

# Improving the removal of systematics in Kepler data

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## ABSTRACT

TBD

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## 1 INTRODUCTION

During almost 5 years of operations, the Kepler space mission produced continuous, high-precision light curves for over 150 000 stars, with a cadence of 29.4 min. This forms a very rich dataset for a wide range of stellar variability studies. However, the light curves also contain instrumental artefacts and systematic trends. Correcting these while preserving ‘real’ astrophysical variability is challenging, but very important for the community to make the most of the Kepler data.

The publicly available Kepler data products (Thompson et al. 2012) include three versions of the time-series data for each target star. The target pixel files contain flux measurements in each of the individual pixels falling within a pre-defined area around each star. These have been corrected for known instrumental effects at the pixel level, but are otherwise ‘raw’. The light curve files contain two versions of the light curve, dubbed SAP (simple aperture photometry) and PDC (pre-search data conditioning). Jenkins et al. (2010); Fanelli et al. (2011) give an overview of the data processing steps involved in producing both the target pixel files and the light curves. The SAP light curves are obtained by summing the flux falling within a subset of the pixels included in the target pixel files, and applying a correction for background flux (Twicken et al. 010a). The PDC light curves result from additional processing steps designed to remove instrumental artefacts and systematics (Stumpe et al. 012a; Smith et al. 2012). As its name indicates, the PDC pipeline is primarily intended to ready the data for planetary transit searches, and is not specifically optimized to preserve other forms of astrophysical variability. The PDC data are nonetheless widely used for variability studies (for example stellar rotation studies, see e.g. Reinhold et al. 2013; Nielsen et al. 2013; McQuillan et al. 013a,b, 2014), because they are the only widely available set of light curves which cover the full baseline of Kepler observations and are relatively free of the systematics which dominate the SAP light curves. However, there are also known problems: different versions of the

PDC pipeline are, to some degree, prone to over-correction (removal of real astrophysical variability and injection of additional noise, see e.g. Roberts et al. 2013). This motivated us to investigate alternative methods to identify and correct systematic trends, with a specific emphasis on retaining real astrophysical variability.

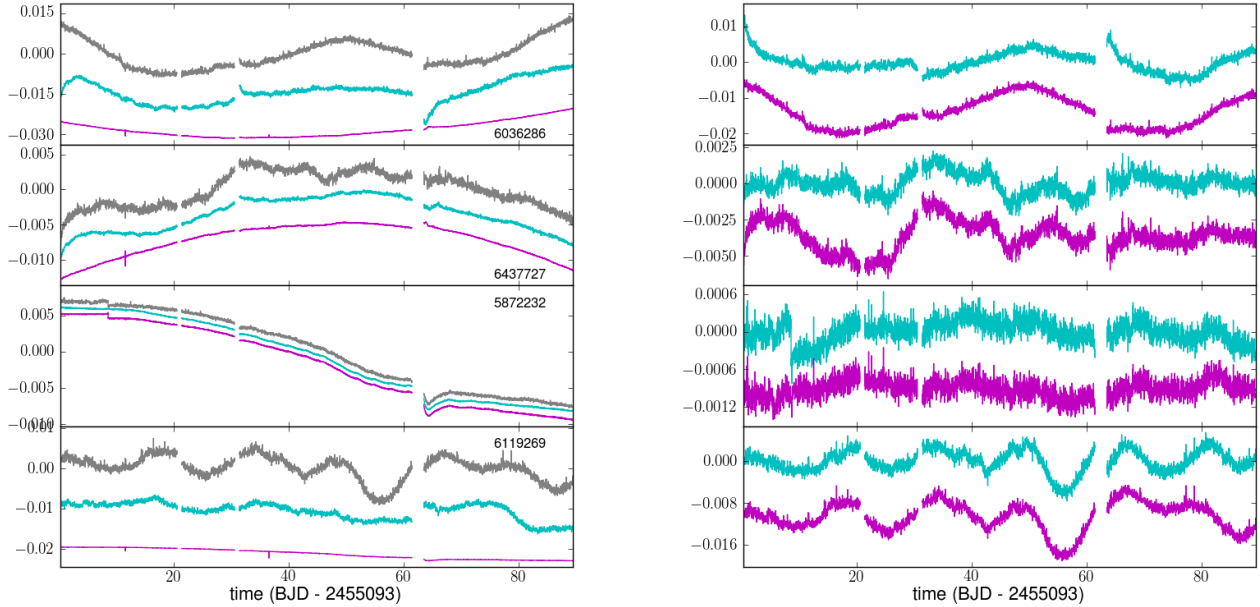
The Kepler detector consists of 21 modules arranged in a  $5 \times 5$  grid with missing corners, each containing two  $4K \times 2K$  CCDs (?). The two halves of each CCD are read out separately, leading to 4 output channels per module. Throughout this paper, we refer to each module plus output channel combination as a modout and identify it as  $X.Y$  where  $X$  is the module number (2 to 24) and  $Y$  the output channel number (1 to 4). For the tests described in this paper, we focussed on 4 modules: 2.1, 7.3, 13.1 and 17.2. These were selected because they are collectively representative of the range of systematic effects which affect Kepler data: 2.1 is located near the upper left corner of the field of view and is particularly sensitive to focus changes, 7.3 is ‘atypically typical’, 13.1 is at the centre of the focal plane, and 17.2 contains some peculiar image artefacts (J. Smith, priv. comm.).

