A Nonlinear Model Predictive Control for an AUV to Track and Estimate a Moving Target Using Range Measurements

R. Praveen Jain(✉), Andrea Alessandretti, A. Pedro Aguiar, and João Borges de Sousa

Faculdade de Engenharia, Universidade do Porto, Porto, Portugal
{praveenjain,andrea.alessandretti,pedro.aguiar,jtasso}@fe.up.pt

Abstract. In this paper, we propose a Nonlinear Model Predictive Control (NMPC) approach that is employed by an Autonomous Underwater Vehicle (AUV) to track and estimate a moving target using range measurements. Due to the nonlinearities in the observation model associated with range-only measurements, there exist state and input trajectories of the AUV that makes the position of the target unobservable. To address this problem, a standard stabilizing NMPC based approach augmented with an economic cost function is utilized to steer the system through highly observable trajectories in order to guarantee a good estimate of the position of the target. The efficacy of the proposed solution is demonstrated through simulations.

Keywords: Nonlinear Model Predictive Control · Target estimation and tracking · AUV · Economic optimization

1 Introduction

Autonomous Underwater Vehicles (AUV) are being used extensively for marine applications such as ocean sampling [8,9], seabed mapping [12], ecological studies [15] to name a few. In such applications, the AUVs with various sensors are deployed for large scale data collection [14] and are often faced with tasks which require tracking of a moving target. One possible application is the case where the AUV needs to return to a possibly mobile base station for recharging, maintenance and so forth, usually performed with manual assistance. Autonomous docking is therefore a problem of interest where the AUVs can dock automatically for routine maintenance and thereby prolong the mission duration. In order to perform such a task the AUV needs to continuously estimate and track the moving target so that it can approach to its vicinity after which a docking process could be initiated. Such a capability is even more desired when operating

This project was supported by the European Unions Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 642153.

A. Ollero et al. (eds.), *ROBOT 2017: Third Iberian Robotics Conference*, Advances in Intelligent Systems and Computing 693, https://doi.org/10.1007/978-3-319-70833-1_14

off-shore. Additionally, limited on-board sensor capabilities and unavailability of reliable communication medium when operating in a marine environment limits the performance abilities of such systems.

In this paper, we consider the problem where an AUV needs to track and estimate a moving or stationary ASV, referred to as 'target', autonomously. Furthermore, we consider the scenario where the only information available to the AUV is the range (geometric distance) to the target, that is obtained by measuring the time-of-flight of an acoustic pulse. The resulting problem is a target estimation and tracking problem using range-only measurements. The position estimation of the target using range-only measurements is a highly nonlinear estimation problem. Additionally, it is possible to show that, there exists undesirable control and state trajectories which result in an unobservable system. Therefore, it is desirable to perform some 'observability based maneuvers', which steers the system through highly observable trajectories in order to guarantee a good estimate of the position of the target.

The works in [7,13] propose solutions based on suitable maneuvers designed to keep the state of the target vehicle observable. Although some of the results are promising, within the proposed framework, it is difficult to asses the quality of the overall estimate obtained during the mission. In this paper, the dual yet conflicting objective of tracking a moving target, while steering the system through highly observable trajectories, is posed as a continuous time sampled-data Nonlinear Model Predictive Control (NMPC) problem with economic optimization [5]. We borrow from [3,4], where the position of the target is estimated in 2D using bearing measurements to the target and extend it to the 3D case with range-only measurements. Specifically, the performance index of the proposed NMPC controller is composed of a stabilizing cost for target tracking and an observability index designed to steer the AUV through highly observable trajectories. We demonstrate the efficacy of the proposed method through simulations.

The paper is organized as follows. Section 2 describes the problem formulation and presents the model used for simulation of the target ASV and the AUV. Section 3 discusses the NMPC based target tracking and estimation method proposed in this paper. The measures of observability are presented and a NMPC problem is formulated which steers the system through highly observable trajectories resulting in good quality estimates. Simulation results are discussed in Sect. 4 followed by conclusions in Sect. 5.

