MA3080-MA4080-MA7080 "Mathematical Modelling"

Task for the miniproject. Chapter 3 draft.
Due till 26.11.2021 23:59
10 marks available

Social Psychology additions to SIR models of COVID

Preliminary work: Read Lecture 5 and additional material. The following notations are used.

States: S - susceptible, I - Infected/infector, R - Removed (recovered or dead)

System of transitions:

$$S \to I \to R$$

For populations we used the same letters S, I, R.

We consider constant population size: $S + I + R = N = const$.

Further we use dimensionless variables:

$$\{S, I, R\} = \{\frac{S}{N}, \frac{I}{N}, \frac{R}{N}\}$$

$$S + I + R = 1$$

Becoming infected depends on contact between S and I. Therefore, assume that intensity of transitions $\mathbf{S} \to \mathbf{I}$ is aI and the flux $\mathbf{S} \to \mathbf{I}$ is aSI, $a = const$.

Assume that the intensity of recovering (or death) is constant (b) and, therefore, the flux $\mathbf{I} \to \mathbf{R}$ is bI.

Now we have the following system of ODE

$$\frac{dS}{dt} = -aSI,$$
$$\frac{dI}{dt} = aSI - bI,$$
$$\frac{dR}{dt} = bI.$$

According to the classical theory of stress and general adaptation syndrome (GAS), GAS is the three-stage process that describes the physiological and psychological changes the organism goes through when under stress. Hans Selye, a medical doctor and researcher, came up with the theory of GAS.

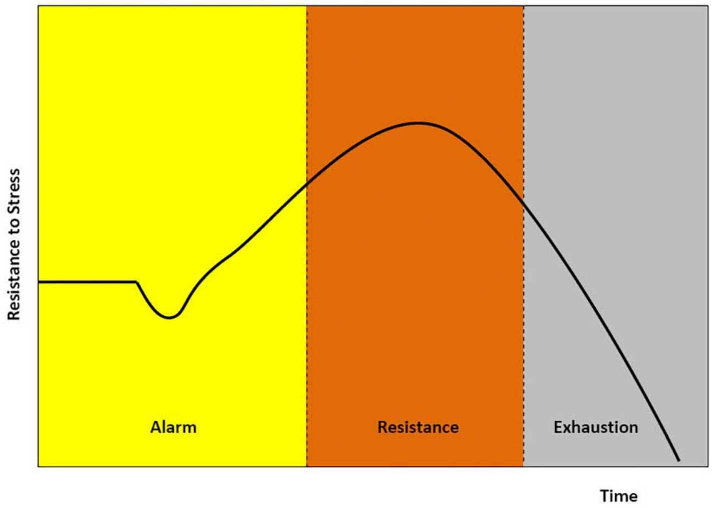
There are three phases of Stress: Alarm, Resistance, and Exhaustion. Let us generalise this approach to analyse the human behaviour under epidemic stress. Assume that there are four types of human behaviour and four subpopulations in S:

S_{ign} – "Ignorant people that do not know anything worrying about the epidemic;

S_{al} – people in "Alarm phase";

S_{res} – people in "Resistance" state, with very rational and save behaviour;

S_{exh} –people in "Exhaustion" state. They are tired of the epidemic, behave unsafe and do not react on alarm stimuli.



$$S = S_{ign} + S_{al} + S_{res} + S_{exh}$$

At the initial state $S(0) = S_{ign}(0)$ and all other components have zero population $S_{al}(0) = S_{res}(0) = S_{exh}(0) = 0$

I propose to reduce the number of variables and include the alarm phase partially in S_{ign}, and partially in S_{res}. This means that we consider 3 modified phases: S_{ign}, S_{res}, and S_{exh}

$$S = S_{ign} + S_{res} + S_{exh}$$

In addition to SIR, $S_{ign} \to I \to R$, we introduce the following transitions

$$S_{ign} \to S_{res} \to S_{exh} \to S_{ign}$$

With the Mass Action Law formalism, SIR with this additions is:
SIR:

$$S_{ign} + I \to 2I; \quad I \to R$$

Stress reaction:

$$S_{ign} + I \to S_{res} + I; \quad S_{res} \to S_{exh}; \quad S_{exh} \to S_{ign}$$

Of course, people in state S_{exh} have the same vulnerability as the ignorant people have and we have to introduce the reaction $S_{exh} + I \to 2I$ with the same reaction rate constants. The difference between S_{ign} and S_{exh} is in the absence of transition $S_{exh} + I \to S_{res} + I$ – people in "Exhaustion" state do not move directly to resistant state.

1. Write kinetic equations for the extended model. For selection of reaction rate constants use the following hints. Reaction rate constant for a reaction $A \to B$ is the inverse life time of the particle A. Try the initial approximation:
   - Reaction rate constant for $S_{res} \to S_{exh}$ is 1/50 (after 50 days in "Resistance" state people become tired);
   - Reaction rate constant for $S_{exh} \to S_{ign}$ is 1/100 (after 100 days in "Exhaustion" state people return to the initial "Ignorant" state and become sensitive to the alarm signals);

- Reaction rate constant for $S_{ign} + I \to S_{res} + I$ is 1 (the transition rate is the product $kS_{ign}I$; we assume that if the proportion of I is close to 1 then ignorant people modify their behaviour to resistant with characteristic time 1 day).
- For SIR model use the parameters you have found in Chapter 2.

All these assumptions are very preliminary, just to start. Use normalised cumulative cases data from the start of epidemic only.

2. Integrate you equations numerically for the initial data you have used in Chapter 2. Compare these calculations to experimental data. Describe similarity and differences. Which hypothesis should be improved, from your point of view? Which constants should be modified?

3. Propose and try several (2-3) modifications, integrate the equations numerically and compare to data. Discuss the results. What can you propose further?

4. **The crowd effect.** Assume that the alarm increases superlinearly and the proper reaction form is $S_{ign} + 2I \to S_{res} + 2I$. The reaction rate is $qS_{ign}I^2$, where q is a new constant. We evaluate q assuming that for some selected proportion of infected I the reaction rate is the same as for the linear reaction. Say, let for $I = I_p = 0.02$ (2% of population) $qS_{ign}I_p^2 = kS_{ign}I_p$. Then $q = k/I_p$ The number I_p characterises the "visibility" of epidemic and depends on activity of mass-media. Write the new modified equation with this crowd effect.

5. Perform Tasks 2 for the modified model with crowd effect. Play with the value of I_p. It could be smaller for some countries or larger. How the second wave was changed?

6. Discuss your experience of modelling of COVID. What should be modified in the models you used? Which idea is nice, from your point of view? What else should we take into account?
   The results should be clearly illustrated by plots and schemes.

The expected length of the report is 2000-3000 words (with all necessary illustrations). That is 8-12 pages with figures. (Please be reasonable and do not try to use more words than necessary.) Do not forget about reasonable references.