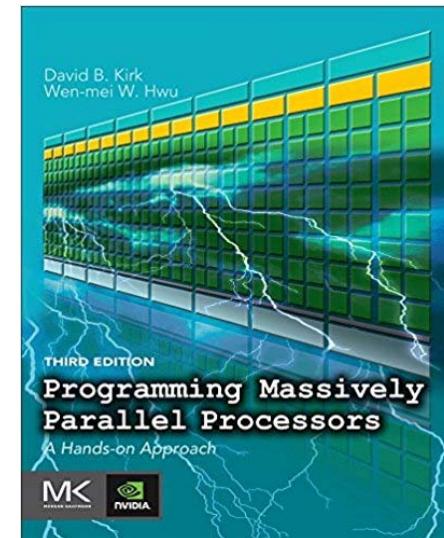


Introduction to CUDA

(4) Memory And Data Locality

Reference

- CUDA C Programming Guide,
 - <https://docs.nvidia.com/cuda/cuda-c-programming-guide/index.html>
- **Programming Massively Parallel Processors**,
 - A Hands-on Approach
 - Third Edition
 - Chapter 4



Content

- Importance of Memory Access Efficiency
 - --Matrix Multiplication
- Memory Types Overview
- Tiling for reduced memory traffic
 - --Matrix Multiplication using Shared Memory

Importance of Memory Access Efficiency

```
for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow) {  
    for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol) {  
  
        int curRow = Row + blurRow;  
        int curCol = Col + blurCol;  
        // Verify we have a valid image pixel  
        if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {  
            pixVal += in[curRow * w + curCol];  
            pixels++; // Keep track of number of pixels in the avg  
        }  
    }  
}
```

The most executed part of the image blurring kernel.

How to evaluate memory access efficiency:

Compute-to-global-memory-access Ratio

Importance of Memory Access Efficiency

```
for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow) {  
    for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol) {  
  
        int curRow = Row + blurRow;  
        int curCol = Col + blurCol;  
        // Verify we have a valid image pixel  
        if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {  
            pixVal += in[curRow * w + curCol];  
            pixels++; // Keep track of number of pixels in the avg  
        }  
    }  
}
```

The most executed part of the image blurring kernel.

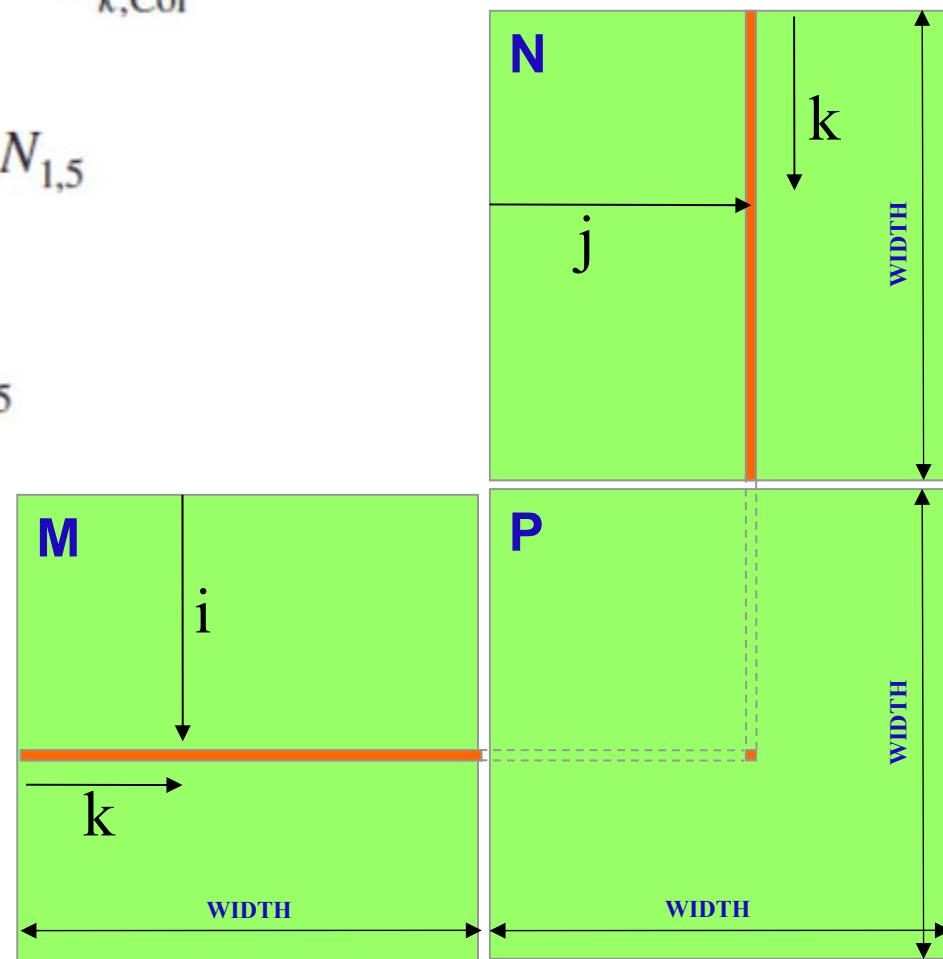
Compute-to-global-memory-access Ratio:

- Global memory bandwidth is 1TB/s, with 4 bytes in each single precision float value,
- No more than $1000/4 = 250$ G single-precision ops can be loaded in one second.
- If Compute-to-global-memory-access ratio is 1, no more than 250GFLOPS can be achieved, only 2% of 12TFLOPS of recent device.
- The ratio need to be improved to 48 or higher.

Matrix Multiplication Example

$$P_{\text{Row},\text{Col}} = \sum M_{\text{Row},k} * N_{k,\text{Col}}$$

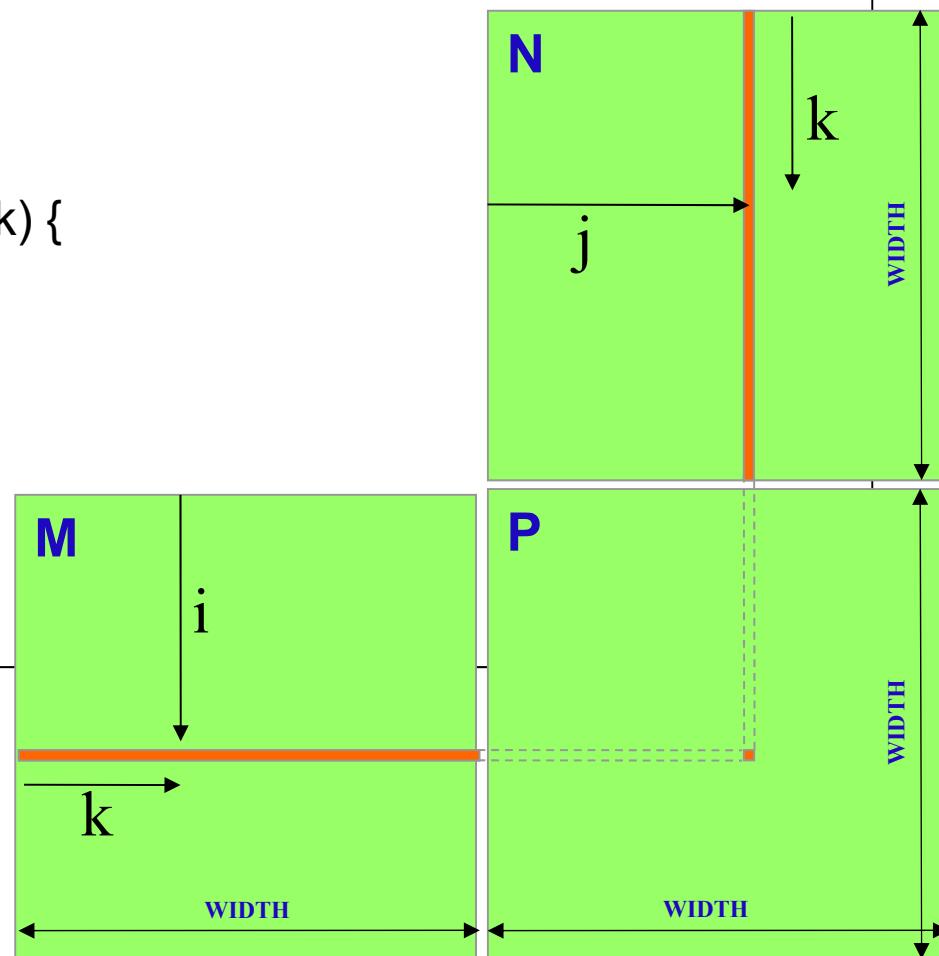
$$\begin{aligned} P_{1,5} &= M_{1,0} * N_{0,5} + M_{1,1} * N_{1,5} \\ &+ M_{1,2} * N_{2,5} + \dots \\ &+ M_{1,\text{Width}-1} * N_{\text{Width}-1,5} \end{aligned}$$



Matrix Multiplication Example

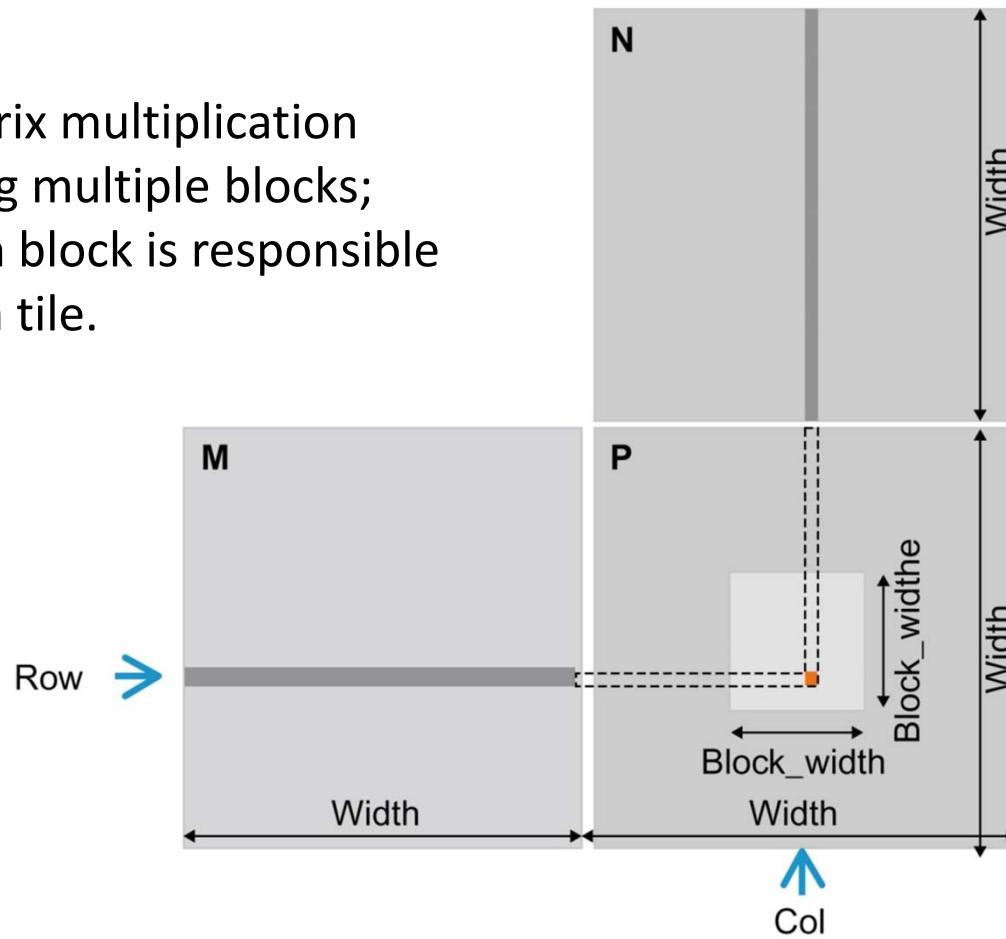
```
// Matrix multiplication on the (CPU) host in single precision
void MatrixMulOnHost(float* M, float* N, float* P, int Width)
{
    for (int i = 0; i < Width; ++i)
        for (int j = 0; j < Width; ++j) {
            float sum = 0;
            for (int k = 0; k < Width; ++k) {
                float a = M[i * Width + k];
                float b = N[k * Width + j];
                sum += a * b;
            }
            P[i * Width + j] = sum;
        }
}
```

A Simple Host Version in C



Initial matrix multiplication version

- Matrix multiplication using multiple blocks;
- Each block is responsible for a tile.



