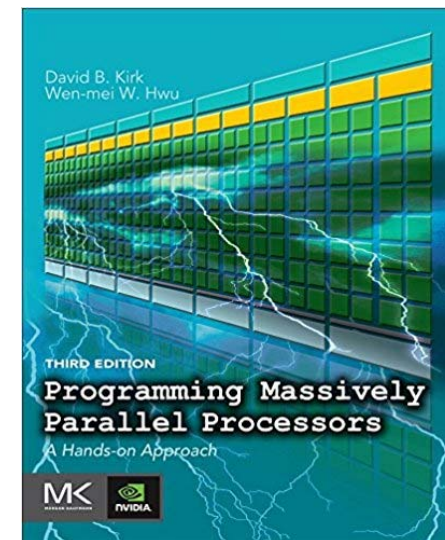


# Introduction to CUDA

## *(6) Parallel Pattern: Convolution*

# Reference

- [CUDA C Programming Guide](https://docs.nvidia.com/cuda/cuda-c-programming-guide/index.html),
  - <https://docs.nvidia.com/cuda/cuda-c-programming-guide/index.html>
- **Programming Massively Parallel Processors,**
  - A Hands-on Approach
  - Third Edition
  - Chapter 7



# Content

- **Convolution**
  - **An important parallel computation pattern**
- Taking advance of
  - Constant memory and caching
- Tiled 1D convolution algorithms
  - Addressing memory bandwidth issue in accessing  $N$
  - Algorithms and analysis
- Tiled 2D convolution with halo cells
  - Algorithms and analysis

# Convolution Applications

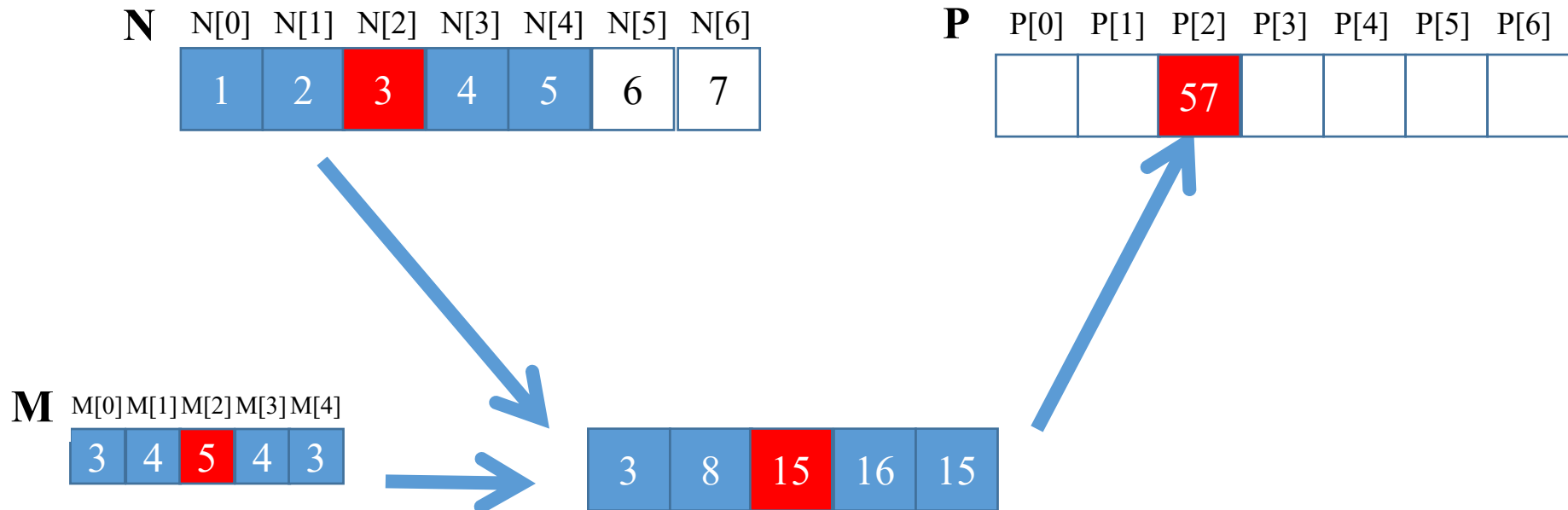
- A popular array operation that is used in various forms in **signal processing, digital recording, image processing, video processing, and computer vision.**
- Convolution is often performed as a **filter** that transforms signals and pixels into more desirable values.
  - Some filters smooth out the signal values so that one can see the big-picture trend.
  - Others like Gaussian filters can be used to sharpen boundaries and edges of objects in images.

# Convolution Computation

- An array operation where each output data element is a weighted sum of a collection of neighboring input elements
- The weights used in the weighted sum calculation are defined by an input mask array, commonly referred to as the *convolution kernel*
  - refer to these mask arrays as convolution masks to avoid confusion( with kernel in CUDA).
  - The same convolution mask is typically used for all elements of the array.

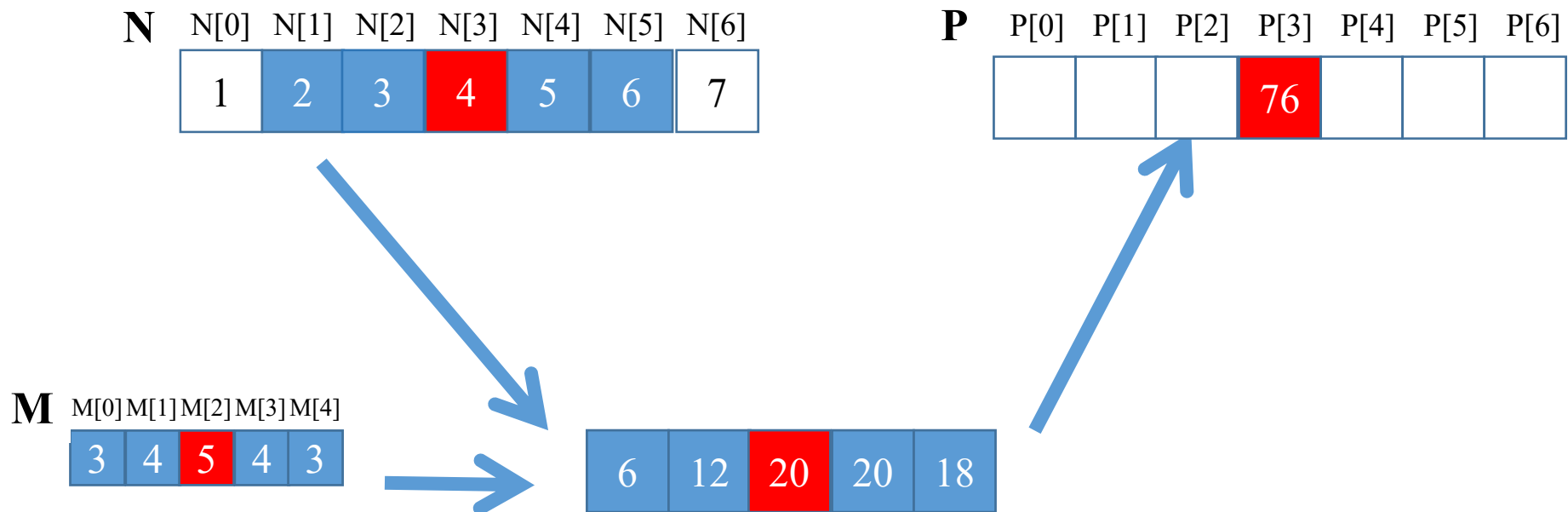
# 1D Convolution Example

- Commonly used for audio processing
  - Mask size is usually an odd number of elements for symmetry (5 in this example)
- Calculation of P[2]



