classmate

Date 30-04-20
Page 1

M. Jishnu

Assignment -2 RollNo: 17309

①

a) There will be total 2^4 code words = 16

uncoded bits	coded bits	uncoded bits	
		1001	1001100
0000	0000000	1010	1010010
0001	0001011	1011	1011001
0010	0010101	1100	1100001
0011	0011110	1101	1101010
0100	0100110	1110	1110100
0101	0101101	1111	1111111
0110	0110011		
0111	0111000		
1000	1000111		

$G = [I | P]$

$$E \quad \underset{1\times4}{(v_0, v_1, v_2, v_3)} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$V_0 = U_0$ $\quad V_4 = U_0 + U_1 + U_2$

$V_1 = U_1$ $\quad V_5 = U_0 + U_1 + U_3$

$V_2 = U_2$ $\quad V_6 = U_0 + U_2 + U_3$

$V_3 = U_3$

b) $CH^T = 0$

$$H^T = \begin{bmatrix} -P \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow [1011100] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= [101]$

Jishnu

Since [1 0 1] is in third position of H^T matrix error is in 3^{rd} position

So correct vector is = [1 0 0 1 1 0 0]

(c) $s = [1 0 1]$

or $s_0 = v_0 + v_1 + v_2 + v_4$ $s_1 = v_0 + v_1 + v_3 + v_5$

$s_2 = v_0 + v_2 + v_3 + v_6$

(2) $v_0 = u_1 + u_2 + u_3$

$v_1 = u_2 + u_1 + u_0$

$v_2 = u_0 + u_1 + u_3$

$v_3 = u_0 + u_2 + u_3$

So we know the code word = $[u_0\ u_1\ u_2\ u_3\ v_0\ v_1\ v_2\ v_3]$

$$G = [I|P] = \left[\begin{array}{cccc|cccc} 1&0&0&0&0&1&1&1 \\ 0&1&0&0&1&1&1&0 \\ 0&0&1&0&1&1&0&1 \\ 0&0&0&1&1&0&1&1 \end{array}\right] \quad P = \begin{bmatrix} 0&1&1&1 \\ 1&1&1&0 \\ 1&1&0&1 \\ 1&0&1&1 \end{bmatrix}$$

$$H = [-P^T/I]$$

$$= \begin{bmatrix} 0&1&1&1&1&0&0&0 \\ 1&1&1&0&0&1&0&0 \\ 1&1&0&1&0&0&1&0 \\ 1&0&1&1&0&0&0&1 \end{bmatrix}$$

Note that column 3, 4, 6 are adding upto zero vector

so $d^* = 4$

$H^T = S$

$$S = [v_0\ v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7] \begin{bmatrix} 0&1&1&1 \\ 1&1&1&0 \\ 1&1&0&1 \\ 1&0&1&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}_{8 \times 4}$$

$s_0 = v_1 + v_2 + v_3 + v_4$

$s_1 = v_0 + v_1 + v_2 + v_5$

$s_2 = v_0 + v_1 + v_3 + v_6$

$s_3 = v_0 + v_2 + v_3 + v_7.$

Scanned with CamScanner

Date
Page

Jishnu

③ code word = $[d_1 d_2 d_3 d_4 \; c_5 c_6 c_7]$

$$G = [I|P] = \left[\begin{array}{cccc|ccc} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{array}\right] \quad P = \begin{bmatrix} 1&1&0 \\ 0&1&1 \\ 1&1&1 \\ 1&0&1 \end{bmatrix}$$

b) $$H^T = \begin{bmatrix} -P \\ I \end{bmatrix} = \begin{bmatrix} 1&1&0 \\ 0&1&1 \\ 1&1&1 \\ 1&0&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}$$

GH^T

$$\Rightarrow \begin{bmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{bmatrix} \begin{bmatrix} 1&1&0 \\ 0&1&1 \\ 1&1&1 \\ 1&0&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix} = 0$$

© The duel code is $n, n-k = 7, 3$ with msg bits $d_1 d_2 d_3$

$$[C] = [d_1 d_2 d_3][H] = [d_1 d_2 d_3] \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&1&0&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix}$$

Msg	Codeword
1) 000	0000000
2) 001	0111001
3) 010	1110010
4) 011	1001011
⑤ 100	1011100
101	1100101
110	0101110
111	0010111

Minimum distance $d_{min} = 1$

(4) The code length n and k are related by $n < 2^{n-k} - 1$. The value of $k = 8$ then $n \le 2^{n-k} - 1$, this equation can be satisfied by $n = 12$ hence it is [12, 8] code.