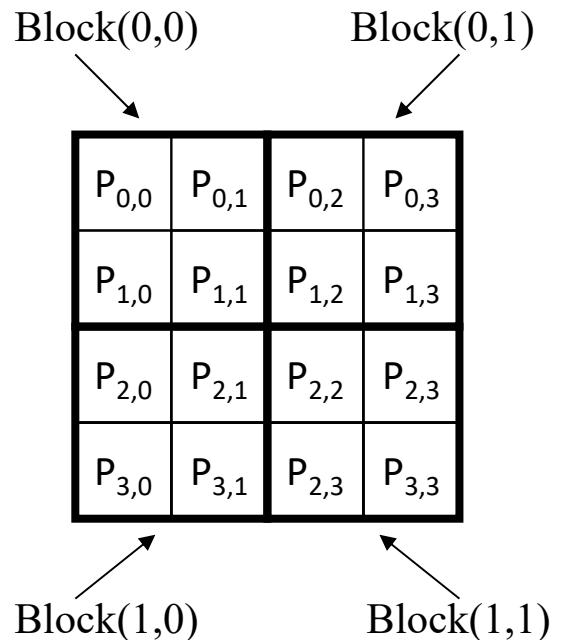
Initial matrix multiplication version

```
__global__ void MatrixMulKernel(float* M, float* N, float* P,
int Width) {
    // Calculate the row index of the P element and M
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
    // Calculate the column index of P and N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;
    if ((Row < Width) && (Col < Width)) {
        float Pvalue = 0;
        // each thread computes one element of the block sub-matrix
        for (int k = 0; k < Width; ++k) {
            Pvalue += M[Row*Width+k]*N[k*Width+Col];
        }
        P[Row*Width+Col] = Pvalue;
    }
}
```

A simple matrix multiplication kernel:
-- using one thread to compute one P element.

Kernel Function - A Small Example

- Have each 2D thread block to compute a $(\text{TILE_WIDTH})^2$ sub-matrix (tile) of the result matrix
 - Each has $(\text{TILE_WIDTH})^2$ threads
- Generate a 2D Grid of $(\text{WIDTH}/\text{TILE_WIDTH})^2$ blocks



$\text{WIDTH} = 4; \text{TILE_WIDTH} = 2$
Each block has $2*2 = 4$ threads

$\text{WIDTH}/\text{TILE_WIDTH} = 2$
Use $2*2 = 4$ blocks

A Slightly Bigger Example

(TILE_WIDTH = 2)

P _{0,0}	P _{0,1}	P _{0,2}	P _{0,3}	P _{0,4}	P _{0,5}	P _{0,6}	P _{0,7}
P _{1,0}	P _{1,1}	P _{1,2}	P _{1,3}	P _{1,4}	P _{1,5}	P _{1,6}	P _{1,7}
P _{2,0}	P _{2,1}	P _{2,2}	P _{2,3}	P _{2,4}	P _{2,5}	P _{2,6}	P _{2,7}
P _{3,0}	P _{3,1}	P _{3,2}	P _{3,3}	P _{3,4}	P _{3,5}	P _{3,6}	P _{3,7}
P _{4,0}	P _{4,1}	P _{4,2}	P _{4,3}	P _{4,4}	P _{4,5}	P _{4,6}	P _{4,7}
P _{5,0}	P _{5,1}	P _{5,2}	P _{5,3}	P _{5,4}	P _{5,5}	P _{5,6}	P _{5,7}
P _{6,0}	P _{6,1}	P _{6,2}	P _{6,3}	P _{6,4}	P _{6,5}	P _{6,6}	P _{6,7}
P _{7,0}	P _{7,1}	P _{7,2}	P _{7,3}	P _{7,4}	P _{7,5}	P _{7,6}	P _{7,7}

WIDTH = 8; TILE_WIDTH = 2
Each block has 2*2 = 4 threads

WIDTH/TILE_WIDTH = 4
Use 4* 4 = 16 blocks

A Slightly Bigger Example (cont.)

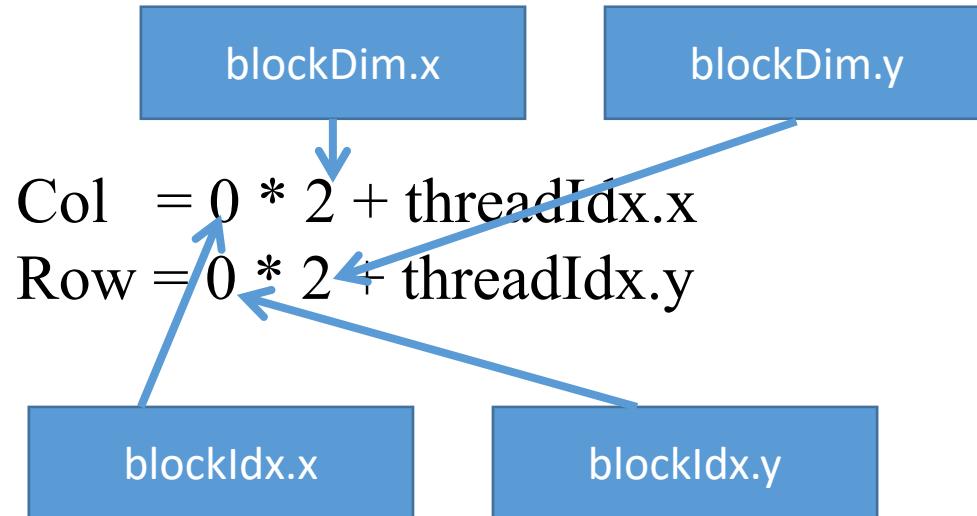
(TILE_WIDTH = 4)

P _{0,0}	P _{0,1}	P _{0,2}	P _{0,3}	P _{0,4}	P _{0,5}	P _{0,6}	P _{0,7}
P _{1,0}	P _{1,1}	P _{1,2}	P _{1,3}	P _{1,4}	P _{1,5}	P _{1,6}	P _{1,7}
P _{2,0}	P _{2,1}	P _{2,2}	P _{2,3}	P _{2,4}	P _{2,5}	P _{2,6}	P _{2,7}
P _{3,0}	P _{3,1}	P _{3,2}	P _{3,3}	P _{3,4}	P _{3,5}	P _{3,6}	P _{3,7}
P _{4,0}	P _{4,1}	P _{4,2}	P _{4,3}	P _{4,4}	P _{4,5}	P _{4,6}	P _{4,7}
P _{5,0}	P _{5,1}	P _{5,2}	P _{5,3}	P _{5,4}	P _{5,5}	P _{5,6}	P _{5,7}
P _{6,0}	P _{6,1}	P _{6,2}	P _{6,3}	P _{6,4}	P _{6,5}	P _{6,6}	P _{6,7}
P _{7,0}	P _{7,1}	P _{7,2}	P _{7,3}	P _{7,4}	P _{7,5}	P _{7,6}	P _{7,7}

WIDTH = 8; TILE_WIDTH = 4
Each block has $4 \times 4 = 16$ threads

WIDTH/TILE_WIDTH = 2
Use $2 \times 2 = 4$ blocks

Work for Block (0,0)



`Col == 0`
`Col == 1`

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

Row = 0
Row = 1

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$	$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$	$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$	$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Work for Block (0,0) Thread(0,0)

During the 0th iteration ($k=0$) :

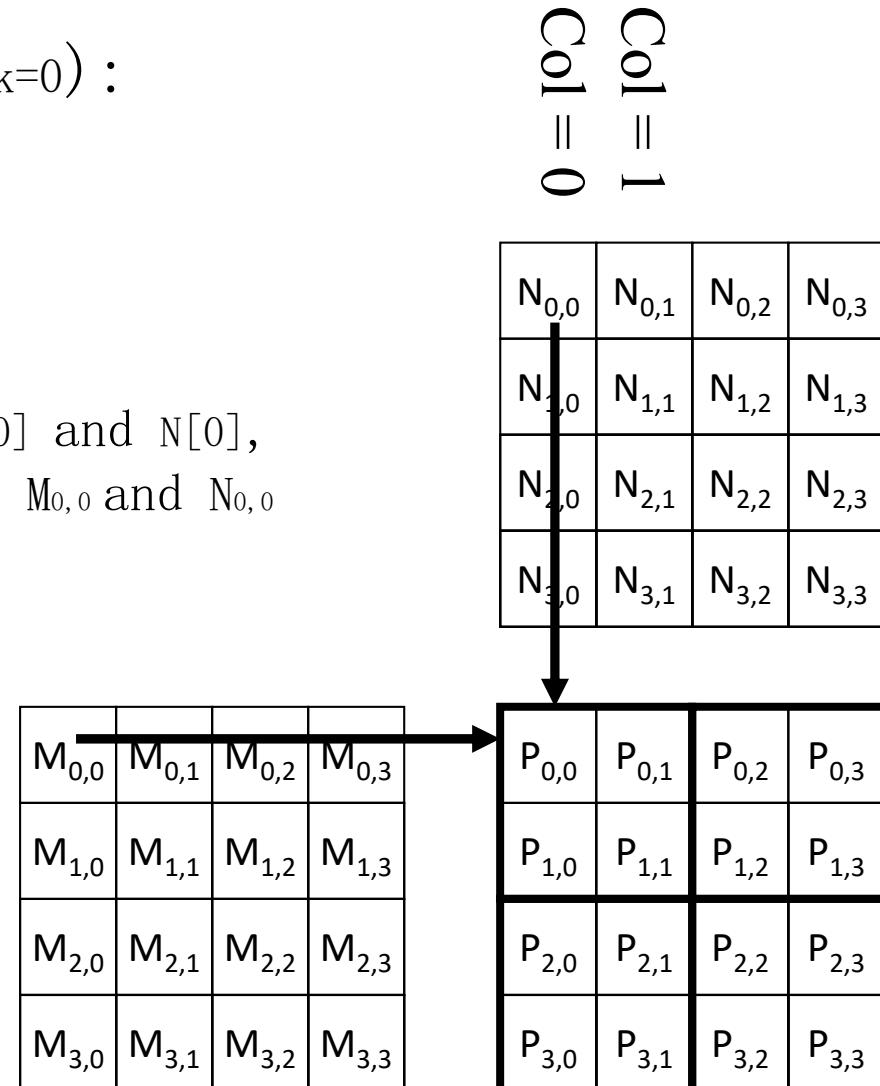
$$\text{Row} * \text{Width} + k = 0 * 4 + 0 = 0$$

$$k * \text{Width} + \text{Col} = 0 * 4 + 0 = 0$$

Therefore, we are accessing $M[0]$ and $N[0]$, which are the 1D equivalent of $M_{0,0}$ and $N_{0,0}$