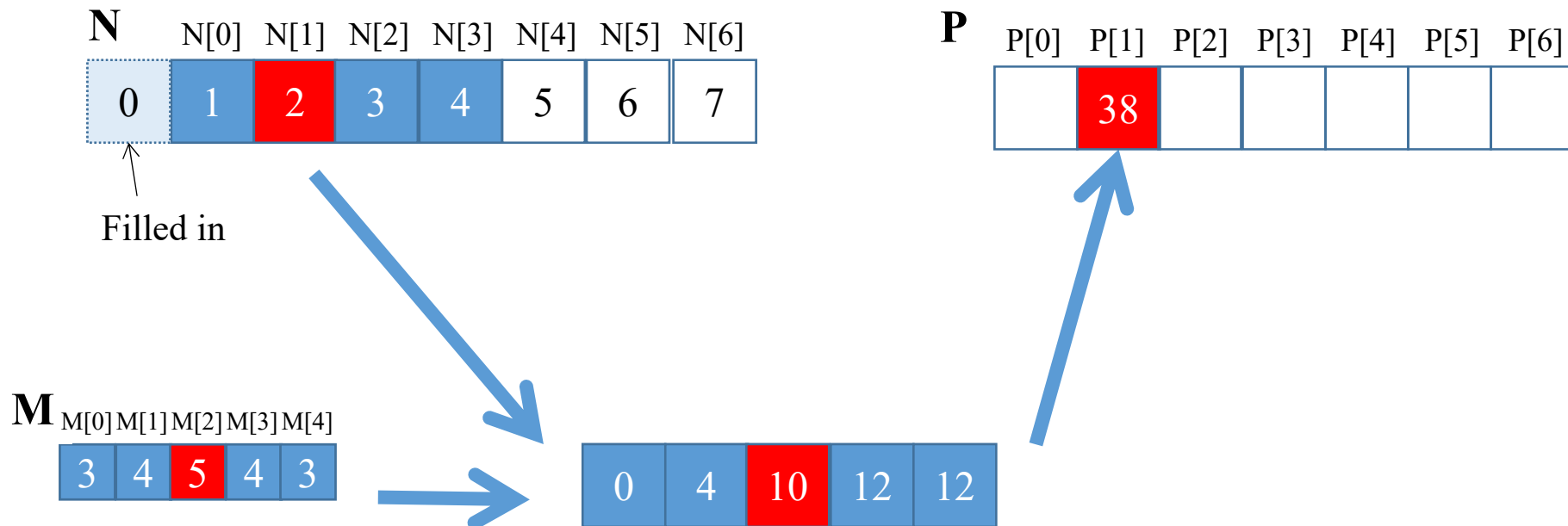
# 1D Convolution Example

- more on inside elements
- Calculation of P[3]



# 1D Convolution Boundary Condition

- Calculation of output elements near the boundaries (beginning and end) of the input array need to deal with “ghost” elements
  - Different policies (0, replicates of boundary values, etc.)





# A 1D Convolution Kernel with Boundary Condition Handling

- This kernel forces all elements outside the valid data index range to 0

```
__global__ void convolution_1D_basic_kernel( float *N, float *M, float *P,
int Mask_Width, int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

    float Pvalue = 0;
    int N_start_point = i - (Mask_Width/2);

    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width) {
            Pvalue += N[N_start_point + j] * M[j];
        }
    }

    P[i] = Pvalue;
}
```

# 2D Convolution

**N**

1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	5	6
5	6	7	8	5	6	7
6	7	8	9	0	1	2
7	8	9	0	1	2	3

