Modeling Queues using Poisson Approximation in IEEE 802.11 Ad-Hoc Networks

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Abstract— **In this paper, we demonstrate that probability distribution of service time intervals of MAC queues in 802.11 ad-hoc networks take a simple exponential form when effects of hidden nodes dominate and when neighborhood "menace" increase in the network. We apply Chen-Stein Poisson approximation method to the superposition of several neighborhood service processes. Using the Chen-Stein bounds on distribution distances, we propose a general solution for service interval distribution and prove that the distribution exhibits Poisson properties. Under saturated conditions, the approximation converges to M/M/1/K discipline, thus greatly reducing the complexity of steady-state MAC queueing analysis in 802.11 ad-hoc networks. Further, we show that inter-arrival distribution of next-hop nodes in a multihop network also take an exponential form owing to the reversibility nature of birth-death service process of previous queue. We validate our analytical model using *ns*2-based simulations of randomly generated topology. Through simulations, we observe that the service intervals are exponentially distributed and the convergence to exponential form occurs as the number of neighbors increase. We believe that our Poisson approximation to service interval distribution is a significant observation established through stochastic superposition of steady-state queues, and it gives a fresh insight into steady-state queueing behavior of 802.11 ad-hoc networks.**

Index Terms – **802.11 MAC, Ad-Hoc networks, Queueing. Poisson distributions, Chen-Steiner method.**

I. Introduction

We are experiencing a revolutionary growth in wireless broadband access networks as we witness an unsurpassed development to provide better end-user experience such as high throughput and seamless mobility. The Wi-Fi alliance and the IEEE 802.11 Task Group n (802.11 Tn) are redefining the IEEE 802.11 Basic Service Set (BSS) to quadruple the physical transfer capacity and to improve the channel efficiency multifold. Simultaneously, the WLAN industry is keen on engineering enterprise-level infrastructure based on this redefined technology (SISO). Their interests are to enable seamless mobility and voice connectivity over WLAN to provide better end-user experience [1].

The technological development towards higher physical rates and improved channel is exciting; however, the importance of modeling such dynamic systems analytically cannot be overstated. Until recently, there has been a growing interest in performance modeling of 802.11 MAC DCF as studied by recent literatures [3], [6]. These models have shown that there exist certain limits and bounds under which the network operates at its best. A similar treatise on 802.11 MAC queueing models is inevitable and essential. Modeling MAC queues is an essential dimension to understand the fine-granular frame-level performance of 802.11 ad-hoc networks. There are however, only a handful of literatures that study the queueing models of 802.11 ad-hoc MAC until last year [4], [5], [6]. Our paper fills this gap.

In this paper, we study the service interval distribution of 802.11 MAC layer that uses DCF basic access CSMA/CA technique. We apply Chen-Stein Poisson approximation method to the superposition of large number of neighborhood MAC service processes. The approximation is possible under the dominant effect of hidden terminals and increased neighborhood "menace". Neighborhood "menace" is caused due to a large number of communicating neighbors contending for the same transmission slot. This increase in the number of communicating neighbors effectively decreases the packet transmission probability significantly because of exponential DCF backoff [3]. In this paper, we prove that several such neighborhood backoff processes can superpose to form an approximation distribution that takes an exponential form of service intervals at every MAC queue. In doing so, we develop the approximation distribution using Chen-Stein method known in the probability community for several years [11], [12]. Chen-Stein method is also widely used in computational biology to model the genetic sequences that occur with unexpected frequencies in the analysis of long DNA sequences. We use the method to identify the existence of Poisson service distribution. Using the Chen-Stein bounds on distribution distances, we propose a general solution for MAC service interval distribution and prove that it exhibits Poisson properties. Under network saturation conditions, the model converges to the well-established M/M/1/K queueing discipline, thus greatly simplifying further analysis of 802.11 MAC queues.

Literatures study three types of MAC queueing disciplines in 802.11 ad-hoc networks – M/G/1 [6], M/MMGI/1/K [5] and G/G/1 [4]. Zhai *et al*. [6] consider the MAC layer service distribution using packet collisions, backoff decrement and successful packet transmission processes. They compare the service time distribution with standard distributions and conclude that log-normal gives a good approximation for high and low collision probabilities and exponential distribution gives a good approximation in the case of high collision probability. We independently observe that exponential service interval distribution gives the best approximation possible under high MAC contention and due to large neighborhood presence. In this paper, we establish this result formally through sound analytical and simulation methods.