Row = 0

Row = 1



Work for Block (0,0) Thread(0,0)

During the 1st iteration ($k=1$) :

$$\text{Row} * \text{Width} + k = 0 * 4 + 1 = 1$$

$$k * \text{Width} + \text{Col} = 1 * 4 + 0 = 4$$

Therefore, we are accessing $M[1]$ and $N[4]$, which are the 1D equivalent of $M_{0,1}$ and $N_{1,0}$

Col = 0
Col = 1

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

Row = 0

Row = 1

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Work for Block (0,0) Thread(0,0)

During the 2nd iteration ($k=2$) :

$$\text{Row} * \text{Width} + k = 0 * 4 + 2 = 2$$

$$k * \text{Width} + \text{Col} = 2 * 4 + 0 = 8$$

Therefore, we are accessing $M[2]$ and $N[8]$, which are the 1D equivalent of $M_{0,2}$ and $N_{2,0}$

Col = 0
Col = 1

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

Row = 0

Row = 1

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Work for Block (0,0) Thread(0,0)

During the 3rd iteration ($k=3$) :

$$\text{Row} * \text{Width} + k = 0 * 4 + 3 = 3$$

$$k * \text{Width} + \text{Col} = 3 * 4 + 0 = 12$$

Therefore, we are accessing $M[3]$ and $N[12]$, which are the 1D equivalent of $M_{0,3}$ and $N_{3,0}$

Row = 0

Row = 1

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

Col = 0
Col = 1

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

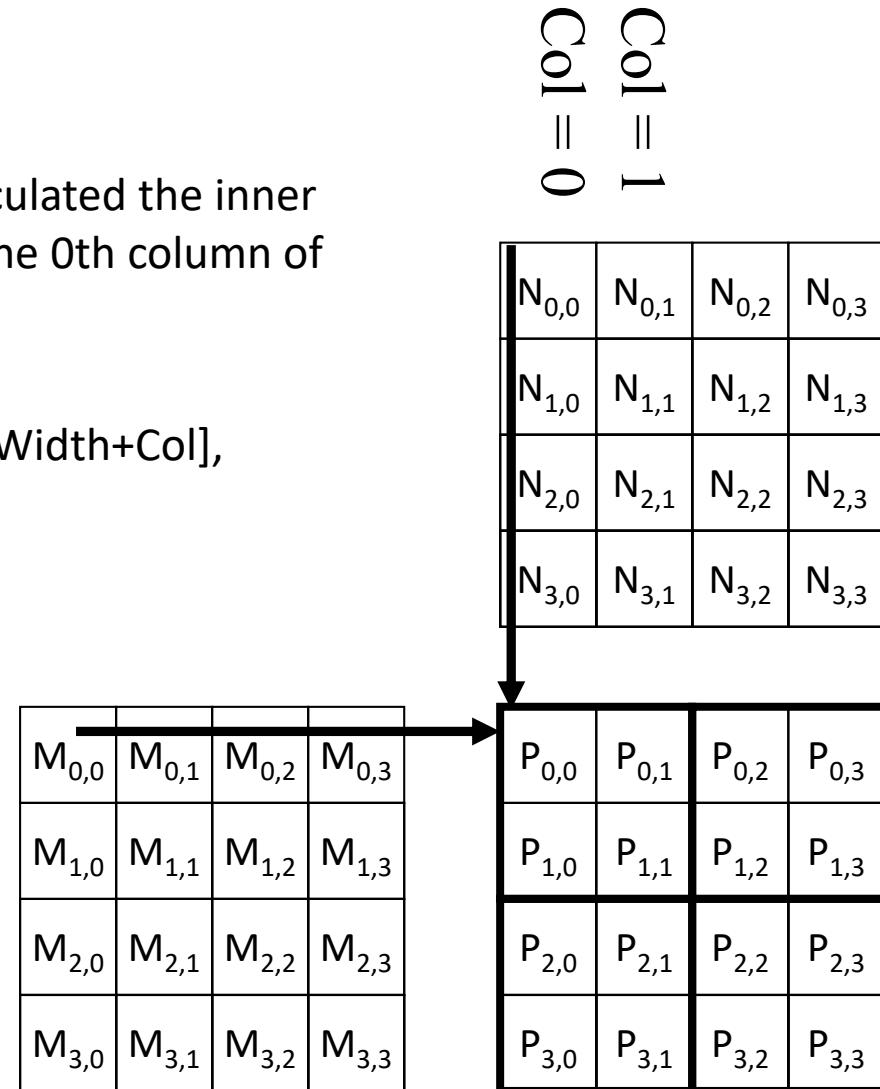
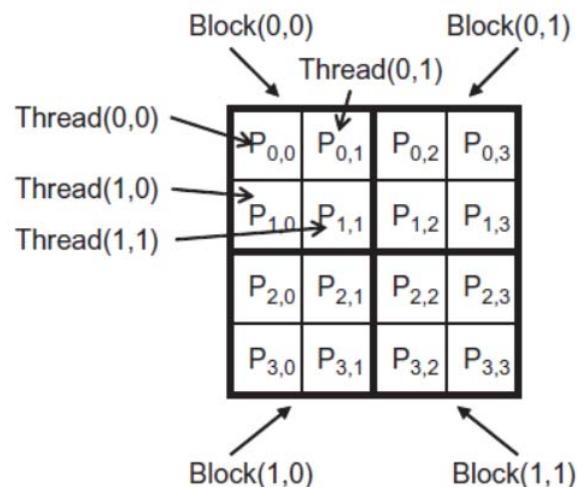
$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Work for Block (0,0) Thread(0,0)

At last:

Thread(0,0) in block(0,0) successfully calculated the inner product between the 0th row of M and the 0th column of N and deposited the result in $P_{0,0}$.

After the loop, the thread writes $P[Row * Width + Col]$, which is $P[0]$, the 1D equivalent of $P_{0,0}$.



Work for Block (0,1)

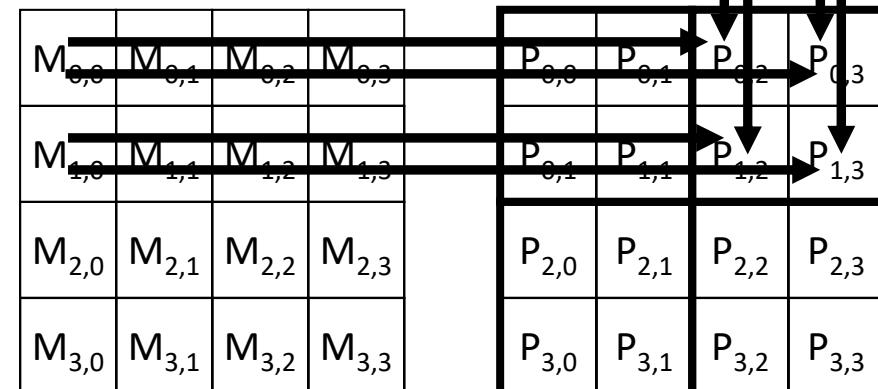
$\text{Col} = 1 * 2 + \text{threadIdx.x}$
 $\text{Row} = 0 * 2 + \text{threadIdx.y}$



Row = 0
Row = 1

Col = 1
||
Col = 2
||
Col = 3

N _{0,0}	N _{0,1}	N_{0,2}	N_{0,3}
N _{1,0}	N _{1,1}	N_{1,2}	N_{1,3}
N _{2,0}	N _{2,1}	N_{2,2}	N_{2,3}
N _{3,0}	N _{3,1}	N_{3,2}	N_{3,3}



Walk through the for loop

```
for (int k = 0; k < Width; ++k)
    Pvalue += d_M[Row*Width+k] * d_N[k*Width+Col];
```

2 global memory access for:

One floating point multiplication
One floating point addition

Compute-to-global-memory-access ratio is 1

Result in less than 2% utilization
of the peak execution speed

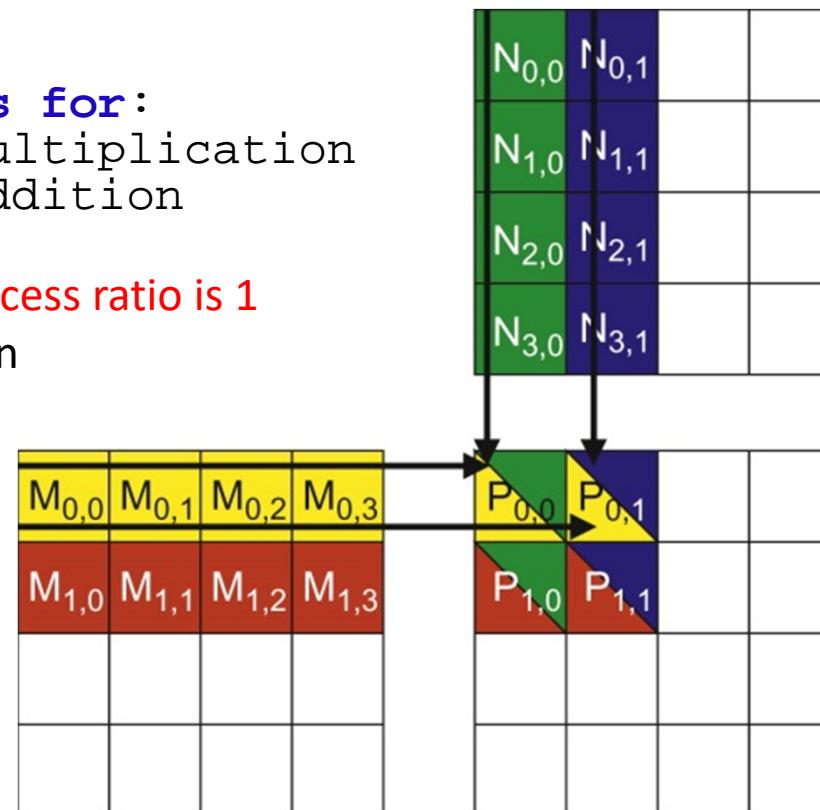
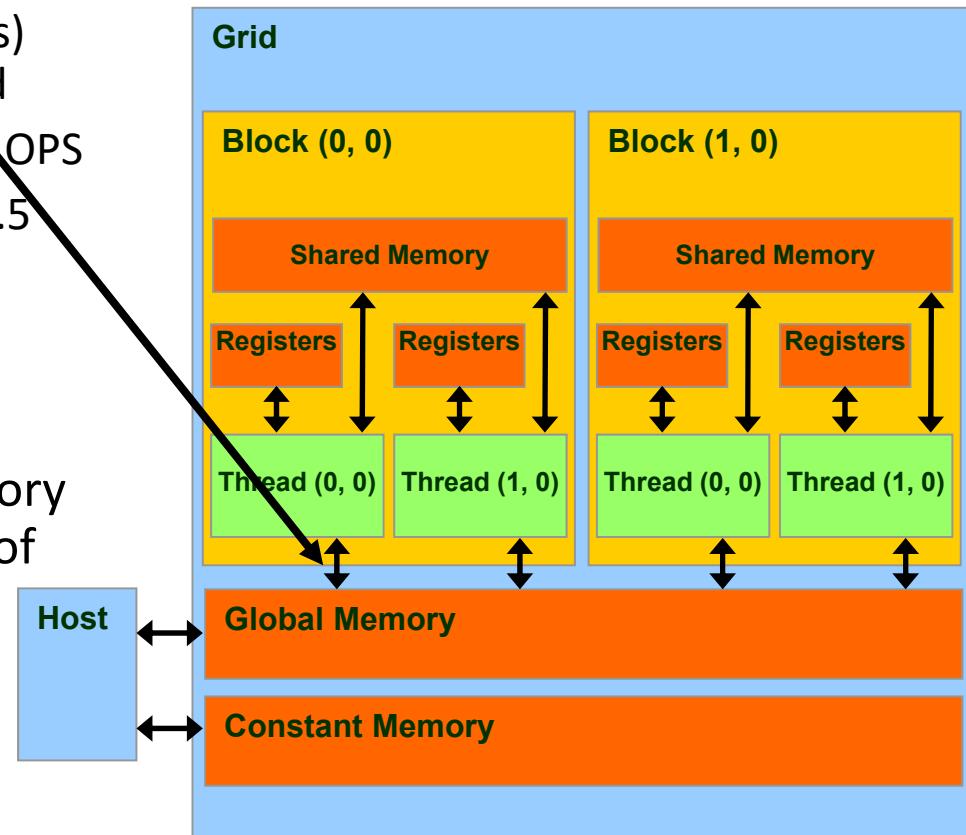