**P**

		321				

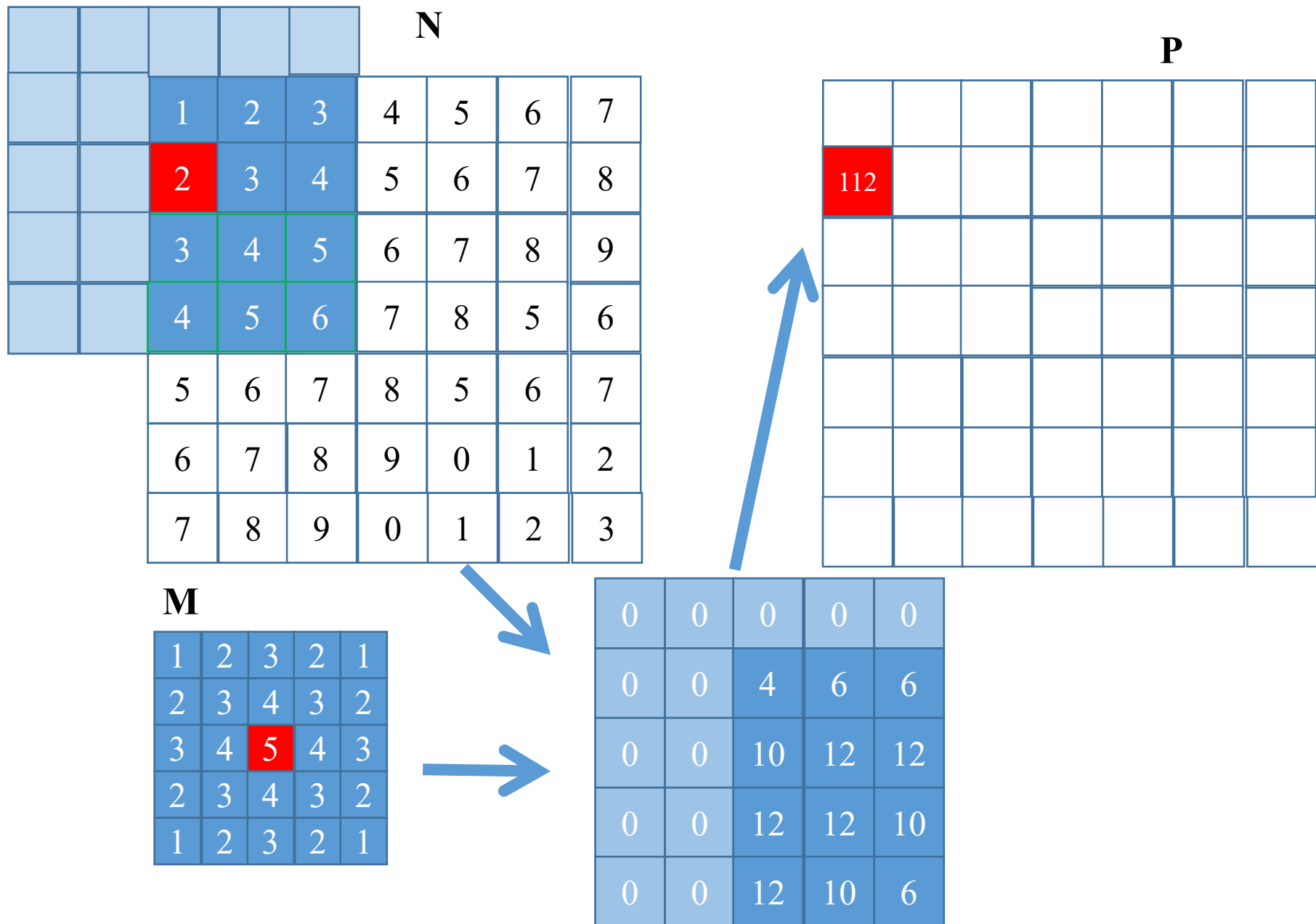
**M**

1	2	3	2	1
2	3	4	3	2
3	4	5	4	3
2	3	4	3	2
1	2	3	2	1

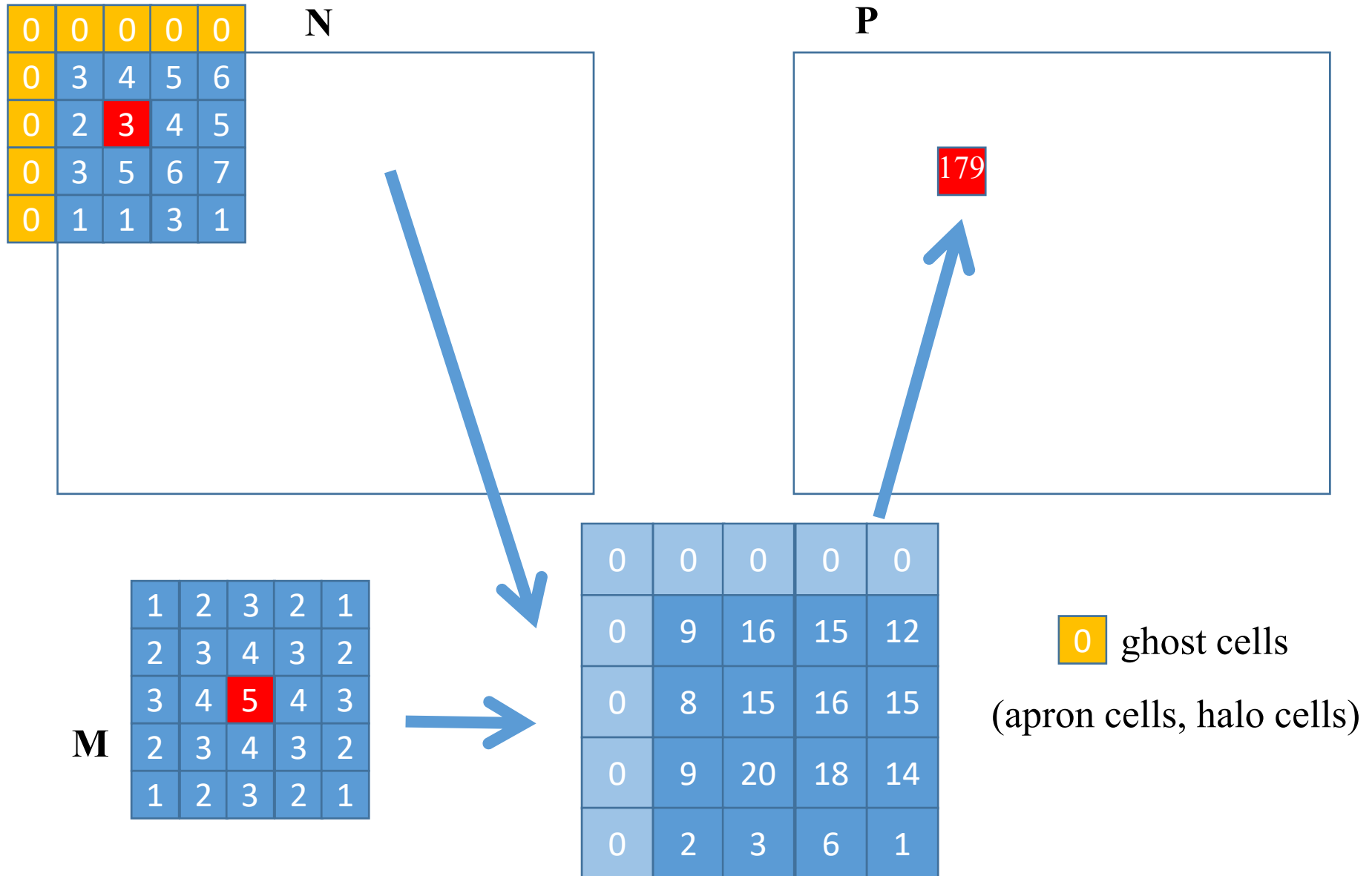


1	4	9	8	5
4	9	16	15	12
9	16	25	24	21
8	15	24	21	16
5	12	21	16	5

# 2D Convolution Boundary Condition



# 2D Convolution – Ghost Cells



# Content

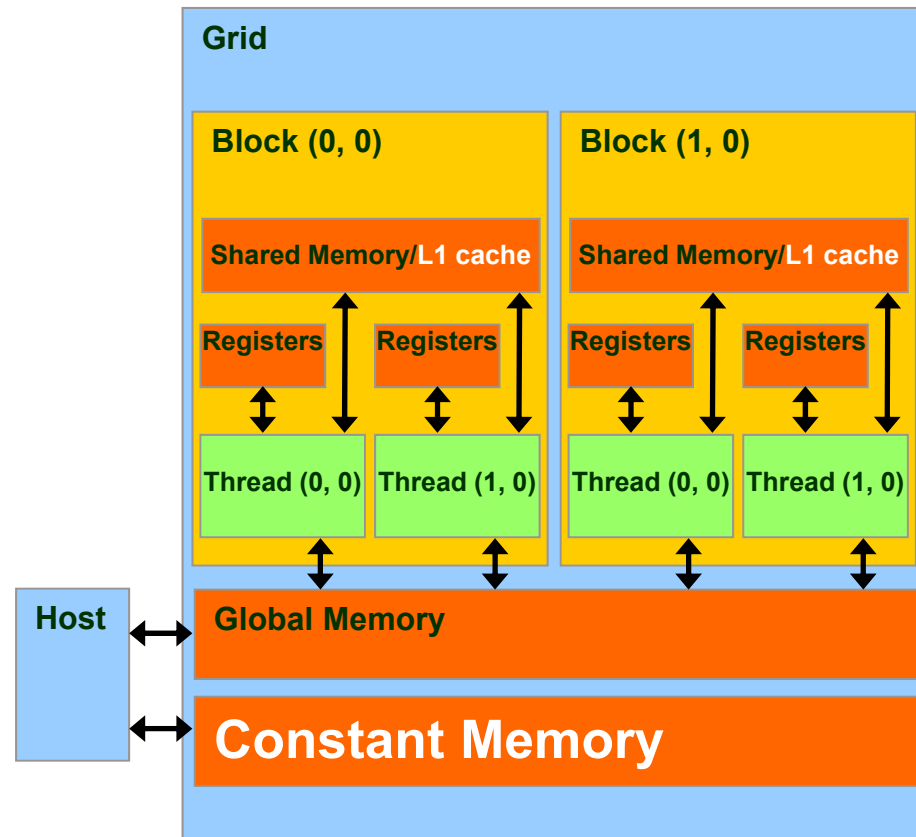
- Convolution
  - An important parallel computation pattern
- **Taking advance of**
  - **Constant memory and caching**
- Tiled 1D convolution algorithms
  - Addressing memory bandwidth issue in accessing  $N$
  - Algorithms and analysis
- Tiled 2D convolution with halo cells
  - Algorithms and analysis

# Access Pattern for M

- M is referred to as mask (a.k.a. kernel, filter, etc.)
  - Elements of M are called mask (kernel, filter) coefficients
- M features:
  - M is small in size
  - All threads need to access mask elements
  - M is not changed during kernel
- Bonus - M elements are accessed in the same order when calculating all P elements
- M is a good candidate for Constant Memory

# Programmer View of CUDA Memories (Review)

- Each thread can:
  - **Read/write per-thread registers (~1 cycle)**
  - Read/write per-block **shared memory (~5 cycles)**
  - Read/write per-grid **global memory (~500 cycles)**
- **Read/only per-grid constant memory (~5 cycles with caching)**



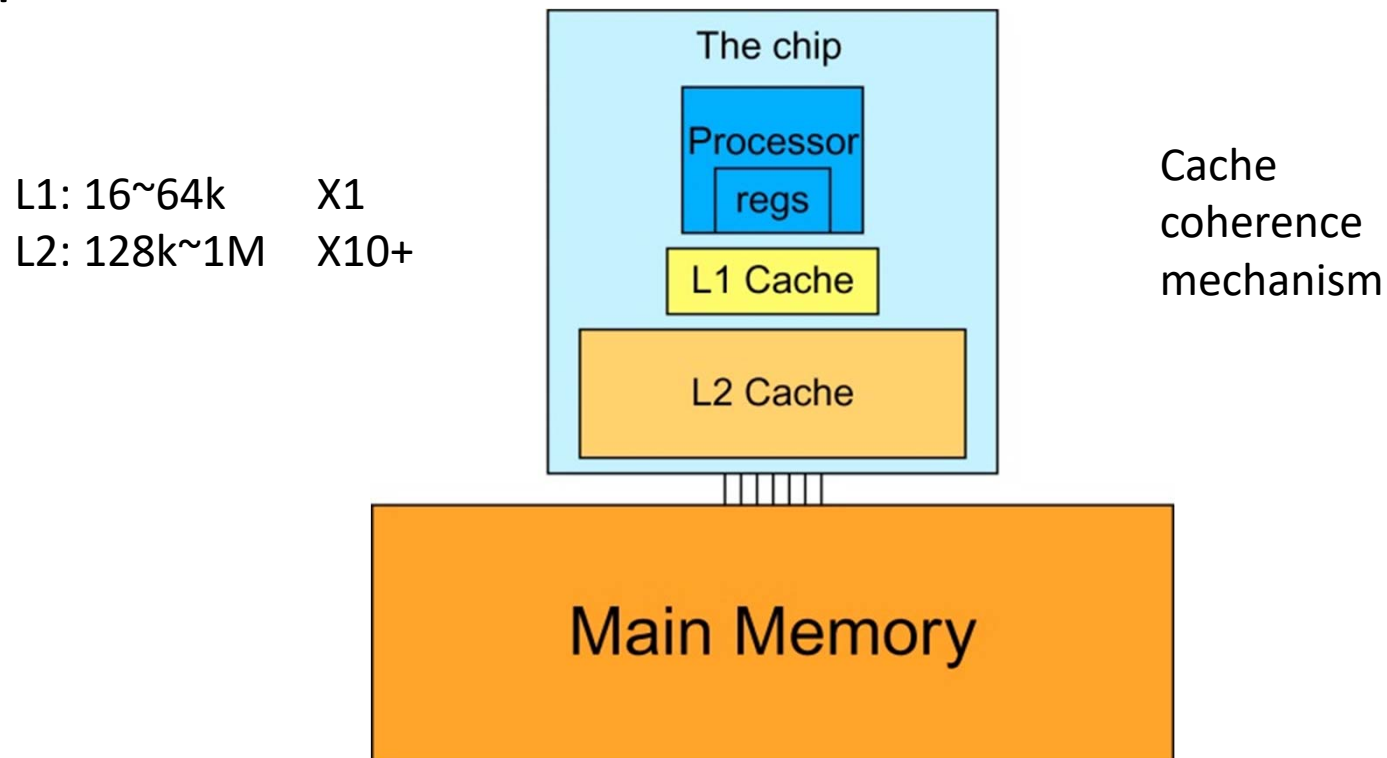
# Memory Hierarchies

- If we had to go to global memory (DRAM) to access data all the time, the execution speed of GPUs would be limited by the global memory bandwidth
- But Constant memory variables also located in DRAM which is known as slow to access.
- How can we benefit from constant memory?
- One solution: **Caches**



