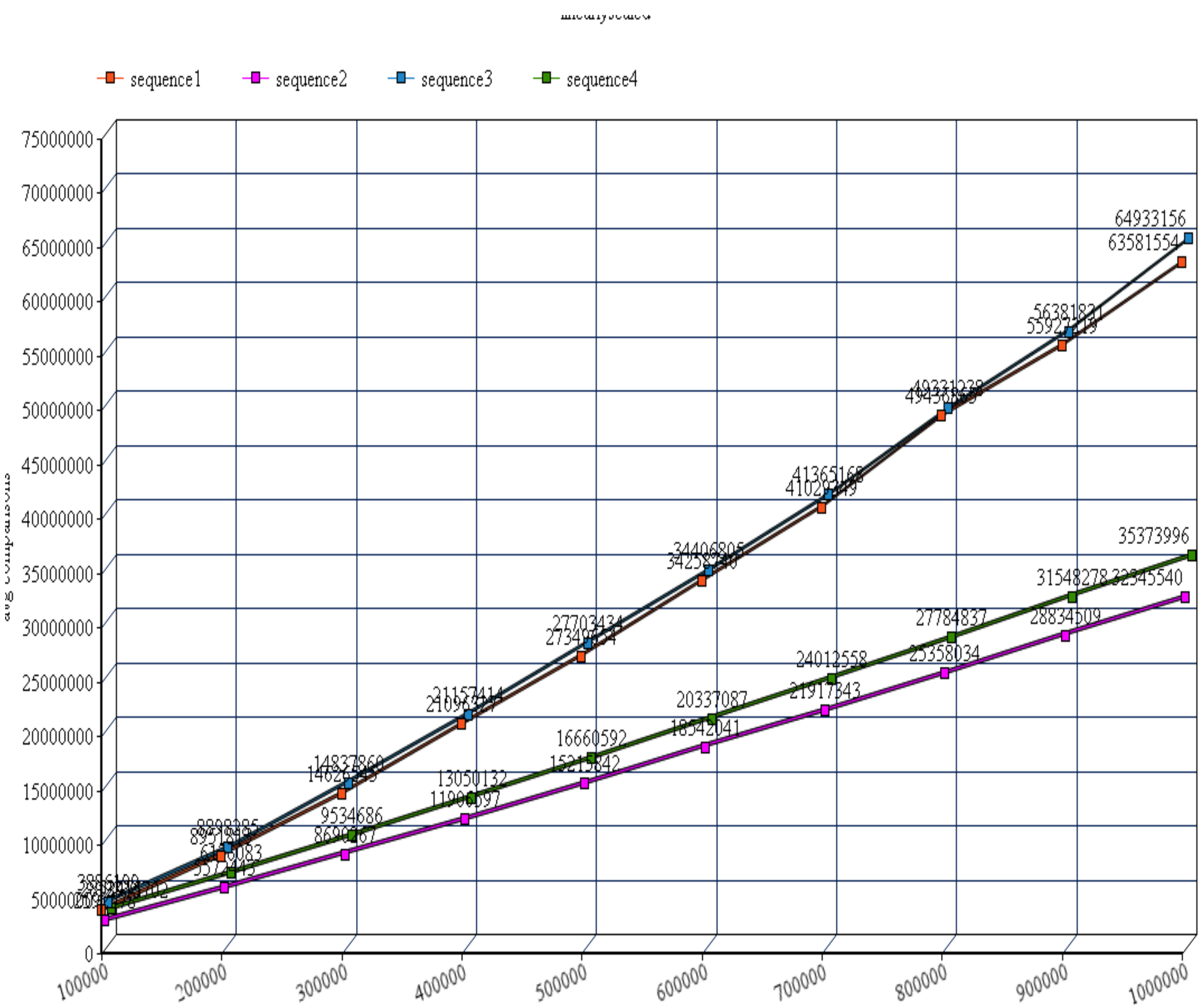
Sequence1: h=2*h+1

Sequence2: $\left\lceil \frac{9^k - 4^k}{5 \cdot 4^{k-1}} \right\rceil$

Sequence3: $(3^k - 1)/2$

My Sequence 4: e^i.