Every three months, the Kepler satellite rolls by  $90^\circ$  about its boresight in order to keep its solar panels pointing towards the Sun. Each 3-month period between rolls is known as a quarter. As each star falls on a different modout in each quarter, the systematics observed vary from quarter to quarter, and their treatment is best carried out separately for each quarter. Each star returns to approximately the same position on the detector every 4 quarters (1 year), so the systematics observed in a given star’s light curve in quarters (say) 3 and 7 tend to be mutually similar. The systematics are easier to correct in some quarters than in others: in this paper we use quarter 3 (Q3) as an example of a relatively well behaved quarter, and quarter 2 (Q2) as an example of a problematic quarter.

### 1.1 Systematics removal in the PDC pipeline

In this subsection, we give a brief overview of the systematics removal methods implemented in the pipeline which

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**Figure 1.** Example light curves before and after systematics correction (random selection from Q6, modout 7.3). The left column shows the SAP light curve for each object in grey, with the corrections applied by the PDC-MAP and by our own pipeline in cyan and magenta, respectively. The right column shows the corrected light curves (PDC-MAP in cyan, this work in magenta). In both columns, arbitrary vertical offsets have been applied to separate the different curves. The PDC-MAP correction is significantly more noisy than the CBV one, particularly in the top three cases. Note also the overcorrection of the intrinsic variability in the bottom case

produces the publicly available PDC light curves. This is a very simplified description, intended only to set the scene for the present paper, and a number of important features have been omitted for the sake of brevity; full details are given in the relevant publications.

The standard approach for systematic trend removal in transit survey data is to model each light curve as a linear combination of systematic trends which are derived either from ancillary engineering and meteorological data (such as telescope pointing, focus, seeing and airmass, see e.g. Bakos et al. 2007) or from the light curves themselves (Tamuz et al. 2005; Kovács et al. 2005). In the latter case, variants of Principal Component Analysis (PCA) are used to construct a reduced basis from the light curves.

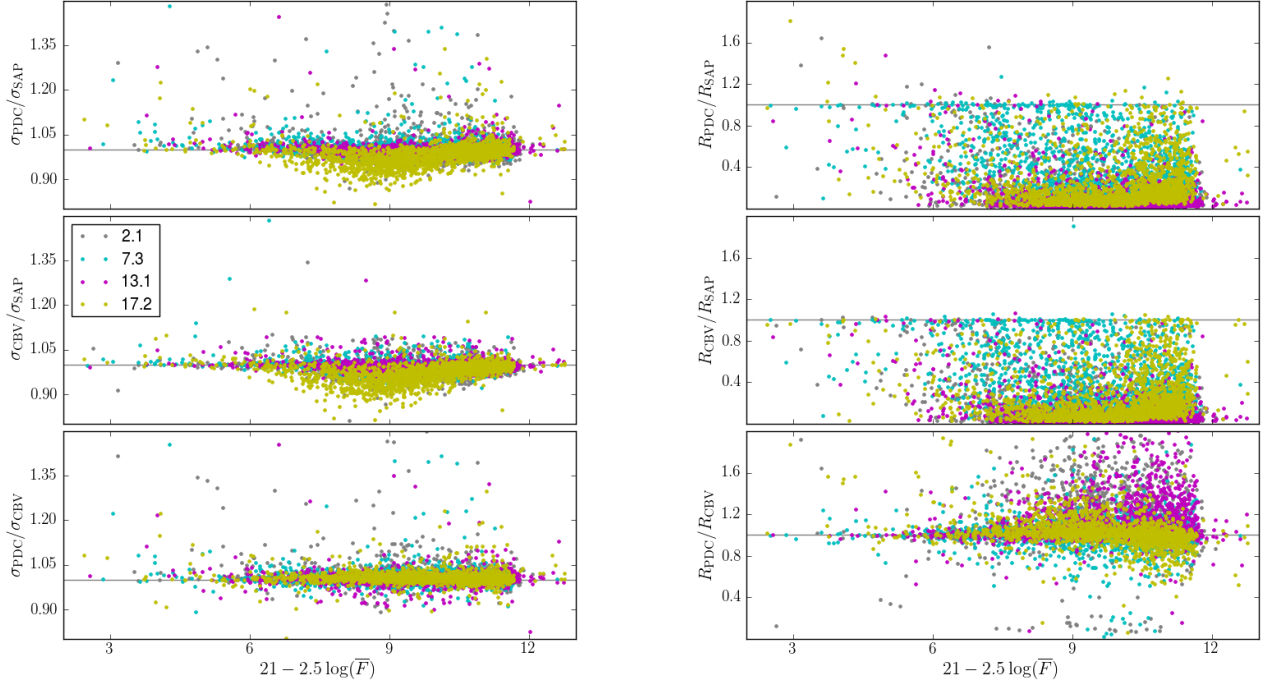
The PDC pipeline broadly follows this paradigm. In its original version (Twicken et al. 010a,b), known as PDC-LS (for least squares), the basis was constructed from ancillary engineering data and the coefficient realting each systematic trend (each term in the basis) to each light curve were derived by least-squares (or maximum likelihood) fitting. However, this method suffered from two problems: overfitting (removal of real astrophysical variability) and injection of noise into the light curves (a side-effect of overfitting combined with noisy basis vectors).

To address these issues, a new version of the PDC was introduced, known as PDC-MAP (for maximum a posteriori, Smith et al. 2012). In the PDC-MAP, the systematic trends (which are known, in Kepler jargon, as cotrending basis vectors, or CBVs), are computed from the light curves themselves, by applying singular value decomposition (SVD) to the 50% of the light curves which show the strongest mutual correlation. This is done separately for each of the 4

output channels in each of the 21 modules composing the Kepler detector. The coefficients linking each CBV to each light curve are then evaluated in a two-step process. First, preliminary estimates are computed using the same least-squares method as in PDC-LS. The resulting coefficients are used to construct prior distributions which are parametrised according to stellar magnitude and position on the sky, and the final coefficients are found by maximising the marginal likelihood for each star, subject to these priors.