FIGURE 4.5: Matrix multiplication actions of one thread block.

How about performance on a device with 150 GB/s memory bandwidth?

- All threads access global memory for their input matrix elements
 - Two memory accesses (8 bytes) per floating point multiply-add
 - 4B/s of memory bandwidth/FLOPS
 - 150 GB/s limits the code at 37.5 GFLOPS
- The actual code runs at about 25 GFLOPS
- Need to drastically cut down memory accesses to get closer to the peak of more than 1,000 GFLOPS

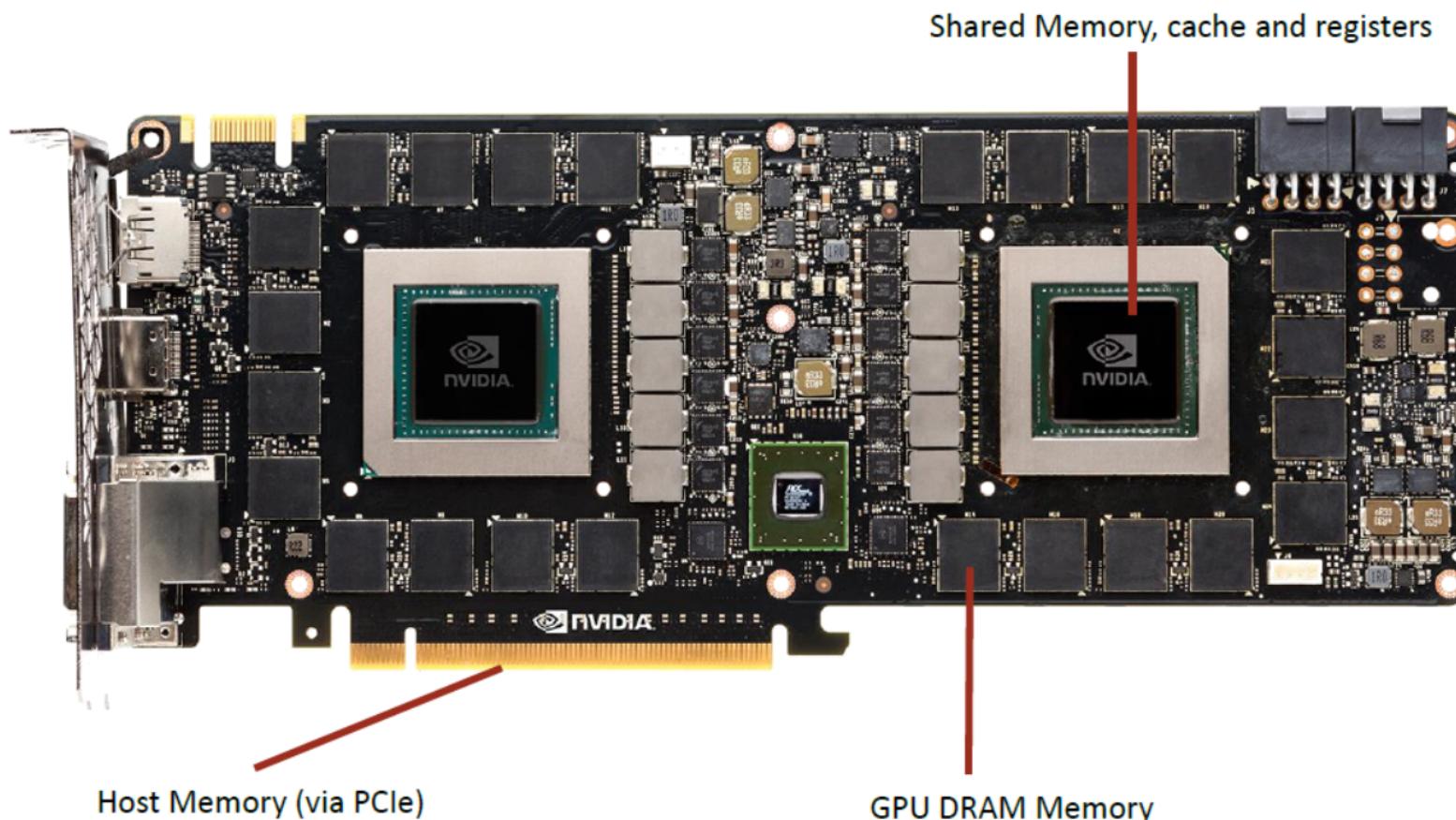


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CUDA Memories

GTX Titan Z



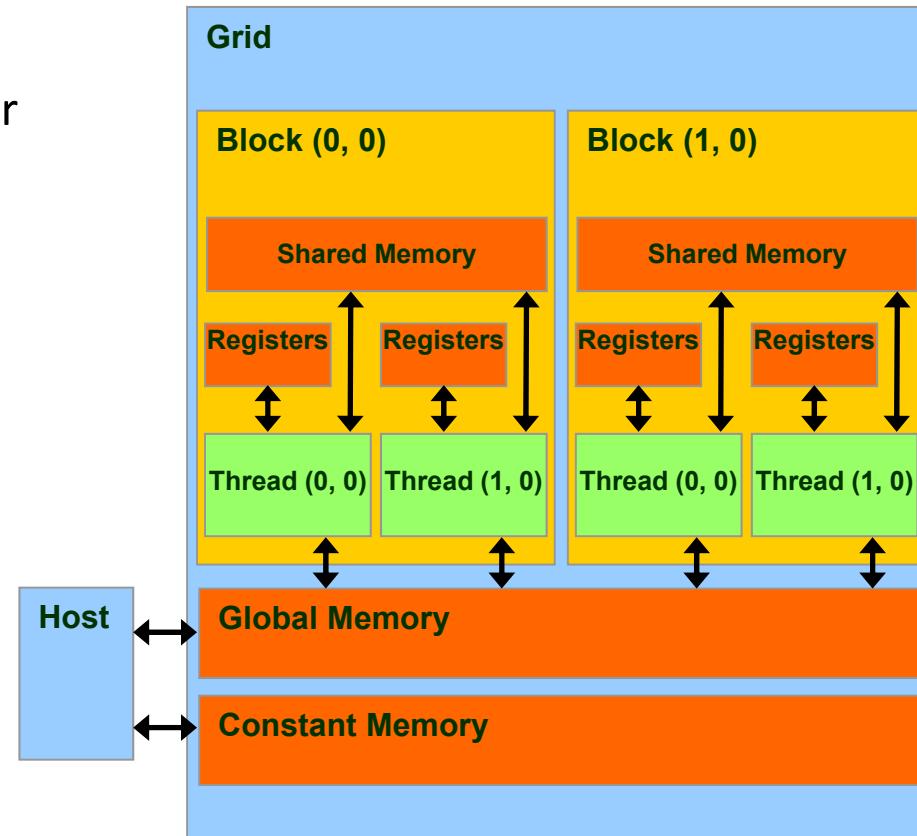
Programmer View of CUDA Memories

Host code can:

- Transfer data to/from per grid **global and constant** memories

Device code can:

- Read/write per-thread
 - **registers**
 - (~1 cycle)
- Read/write per-block
 - **shared memory**
 - (~5 cycles)
- Read/write per-grid
 - **global memory**
 - (~500 cycles)
- Read/only per-grid
 - **constant memory**
 - (~5 cycles with caching)



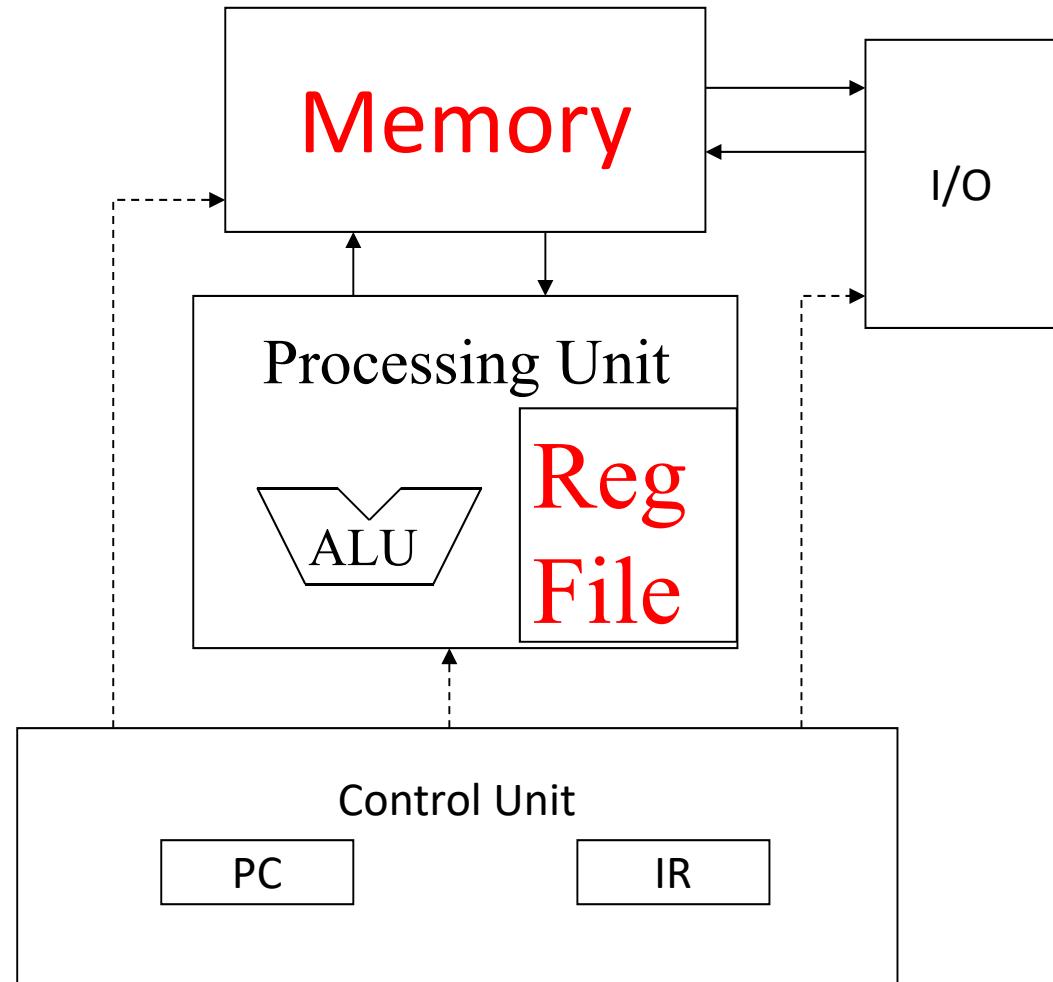
The Von-Neumann Model

Global Memory:

- Off-chip
- implies long access latencies and relatively low access bandwidth.