Tickoo *et al*. [4] model ad-hoc MAC queues as discrete G/G/1 discipline and derive the service distribution under non-saturated conditions. They use spectral analysis to derive a general solution to MAC service time, which is a function of delay due to other stations transmitting and variable packet size. Özdemir *et al*. [5] develop a M/MMGI/1/K discipline in which the service distribution is derived from the probability of number of busy nodes contending for the same slot. Using the second moment of service distribution, they obtain bounds on performance parameters, but the study offers no insight on the form of distribution itself. Furthermore, all the above models assume Poisson arrivals but do not seek to explain as to why the arrivals should take Poisson form in a series of MAC queues.

The structure of our paper is as follows. In section II, we develop an analytical foundation of 802.11 Distributed Coordinated Function (DCF) basic-access scheme taking contention probabilities due to hidden nodes and due to neighbors into account. In section III, we give a background of Chen-Stein Poisson approximation method. Using the Chen-Stein bound on distribution distance, we prove that superposition service process exhibits Poisson properties. In section IV, we validate our analytical model using *ns*2-based simulations of randomly generated topology.

II. 802.11 Distributed Coordinated Function

IEEE 802.11 Media Access Control (MAC) layer specification [2] describes Distributed Coordinated Function (DCF) as a contention-based CSMA/CA access mechanism for shared wireless channel. At 802.11 physical layer (PHY), channels are divided into multiple discrete “time slots”, that is the least time needed for any station to detect a transmission or transmit a MAC frame. There is a propagation delay associated with each bit transmission and it is the time needed to switch from receiving to transmitting state (in the order of few microseconds).

Several literatures provide a timing diagram representation of DCF basic access and RTS/CTS schemes (see Bianchi [3], for example). In this section, we provide an alternative state transition diagram of various states and events possible in DCF. In Fig. 1, the circles represent the possible states that the DCF protocol undergoes and the arrows represent probabilistic events.

DCF defines two types of handshaking techniques for MAC transmissions – two-way and four-way handshaking. Both the techniques verify the current state of channel whenever it receives a packet to transmit from higher layers, and remains in this state until it detects an idle slot. When the channel is sensed idle for Distributed Interframe Space (DIFS+) period, the protocol initiates a first backoff. Both DCF techniques use random exponential backoff scheme with backoff time uniformly chosen in the range of $(0, w-1)$, where w is the MAC channel contention window that takes integer values in the range $(CW_{\min}, CW_{\max})$. This is represented by the state transition $2 \rightarrow 4$ in Fig. 1. In DCF basic access scheme, whenever the backoff counter becomes zero, data is transmitted successfully provided no collisions occur. If collision takes place, the frame is scheduled for retransmission, thus undergoing the backoff process over again. These are seen by state transitions $4 \rightarrow 5$ and $4 \rightarrow 3 \rightarrow 2 \rightarrow 4$ respectively. As soon as the data frames are received by the destination, an ACK is sent back within Short Interframe Spacing (SIFS) slot, thus preventing ACK collision with other data frames.

The four-way RTS/CTS handshake procedure is same as described above except that the sending station transmits a special request-to-send (RTS) signal and awaits clear-to-send (CTS) from the receiving station before new data transmission can begin. This effectively indicates channel reservation until all data and ACK frames are exchanged collision-free.