The PDC-MAP performs significantly better than the PDC-LS, and it is less susceptible to overfitting and noise injection, although these effects do remain apparent in some of the light curves. Figure ?? shows a few representative examples where the PDC-MAP pipeline clearly introduces noise and/or overcorrects astrophysical variability. This is also illustrated in a statistical manner in the top row of Figure 2, which illustrates the change in the white noise level and light curve amplitude before and after correction by the PDC-MAP pipeline. We measure the white noise level  $\sigma$  as the standard deviation of the first difference of each light curve (throughout this paper, all light curves are assumed to be normalised by dividing them by the median flux value). Using the first difference ensures that any long-term trends are taken out of the equation. The PDC-MAP leaves  $\sigma$  unchanged in most cases and can even reduce it in certain modouts, but there is always a significant number of cases where  $\sigma_{\text{PDC}} \gg \sigma_{\text{SAP}}$ , i.e. PDC-MAP has injected significant amounts of noise into the light curve.

We use the range  $R$ , first introduced by ??, to describe the light curve amplitude.  $R$  is defined as the interval between the 5<sup>th</sup> and the 95<sup>th</sup> percentiles of the normalised flux values. It captures variations on a wide range of time-



**Figure 2.** Statistical comparison of the SAP, PDC-MAP and CBV light curves for Q3. The left column shows the short-term scatter,  $\sigma$  and the right hand column shows the range,  $R$  (see text for definitions). In all cases, the x-axis shows a magnitude-like quantity based on the median flux in each light curve. The different colours correspond to different modouts (see legend in middle left panel). The detailed behaviour of the different modouts is discussed in Section 4.2.

scales, including both systematics and intrinsic variability. Most light curves are initially systematics-dominated, so the PDC-MAP correction should – and does – reduce  $R$  significantly in most cases. However, the PDC-MAP correction *increases*  $R$  for a few bright stars in each modout (Figure 2, top-right panel). This behaviour is not what would be expected if the correction was actually removing systematics.

In addition to the general issues already mentioned, some quarters remain particularly problematic. [illustrate this with a plot of a full light curve, over Q0-Q17, showing bad quarters]

The most recent data releases employ yet another PDC version, PDC-msMAP (for multi-scale MAP, Stumpe et al. 012b), where a band-splitting process is used to separate long, medium and short-term effects, which are treated in different ways. Long-term effects are removed using least-squares (PDC-LS), which results in the near-complete removal of all variability, whether systematic or real, on timescales longer than 20 days. Medium-term effects are treated using the PDC-MAP approach, and short-term effects are not removed at all. The motivation for this is to minimise the disruption of transit signals by the PDC pipeline, but the side effect is that PDC-msMAP data are unsuitable for variability studies. On the other hand, the PDC-MAP CBVs (without band-splitting) continue to be made available together with the SAP and PDC-msMAP light curves.

## 1.2 Alternative systematics removal methods

[Flesh out this section later – mention methods using the target pixel files, and also the methods used in the KASC].

In Roberts et al. (2013, hereafter Paper I), we presented a new method to identify and correct common-mode systematics (trends present, to a greater or lesser extent, in the majority of light curves), which we called the ARC (Astrophysically Robust Correction) method, as it was specifically designed to minimize the risk of removing or altering astrophysical variability along with the instrumental systematics. Like the PDC, the ARC is based on decomposing each light curve into a linear superposition of systematic trends, plus a unique vector representing intrinsic variability and random noise, and like in PDC-MAP the systematic trends are identified from the light curves themselves, for each output channel. The main differences are a) that the ARC uses an information entropy criterion to identify and retain only candidate trends that are genuinely systematic, i.e. composed of small contributions from many light curves b) that the trends are smoothed before they are corrected, to minimise the risk of injecting noise into the corrected light curves, and c) that adaptive, zero-mean (shrinkage) priors were used to evaluate coefficients linking each systematic trend to each light curve. The ARC uses a variational inference framework to carry out Bayesian inference under the linear basis model (see the Appendix of paper I for full details), which is extremely fast: once the basis vectors have been identified, correcting all the light curves takes only a few CPU hours per quarter.

The trend smoothing and the use of shrinkage priors

mean that the ARC should be even less prone to overfitting and noise injection than the PDC-MAP, which is indeed what we found when comparing the two methods on quarter 1 (Q1) data. However, before the ARC can be applied to later quarter, the smoothing step must be revisited: in Paper 1, we used a method called empirical mode decomposition (EMD, [ref]) to decompose each trend into what are essentially distinct oscillatory modes with different numbers of turning points, and retained only the mode with the largest variance. Later quarters contain discontinuities and sharp decays following each  $\sim$  monthly data down-link event, which cannot be captured adequately with EMD. We did investigate – and are still looking into – alternative smoothing methods, in particular modelling each trend with a Gaussian process (GP). If desired, the GP model can be tuned to reflect any a priori knowledge on the characteristics of the systematic effects which are expected to be present (such as time-scale, quasi-periodic nature, etc...). However, so far we did not converge on a completely satisfactory trend-smoothing procedure.

On the other hand, we noted that the first few systematic trends identified by the ARC (before de-noising) and the CBV tend to be mutually quite similar, so that any differences in performance are more likely to result from the trend smoothing and coefficient fitting parts of the process than from the trend identification process. Furthermore, both the first few ARC trends and the first few PDC-MAP CBVs contain some high-frequency structure which appears to be real, in the sense that it is also present in the light curves of bright stars (where the photon noise does not mask these effects). This is visible in the top left panel of Figure ?? for modout 17.2 (yellow symbols). The PDC-MAP actually reduces the white noise in most light curves, particularly those in the middle of the magnitude range. In other words, not all of the short-term variation in the CBVs is noise, which suggests that smoothing the trends may not be optimal.