2 Problem Formulation

Consider right handed frame of references, namely, the Inertial frame $\{I\}$ which is a fixed reference frame, a reference frame $\{T\}$ attached to the body of the target (ASV) and a reference frame $\{F\}$ attached to the AUV. Let $\mathbf{p}_t = [x_t,\ y_t]^T \in \mathbb{R}^2$ and $\mathbf{p}_f = [x_f,\ y_f,\ z_f]^T \in \mathbb{R}^3$ denote the position of the target and the AUV respectively, expressed in frame $\{I\}$. The position of the target along the z-direction is considered to be the reference depth and it is assumed that $z_t = 0$ remains constant throughout for the sake of simplicity. The motion of the target ASV is captured by the unicycle model,

$$\begin{aligned}\dot{\mathbf{p}}_t &= R_T(\psi_t)\mathbf{v}_t \\ \dot{\psi}_t &= \omega_t\end{aligned} \tag{1}$$

where $\mathbf{v}_t = [v_t,\ 0]^T$ is the linear velocity of the target in frame $\{T\}$ (v_t is the longitudinal velocity), ω_t is the angular velocity of the target, and $R_T(\psi_t) \in \mathcal{SO}(2)$ is the rotation matrix parameterized using the heading angle of the target ψ_t. Similarly, the AUV is modeled as a 6DOF rigid body kinematic model given as

$$\begin{bmatrix}\dot{\mathbf{p}}_f \\ \dot{\mathbf{\Theta}}_f\end{bmatrix} = \begin{bmatrix}R_F(\phi_f, \theta_f, \psi_f) & 0 \\ 0 & W(\phi_f, \theta_f)\end{bmatrix}\begin{bmatrix}\mathbf{v}_f \\ \boldsymbol{\omega}_f\end{bmatrix} \tag{2}$$

where the attitude of the AUV is given by the rotation matrix $R_F(\phi_f, \theta_f, \psi_f)$ parameterized using ZYX - Euler angles $\mathbf{\Theta} = [\phi_f,\ \theta_f,\ \psi_f]^T$ and

$$W(\phi_f, \theta_f) = \begin{bmatrix}1 & \sin\phi_f \tan\theta_f & \cos\phi_f \tan\theta_f \\ 0 & \cos\phi_f & -\sin\phi_f \\ 0 & \sin\phi_f / \cos\theta_f & \cos\phi_f / \cos\theta_f\end{bmatrix} \tag{3}$$

relates the body fixed angular velocities $\boldsymbol{\omega}_f$ to the rate of change of Euler angles. We consider that the target vehicle moves along an unknown path $\mathbf{p}_t(t)$ for all $t \in [0, \infty)$. Furthermore, the only available information regarding the target is the range measurement from the AUV to the target $r \in \mathbb{R}_{>0}$, defined as

$$r = \|\mathbf{q}_t - \mathbf{p}_f\| \tag{4}$$

where $\mathbf{q}_t = [\mathbf{p}_t^T,\ z_t]^T$ is the position of the target in 3D with $z_t = 0$. It is assumed that the AUV is equipped with a navigation system that provides estimates of its attitude and position. The problem of target estimation and tracking is stated as follows.

Problem 1 (Target Estimation and Tracking). *Given the dynamical model of the target* (1)*, the dynamical model of the AUV* (2)*, and the relative range measurement model* (4)

1. *Estimate the state of the target* $\hat{\mathbf{p}}_t$.
2. *Design a predictive control law for the AUV control inputs* $\mathbf{u}_f(t) = [v_f,\ \boldsymbol{\omega}_f^T]^T$ *which steers the AUV near the vicinity of the target while ensuring that the resulting state and input trajectories are highly observable.* □

Remark 1. In order to obtain high quality estimates of the target position using range-only measurements, it is necessary to drive the system through highly observable state and input trajectories.

3 Target Estimation and Tracking Controller

In this section, the various steps involved in the target estimation and tracking control design are discussed and the design choices are motivated. The methodology adopted is to design the predictive control law in state feedback form, assuming that the states of the AUV and the target are known. Next, the states of the overall system are estimated using an observer such as an Extended Kalman Filter with the available measurements. The estimation process assumes that the underlying system is observable. In order to avoid unobservable trajectories, we incorporate a measure of unobservability in the cost function of the predictive controller. This allows to obtain a good quality estimate of the target states.