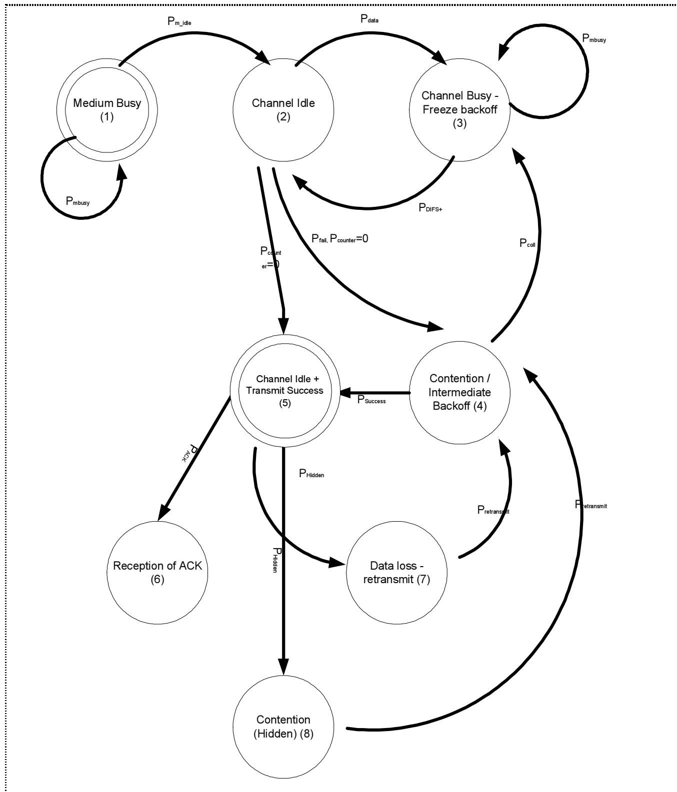

Fig. 1. General State Transition Diagram of 802.11 MAC DCF

A. System Description

Our system consists of 802.11 mobile ad-hoc multihop network with a total of N nodes. Nodes are randomly distributed initially on a two-dimensional topography of area A. Ad-hoc nodes are set to constant motion in random direction to a randomly chosen destination point on the topography. Nodes use standard communication protocols to communicate with each other. Further, nodes that do not have a direct connectivity use ad-hoc routing protocols (such as DSDV) to forward packets along the path.

In this paper, we make extensive use of the model and representations developed by Bianchi [3], but with few changes. We explicitly model hidden nodes in our collision probability. Our model differs in that the *conditional collision probability* p is different for MAC data and ACK frames. In our model, p refers to the conditional collision probability of data frames only. This is because the ACK frames are dependent on the successful transmission of previous data frames and hence it violates the constant and independent transmission probability assumed for modeling. However, we do not explicitly model the collision of ACK frames themselves in this paper.

Finally, we consider the network in saturation state at all times. That is, the MAC queue always has at least one packet to service, either from higher layers or the packets that needs be forwarded to other nodes in a multihop environment.

B. Effects Due to Hidden Terminals

A set of k_1 neighbors (with respect to a given node) that are not directly communicating (non-interfering) with a set of k_2 neighbors, such that, transmission between nodes in either set during the same transmission slot results in a hidden terminal contention [10].

The detrimental effects of hidden terminals due to transmission interference are well studied in [7], [8], [9], and [10]. In [9], [10], the authors observe severe MAC performance degradation as the number of hidden node presence increase. Other literatures [7], [8] survey the interference effects due to hidden terminals in multi-rate ad-hoc networks and conclude that the presence of hidden nodes effectively shorten the successful transmission range.

In our model, we consider a homogeneous multihop ad-hoc network in which the transmission radius R (a circular cell) is finite and same for all the nodes. We assume that sum of cardinalities of hidden terminal sets k_1 and k_2 remain a constant k. That is, $|k_1| + |k_2| = k \le N_i$, where N_i is the total neighbors of node i. This assumption simplifies the calculation of hidden terminal probability as seen below. Note that not all neighbors of node i are a part of hidden terminal set k_1 or k_2.

In this paper, we represent the probability of existence of hidden nodes at node i by P_h^i. For a given pair of neighbors of node i, we define *hidden terminal probability* P_h^i as the probability of existence of two nodes within the transmission radius R of node i and the probability that the neighbors do not interfere (out of their transmission ranges). If the distance between nodes A and B is D, then the probability of existence of a hidden node H within the transmission range of node A but out of range of B is given by:

$$P(H_x = x) = \int_R^x \frac{2y}{D^2 - R^2} dy , \tag{1}$$

where H_x is the distance of hidden node H from A [7]. In our model, we assume that hidden nodes are uniformly distributed in the range $H_x \in (0, 2R)$. Using the above definition, the overall probability of existence of hidden terminal for node i is given by

$$\begin{aligned} P_h^i &= P\{k_1 \text{ out of } N_i \le 2R\} + \\ & P\{k_2 \text{ out of } N_i \le 2R\} \times \prod_{j=1}^{j=|k_1|} \prod_{m=1}^{m=|k_2|} P\{D(j,m) \ge 2R\} \end{aligned} .$$

Simplifying, this yields

$$P_h^i = C' \frac{k}{N_i} \left(\frac{1}{2R} \right) |k_1| \, |k_2| \left(1 - \frac{1}{2R} \right), \tag{2}$$

where C' is a normalization constant. In a given topography, expression (2) shows that the effect of hidden terminals as the number of neighborhood nodes increase is roughly parabolic.