On the other hand, we have also seen that the PDC-MAP *increases* the white noise and appears to remove astrophysical variability<sup>1</sup> for a small but significant fraction of the light curves. As we have shown in Paper 1, this problem is less prominent for the ARC because of the use of adaptive, shrinkage priors. This suggests a potentially ‘quick and easy’ way to improve the correction of systematic trends in Kepler data using existing resources: by using the existing PDC-MAP CBVs, but the ARC shrinkage prior approach. This is the subject of the present paper.

An additional motivation for developing an independent correction method based on the PDC-MAP CBVs is that it enables us to quantify the effect of the correction on stellar signals using injection tests (where simulated signals are injected into the light curve before correction and our ability to recover and characterise them after correction is evaluated). We cannot do this for the PDC-MAP correction itself, as we do not have access to the pipeline, only the data products.

<sup>1</sup> This is a subjective statement, since we do not know which components of the variability in a given light curve are actually real, and which are systematic, but after visually examining many hundreds of Kepler light curves, one begins to build up a fairly robust prior on what kind of variations are systematic and what kind are not.

The paper is structured as follows. In Section 2 we describe and test a method for identifying and correcting isolated discontinuities in individual light curves. The correction of these so-called sudden pixel sensitivity dropouts (SPSDs) is done as part of the PDC, but the published data products do not provide enough information to separate this from the systematic trend correction, so it was necessary for us to develop our own SPSP correction. In Section 3 we describe our method for applying the PDC-MAP CBVs to the light curves, which we call the CBV correction (to distinguish it from the PDC-MAP correction). In particular we present a simple way of deciding, for each light curve, how many CBVs should be used in the correction. This Section also includes links to the corrected data and the code used to produce them. In Section 4 we evaluate the performance of our method using injection tests, and compare it to the PDC-MAP method. [I may or may not add an additional section illustrating some potential applications]. We discuss the potential applications of this correction and summarise our conclusions in Section 5.

## 2 DETECTION AND CORRECTION OF INDIVIDUAL DISCONTINUITIES

[Hannu to provide text and plots for this section]

## 3 OPTIMIZED USE OF THE CBVS

### 3.1 CBV fitting using Variational Bayes

We fit each light curve using the standard linear basis model:

$$F_j^{(i)} = \sum_{k=1}^K w_k^{(i)} C_{kj} + \epsilon_j^{(i)} \quad (1)$$

where  $F_j^{(i)}$  is the flux measured for star  $i$  in observation  $j$ ,  $C_{kj}$  is the value of the  $k^{\text{th}}$  basis vector (systematic trend, or CBV) in observation  $j$ ,  $w_k^{(i)}$  is the coefficient, or weight, relating basis vector  $k$  to light curve  $i$ , and  $\epsilon_j^{(i)}$  represents the residuals of the correction for star  $i$  in observation  $j$ . In the remainder of this section, we omit the superscript  $^{(i)}$  for simplicity – the analysis is done separately for each light curve. Note that  $\epsilon$  contains both intrinsic variability and noise, but is treated as pure noise for the purposes of the systematics correction.

In a simple least-squares framework (such as PDC-LS), one seeks the set of  $w$ ’s which minimizes the total squared residuals,  $\sum_j \epsilon_j^2$ . If the residuals are assumed to be drawn independently from a Gaussian distribution with known precision (inverse variance)  $\beta$  this is equivalent to maximising the likelihood of the model:

$$p(\mathbf{F}|\mathbf{w}, \mathbf{C}) = \mathcal{N}(\boldsymbol{\epsilon}; 0, \beta^{-1}\mathbf{I}). \quad (2)$$

(Note that strict equivalence only holds if  $\beta$  is known, if it is a free parameter then the full likelihood expression must be used.)

Maximum likelihood linear basis models are notoriously prone to overfitting. Better results can be obtained in a Bayesian framework, by using priors over the  $w$ ’s to encapsulate any external information available over the expected

values for the weights, and maximise the posterior distribution instead of the likelihood.

$$p(\mathbf{w}|\mathbf{F}, \mathbf{C}) = \frac{p(\mathbf{F}|\mathbf{w}, \mathbf{C}) p(\mathbf{w})}{\int p(\mathbf{F}|\mathbf{w}, \mathbf{C}) p(\mathbf{w}) d\mathbf{w}}, \quad (3)$$

where the normalisation constant in the denominator is the model evidence  $p(\mathbf{F}|\mathbf{C})$ . In the PDC-MAP, the priors over the  $w$ 's are based on the distribution of the coefficients derived in the maximum likelihood case (i.e. in the absence of priors), parametrised as a function of star position and magnitude. This reflects the belief that stars which are near each other on the detector and have similar brightnesses should also display similar systematics. As we have seen, it does reduce the overfitting problems which had been noted in PDC-LS, but does not entirely do away with them. One plausible explanation for this is that the PDC-MAP priors themselves are affected by overfitting in the initial, maximum likelihood step.