# Memory Hierarchies

- Simplified view of the cache hierarchy of modern processors:



# Cache

- A cache is an “array” of cache lines
  - A cache line can usually hold data from several consecutive memory addresses
- When data is requested from the global memory:
  - an entire cache line that includes the data being accessed is loaded into the cache, in an attempt to reduce global memory requests;
  - The data in the cache is a “**copy**” of the original data in global memory;

# Caches - Cont'd

Some definitions:

- ***Spatial locality***: when the data elements stored in consecutive memory locations are access consecutively
- ***Temporal locality***: when the same data element is access multiple times in short period of time
- Both spatial locality and temporal locality improve the performance of caches

# Scratchpad vs. Cache

- Scratchpad (**shared memory** in CUDA) is another type of temporary storage used to relieve main memory contention.
  - In terms of distance from the processor, scratchpad is **similar to L1 cache**.
- Unlike cache, scratchpad does not necessarily hold a copy of data that is also in main memory
  - Scratchpad **requires explicit data transfer** instructions into locations in the scratchpad , whereas cache doesn't

# Constant Cache in GPUs

- Modification to cached data needs to be (eventually) reflected back to the original data in global memory
  - Requires logic to track the modified status, etc.
- **Constant cache** is a special cache for constant data:
  - Data declared in the constant memory will not be modified during kernel execution.
  - Constant cache can be accessed with higher throughput than L1 cache for some common patterns

# How to Use Constant Memory

- Host code allocates, initializes variables the same way as any other variables that need to be copied to the device
- Use `cudaMemcpyToSymbol(dest, src, size)` to copy the variable into the device memory
- This copy function tells the device that **the variable will not be modified by the kernel and can be safely cached.**

# A 1D Convolution Kernel using constant memory

- This kernel forces all elements outside the valid data index range to 0

```
__global__ void convolution_1D_basic_kernel( float *N, float *P,
int Mask_Width, int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

    float Pvalue = 0;
    int N_start_point = i - (Mask_Width/2);
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width) {
            Pvalue += N[N_start_point + j] * M[j];
        }
    }
    P[i] = Pvalue;
}
```

# Host Code

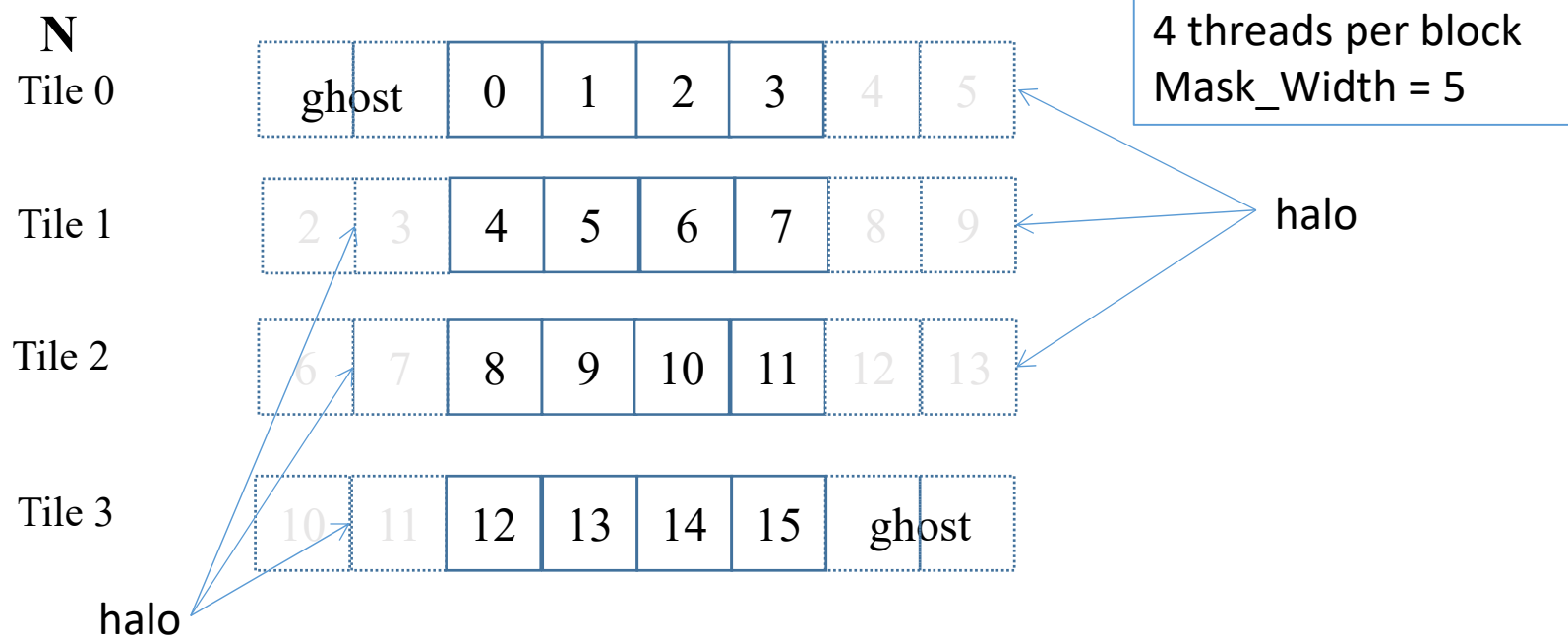
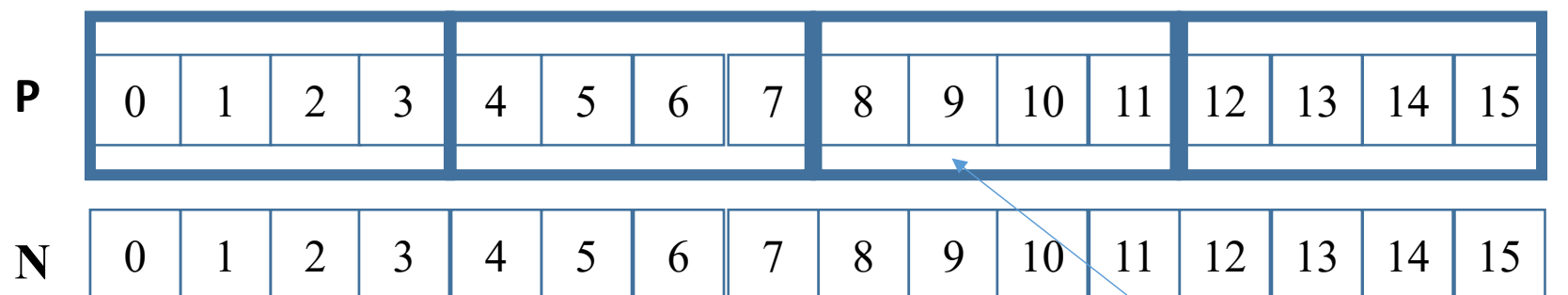
```
// global variable, outside any kernel/function
__constant__ float Mc[MASK_WIDTH][MASK_WIDTH];
...
// allocate N, P, initialize N elements, copy N to Nd
Matrix M;
M = AllocateMatrix(MASK_WIDTH, MASK_WIDTH, 1);
// initialize M elements
...
cudaMemcpyToSymbol(Mc, M.elements,
    MASK_WIDTH*MASK_WIDTH*sizeof(float));
ConvolutionKernel<<<dimGrid, dimBlock>>>(Nd, Pd);
```