C. Markov Model for 802.11 DCF

Consider the stochastic backoff counter process $b(t)$ that counts the backoff time at the beginning of each slot of a node i at time t. We define backoff stage $s(t)$ of a node i at time t as the current transmission attempt of total retransmissions allowed. Bianchi [3] shows that the stochastic process pair $\{s(t), b(t)\}$ forms a discrete Markov chain so long as each MAC frame collides with constant and independent probability. In this paper, we extensively use the model and representations developed in [3], but we also take the contention due to hidden terminals into account.

A critical assumption by Bianchi [3] is the constant and independent packet collision probability at each transmission slot. That is, collision probability at ad-hoc nodes is neither dependent on the content of data frames nor on the number of retransmissions suffered by the frame at the head of MAC queue. In the first case, the dependency of data generated (and leading) to another transmission (such as a new TCP segment after TCP ACK) is eliminated because we consider the MAC queues in saturated state at all times. That is, there is always at least one packet to transmit. In the second case, frames that suffer retransmissions are not any different from new data frames waiting for transmission for the first time and hence, they undergo the states transitions 2, 3, 4 and 5 as in Fig. 1. Together, these two conditions make the MAC transmission event, within a node and between nodes an independent event – a necessary property for Markov chain.

Using a two-dimensional discrete Markov chain developed in [3], the one-step transition probabilities at each node i is given by

$$\begin{cases} P\{i,k \mid i,k+1\} = 1 & k \in (0, W_i - 2),\ i \in (0,m) \\ P\{0,k \mid i,0\} = (1-p)/W_0 & k \in (0, W_0 - 1),\ i \in (0,m) \\ P\{i,k \mid i-1,0\} = p'/W_i & k \in (0, W_i - 1),\ i \in (1,m) \\ P\{m,k \mid m,0\} = p'/W_m & k \in (0, W_m - 1) \end{cases} , \tag{3}$$

where, $p' = p + P_h^i$ is the hidden terminal probability explicitly taken into consideration for node i. Note that p' is applicable only in the third and fourth expressions as they represent unsuccessful transmission due to contention with hidden terminals as well as due to neighboring nodes. Imposing the normalization conditions and simplifying (3), we get the following for steady-state probability of initial state $b_{0,0}$.

$$b_{0,0} = \frac{2(1-2p')(1-p)}{(1-2p')(W+1) + pW(1-(2p')^m)} \tag{4}$$

The steady-state probability τ that MAC transmits in a given transmission slot with contention due to neighbors and hidden nodes is derived from expression (4) and is given by

$$\begin{aligned} \tau &= \sum_{i=0}^{k} b_{i,0} = \frac{b_{0,0}}{(1-p)} \\ \tau &= \frac{2(1-2p')}{(1-2p')(W+1) + pW(1-(2p')^m)} \end{aligned} . \tag{5}$$

III. Analytical Modeling of 802.11 Queues

In this section, we develop an analytical model of 802.11 MAC queues in steady-state condition by making use of the DCF transmission and backoff probabilities derived in section II.C. One of the reasons that existing literatures [4], [5], [6] struggle to analytically model MAC queues is due to the lack of a known distribution of service and arrival disciplines. We observe the following from the current literature.

- Studies on MAC queues [4], [5], [6] typically assume Poisson arrivals for simplicity and tractability. A natural question arises as to whether their models function as claimed in high contention conditions and large neighborhood presence [5], [6] with Poisson arrivals.
- Modeling using general service distribution requires stochastic spectral analysis through complex Lindley's integrals. The result obtained is only a bound on queue parameters such as queue size and delays, as shown by Tickoo *et al*. in [4]. The procedure to analyze series of queues using general distribution is complex and not well-understood.
- Modeling MAC service distribution is key to understand inter-arrival distribution and hence the MAC network of queues. Multihop analysis of ad-hoc networks is possible only when series of queues is well understood.

We are motivated to study the existence of an exponential form of service intervals as observed by Zhai *et al*. in [6] because it gives a convenient and accurate model of entire queueing discipline (including multihop analysis) under steady-state conditions. We have independently observed that exponential service interval distribution provides the best approximation under large neighborhood and high contention conditions. Here, we establish this result formally through analytical and simulation methods.