To reduce the risk of overfitting further, we propose to use priors which specifically penalise larger weights, and make it less likely that one basis vector will compensate for another. A natural choice for this is to use zero-mean Gaussian priors:

$$p(w_j|\alpha_j) = \frac{\alpha_j}{\sqrt{2\pi}} \exp(-\alpha_j w_j^2/2) \quad (4)$$

for each  $j$  (the individual prior weights are treated as mutually independent). Furthermore, we do not fix the priors, but instead treat the inverse variances,  $\alpha$ , as parameters themselves, subject to their own prior  $p(\alpha)$ , for which we use a Gamma distribution<sup>2</sup>. Unless there is strong evidence for a non-zero weight for particular light curve / basis vector combination, the Gamma prior over  $\alpha$  will tend to make the prior over  $w$  collapse to a delta function centred on zero, so most basis vectors will have zero weight in most light curves. This is sometimes referred as automatic relevance determination (ARD) or shrinkage.

We then seek to evaluate the posterior distribution over the weights  $\mathbf{w}$ , marginalised over the prior precisions  $\alpha$ :

$$p(\mathbf{w}|\mathbf{F}, \mathbf{C}) \propto \int p(\mathbf{F}|\mathbf{w}, \mathbf{C}) p(\mathbf{w}|\alpha) p(\alpha) d\alpha, \quad (5)$$

and to maximise it with respect to  $\mathbf{w}$ . In general, the posterior distribution is unknown and is not analytically tractable. Numerically evaluating and optimizing the posterior would require evaluating the likelihood over a very large number of  $(\mathbf{w}, \alpha)$  combinations, which is unfeasible, especially as it needs to be done for every Kepler light curve.

An elegant workaround consists in approximating the posterior with a proposal distribution which is analytically tractable, and iteratively refining the latter so that it approaches the true posterior. This class of methods is known as approximate inference. More specifically, one can restrict oneself to proposal distributions which belong to the exponential family: refining the proposal then consists in optimizing an integral with respect to a functional, which is typically done using the calculus of variations. This approach is thus

known as variational inference, or variational Bayes (VB). A detailed description of the VB method as applied to our linear basis model was given in the appendix of Paper I, so we do not repeat it here. The algorithm essentially consists in cycling through a set of update equations for the weights  $\mathbf{w}$ , the prior precisions  $\alpha$  and the noise precision  $\beta$ . Importantly, convergence is guaranteed, and typically occurs after just a few iterations. The computational requirement scales as  $JK^2$ , where  $J$  is the number of observations and  $K$  is the number of basis vectors. Our MATLAB batch implementation of the method runs in  $< 3$  minutes on [describe CPU] per output channel (1500–2000 stars) and per quarter ( $\sim 4300$  observations) for up to 8 CBVs.

### 3.2 How many CBVs?

Despite the measures described in the previous section to minimize the risk of overfitting, the results still depend, in some cases, on the number of basis vectors used. The PDC-MAP CBVs result from a singular value decomposition of a subset of the light curves, and therefore the first CBV represents a larger fraction of the overall variance of the light curves, and so on... The PDC-MAP pipeline saves 16 CBVs but only uses at most 8 to perform the correction. Additionally, a signal-to-noise ratio (SNR) criterion is used to exclude CBVs which are deemed to contribute more noise than useful information (see Smith et al. 2012 for details), so the actual number of CBVs used ranges from 5 to 8.

As the VB method is very fast, it is feasible to run it for every possible number of CBVs,  $K$  (from 1 to 16). We first did this for a few representative quarters (3 to 6) and output channels, and performed a visual comparison of the results on a random selection of light curves.

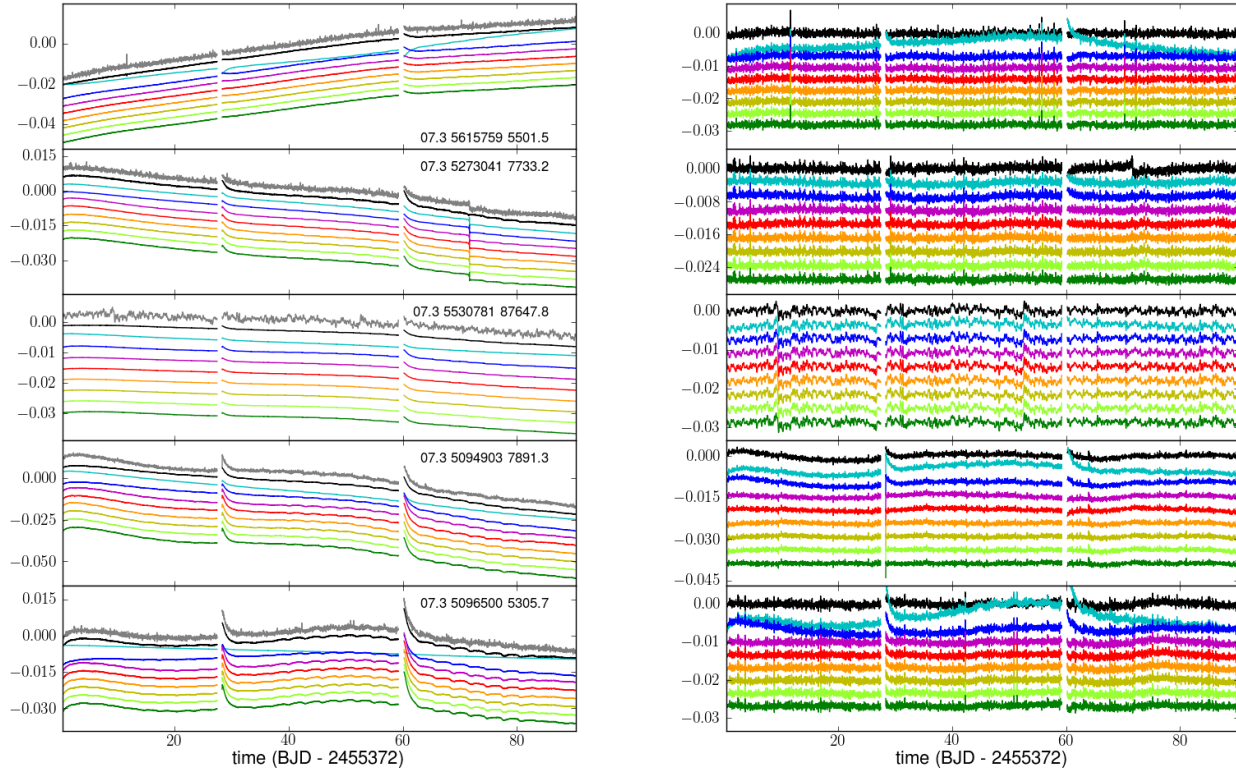
We plotted the original (SAP) light curve, the correction applied and the corrected light curve, for a few tens of stars selected at random in each modout. A few examples are shown in Figure 3. In most cases, the overall shape of the correction is fairly insensitive to the number of CBVs used, so long as it is at least 2 or 3. On the other hand, as the CBVs are increasingly noisy, using more CBVs introduces more noise into the light curves particularly for the brighter stars. It is therefore important to use the smallest number of CBVs which gives an adequate correction. Importantly, the examples we examined suggest that this number differs from star to star: in some cases, including the 4<sup>th</sup> and 5<sup>th</sup> CBVs removes features which appear systematic in nature, and which were not removed when using only the first two. In other cases, even using the 2<sup>nd</sup> CBV appears to have a detrimental effect.