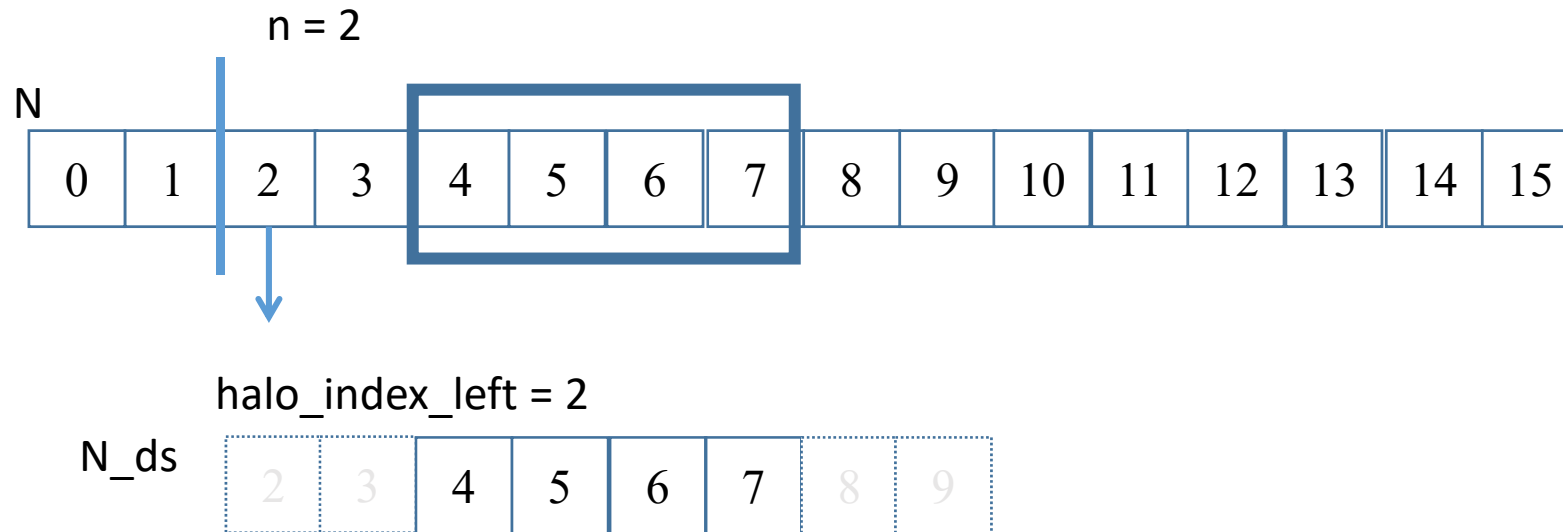
# Content

- Convolution
  - An important parallel computation pattern
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- **Tiled 1D convolution algorithms**
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# Tiled 1D Convolution Basic Idea



# Loading the left halo

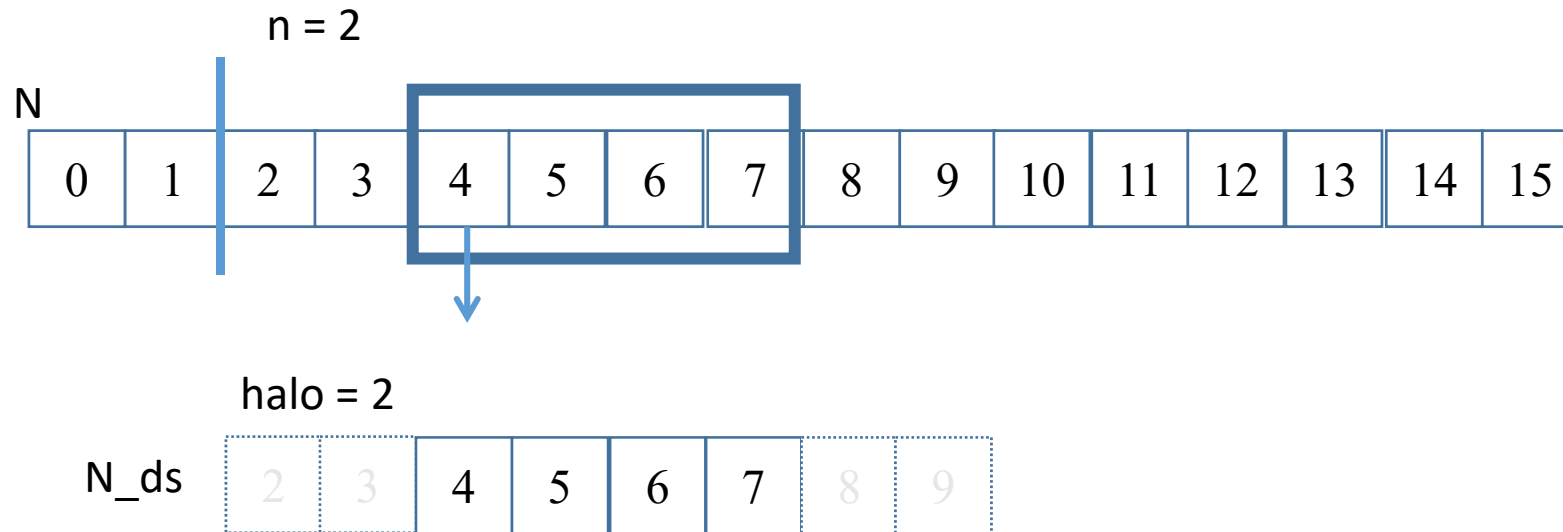


```
int n = Mask_Width/2;
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
if (threadIdx.x >= blockDim.x - n) {
    N_ds[threadIdx.x - (blockDim.x - n)] =
        (halo_index_left < 0) ? 0 : N[halo_index_left];
}
```

Previous tile

Last n

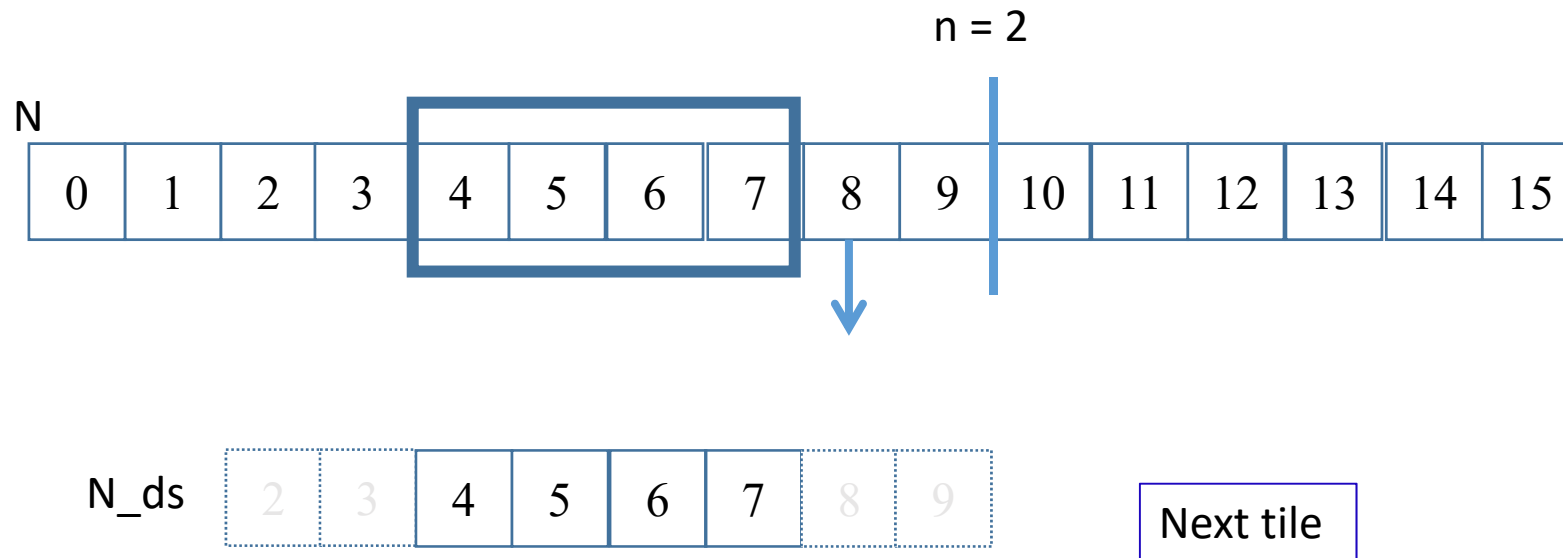
# Loading the internal elements



```
N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];
```

Shift  $n$ , number of left halo

# Loading the right halo



```
int halo_index_right = (blockIdx.x+1)*blockDim.x + threadIdx.x;
if (threadIdx.x < n) {
    N_ds[n + blockDim.x + threadIdx.x] =
        (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```

```

__global__ void convolution_1D_tiled_kernel(float *N, float *P, int Mask_Width,
int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;
    __shared__ float  N_ds[TILE_SIZE + MAX_MASK_WIDTH - 1];

    int n = Mask_Width/2;

    int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x >= blockDim.x - n) {
        N_ds[threadIdx.x - (blockDim.x - n)] =
            (halo_index_left < 0) ? 0 : N[halo_index_left];
    }

    N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];