6)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9534686

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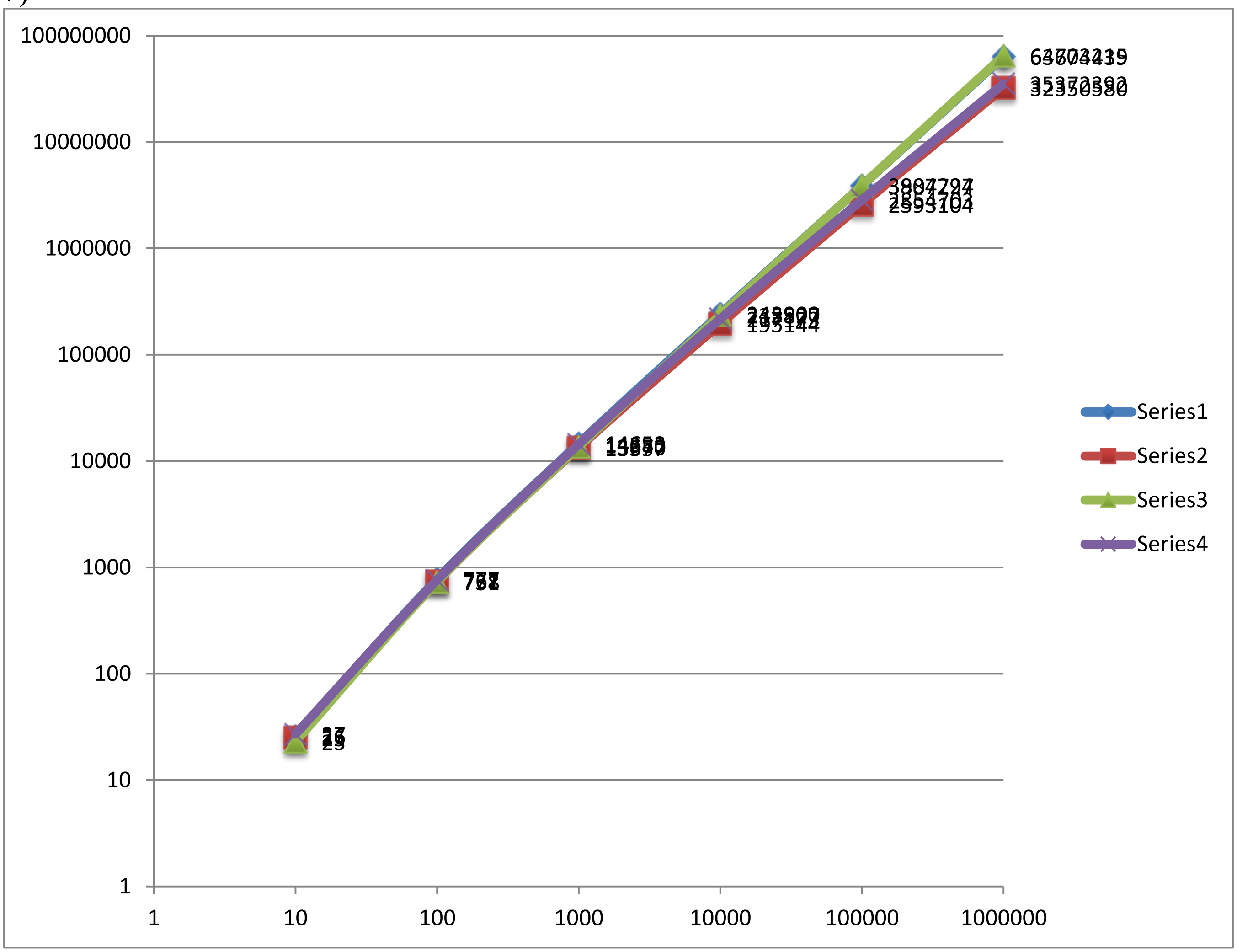
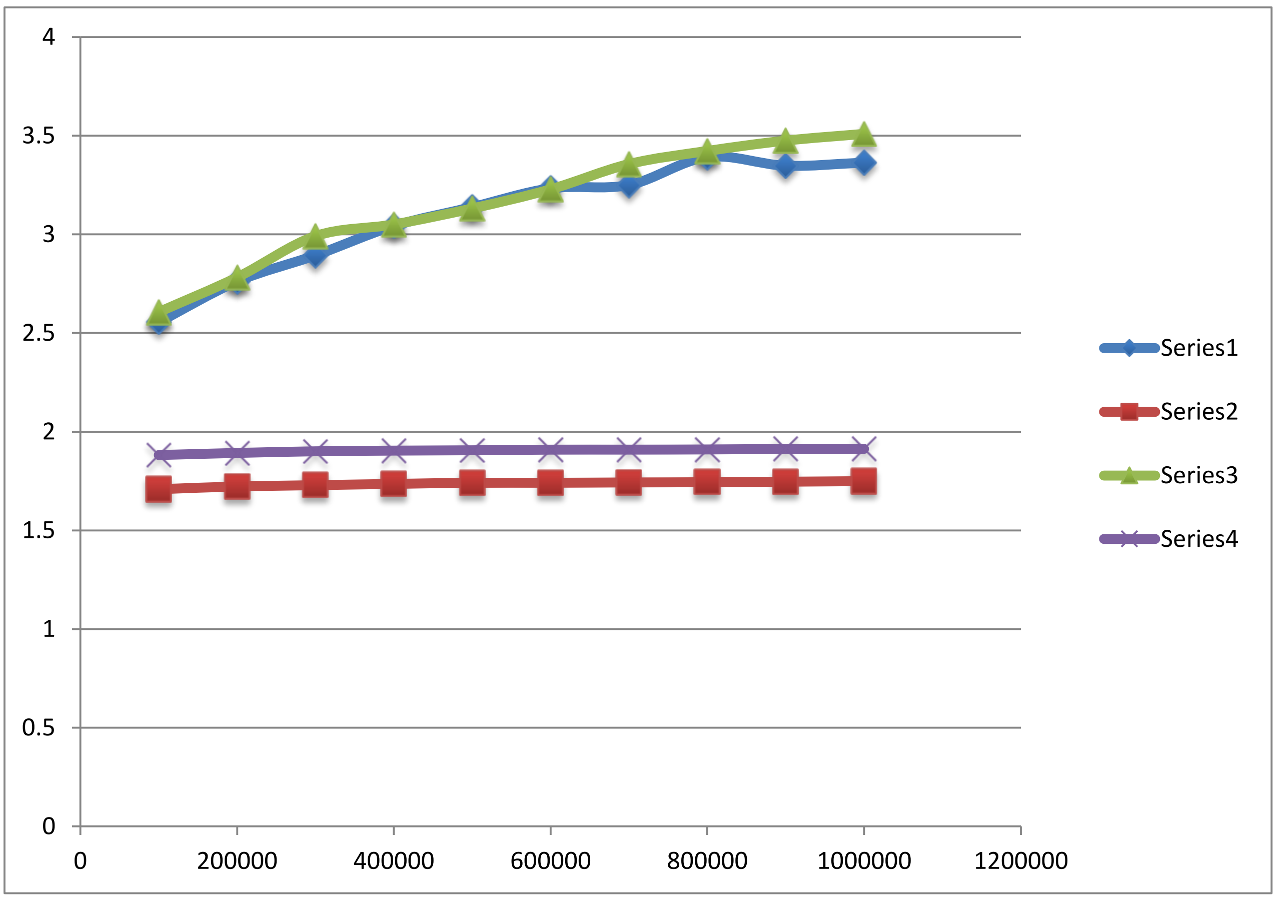
7)



8)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2.5
2
1.5
1
0.5
0
0
200000
400000
600000
800000
1000000
1200000
Series1
Series2
Series3
Series4

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9)