Rather than specifying an overall value for  $K$  for – say – each quarter and modout, we therefore decided to select the optimal number of CBVs to use a posteriori, on a light curve by light curve basis, based on a statistical comparison of the light curve properties before and after correction, using the simple statistics  $R$  and  $\sigma$ .

If most light curves are initially dominated by systematics, the dependence of  $R$  on  $K$  tracks the extent to which the correction of systematics is improved by adding more CBVs. Typically,  $R$  decreases rapidly for low  $K$  but then reaches a plateau for larger  $K$ . Within this plateau regime, increasing  $K$  beyond does not usually improve the correction significantly but it does introduce more noise, so one might

<sup>2</sup> Our choice of priors over  $\mathbf{w}$  and  $\alpha$  is also mathematically convenient, because they are conjugate with each other and with the likelihood (which is Gaussian), enabling a number of the integrals involved in the inference to be performed analytically.





**Figure 3.** Comparison of the systematics correction using 1 to 8 CBVs, for a random selection of stars in Q3, modout 17.3. The left panel shows the original (SAP) light curve in grey and the corrections applied using 1 to 8 CBVs in different colours (cyan to green, and top to bottom). The PDC-MAP correction is also shown in black, for comparison. The right panel shows the corrected light curves, using the same colour-coding. In both panels, the different light curve versions have been offset by an arbitrary amount, for clarity. [Will find better, more illustrative examples when I get the chance]

simply opt for the lowest value of  $R$  at which the plateau is reached. We initially define  $K_{\text{opt}}$  as the smallest value of  $K$  for which  $R(K) < \langle R \rangle + 3\sigma_R$ , where  $\langle R \rangle$  and  $\sigma_R$  are the median and standard deviation of  $R$  over all values of  $K$ , for a given light curve.

However, when the initial amplitude of systematics is small relative to that of the intrinsic variability, this criterion alone is insufficient:  $R$  can decrease with increasing  $K$  not because more systematics are being removed, but because real variations are actually being removed. This is particularly frequent for bright, variable stars. One way of testing for this would be to inject realistic simulated variability signals into the light curve before correction, and check how well they are recovered post-correction. However, doing this for every light curve would be prohibitively expensive. We do use such injection tests can be used to evaluate the overall ability of our correction method to preserve astrophysical signals, but only on a subset of the data (see Section 4.1).

Fortunately, there is an easier way to identify these problematic cases: visual inspection shows that, when the correction removes what looks like real variability, it also introduces significant amounts of high frequency noise into the light curves. This can be diagnosed by tracking the dependence of  $\sigma$  on  $K$ . Specifically, if  $\sigma(K_{\text{opt}})/\sigma_0 > 1.1$ , where  $K_{\text{opt}}$  is determined from the behaviour of  $R$  as described above, and  $\sigma_0$  is the short-term scatter before correction,

$K_{\text{opt}}$  is decreased further until the scatter ratio falls below the 1.1 threshold. The choice of threshold is somewhat arbitrary, but its exact value is not critical: when it is exceeded it is usually by a fairly wide margin.

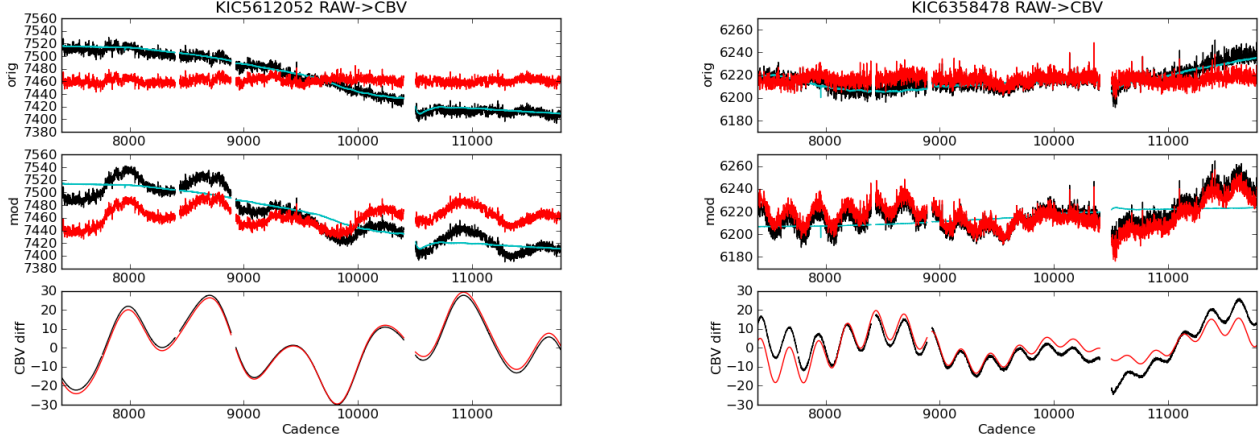
[include a figure illustrating the selection of  $K_{\text{opt}}$  for a straight-forward case, and one where the  $\sigma$  threshold had to be used.]