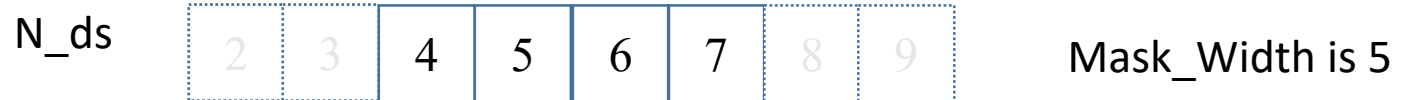
    int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x < n) {
        N_ds[n + blockDim.x + threadIdx.x] =
            (halo_index_right >= Width) ? 0 : N[halo_index_right];
    }

    __syncthreads();

    float Pvalue = 0;
    for(int j = 0; j < Mask_Width; j++) {
        Pvalue += N_ds[threadIdx.x + j]*M[j];
    }
    P[i] = Pvalue;
}

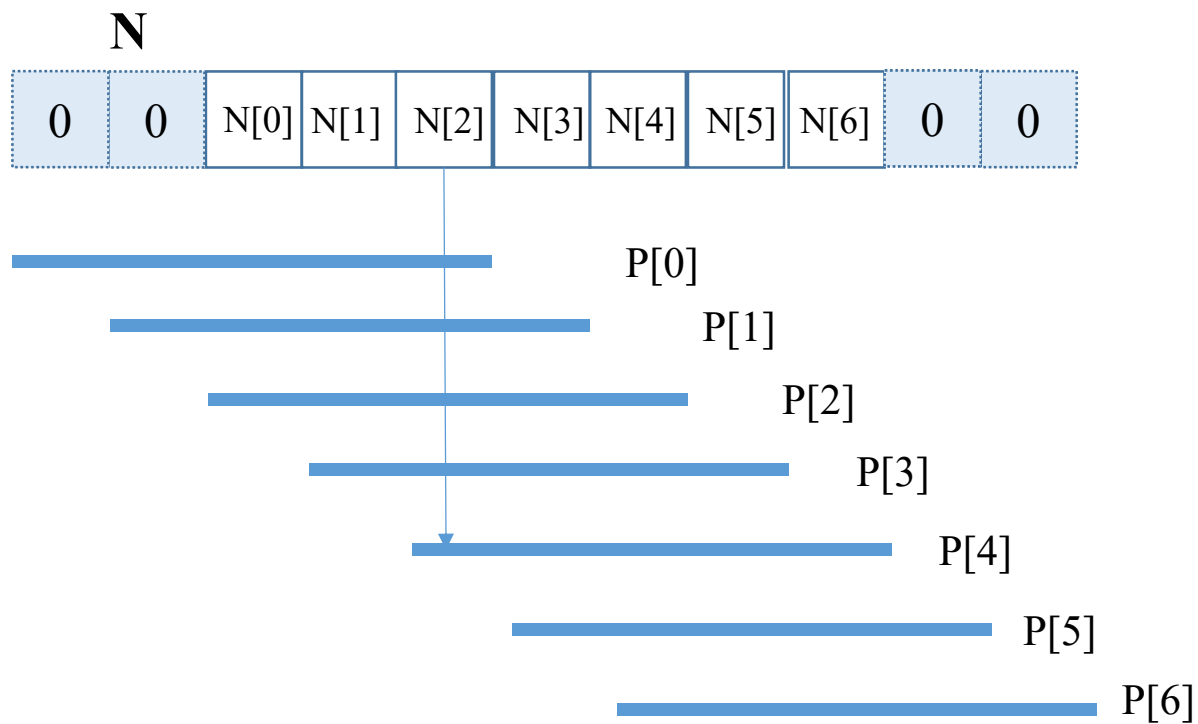
```

# Shared Memory Data Reuse



- Element 2 is used by thread 4 (1X)
- Element 3 is used by threads 4, 5 (2X)
- Element 4 is used by threads 4, 5, 6 (3X)
- Element 5 is used by threads 4, 5, 6, 7 (4X)
- Element 6 is used by threads 4, 5, 6, 7 (4X)
- Element 7 is used by threads 5, 6, 7 (3X)
- Element 8 is used by threads 6, 7 (2X)
- Element 9 is used by thread 7 (1X)

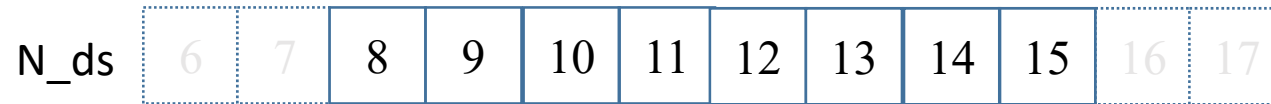
# Shared Memory Data Reuse



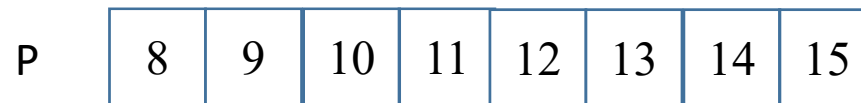


# A Small 1D Example

TILE\_SIZE = 8, Mask\_Width=5



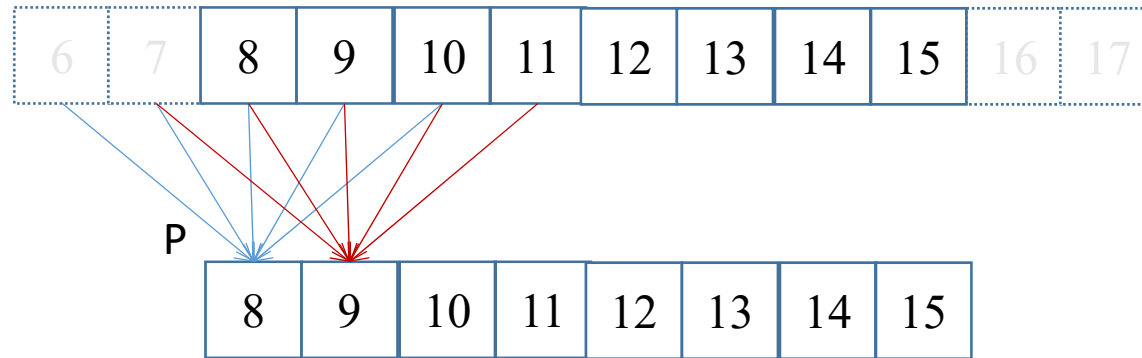
Mask\_Width is 5



- output and input tiles for block 1
- For Mask\_Width = 5, each block loads  $8+5-1 = 12$  elements (12 memory loads)

# Each output P element uses 5 N elements (in N\_ds)

N\_ds



Mask\_Width is 5

- $P[8]$  uses  $N[6]$ ,  $N[7]$ ,  $N[8]$ ,  $N[9]$ ,  $N[10]$
- $P[9]$  uses  $N[7]$ ,  $N[8]$ ,  $N[9]$ ,  $N[10]$ ,  $N[11]$
- $P[10]$  uses  $N[8]$ ,  $N[9]$ ,  $N[10]$ ,  $N[11]$ ,  $N[12]$
- ...
- $P[14]$  uses  $N[12]$ ,  $N[13]$ ,  $N[14]$ ,  $N[15]$ ,  $N[16]$
- $P[15]$  uses  $N[13]$ ,  $N[14]$ ,  $N[15]$ ,  $N[16]$ ,  $N[17]$

A Total of  $8 * 5$  N elements are used for the output tile.

# A simple way to calculate tiling benefit

- For internal tiles:
  - $(8+5-1)=12$  elements loaded
  - $8*5$  global memory accesses replaced by shared memory accesses
  - This gives a bandwidth reduction of  $40/12=3.3$
- For a **boundary tile**:
  - $(8+(5-1)/2)=10$  elements loaded
  - Total accesses is  $40-3= 37$

# In General for 1D, internal tiles

- The total number of global memory accesses to the  $(\text{TILE\_SIZE} + \text{Mask\_Width} - 1)$  N elements replaced by shared memory accesses is

$$\begin{aligned} & 1 + 2 + \dots + \text{Mask\_Width} - 1 + \text{Mask\_Width} * (\text{TILE\_SIZE} - \text{Mask\_Width} + 1) + \\ & \quad \text{Mask\_Width} - 1 + \dots + 2 + 1 \\ &= ((\text{Mask\_Width} - 1) * \text{Mask\_Width}) / 2 + \text{Mask\_Width} * (\text{TILE\_SIZE} - \text{Mask\_Width} + 1) + \\ & \quad ((\text{Mask\_Width} - 1) * \text{Mask\_Width}) / 2 \\ &= (\text{Mask\_Width} - 1) * \text{Mask\_Width} + \text{Mask\_Width} * (\text{TILE\_SIZE} - \text{Mask\_Width} + 1) \\ &= \text{Mask\_Width} * (\text{TILE\_SIZE}) \end{aligned}$$

In General, for 1D convolution kernel

- For internal thread blocks, the ratio of memory accesses between the basic and the tiled 1D kernel:

$$(\text{blockDim.x} * (2n+1)) / (\text{blockDim.x} + 2n)$$

- whereas the ratio for boundary blocks is:

$$(\text{blockDim.x} * (2n+1) - n(n+1)/2) / (\text{blockDim.x} + n)$$

- For most situations, blockDim.x is much larger than n.

$$\begin{aligned} & (\text{blockDim.x} * (2n+1) / \text{blockDim.x} \\ & = 2n+1 = \text{Mask\_Width} \end{aligned}$$

# Bandwidth Reduction for 1D

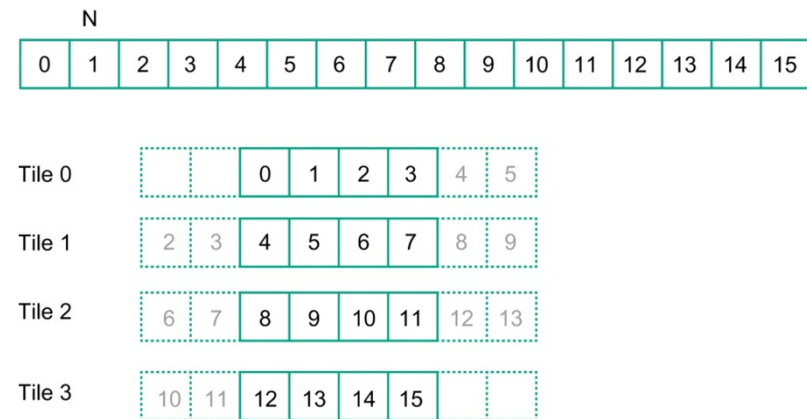
- The reduction is

$$\text{Mask\_Width} * (\text{TILE\_SIZE}) / (\text{TILE\_SIZE} + \text{Mask\_Width} - 1)$$

TILE_SIZE	16	32	64	128	256
Reduction Mask_Width = 5	4.0	4.4	4.7	4.9	4.9
Reduction Mask_Width = 9	6.0	7.2	8.0	8.5	8.7

# Simpler Tiled 1D convolution

- Using general caching:
  - Modern GPU like Fermi has L1 and L2 caches:
  - L1 is private to each SM
  - L2 is shared among all SMs



- Blocks can take advantages of the fact that their halo cells maybe available in L2 cache.
- Simpler version that only load the internal elements of each tile into the shared memory.

