MA3080-MA4080-MA7080 "Mathematical Modelling"

Task for the miniproject. Chapter 2 draft.
Due till 12.11.2021 23:59
10 marks available

SIR models for COVID

Preliminary work: Read Lecture 5 and additional material. The following notations are used.

States: S - susceptible, I - Infected/infector, R - Removed (recovered or dead)

System of transitions:

$$S \to I \to R$$

For populations we used the same letters S, I, R.

We consider constant population size: $S + I + R = N = const$.

Further we use dimensionless variables:

$$\{S, I, R\} = \{\frac{S}{N}, \frac{I}{N}, \frac{R}{N}\}$$

$$S + I + R = 1$$

Becoming infected depends on contact between S and I. Therefore, assume that intensity of transitions $\mathbf{S} \to \mathbf{I}$ is aI and the flux $\mathbf{S} \to \mathbf{I}$ is aSI, $a = const$.

Assume that the intensity of recovering (or death) is constant (b) and, therefore, the flux $\mathbf{I} \to \mathbf{R}$ is bI.

Now we have the following system of ODE

$$\frac{dS}{dt} = -aSI,$$
$$\frac{dI}{dt} = aSI - bI,$$
$$\frac{dR}{dt} = bI.$$

1. Use normalised data for countries which you used in the first chapter:
   - Remove initial fragment before epidemic start.
   - Divide cumulative cases by the population of country.
2. Identify period of exponential grows and define parameters of exponential grows
   - Plot logarithms of normalised cumulative data.
   - Identify the period of (approximate) exponential growth (linear growth in logarithms). What is S at the beginning and at the end of this period?
   - Find the exponent r by linear regression for normalised cumulative data ($\log P \approx c + rt$). Is it the same r that you have found in your first chapter? Why?

3. Define parameters of SIR model
   - When S is close to 1, this exponent is $r \approx a - b$, according to SIR model.
   - Take $b = 0.1$ (time is measured in days; $b = 1/\tau$, where τ is, approximately, the time of virus spreading by an infected person; we take here $\tau \approx 10$ days).
   - Thus, we know $b = 0.1$ and $a = r + b$. Calculate all SIR coefficients.
4. Write SIR equations for each country. Define initial values $S(0), I(0), R(0)$. Explain your choice of initial values. Numerically integrate system of ODE. Play with different initial values of I ($I(0)$).
5. Compare the results with the cumulative data about COVID patients (the hint – normalised cumulative data are $P = 1 - S$ that is the decrease in the proportion of susceptible people in the population).
6. Discuss this comparison in detail. What is good and what is bad in the comparison of SIR model with your data?
7. How can you improve the model? What effects should be considered when describing the "second wave" of COVID?
8. Read the review "The Mathematics of Infectious Diseases" and discuss which models from this review can improve the agreement between SIR and empirical data.

The expected length of the report is 2000-3000 words (with all necessary illustrations). That is 8-12 pages with figures. (Please be reasonable and do not try to use more words than necessary.) Do not forget about reasonable references.