Having defined a method for selecting  $K_{\text{opt}}$ , we processed all the Kepler data publicly available at the time of writing, i.e. quarters 0 to 17. The resulting corrected light curves are publicly available at [include URL], along with the code used to produce them. There are two versions of the code: a batch version, written in MATLAB, which processes entire modouts, and a single-light curve version, written in PYTHON, which works directly on the FITS light curve and CBV files produced by the Kepler pipeline. In the next Section, we test the performance of the CBV correction using injection tests, and compare it to the PDC-MAP results.

## 4 PERFORMANCE TESTS

### 4.1 Injection tests

The method we use to select the number of CBVs used, as described in the previous Section, is intended to remove systematics as effectively as possible while minimizing the risk



**Figure 4.** Injection test examples (Q3, modout 17.2). Top: original and corrected light curve (black and red, respectively) with the correction shown in cyan. Middle: same after injecting the simulated signal. Bottom: injected and recovered signals (red and black, respectively).

of overfitting, i.e. removing astrophysical variability, and of introducing extra noise into the light curves due to the noisy nature of the CBVs themselves. To test the extent to which our systematics removal method affects stellar variability, we now perform a series of injection tests. Specifically, we are interested in variability caused by rotational modulation of surface inhomogeneities such as star-spots, since this is a powerful diagnostic of stellar rotation rates and hence angular momentum evolution.

We simulate rotation-like signals, consisting of between 1 and 5 coadded sinusoidal variations, with periods randomly drawn from a log uniform distribution ranging from 5 to 60 days, and amplitudes drawn from a log normal distribution with mean  $10^{-3}$  and standard deviation 0.5 dex. These were added to 200 randomly select light curves in each modout, which were then corrected for systematics as described in the previous section. The difference between the corrected light curves with and without injected signal, hereafter referred to as the *recovered* signal, is then compared to the injected signal itself: any discrepancies arise because the correction is affecting the injected signal. Figure ?? illustrates this process for a few example light curves. Of course, the light curves into which the simulated signals are injected already contain astrophysical variability, which themselves may have been affected by the correction, and this may contribute to the differences between the injected and recovered signals. However, we opted for this approach rather than attempting to generate light curves with simulated stellar signals *and* systematics, because we do not have a good generative model for the latter.

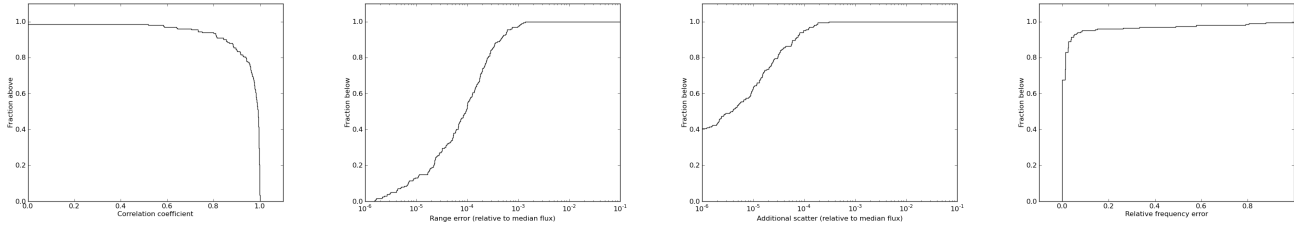
We quantify the effect of the correction on the simulated signals by measuring the Pearson correlation coefficient  $P$  between the injected and recovered signal, and by comparing the range  $R$ , scatter  $\sigma$  (as defined earlier) and frequency  $F$  of the injected and recovered signals. The results are shown for a representative modout in Figure ?.  $F$  was measured by simple least squares fitting of a sinusoid over a grid of equally spaced trial frequencies. These plots show that the injected signals are remarkably well preserved:  $P$  is  $> 0.9$  and the error in  $R$  is  $< 500$  ppm 83% of the time,

while the added scatter is  $< 100$  ppm and the relative frequency error  $< 0.05$  92% of the time. These plots shown are for modout 17.2 in Q3, but the same tests were performed for all 4 of the test modouts in Q3–Q6 inclusive, and gave similar results. We also checked for any correlation between the metrics discussed above and the amplitude or frequencies of the injected signal compared to the range, scatter and frequency of the original light curve after correction, but did not find any significant trends.