```

__global__ void convolution_1D_tiled_cache_kernel(float *N, float *P,
int Mask_Width, int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;
    __shared__ float N_ds[TILE_SIZE];

    N_ds[threadIdx.x] = N[i];

    __syncthreads();

    int This_tile_start_point = blockIdx.x * blockDim.x;
    int Next_tile_start_point = (blockIdx.x + 1) * blockDim.x;
    int N_start_point = i - (Mask_Width/2);
    float Pvalue = 0;
    for (int j = 0; j < Mask_Width; j++) {
        int N_index = N_start_point + j;
        if (N_index >= 0 && N_index < Width) {
            if ((N_index >= This_tile_start_point)
                && (N_index < Next_tile_start_point)) {
                Pvalue += N_ds[threadIdx.x+j-(Mask_Width/2)]*M[j];
            } else {
                Pvalue += N[N_index] * M[j];
            }
        }
    }
    P[i] = Pvalue;
}

```

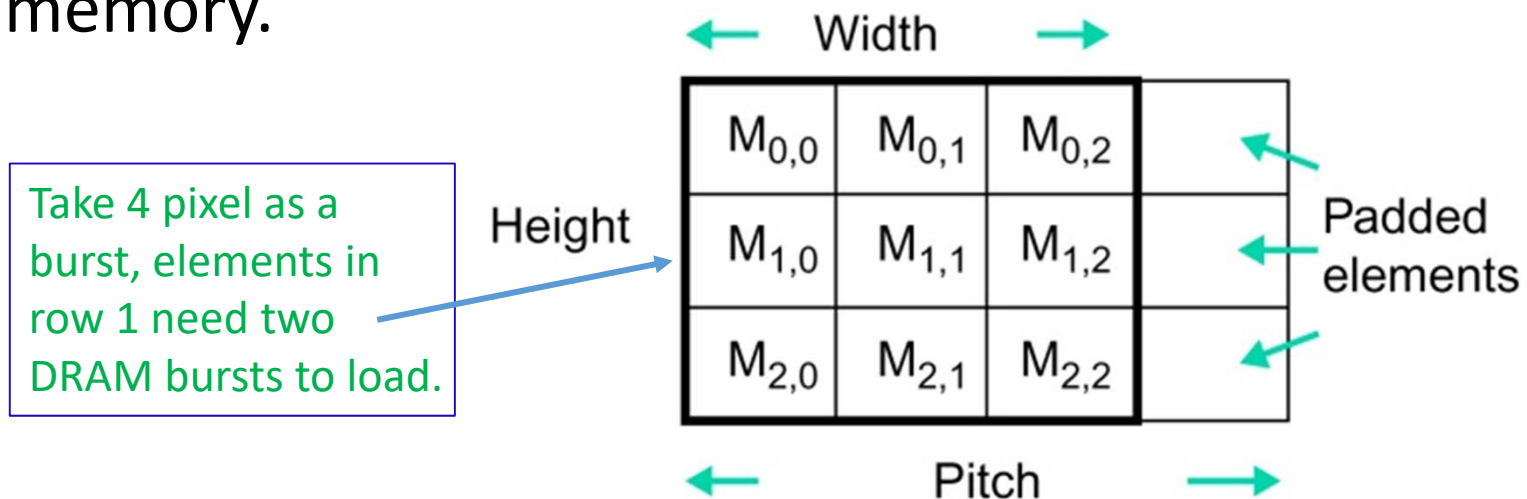


# Content

- Convolution
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  - Algorithms and analysis
- **Tiled 2D convolution with halo cells**
  - **Algorithms and analysis**

# Padded format

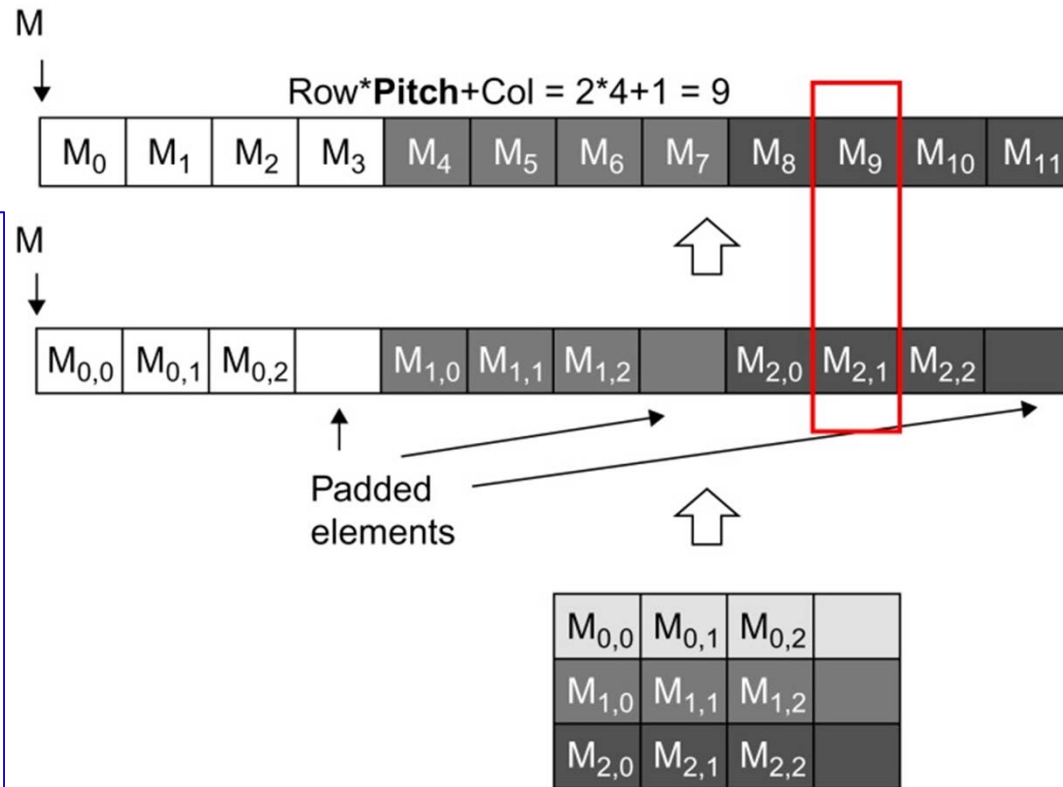
- Images come in all size and shapes, and stored in row-major layout when reading from files to memory.



- If the width of the image in terms of bytes is not a multiple of DRAM burst size, poor utilization of DRAM band-width.

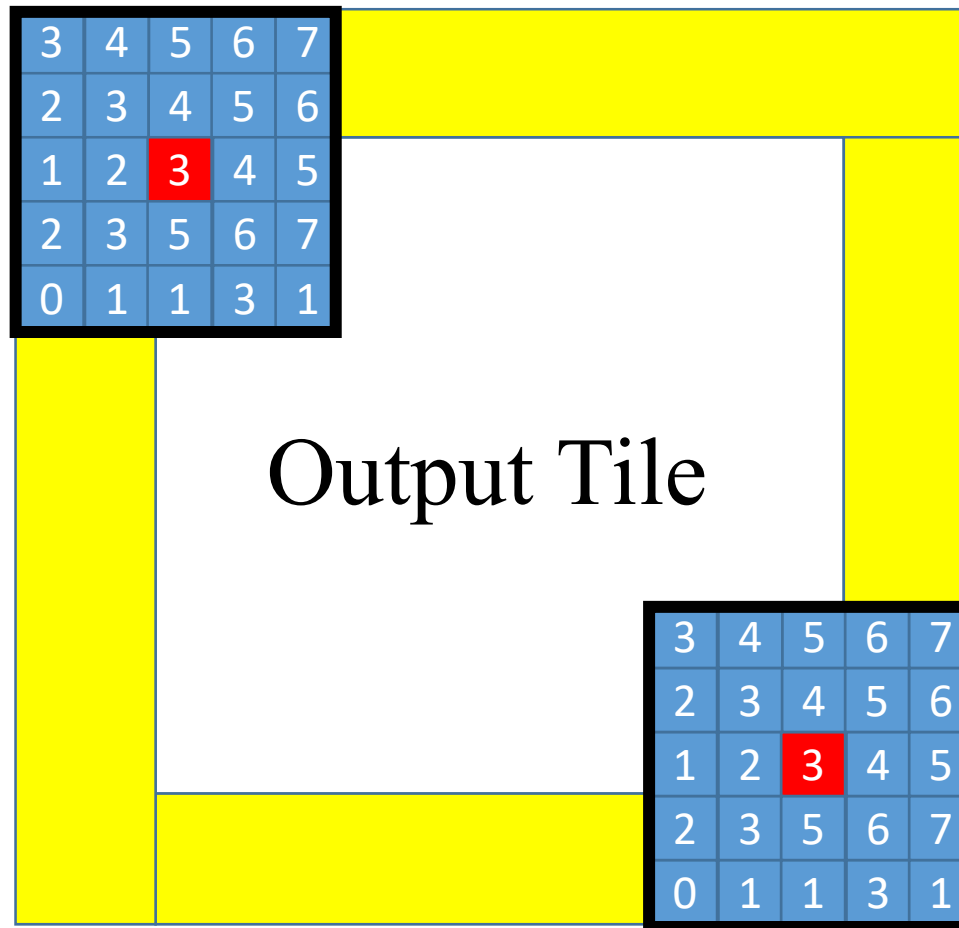
# Padded format