3.1 Target-Follower System Description

As described earlier, the only information of the target available to the AUV is the range of the target from the AUV. Since it is assumed that the dynamics of the target is unknown from the perspective of the AUV, we propose to model the target as a single integrator model with inputs $\mathbf{v}_t = [v_{t_x},\ v_{t_y}]^T$.

$$\dot{\mathbf{p}}_t = \mathbf{v}_t \tag{5}$$

Additionally, we also assume that the control inputs applied to the target are unknown and hence these inputs need to be estimated. One possible strategy is to consider that the target inputs (linear velocities) are slowly varying. This allows us to assume that the target linear velocities are constant over a finite prediction horizon, that is,

$$\dot{\mathbf{v}}_t = 0 \tag{6}$$

The dynamics of the AUV (2) is used in the process of control design and estimation.

Motion Model: Therefore, the system model used for prediction and estimation can be written as

$$\begin{aligned} \dot{\mathbf{p}}_f &= R_F(\phi_f, \theta_f, \psi_f)\mathbf{v}_f \\ \dot{\boldsymbol{\Theta}}_f &= W(\phi_f, \theta_f)\boldsymbol{\omega}_f \\ \dot{\mathbf{p}}_t &= \mathbf{v}_t \\ \dot{\mathbf{v}}_t &= 0 \end{aligned} \tag{7}$$

Measurement Model: We assume that the attitude and the position of the AUV is known through the on-board navigation system, i.e., $\mathbf{p}_f$ and $\boldsymbol{\Theta}_f$ measurements are always available. The measurement model is given as follows:

$$\begin{aligned} \mathbf{p}_f &= \mathbf{p}_f \\ \boldsymbol{\Theta}_f &= \boldsymbol{\Theta}_f \\ r &= \sqrt{(\mathbf{q}_t - \mathbf{p}_f)'(\mathbf{q}_t - \mathbf{p}_f)} \end{aligned} \tag{8}$$

Since all the states of the AUV are considered to be measured, the observability of the system is equivalent to observability of target states $(\mathbf{p}_t, \mathbf{v}_t)$ from the range measurements r. Consequently, the measure of observability, discussed later, is derived from the measurement model related to the range measurement only.

3.2 Predictive Controller

In this section, we propose a continuous time sampled-data NMPC law to drive the AUV near the vicinity of the target, while actively estimating the position of the target. We use the trajectory tracking NMPC presented in [2] and apply it to the target tracking problem. The controller uses the estimates of the states as input and solves an open-loop optimal control problem over a finite horizon of T seconds. A part of the optimal control input is applied to the system and the optimization step is repeated every sampling time of δ seconds with latest received estimates of the states as initial condition, resulting in a feedback controller. We define the optimal control problem next, followed by discussion of the design considerations. For a given signal $x(.)$, $x([t_1, t_2])$ denotes the evolution of the signal over the time interval $[t_1, t_2]$. We use the notation t, to denote the evolution of time during simulations and τ to denote the time in prediction phase of the NMPC. Similarly the internal variable associated with the controller is denoted by a bar symbol over the variable, for example, $\bar{x}(t)$.

The target tracking NMPC controller is defined as follows.

Definition 1 (Target tracking NMPC). *Given the initial condition vector $z(t)$, a horizon length $T \in \mathbb{R}_{>0}$, the target-follower system dynamics (7), the target tracking NMPC at time t consists of finding the optimal control signal $\bar{\mathbf{u}}_f^*([0,T]) \in \mathcal{PC}(0,T)$, where $\mathcal{PC}(0,T)$ denotes space of piecewise continuous signals, by solving the following open-loop optimization problem,*

$$\min_{\bar{\mathbf{u}}_f([0,T])} \quad \int_0^T \|\mathbf{e}(\tau)\|_Q^2 \, d\tau + a_2 \|\bar{\mathbf{e}}(T)\|^2 \tag{9}$$