Register File :

- on processor chip
- very short access latency and drastically higher access bandwidth



Going back to the program

- Every instruction needs to be fetched from memory, decoded, then executed.
 - The decode stage typically accesses register file
- Instructions come in three flavors:
 - Operate, Data transfer, and Program Control Flow.
- An example instruction cycle is the following:
Fetch | Decode | Execute | Memory

Operate Instructions

- Arithmetic instructions in most modern processors have “built-in” register operands.
- Example of an operate instruction:

fadd r1, r2, r3
↓
Register numbers

- when an operand of an arithmetic instruction is in a register, no additional instruction is required to make the operand value available to ALU.

Memory Access Instructions

- if an operand value is in the global memory:

load r2, r4, offset

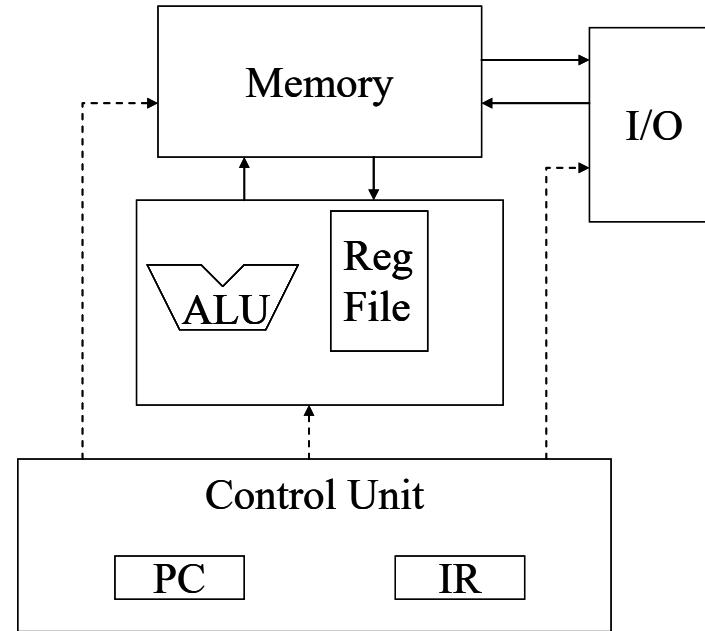
fadd r1, r2, r3



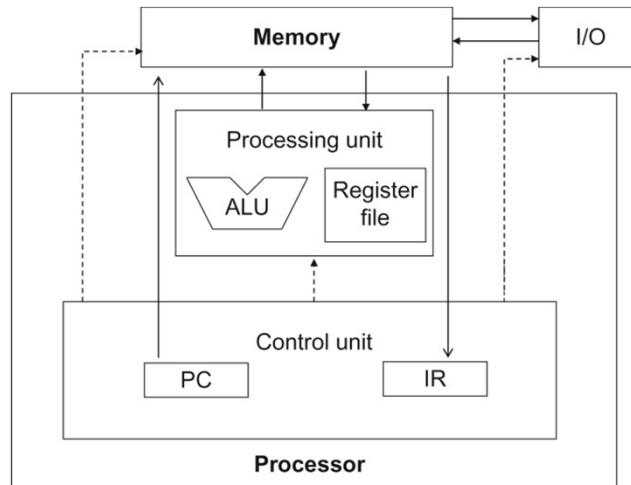
- It takes much more cost to load data from global memory than registers:
 - More clock cycles;
 - More energy;

Registers vs Memory

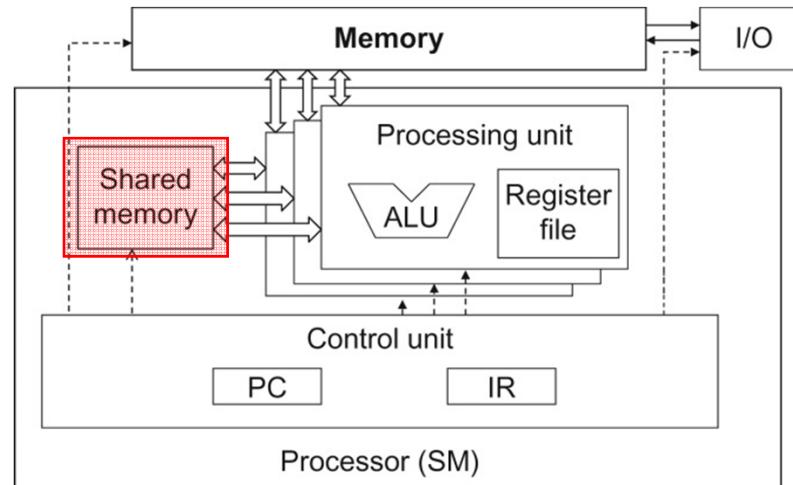
- Registers are “free”
 - No additional memory access instruction
 - Low energy consumption
 - Very fast to use, however, there are very few of them
- Memory is expensive (slow), but very large



Registers vs Memory



von Neumann model



CUDA device SM

Differences among registers, shared memory, global memory:

- Location
- Latency
- Bandwidth
- Access scope

CUDA Variable Type Qualifiers

Variable declaration	Memory	Scope	Lifetime
Automatic variables <code>int LocalVar;</code>	Register*	thread	kernel
<code>__device__ __shared__</code> <code>int SharedVar;</code>	shared	block	Kernel
<code>__device__</code> <code>int GlobalVar;</code>	global	grid	application
<code>__device__ __constant__</code> <code>int ConstantVar;</code>	constant	grid	application

- **`__device__`** is optional when used with **`__shared__`**, or **`__constant__`**
- **Automatic variables**
 - without any qualifier reside in a **register**
 - Private copy for each thread (do not exceed limit)
 - *Except per-thread arrays that reside in global memory

CUDA Variable Type Qualifiers

Variable declaration	Memory	Scope	Lifetime
<code>Automatic variables int LocalVar;</code>	Register*	thread	kernel
<code>__device__ __shared__ int SharedVar;</code>	shared	block	Kernel
<code>__device__ int GlobalVar;</code>	global	grid	application
<code>__device__ __constant__ int ConstantVar;</code>	constant	grid	application

- **Shared variables**
 - All threads in a block see the same version
 - Private copy for each block
 - Efficient means for threads in block to collaborate
 - Hold portion of global memory data that heavily used in kernel execution phase.

CUDA Variable Type Qualifiers

Variable declaration	Memory	Scope	Lifetime
<code>Automatic variables int LocalVar;</code>	Register*	thread	kernel
<code>__device__ __shared__ int SharedVar;</code>	shared	block	Kernel
<code>__device__ int GlobalVar;</code>	global	grid	application
<code>__device__ __constant__ int ConstantVar;</code>	constant	grid	application

- **Global variables**
 - Slow, but recent devices use cache to improve latency and throughput
 - Can be used for threads to collaborate across blocks,
 - Or pass information from one kernel invocation to another kernel invocation.

CUDA Variable Type Qualifiers

Variable declaration	Memory	Scope	Lifetime
<code>Automatic variables int LocalVar;</code>	Register*	thread	kernel
<code>__device__ __shared__ int SharedVar;</code>	shared	block	Kernel
<code>__device__ int GlobalVar;</code>	global	grid	application
<code>__device__ __constant__ int ConstantVar;</code>	constant	grid	application

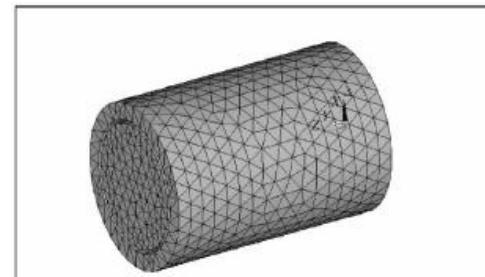
- **Constant variables**
 - In global memory but cached for efficient access.
 - Must outside any function body
 - All threads in all grids see the same version
 - Often used for variables that provide input values to kernel functions.

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A Common Programming Strategy

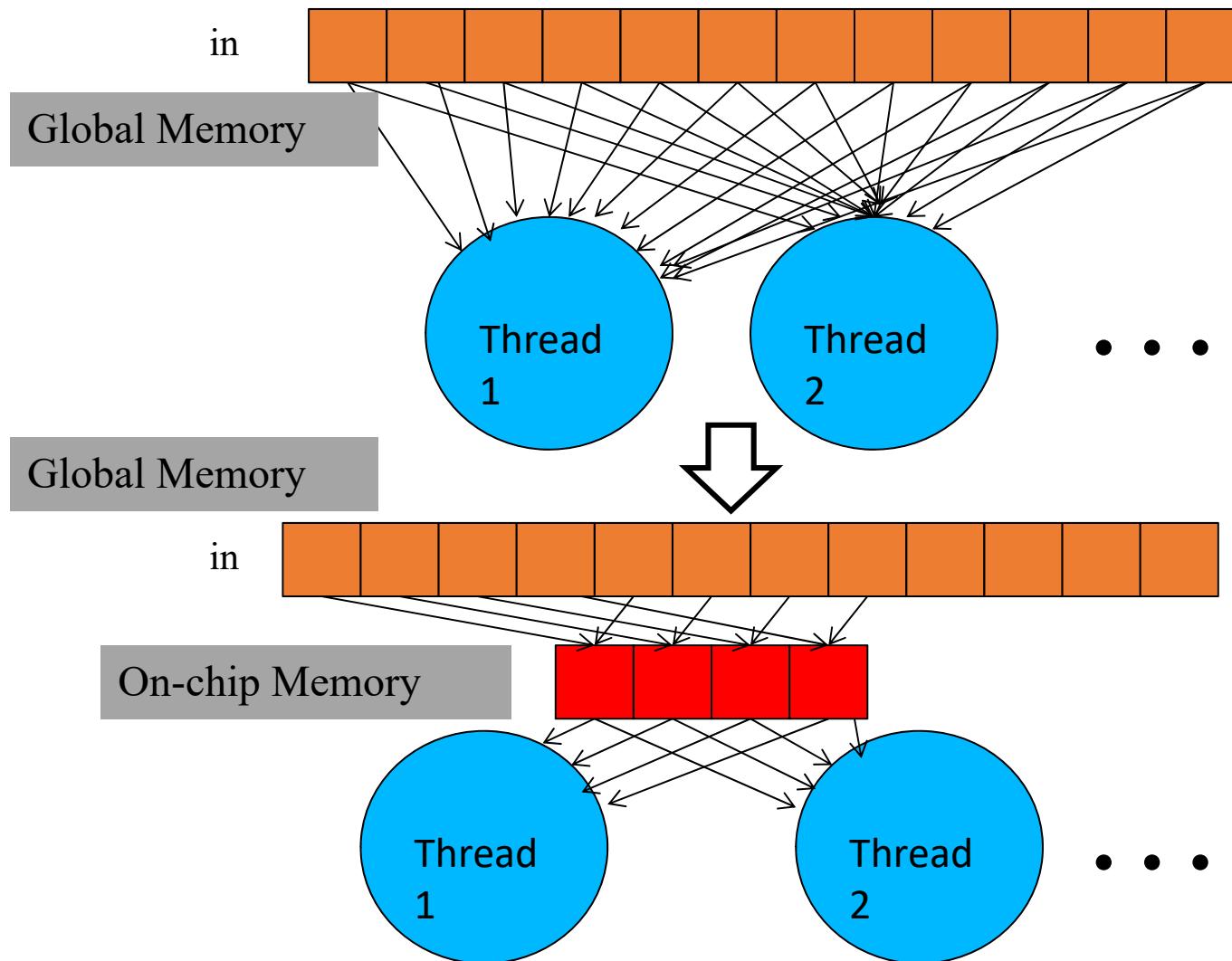
- Tradeoff of device memories in CUDA:
 - the global memory is large but slow,
 - whereas the shared memory is small but fast.
- A common strategy:
 - partition the data into subsets called *tiles*
 - so that each tile fits into the shared memory.