A. Service Interval Distribution

Consider the DCF state transition diagram in Fig. 1. After a successful transmission and detection of channel idle for a DIFS+ period, the MAC initiates a random exponential backoff. Whenever the backoff counter becomes zero, two possible events can happen – collision or successful transmission. These events are represented by probabilities $P_{counter}=0$ and P_{fail} that correspond to state transitions $2\rightarrow 4$ and $2\rightarrow 5$ respectively. Once collision is detected, the process is repeated over again, after the channel is sensed idle – represented by P_{coll} corresponding to state transition $4\rightarrow 3$. Thus, the interval between successful transmissions is dependent on three probabilities – transmission slot being empty P_{m_idle}, transmission being successful $(P_{counter}, P_{success})$ and collision occurs and transmission deferred P_{coll} [3]. The probability P_{m_idle} that there is at least one transmission in a given MAC transmission slot with n neighbors contenting is given by

$$P_{tr}=1-(1-\tau)^n . \quad (6)$$

The probability that a slot becomes idle is thus given by $P_{m\_idle}=1-P_{tr}=(1-\tau)^n$. Similarly, a successful transmission $P_{success}$ occurs with a probability that exactly one node transmits on the channel, conditioned on the probability that at least one node transmits. That is,

$$P_{success}=\frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n} . \quad (7)$$

Further, the probability that transmission occurs followed by a contention is derived from (6) and (7) above. That is,

$$\begin{aligned} P_{coll}&=(1-P_{m\_idle})\times(1-P_{success}) \\ P_{coll}&=(1-(1-\tau)^n)\times P_{success} \end{aligned} . \quad (8)$$

As in [3], a successful MAC service probability is given by $P_{service}=P_{m\_idle}\times(P_{tr}\times P_{success})\times(1-P_{coll})$. Simplifying this using (6)–(8), yields us the following form.

$$P_{service}=C'\times n\tau(1-\tau)^{3n-2}(1-\tau-n\tau) , \quad (9)$$

where C' is a normalization constant. We expect that the servicing probability $P_{service}$ reduce significantly due to increased MAC contention, as the number of neighbors increase in the network. This is indeed observed from (9) in which $P_{service}\le n\tau(1-\tau)^{3n-2}\rightarrow 0$, as $n\rightarrow\infty$.

From (6)–(9), the service interval expectation $E(S_i)$ of a successful transmission at node *i* is given by

$$E(S_i)=(1-P_{tr})\sigma+P_{tr}P_sT_s+P_{tr}(1-P_s)T_c , \quad (10)$$

where σ is the duration of an idle slot, T_s is the average time channel is sensed busy, and T_c is the average time channel is sensed busy by each node during collision. These constants are defined in the simulations section IV. The average time MAC channel is sensed busy during collision is taken into consideration in (10) (as per the definition of service slot time), but not in deriving the successful service probability of MAC frames in (9). Further, note that (10) precisely is the expected slot time of a MAC transmission as derived by Bianchi [3].

B. Chen-Stein Poisson Approximation Method

Let $I_1, I_2, ..., I_n$ be a set of independent and identically distributed (i.i.d.) Bernoulli random variables. A classical result in probability is the law of small numbers that states that the distribution of sum $W=\sum_{j=1}^{n} I_j$ converges to exponential with parameter λ provided: $p_j=P[I_j=1]$ are all small, none of I_j is dominating in the sum and none of I_j are strongly dependent on each other. Chen-Stein approximation method uses the law of small numbers principle and establishes bounds on total variation (distribution) distance between the known distribution sum W and an unknown Poisson P_λ with parameter λ [11], [12]. Formally, the total variation distance between the sum process W and P_λ is defined as:

$$d_{TV}(W,P_\lambda)=\sup\{|W(A)-P_\lambda(A)| : A\subset \mathrm{Z}^+\}, \quad (11)$$

where $A\subset \mathbf{Z}^+$ is a measurable discrete space of operation of W and P_λ [11]. Furthermore, the distribution of W converges to P_λ with parameter λ if and only if $d_{TV}(W,P_\lambda)\rightarrow 0$ as $n\rightarrow\infty$. In this case, the parameter λ is given by

$$\lambda=E[W]=\sum_{j=1}^{n} P[I_j=1]=\sum_j p_j . \quad (12)$$

In addition to this, Stein proved that a discrete random variable $X \sim P_\lambda$, if and only if for all bounded non-negative function $g(X)$, the following holds well.