$$\begin{aligned}
\text{subject to} \quad & \dot{\bar{\mathbf{p}}}_f(\tau) = \bar{R}_F(\tau)\bar{\mathbf{v}}_f(\tau) && \forall \tau = [0,T] \\
& \dot{\bar{\boldsymbol{\Theta}}}_f(\tau) = \bar{W}(\tau)\bar{\boldsymbol{\omega}}_f(\tau) && \forall \tau = [0,T] \\
& \dot{\bar{\mathbf{p}}}_t(\tau) = \bar{\mathbf{v}}_t(\tau) && \forall \tau = [0,T] \\
& \dot{\bar{\mathbf{v}}}_t(\tau) = 0 && \forall \tau = [0,T] \\
& \bar{\mathbf{e}}(\tau) = \bar{R}'_F(\tau)(\bar{\mathbf{p}}_f(\tau) - \bar{\mathbf{q}}_t(\tau)) + \boldsymbol{\epsilon} && \forall \tau = [0,T] \\
& \bar{\mathbf{u}}_f(\tau) \in \mathcal{U}, \ \bar{\mathbf{e}}(T) \in \mathcal{E}_f && \forall \tau = [0,T] \\
& \big(\bar{\mathbf{p}}_f(0), \ \bar{\mathbf{p}}_t(0), \ \bar{\mathbf{v}}_t(0), \ \bar{\boldsymbol{\Theta}}_f(0)\big) = z(t)
\end{aligned}$$

where the initial condition vector is initialized as $z(t) = \Big(\hat{\mathbf{p}}_f(t), \hat{\mathbf{p}}_t(t), \hat{\mathbf{v}}_t(t), \hat{\boldsymbol{\Theta}}_f(t)\Big)$, $\mathcal{U}$ defines the input constraint set, and $\mathcal{E}_f$ is the terminal constraint set. □

In a receding horizon strategy, the control input is computed at time instants t_k with sampling time of δ seconds. The input applied to the system is given as $\mathbf{u}_{\text{mpc,f}}(t) = \bar{\mathbf{u}}_f^*(t - t_k; z(t_k))$ for all $t \in [t_k, t_{k+1})$.

Remark 2. The terminal cost function $a_2\|\bar{\mathbf{e}}(T)\|^2$ and the terminal constraint set $\mathcal{E}_f$ are necessary for design of stabilizing NMPC [10], assuming that the optimization problem (9) is feasible at initial time $t = 0$. For detailed discussion on design of these terms see [2]. Roughly speaking, the design of terminal set and the terminal cost relates to existence of a nonlinear control law valid over the terminal set $\mathcal{E}_f \subseteq \mathbb{R}^3$.

Remark 3. The tracking error variable $\mathbf{e}(t) = R'_F(\mathbf{p}_f - \mathbf{q}_t) + \boldsymbol{\epsilon}$ is defined in [1]. It must be noted that if $\boldsymbol{\epsilon} = 0$, we do not have direct control over the attitude of the AUV. Hence, it is necessary to have the condition $\boldsymbol{\epsilon} \neq 0$ satisfied for existence of nonlinear auxiliary control law which is a necessary requirement for design of stabilizing NMPC.

3.3 Observability Index

The NMPC law of previous section assumes that the states (or its estimates) of the system are available. As mentioned previously, the estimation of the states of the target using range-only measurements is a highly nonlinear estimation problem with existence of state and control trajectories which results in an unobservable system. In order to mitigate this problem we follow [4], and modify the MPC optimization problem of the previous section. Specifically, we modify the stage cost of the optimization problem to include an index of observability l_o as

$$\min_{\bar{\mathbf{u}}_f([0,T])} \quad \int_0^T \left(\|\mathbf{e}(\tau)\|_Q^2 + l_o(\bar{\mathbf{q}}_t, \bar{\mathbf{p}}_f, \bar{\mathbf{v}}_t, \dot{\bar{\mathbf{p}}}_f) \right) d\tau + a_2 \|\bar{\mathbf{e}}(T)\|^2 \tag{10}$$

with the objective of avoiding weakly observable/non observable closed loop trajectories resulting in an effective target estimation and tracking controller.

In this section we propose an index of observability. Consider the observability matrix of the target-follower system (7), associated with the measured output r,

$$\mathcal{O} = \frac{\partial}{\partial x} \left[r', \dot{r}', \ddot{r}', \cdots, r^{\{l\}'} \right]' \tag{11}$$

where $r^{\{l\}}$ denotes the l^{th} derivative of the output r with respect to time t. From the properties of the observability matrix, given $l \in \mathbb{N}_{>0}$ the state of the target system $(\mathbf{p}_t, \mathbf{v}_t)$ is locally observable at a given state and input of the target-follower system (7), if the matrix $\mathcal{O}$, is full rank. For general nonlinear systems, the number of derivatives l to be considered is not known a priori. An intuitive procedure to select l consists in increasing it until the observability matrix becomes full rank for some values of the state and input vectors. Then, driving the system through those values is enough to guarantee observability.