A Common Programming Strategy

- Tile the input data to take advantage of fast shared memory.
- Handle each data subset with one thread block by:
 - Loading the subset from global memory to shared memory, using multiple threads to exploit memory-level parallelism
 - Performing the computation on the subset from shared memory; each thread can efficiently multi-pass over any data element
 - Copying results from shared memory to global memory

Shared Memory Blocking Basic Idea



Outline of Technique

- Identify a tile of global data that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Have the multiple threads to access their data from the on-chip memory
- Move on to the next block/tile

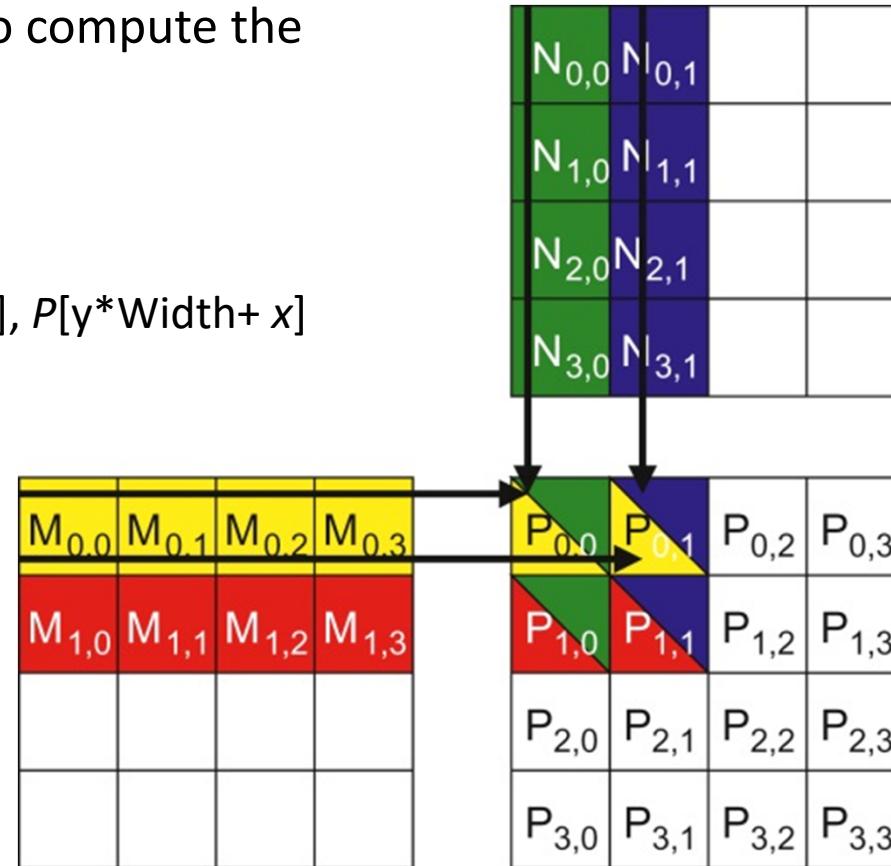
The small example

Assumes four 2×2 blocks to compute the P matrix.

For brevity:

$M[y * \text{Width} + x]$, $N[y * \text{Width} + x]$, $P[y * \text{Width} + x]$
as

$M_{y,x}$ $N_{y,x}$ $P_{y,x}$.



4 threads of block(0, 0) compute for $P_{0,0}$, $P_{0,1}$, $P_{1,0}$, and $P_{1,1}$.

Thread(0, 0) reads $M_{0,0}$ and $N_{0,0}$, followed by $M_{0,1}$ and $N_{1,0}$, ...

The small example

Memory access of Block(0,0)

Access order					
thread _{0,0}	M _{0,0} * N _{0,0}	M _{0,1} * N _{1,0}	M _{0,2} * N _{2,0}	M _{0,3} * N _{3,0}	
thread _{0,1}	M _{0,0} * N _{0,1}	M _{0,1} * N _{1,1}	M _{0,2} * N _{2,1}	M _{0,3} * N _{3,1}	
thread _{1,0}	M _{1,0} * N _{0,0}	M _{1,1} * N _{1,0}	M _{1,2} * N _{2,0}	M _{1,3} * N _{3,0}	
thread _{1,1}	M _{1,0} * N _{0,1}	M _{1,1} * N _{1,1}	M _{1,2} * N _{2,1}	M _{1,3} * N _{3,1}	

Every M and N element is accessed twice.

The potential reduction in global memory traffic is proportional to the dimension of blocks used.

If we use 16x16 blocks, traffic will be reduced to _____ ?

Compute-to-global-memory-access Ratio increased from 1 to 16,
peak performance approach 600GFLOPS.



**Tiled algorithm are highly similar to carpooling arrangement.
With extra effort.**

Carpooling requires synchronization

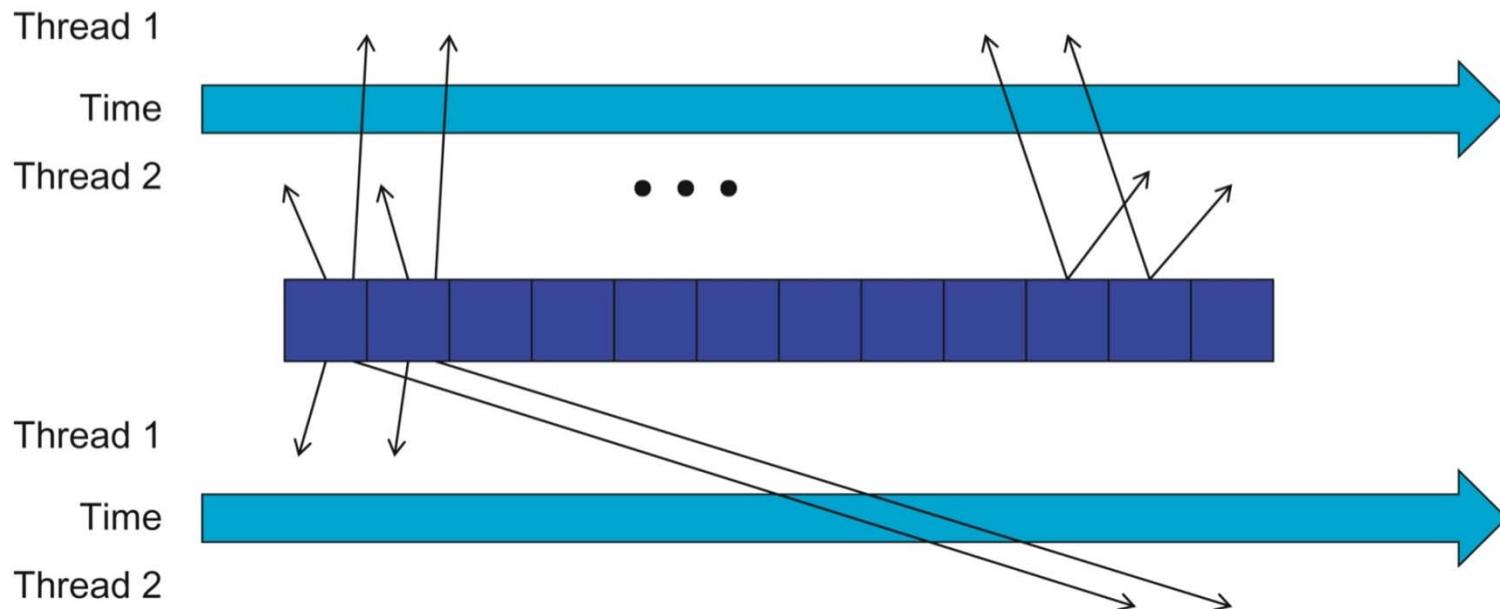
Good – people have similar schedules



Bad – people have very different schedules



Good — threads have similar access timing



Bad — threads have very different timing

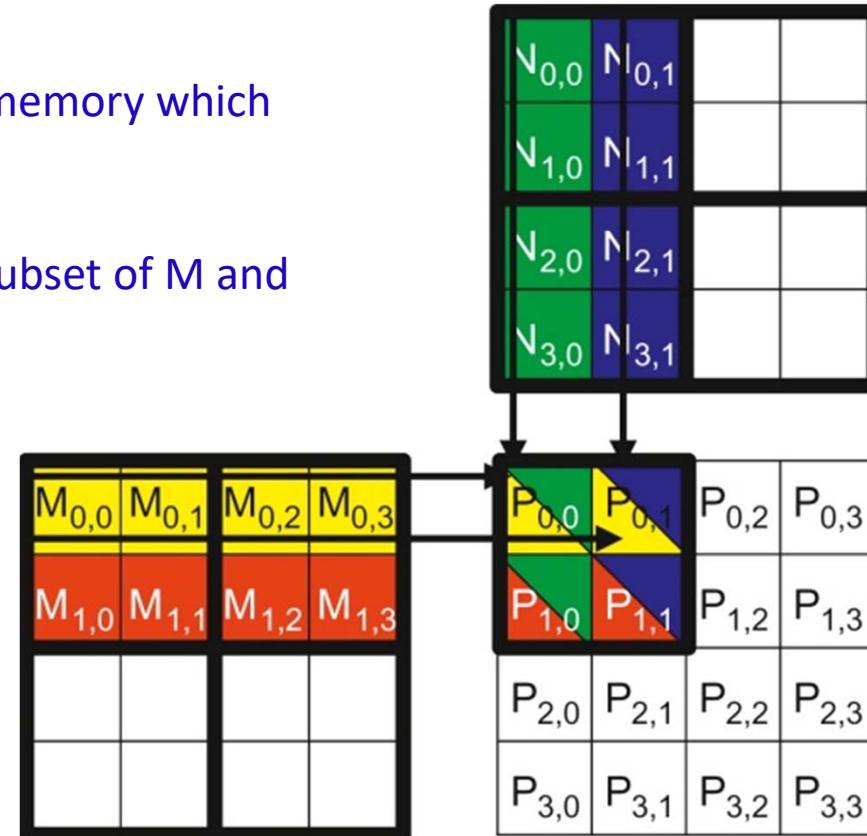
FIGURE 4.13: Tiled Algorithms require synchronization among threads.