$$E[\lambda \times g(X+1)) - X \times g(X)] = 0\,, \qquad (13)$$

where $g : \mathbf{Z}^+ \to \mathbf{R}$ [12]. Expression (13) gives the measure of "closeness" of a function of random variable *X* against the Poisson with mean λ, in terms of function *g*. Establishing that Chen-Stein distance bound converges to limit zero proves by itself that the approximated distribution is Poisson.

Chen-Stein method is applicable only when the situation generalizes to dependent, low probability random variables so long as the dependencies of variables are negligible as $n \to \infty$ [11]. The importance of method in (11)–(13) is that, although no explicit form of distribution P_λ (with a known mean λ) is available, the structure of sum *W* permits to prove that P_λ indeed is Poisson. Further, Chen-Stein method differs from other approximation methods such as Fourier techniques, central limit theorem, weak law of large numbers, etc., in that the Chen-Stein method gives an immediate access to the distribution bound [11], [12].

C. Poisson Service Distribution

In this section, we propose a Chen-Stein variation distance bound and prove that MAC transmission intervals satisfies the condition that the bound tends to limit zero as the number of neighbors $n \to \infty$. Consider the sum process *W*, which is the superposition of several neighborhood MAC service processes.

Theorem: Under large neighborhood presence and under network saturation conditions, the Chen-Stein distribution distance between the service interval distribution of an ad-hoc MAC queue and an exponential distribution with parameter λ is bounded by $d_{TV}(W, P_\lambda) \le \frac{(1-e^{-\lambda})}{\lambda} \sum_{j=1}^{n} p_j^{\,2}$.

Proof: Consider a collection of Chen-Stein Bernoulli indicator variables I_j where $j \in (1, n)$ for *n* neighborhood nodes, such that the indicator variables are non-decreasing functions. We define the indicator variables I_j to be a binary event of successful transmission of a MAC frame in a given transmission slot σ. That is, $I_j = 1$ if the transmission event is successful. As studied in section III.A, the probability of a successful transmission of a MAC frame is the probability of a successful service given by (9). Thus, the indicator variables take value 1 with probability given by $P\{I_j = 1\} = P_{service}$.

$$P\{I_j = 1\} = (1 - P_{m\_idle}) \times (P_{tr} \times P_{success}) \times (1 - P_{coll})\,. \qquad (14)$$

Let $W = \sum_{j=1}^{n} I_j$ be the summation process of all successful MAC transmission indicator variables I_j. We prove the theorem in four steps. First, in order to apply the Chen-Stein method, the individual probabilities of indicator variables $P\{I_j = 1\} \to 0$, as $n \to \infty$. That is, none of I_j is dominating in the sum *W*. Using (9), we find that

$$\begin{aligned} P\{I_j = 1\} = P_{service} &\le n(1-\tau-n\tau)(1-\tau)^{3n-2} \\ &\to 0 \;\text{ as }\; n \to \infty \end{aligned}\,. \qquad (15)$$

Second, we prove that Stein's expression (13) holds good for function $g(W)$ and for discrete random variable *W*. Here, discreteness of random variable *W* is due to the slotted intervals σ during which transmissions can occur. For any non-negative bounded function *g*, expression (13) holds well so long as the bound below also holds good.

$$\left|P(W) - P_\lambda\right| \le \sup_{k \ge 1}\left|g(k+1) - g(k)\right| \times \sum_{j=1}^{n} p_j^{\,2} \qquad (16)$$

Details of the proof are discussed in Chen-Stein literature [11], [12]. Third, we prove that the distribution distance variation tends to limit zero: $d_{TV}(W, P_\lambda) \to 0$ as $n \to \infty$. To prove this, we make use of the bounds derived from expression (16) and the limit obtained from (15). That is,

$$\begin{aligned} \left|P(W) - P_\lambda\right| &\le \sup_{k \ge 1}\left|g(k+1) - g(k)\right| \times \sum_{j=1}^{n} p_j^{\,2} \\ &\le \frac{(1-e^{-\lambda})}{\lambda} \times \sum_{j=1}^{n} p_j^{\,2} \\ &\le \frac{(1-e^{-\lambda})}{\lambda} \times \sum_j \left(n(1-\tau-n\tau)(1-\tau)^{3n-2}\right)^2 \\ &\to 0 \;\text{ as }\; n \to \infty \end{aligned}\,.$$