Let $\sigma_{\min}(A)$ and $\sigma_{\max}(A)$ denote the minimum and maximum singular value of a generic matrix A. To obtain a measure of the degree of observability, one possibility is to use the index $1/\sigma_{\min}(\mathcal{O})$ that increases as $\mathcal{O}$ gets close to singularity and becomes infinity when $\mathcal{O}$ loses rank. Another index of interest is the condition number of $\mathcal{O}$, i.e., $\kappa(\mathcal{O}) := \sigma_{\max}(\mathcal{O})/\sigma_{\min}(\mathcal{O})$, which broadly speaking, provides a measure of the difference of the "quality" of observability of the state components, where $\kappa(\mathcal{O}) = 1$ if all the state components have the same "quality" of observability. Prompted by these observations, we select the following observability index:

$$l_o(\bar{\mathbf{q}}_t, \bar{\mathbf{p}}_f, \bar{\mathbf{v}}_t, \dot{\bar{\mathbf{p}}}_f) = k \arctan\left(\frac{1}{k}\left(\frac{\alpha_1}{\sigma_{\min}(\mathcal{O})} + \alpha_2(\kappa(\mathcal{O}) - 1)^2\right)\right) \tag{12}$$

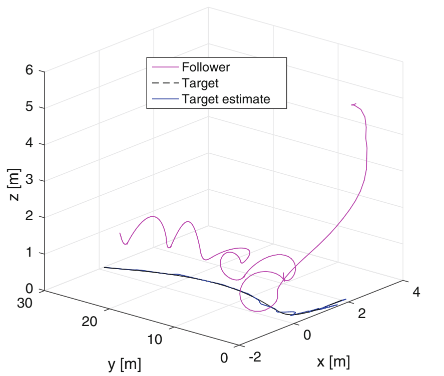
for some positive constants $\alpha_1 > 0$ and $\alpha_2 > 0$, where the positive constant $k > 0$ defines the width of the region where the nonlinearity arctan, is used as smooth saturation-like function, behaves almost linearly. The proposed performance index is saturated in order to guarantee that the system is not driven to unstable behaviors [5]. Note that the observability matrix is not the only method to define the index of observability and other mathematical tools, e.g., the determinant or the trace of the Fisher Information Matrix (FIM), can be exploited in a similar fashion. Assuming that the resulting NMPC steers the target-follower system, through observable state and input trajectories, an Extended Kalman Filter (EKF) is used to estimate the states.

4 Simulation Results

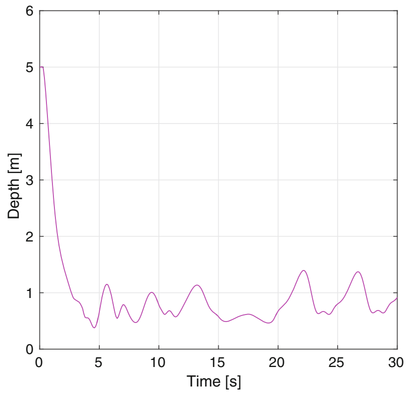
The target estimation and tracking approach presented in the previous section is validated through simulations using the VirtualArena Matlab toolbox [6]. The target ASV follows a trajectory specified as $\mathbf{p}_{t_d} = (\sin(0.1t), t)'$ and $\dot{\mathbf{p}}_{t_d} = (0.1\cos(0.1t), 1)'$ which is not known to the AUV a priori. The only measurement AUV has access to is the range measurement $r = \|\mathbf{q}_t - \mathbf{p}_f\|$. Simulation was conducted for 30 s with a sampling period of 0.1 s. The parameters associated with the NMPC are $Q = I$, $T = 0.3$ s, and $\boldsymbol{\epsilon} = [-1, 0, -1]'$. It is assumed that the initial states of the AUV and the target are known a priori and were chosen as $\mathbf{p}_f = [3, 3, 5]'$ meters, $\boldsymbol{\Theta}_f = [0, 0, 0]'$ radians, $\mathbf{q}_t = [2, 1, 0]'$ meters. Same initial conditions were provided to both the NMPC and the EKF estimator in order to prove the concept and prevent the divergence of EKF. The assumption that the initial states of the target are known is realistic as the AUV could be provided with the necessary information before the start of the mission. The AUV could then estimate the target's position continuously during the mission.