The small example

Tiling M and N to utilize shared memory which is fast but small.

All threads collaboratively load subset of M and N before dot product.

Divide M and N into 2×2 tiles:
dot product by each thread
are now divided into phases.



In each phase, all threads in a block collaborate to load a tile of M and a tile of N into the shared memory.

Every thread in a block load one M element and one N element.

The small example

Execution phases of a tiled matrix multiplication.

	Phase 1			Phase 2		
thread _{0,0}	M_{0,0} ↓ Mds _{0,0}	N_{0,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}	M_{0,2} ↓ Mds _{0,0}	N_{2,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}
thread _{0,1}	M_{0,1} ↓ Mds _{0,1}	N_{0,1} ↓ Nds _{1,0}	PValue _{0,1} += Mds _{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}	M_{0,3} ↓ Mds _{0,1}	N_{2,1} ↓ Nds _{0,1}	PValue _{0,1} += Mds _{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}
thread _{1,0}	M_{1,0} ↓ Mds _{1,0}	N_{1,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}	M_{1,2} ↓ Mds _{1,0}	N_{3,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}
thread _{1,1}	M_{1,1} ↓ Mds _{1,1}	N_{1,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}	M_{1,3} ↓ Mds _{1,1}	N_{3,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}

time →

The shared memory array for the M elements is called Mds

Each value in the shared memory is used twice; e. g. $M_{1,1}$

locality

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width){  
1.    __shared__ float subTileM[TILE_WIDTH][TILE_WIDTH];  
2.    __shared__ float subTileN[TILE_WIDTH][TILE_WIDTH];  
3.    int bx = blockIdx.x;    int by = blockIdx.y;  
4.    int tx = threadIdx.x;  int ty = threadIdx.y;  
    // Identify the row and column of the P element to work on  
5.    int Row = by * TILE_WIDTH + ty;  
6.    int Col = bx * TILE_WIDTH + tx;  
7.    float Pvalue = 0;  
    // Loop over the M and N tiles required to compute the P element  
8.    for (int m = 0; m < Width/TILE_WIDTH; ++m) {  
        // Collaborative loading of M and N tiles into shared memory  
9.        subTileM[ty][tx] = M[Row*Width + m*TILE_WIDTH+tx];  
10.       subTileN[ty][tx] = N[(m*TILE_WIDTH+ty)*Width+Col];  
11.       __syncthreads();  
12.       for (int k = 0; k < TILE_WIDTH; ++k)  
13.           Pvalue += subTileM[ty][k] * subTileN[k][tx];  