The transpose of $H = \begin{bmatrix} P_{8\times4} \\ S_4 \end{bmatrix}$ there are 2^{n-k} combinations so 2^4 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111 in which 0000 is excluded and 0001, 0010, 0100, 1000 are identity matrix.

$$H^T = \begin{bmatrix} 0011 \\ 0101 \\ 0110 \\ 0111 \\ 1001 \\ 1011 \\ 1100 \\ 1111 \\ 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$$

$$H = \left[\begin{array}{c|c} 00001111 & 1000 \\ 01110001 & 0100 \\ 10111110 & 0010 \\ 11011010 & 0001 \end{array}\right] \qquad G = \left[\begin{array}{c|c} 10000000 & 0011 \\ 01000000 & 0101 \\ 00100000 & 0110 \\ 00010000 & 0111 \\ 00001000 & 1001 \\ 00000100 & 1010 \\ 00000010 & 1011 \\ 00000001 & 1100 \end{array}\right]$$

Jishnu

(b) (6, 3) linear block code

a) $n = 6, k = 3, 2^3 = 8$ possible msgs

$u = [000, 100, 010, 110, 001, 101, 011, 111]$

$$G = [I|P] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$v = [v_0, v_1, v_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$v_0 = u_0$ $\quad v_4 = u_1 + u_2$

$v_1 = u_1$ $\quad v_5 = u_0 + u_1$

$v_2 = u_2$

$v_3 = u_0 + u_2$

1	000	000000
2	001	001110
3	010	010011
4	011	011101
5	100	100101
6	101	101011
7	110	110110
8	111	111000

$$H^T = \begin{bmatrix} P \\ I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= [1\,1\,1]$.

classmate

Date

Page

Jishnu

⑥ $[-P^T | I]$ or $[I | P^T]$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

②

So $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $\quad G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$

(b) Because

$$CH^T = [v_0\ v_1\ v_2\ v_3\ v_4\ v_5] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$S_0 = v_0 + v_4 + v_5$

$S_1 = v_1 + v_3 + v_5$

$S_2 = v_2 + v_3 + v_4$

Because K is 3. And $n-k=3$ So Code word weight must be greater than 3 since $d_{min}=3$.

⑦ (a) Reduce G to standard form (5,3)

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_3} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Jithin

(b) $H = [I / P^T] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

(c) $C^{\perp} = \{00000, 10100, 01111, 11011\}$

(Q) a) $m = n - k = 6 - 3 = 3$

$k = 2^m - m = 2^3 - 3 - 1 = 4 \neq 3$

$n = 2^m - 1 = 8 - 1 = 7 \neq 6$

Hence (6,3) is not a hamming code

$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

b)

I/P	Code word	Weight
000	000000	0
001	001101	3
010	010011	3
011	011110	4
100	100110	3
101	101011	4
110	110101	4
111	111000	3

Jishnu

(c) $d_{min} = 3$ from above table $c = d_{min} - 1 = 2$
Hence (6,3) linear block code can detect 2 bit errors and correct 1 bit error

(d) $s_0 = v_0 + v_2 + v_3$ $s_1 = v_0 + v_1 + v_4$ $s_2 = v_1 + v_2 + v_5$

decoding table

error	syndrome
000000	000
1000000	110
010000	011
001000	101
000100	100
000010	010
000001	001

$$S = rH^T$$

$$[1\,1\,1\,0\,0\,1]\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$s = [0\,0\,1]$

$e = [0\,0\,0\,0\,0\,1]$

$c = r + e = [1\,1\,1\,0\,0\,1] + [0\,0\,0\,0\,0\,1]$
$= [1\,1\,1\,0\,0\,0]$