Figure 1 shows the simulation results of NMPC for target tracking and estimation of a moving target. From Fig. 1a, it can be seen that the AUV approaches the target while executing maneuvers that enhance the observability of the system. The AUV stays within the vicinity of the target as is evident from the Fig. 1b and c. The size of the vicinity around the target is defined by the size of $\boldsymbol{\epsilon}$ used in the NMPC formulation (9). Ideally, the size of the vicinity around the target could be made arbitrarily small, however we choose a larger region so as to drive the AUV 'near' the target. Once near the target, AUV could initiate a docking maneuver (not discussed in this paper).

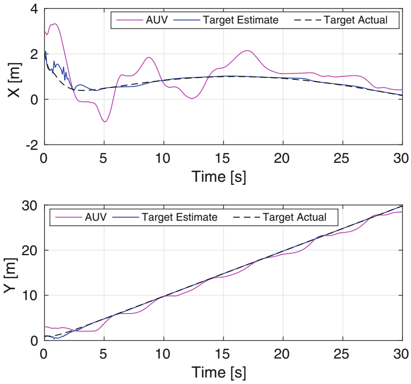
Figure 1c shows the path taken by the AUV and the path followed by the target. Estimated position of the target is also shown and it can be seen that the EKF along with the observability based cost function in NMPC, allows one to obtain good quality estimates of the target. Figure 1d shows the plot of



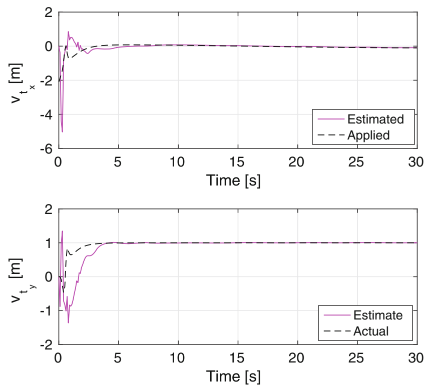
(a) 3D position of the AUV and the target



(b) Depth reached by the AUV



(c) Position of the AUV and target (actual and estimated)



(d) Linear velocity of the target

Fig. 1. Performance of the Nonlinear Model Predictive controller for target estimation and tracking

achieved linear velocity of the target and the velocities estimated by the EKF on the AUV. Clearly, using range-only measurements, AUV is able to successfully reconstruct the target position and velocity. Although, the simulations point to the successful reconstruction of the target states, it must be noted that in general, it is not possible to guarantee global convergence of EKF, and thus, the results are only valid locally for small errors in initial conditions. Stability properties of NMPC with economic optimization under state feedback has been proven in [4,5]. Despite the latter result provides ISS guarantees only for the case of state feedback, the simulation results presented in this paper serves as a proof-of-concept for target estimation and tracking problems using range-only measurements where the state is estimated using an EKF.

5 Conclusions

In this paper, a NMPC approach to target estimation and tracking problem was considered. The problem was addressed using range-only measurements. In order to mitigate the observability based issues with the target estimation using range-measurements, an economic cost term was included in the standard stabilizing cost function which serves to satisfy the dual objective of target tracking and performing maneuvers which steers the system through highly observable trajectories. It was shown under certain conditions that, the EKF is able to accurately estimate the position and velocity of the target. However, no formal guarantees were provided regarding the convergence of the filter or the stability of the NMPC with an estimator in the feedback loop. This is still an open research problem and developing methods which guarantee stability and convergence of NMPC based output-feedback control could be investigated. The simulation results serve as the proof of concept and several further extensions to the current work could be made. As mentioned earlier, observability matrix is not the only measure of observability and other means such as the Fisher Information Matrix could be exploited.

In this paper, external disturbances such as ocean currents, noise sources on the actuators or the sensors were not considered. The logical extension of this work would be to consider a non-ideal scenario with these external influences. Furthermore, we have considered that the range measurements are obtained continuously which is usually not true in practice. It might therefore be interesting to consider the scenario where the measurements are discrete and available intermittently. NMPC being an computationally expensive operation, state-of-the-art fast solvers such as ACADO [11] could be used for experimental validation.