14.       __syncthreads();  
15.    }  
16.    P[Row*Width+Col] = Pvalue;  
}
```

Loading an Input Tile 0

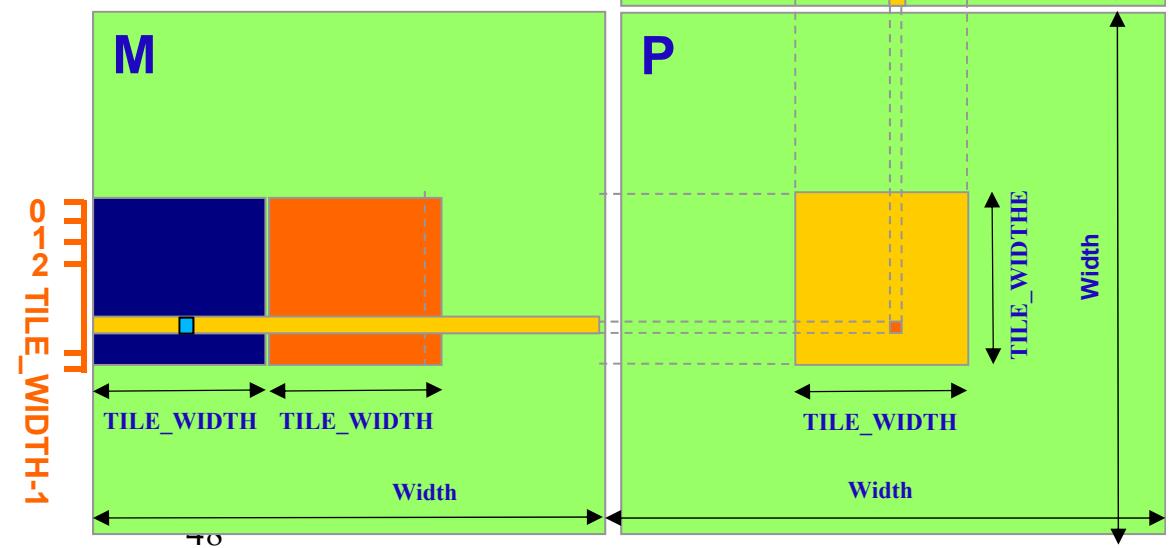
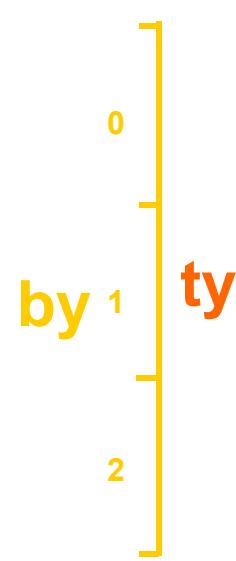


tx
0 1 2
TILE_WIDTH-1

Accessing tile 0 2D indexing:

M[Row][tx]

N[ty][Col]



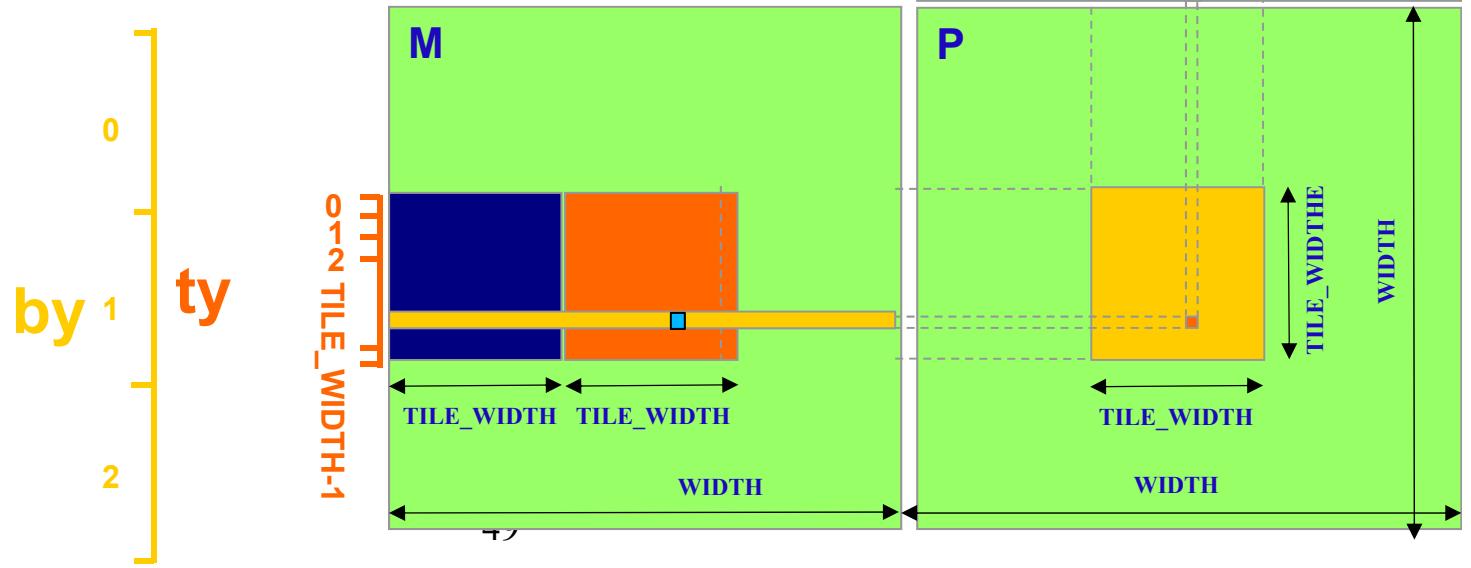
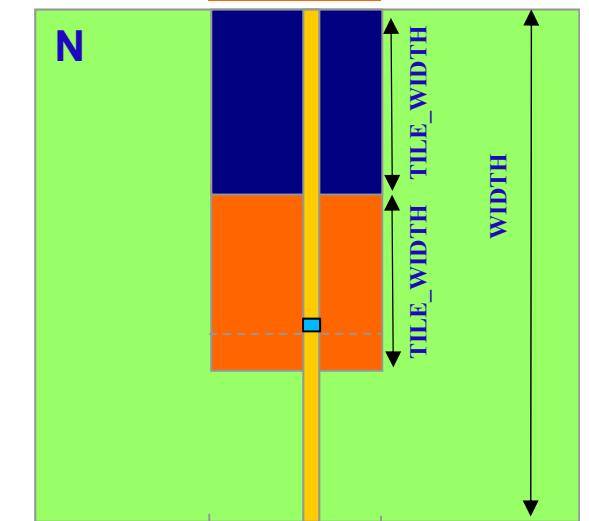
Loading an Input Tile 1



tx
012 $TILE_WIDTH-1$

Accessing tile 1 in 2D indexing:

$M[Row][1*TILE_WIDTH+tx]$
 $N[1*TILE_WIDTH+ty][Col]$



ty
0
1
2

Loading an Input Tile m



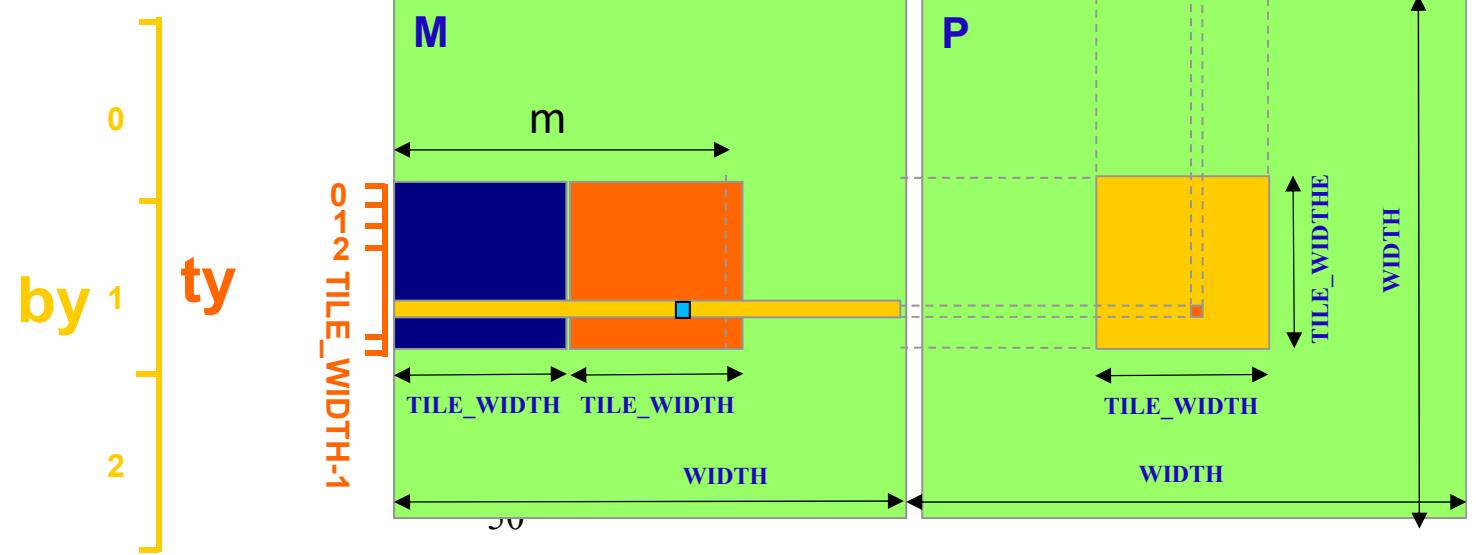
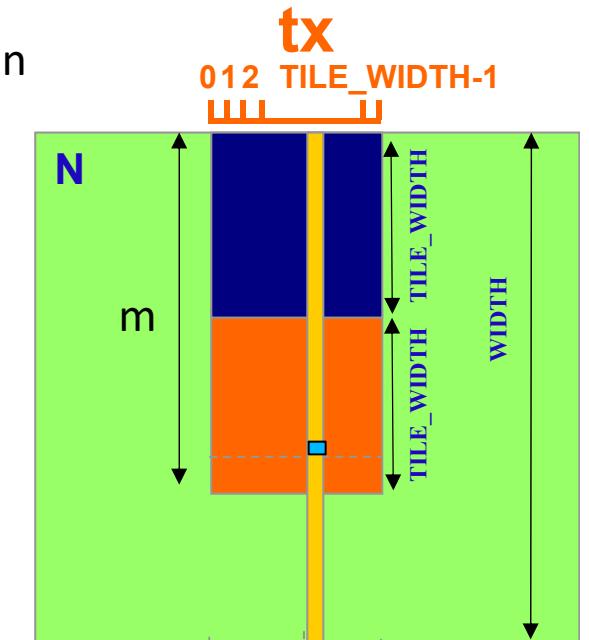
However, recall that M and N are dynamically allocated and can only use 1D indexing:

$M[\text{Row}][m * \text{TILE_WIDTH} + tx]$

$M[\text{Row} * \text{Width} + m * \text{TILE_WIDTH} + tx]$

$N[m * \text{TILE_WIDTH} + ty][\text{Col}]$

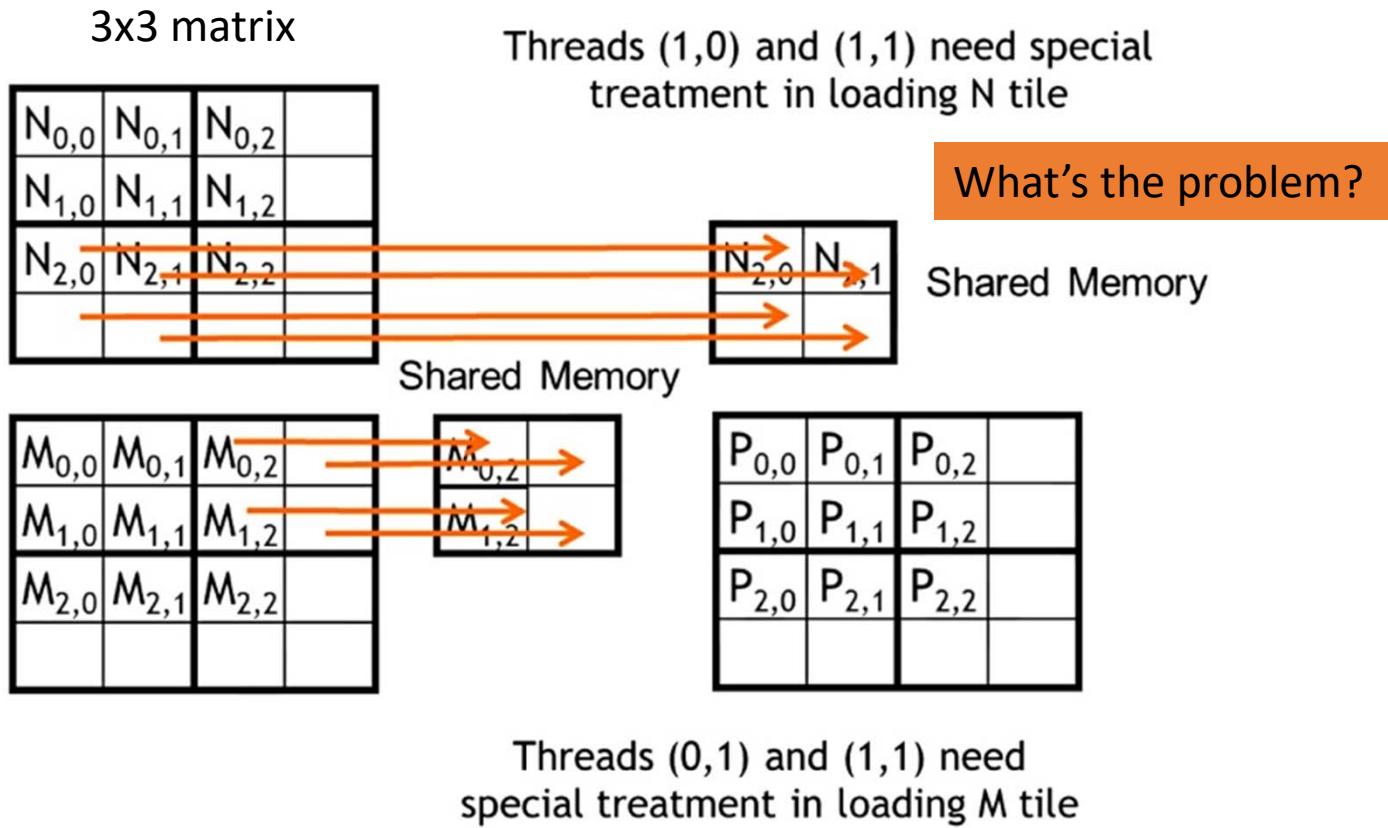
$N[(m * \text{TILE_WIDTH} + ty) * \text{Width} + \text{Col}]$



Barrier Synchronization

- An API function call in CUDA
 - `__syncthreads()`
- All threads in the same block must reach the `__syncthreads()` before any can move on
- Best used to coordinate tiled algorithms
 - To ensure that all elements of a tile are loaded
 - To ensure that all elements of a tile are consumed

Boundary Checks



Loading input matrix elements that are close to the edge— phase 1 of Block0,0.

Boundary Checks

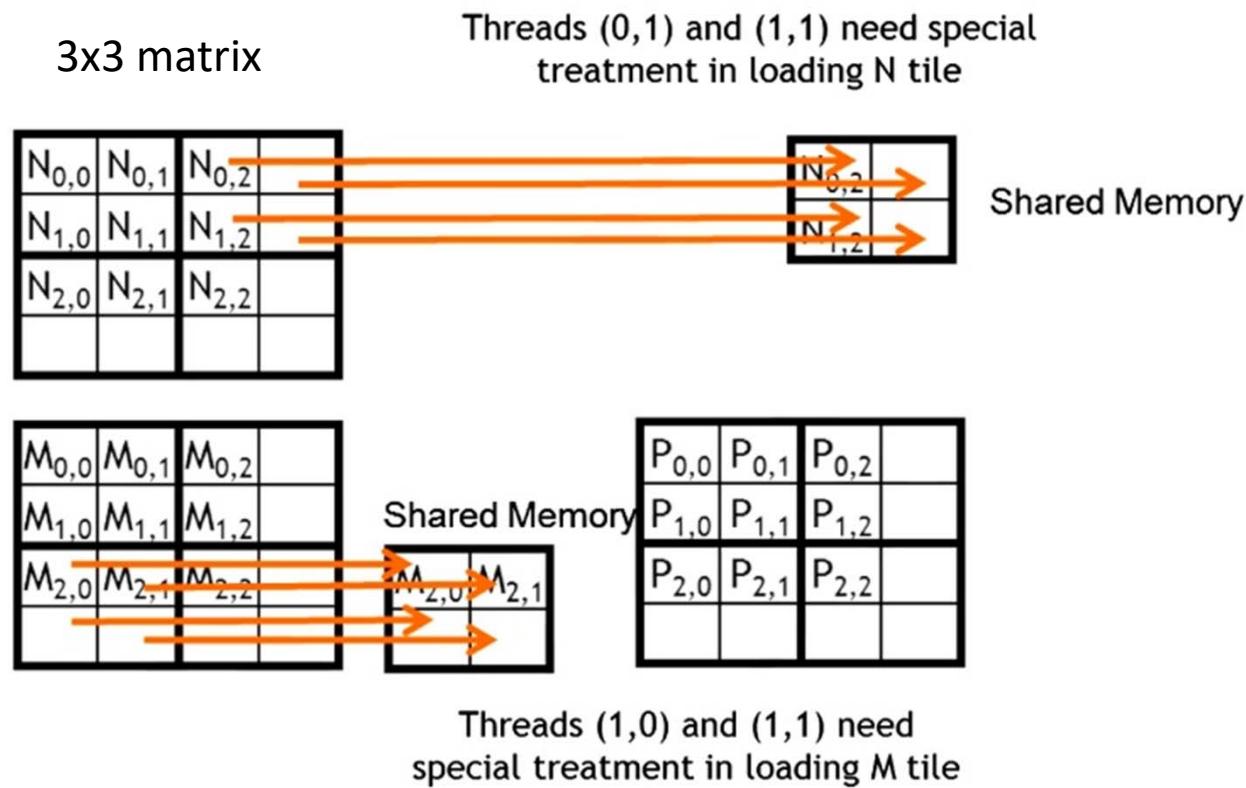


FIGURE 4.19: Loading input elements during phase 0 of block1,0.

Boundary Checks

```
// Loop over the M and N tiles required to compute P element
8. for (int ph = 0; ph < ceil(Width/(float)TILE_WIDTH); ++ph) {
    // Collaborative loading of M and N tiles into shared memory
9. if ((Row< Width) && (ph*TILE_WIDTH+tx)< width)
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH + tx];
10. if ((ph*TILE_WIDTH+ty)<width && Col<Width)
    Nds[ty][tx] = N[(ph*TILE_WIDTH + ty)*Width + Col];
11. __syncthreads();
12. for (int k = 0; k < TILE_WIDTH; ++k) {
13.     Pvalue += Mds[ty][k] * Nds[k][tx];
    }
14. __syncthreads();
}
15. if ((Row<Width) && (Col<Width)) P[Row*Width + Col] = Pvalue;
```

Tiled matrix multiplication kernel with boundary condition checks.

Summary

- Importance of Memory Access Efficiency
 - --Matrix Multiplication
- Memory Types Overview
- Tiling for reduced memory traffic
 - --Matrix Multiplication using Shared Memory