Finally, the expectation of sum *W* is given by $E[W] = \sum_j I_j P[I_j = 1] = nE[S_i]$, where $E(S_i)$ is the expectation of service interval at node *i*. That is, the expectation of *W* is the mean service interval of a MAC queue, as given by (10) in section III.B. Since the service processes are all independent and identically distributed (i.i.d.), the mean service interval is the product of number of neighbors *n* and the expectation of a successful transmission interval, as $n \to \infty$. Thus, the mean service rate of a MAC queue is given by

$$\frac{1}{\lambda} = \frac{1}{n\left((1-P_{tr})\sigma + P_{tr}P_sT_s + P_{tr}(1-Ps)T_c\right)}\,. \qquad (17)$$

This completes the proof of the theorem. ■

Further, the process *W* exhibits all necessary and sufficient (general) conditions of a Poisson process – *W* has stationary and independent increments. We do not show this here due to space limitations.

D. Inter-Arrival Distribution

Consider the 802.11 ad-hoc network as a series of MAC queues. In traditional analytical modeling of series of queues, the inter-arrival distribution of a queue is a function of service interval distribution at the previous queue and the arrivals due to internal queues at that node. Such series of queues are termed open-loop Jackson networks.

From first principles, the service time distribution of a MAC queue in an open-loop Jackson network is identical to the inter-arrival time distribution with the mean arrival interval λ. Note that this result permits modeling of each node individually as an independent M/M/1 queue.

IV. SIMULATIONS

A. Simulation Setup

We validate our analytical model through *ns*2-based experiments of 802.11 wireless ad-hoc network. Our topography is 200×200m rectangular 2-dimensional grid with several mobile nodes at random. The configuration of nodes used is same as Lucent Orinoco 802.11b WLAN PCMCIA card. The nodes use omni-antenna and two-ray ground radio propagation model with power equivalent to a maximum range of 22.5m. Nodes operate with a channel capacity of 2 Mbps. Each node has a random initial position and is set in motion to another random point in the grid. Nodes move at a constant speed of 5m/s. Further, each node acts as a source and a sink of CBR traffic with connected nodes. CBR rate is set to greater than 2Mbps so that there is always at least one frame to transmit.

B. Discussion

We perform three sets of experiments with two ad-hoc topologies – chain and random. In the chain, we observe that even though the MAC encounters several contentions (due to CBR traffic far higher than channel capacity), the service intervals still remain non-exponential as in Fig. 2. In the second, the nodes move randomly and encounter higher contention that leads to a greater superposing effect of backoff processes. During the random motion, the nodes also encounter several hidden terminals that boost this effect. This is demonstrated in Fig. 3 in which the service interval tends to an exponential form for N=5. We observe the same with N=10 nodes in Fig. 4. Note that N=10 nodes is sufficient for the superposed service intervals to converge to an exponential form. Further, the mean service rate of random topology experiments matches well with our analytical results in (17).

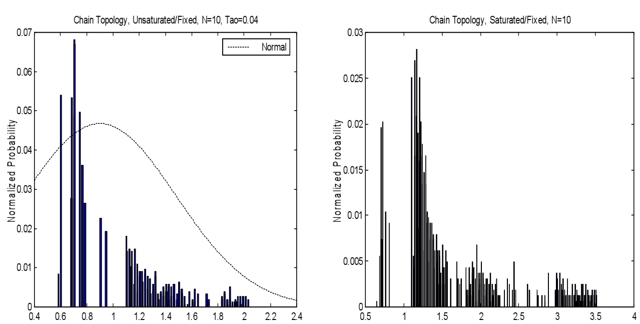

Fig. 2. Normal Service Distribution of Chain Topology (N=10)