References

1. Aguiar, A.P., Hespanha, J.P.: Trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty. IEEE Trans. Autom. Control **52**(8), 1362–1379 (2007). doi:10.1109/TAC.2007.902731
2. Alessandretti, A., Aguiar, A.P., Jones, C.N.: Trajectory-tracking and path-following controllers for constrained underactuated vehicles using model predictive control. In: 2013 European Control Conference (ECC) July 17–19, 2013, Zurich, Switzerland, pp. 1371–1376 (2013)
3. Alessandretti, A., Aguiar, A.P., Jones, C.N.: Optimization based control for target estimation and tracking via highly observable trajectories. In: Proceedings of 11th Portuguese Conference on Automatic Control, CONTROLO 2014, EPFL-CONF-197746 (2014)
4. Alessandretti, A., Aguiar, A.P., Jones, C.N.: A model predictive control scheme with ultimate bound for economic optimization. In: 2015 American Control Conference (ACC), pp. 1653–1658 (2015). doi:10.1109/ACC.2015.7170970
5. Alessandretti, A., Aguiar, A.P., Jones, C.N.: An input-to-state-stability approach to economic optimization in model predictive control. IEEE Trans. Autom. Control **PP**(99), 1 (2017). doi:10.1109/TAC.2017.2700388

6. Alessandretti, A., Aguiar, A.P., Jones, C.N.: VirtualArena: an object-oriented MATLAB toolkit for control system design and simulation. In: Proceedings of the 2017 International Conference on Unmanned Aircraft Systems (ICUAS), Miami, USA (2017)
7. Cochran, J., Krstic, M.: Nonholonomic source seeking with tuning of angular velocity. IEEE Trans. Autom. Control **54**(4), 717–731 (2009). doi:10.1109/TAC.2009.2014927
8. Curtin, T.B., Bellingham, J.G., Catipovic, J., Webb, D.: Autonomous oceanographic sampling networks. Oceanography **6**(3), 86–94 (1993)
9. Fiorelli, E., Leonard, N.E., Bhatta, P., Paley, D.A., Bachmayer, R., Fratantoni, D.M.: Multi-auv control and adaptive sampling in monterey bay. IEEE J. Oceanic Eng. **31**(4), 935–948 (2006). doi:10.1109/JOE.2006.880429
10. Fontes, F.A., Fontes, D.B., Caldeira, A.C.: Model predictive control of vehicle formations. In: Optimization and Cooperative Control Strategies, pp. 371–384. Springer (2009)
11. Houska, B., Ferreau, H., Diehl, M.: ACADO toolkit - an open source framework for automatic control and dynamic optimization. Optim. Control Appl. Methods **32**(3), 298–312 (2011)
12. Kapoutsis, A.C., Chatzichristofis, S.A., Doitsidis, L., de Sousa, J.B., Pinto, J., Braga, J., Kosmatopoulos, E.B.: Real-time adaptive multi-robot exploration with application to underwater map construction. Auton. Robots **40**(6), 987–1015 (2016)
13. Namaki-Shoushtari, O., Pedro Aguiar, A., Khaki-Sedigh, A.: Target tracking of autonomous robotic vehicles using range-only measurements: a switched logic-based control strategy. Int. J. Robust Nonlinear Control **22**(17), 1983–1998 (2012). doi:10.1002/rnc.1806
14. Pinto, J., Dias, P.S., Sousa, J.B., Pereira, F.L.: Large scale data collection using networks of heterogeneous vehicles and sensors. In: OCEANS 2009-EUROPE, pp. 1–6 (2009). doi:10.1109/OCEANSE.2009.5278239
15. Sousa, L.L., Lpez-Castejn, F., Gilabert, J., Relvas, P., Couto, A., Queiroz, N., Caldas, R., Dias, P.S., Dias, H., Faria, M., Ferreira, F., Ferreira, A.S., Fortuna, J., Gomes, R.J., Loureiro, B., Martins, R., Madureira, L., Neiva, J., Oliveira, M., Pereira, J., Pinto, J., Py, F., Queirs, H., Silva, D., Sujit, P.B., Zolich, A., Johansen, T.A., de Sousa, J.B., Rajan, K.: Integrated monitoring of mola mola behaviour in space and time. PLOS ONE **11**(8), 1–24 (2016). doi:10.1371/journal.pone.0160404