## 4.2 Comparison to PDC-MAP

Figures 1 and 2 show a comparison of the PDC-MAP and CBV corrections using illustrative examples and quantitative statistics, respectively. The most important feature, in both cases, is that the corrected light curves and associated statistics are generally similar. This is not altogether surprising, since both corrections are based on the same set of basis vectors, but it is reassuring: it indicates that the different choices of priors do not usually have too strong an effect on the results. Nonetheless, for a subset of the cases, the PDC-MAP correction is significantly more noisy than the CBV one, as illustrated by the left panel panel of Figure 1 (particularly rows 1, 2 and 4) and the bottom left panel of Figure 2. The number of cases where the scatter is significantly increased after correction is also significantly lower in the CBV case than in the PDC-MAP one. In quantitative terms, the median value of  $\sigma_{\text{PDC}}/\sigma_{\text{CBV}}$  was 1.004 for the 4 modouts considered in Q3, but the number of cases where  $\sigma_{\text{PDC}} > 1.1\sigma_{\text{CBV}}$  was 115 (out of 7935), whereas the number of cases where  $\sigma_{\text{PDC}} < 0.9\sigma_{\text{CBV}}$  was only 19.

As an aside, we note that comparing  $\sigma$  before and after correction can be used as a useful diagnostic of problematic light curves: when the CBV correction increases  $\sigma$  by more than 10%, visual examination of the light curves often reveals abnormal behaviour, apparently associated with image artefacts, stars located near the edge of a CCD, or very high proper motion stars (where the fraction of the flux collected within the photometric aperture might change significantly during a quarter). [Need to include a figure illustrating some



**Figure 5.** Cumulative distributions of the Pearson correlation coefficient between injected and simulated signals (left), the range error (difference between ranges of injected and recovered signal, middle left), the scatter injected by the correction (scatter of recovered signal minus scatter of injected signal, middle right) and the relative frequency error (difference between the measured frequencies in the injected and recovered signals, divided by the former, right).

of these]. We therefore flag these light curves so that they can be identified readily.

We now examine the behaviour of the range parameter  $R$  (right hand-panel of Figure 2). The CBV correction never increases  $R$ . This is so by design: the only circumstance in which removing systematics should increase  $R$  is one where the systematics had somehow ‘conspired’ to ‘cancel out’ some of the real variability, which is unlikely. Aside from that, the before and after range comparison looks similar for the CBV and the PDC-MAP correction. Direct comparison between the two shows broad similarity (the median value of  $R_{\text{PDC}}/R_{\text{CBV}}$  is 1.020 for the 4 modouts considered in Q3) but with significant modout-to-modout differences.

[Include discussion of individual modout behaviour]

## 5 DISCUSSION AND CONCLUSIONS

This paper has been typeset from a  $\text{\LaTeX}$  file prepared by the author.

## REFERENCES

- Bakos G. Á., Kovács G., Torres G., Fischer D. A., Latham D. W., Noyes R. W., Sasselov D. D., Mazeh T., Shporer A., Butler R. P., Stefanik R. P., Fernández J. M., et al. 2007, *ApJ*, 670, 826
- Fanelli M., Jenkins J., Haas M., Gautier T., , 2011, *Kepler Data Processing Handbook*
- Jenkins J. M., Caldwell D. A., Chandrasekaran H., Twicken J. D., Bryson S. T., Quintana E. V., Clarke B. D., Li J., Allen C., Tenenbaum P., Wu H., Klaus T. C., Middour C. K., et al. 2010, *ApJL*, 713, L87
- Kovács G., Bakos G., Noyes R. W., 2005, *MNRAS*, 356, 557
- McQuillan A., Aigrain S., Mazeh T., 2013a, *MNRAS*, 432, 1203
- McQuillan A., Mazeh T., Aigrain S., 2013b, *ApJL*, 775, L11
- McQuillan A., Mazeh T., Aigrain S., 2014, *ApJ*, in press
- Nielsen M. B., Gizon L., Schunker H., Karoff C., 2013, *A&A*, 557, L10
- Reinhold T., Reiners A., Basri G., 2013, *A&A*, 560, A4
- Roberts S., McQuillan A., Reece S., Aigrain S., 2013, *MNRAS*, 435, 3639

- Smith J. C., Stumpe M. C., Van Cleve J. E., Jenkins J. M., Barclay T. S., Fanelli M. N., Girouard F. R., Kolodziejczak J. J., McCauliff S. D., Morris R. L., Twicken J. D., 2012, *PASP*, 124, 1000
- Stumpe M. C., Smith J. C., Van Cleve J., Jenkins J. M., Barclay T. S., Fanelli M. N., Girouard F., Kolodziejczak J., McCauliff S., Morris R. L., Twicken J. D., 2012b, in *American Astronomical Society Meeting Abstracts #220 Vol. 220 of American Astronomical Society Meeting Abstracts, Multiscale Systematic Error Correction via Wavelet-Based Band Splitting and Bayesian Error Modeling in Kepler Light Curves*. p. 330.04
- Stumpe M. C., Smith J. C., Van Cleve J. E., Twicken J. D., Barclay T. S., Fanelli M. N., Girouard F. R., Jenkins J. M., Kolodziejczak J. J., McCauliff S. D., Morris R. L., 2012a, *PASP*, 124, 985
- Tamuz O., Mazeh T., Zucker S., 2005, *MNRAS*, 356, 1466
- Thompson S., Fraquelli D., Nevitt R., Chow C., Gilliland R., Haas M., , 2012, *Kepler Archive Manual*
- Twicken J. D., Chandrasekaran H., Jenkins J. M., Gunter J. P., Girouard F., Klaus T. C., 2010a, in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series Vol. 7740 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Presearch data conditioning in the Kepler Science Operations Center pipeline*
- Twicken J. D., Clarke B. D., Bryson S. T., Tenenbaum P., Wu H., Jenkins J. M., Girouard F., Klaus T. C., 2010b, in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series Vol. 7740 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Photometric analysis in the Kepler Science Operations Center pipeline*