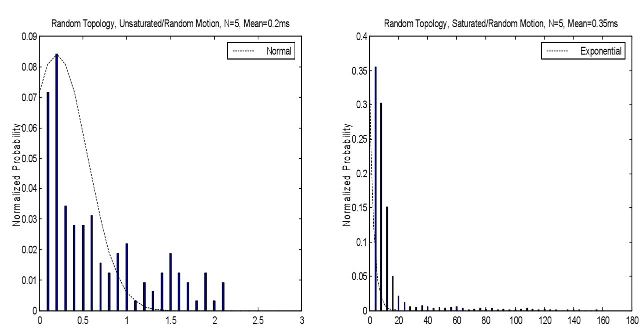

Fig. 3. Normal and Exponential Service Distribution of Random Topology (N=5)

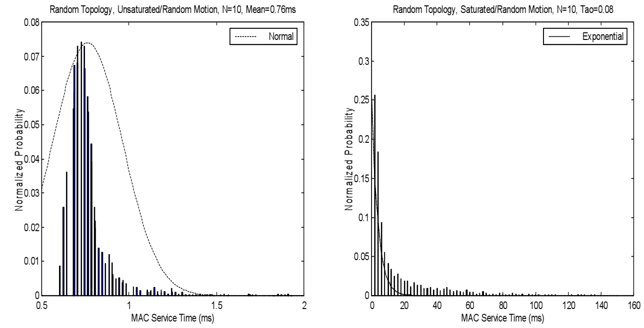

Fig. 4. Normal and Exponential Service Distribution of Random Topology (N=10)

Constants used in simulations: $\sigma = 50\mu s, T_s = 9ms, T_c = 8.7ms.$

V. CONCLUSIONS

In this paper, we established analytically that Poisson service distribution fits well under network saturated conditions when the number of neighbors becomes large. Using the Chen-Stein bounds on distribution distances, we proved that the convergence to exponential form occurs due to superposition of several neighborhood service processes. This leads to several simplifications in ad-hoc MAC queue modeling. We believe that this is a significant observation proved through sound analytical modeling and it can open up several areas of study on stabilizing MAC queues in 802.11 ad-hoc networks.

REFERENCES

[1] P. Schmitz, "The Wireless Office: Laying the Foundation for a Wireless Enterprise", Technology@Intel Magazine, Feb 2005. Available at http://www.intel.com/technology/magazine/communications/Wireless-office-0405.pdf

[2] IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, Nov. 1997. P802.11.

[3] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function", IEEE Journal on Selected Areas in Communications, Vol. 18, No. 3, March 2000.

[4] O. Tickoo and B. Sikdar, "Queueing Analysis and Delay Mitigation in IEEE 802.11 Random Access MAC based Wireless Networks", INFOCOM 2004.

[5] M. Özdemir and A. B. McDonald, "A Queuing Theoretic Model for IEEE 802.11 DCF using RTS/CTS", The 13th IEEE Workshop on Local and Metropolitan Area Networks (LANMAN), April 2004.

[6] H. Zhai, Y. Kwon, and Y. Fang, "Performance Analysis of IEEE 802.11 MAC Protocols in Wireless LANs", Wireless Communication and Mobile Computing, Vol. 4, 2004.

[7] J. Yoo and C. - K. Kim, "On the Hidden Terminal Problem in Multi-Rate Ad Hoc Wireless Networks", In Lecture Notes in Computer Science (LNCS), Number 3391, Springer-Verlag, January 2005, pp 479-488. Also presented at International Conference on Information Networking (ICOIN) 2005.

[8] M. Borgo, A. Zanella, P. Bisaglia, and S. Merlin, "Analysis of the Hidden Terminal Effect in Multi-rate IEEE 802.11b Networks", In Proc. of WPMC04, Sept. 2004.

[9] K. C. Huang and K. C. Chen, "Interference analysis of non-persistent CSMA with hidden terminals in multi-cell wireless data networks," in Proc. IEEE PIMRC, Toronto, Canada, Sept. 1995.

[10] F. Tobagi, and L. Kleinrock, "Packet Switching in Radio Channels: Part II--the Hidden Terminal Problem in Carrier Sense Multiple-Access and the Busy-Tone Solution", IEEE Transactions on Communications, Vol. COM-23, No. 12, pp. 1417-1433, December 1975.

[11] C. Goldschmidt, "The Chen-Stein Method for Convergence of Distributions", Masters-level essay, University of Cambridge, UK, 2000. Available at http://www.statslab.cam.ac.uk/~cag27/chen-stein.ps.gz.

[12] D. Barbour, L. Holst, and S. Janson, "Poisson Approximation", Oxford University Press, Feb 1992.