```
//Image Matrix  
declaration  
typedef struct {  
    int width;  
    int height;  
    int pitch;  
    float* data;  
} *wbImage_t;
```



Linearized 1D index = row \* pitch + column

Input tiles need to be larger than output tiles.

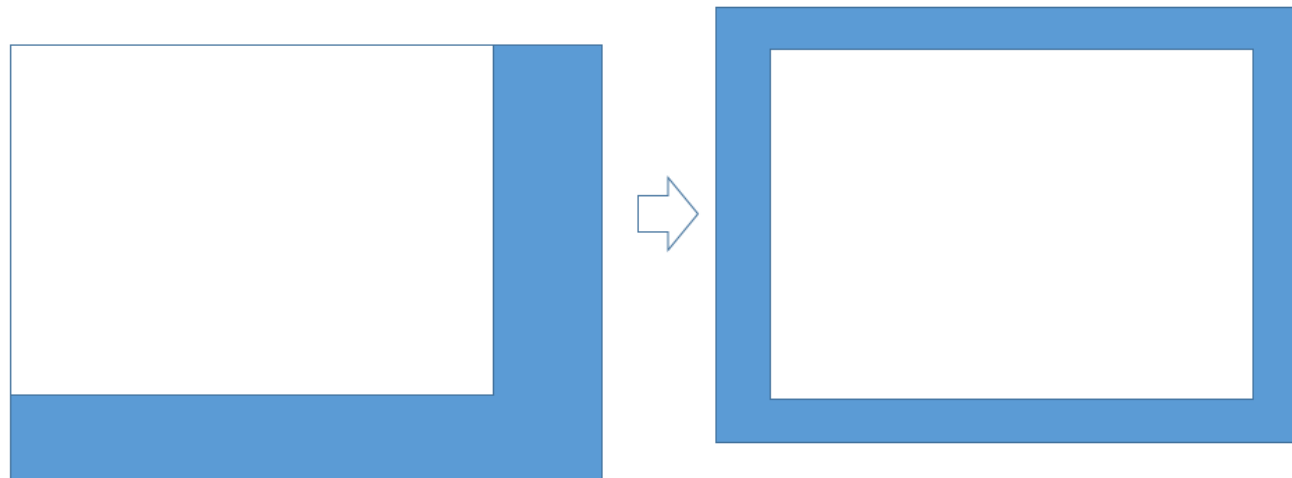
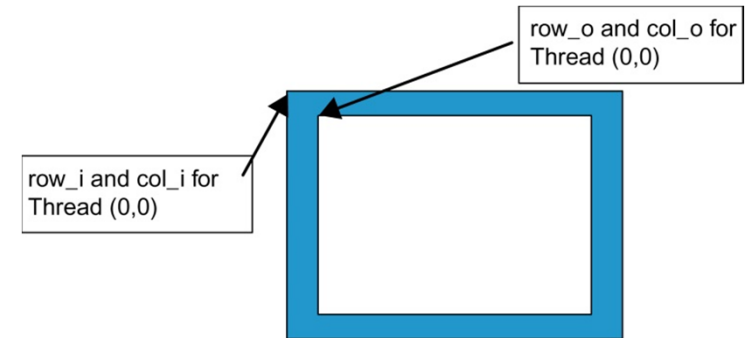


← Input Tile

We will use a strategy where the input tile will be loaded into the shared memory.

# Input v.s. Output tile

```
int tx = threadIdx.x;  
int ty = threadIdx.y;  
int row_o = blockIdx.y * O_TILE_SIZE + ty;  
int col_o = blockIdx.x * O_TILE_SIZE + tx;  
  
int row_i = row_o - Mask_Width / 2;  
int col_i = col_o - Mask_Width / 2;
```

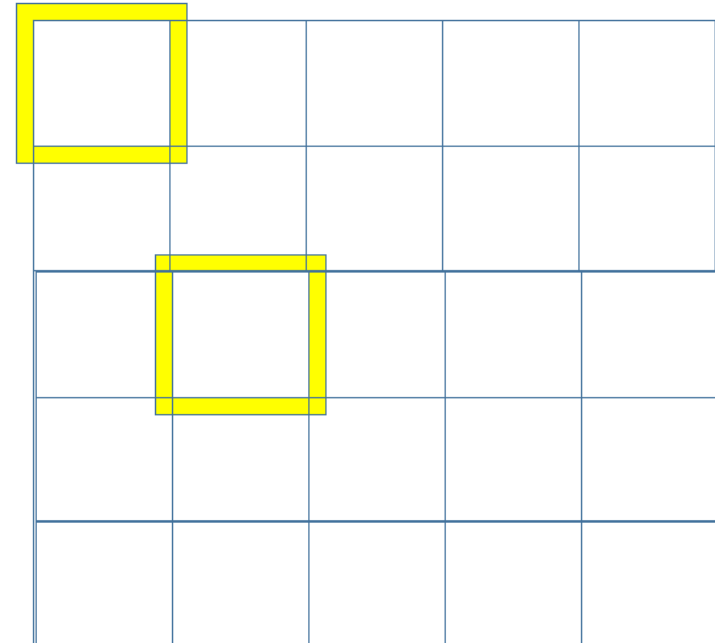


# Input v.s. Output tile

- Use a thread block that matches input tile
  - Each thread loads one element of the input tile
  - Some threads do not participate in calculating output
- There will be if statements and control divergence to make sure read in valid elements.

# Load Input Tile

```
__shared__ float N_ds[TILE_SIZE +  
MAX_MASK_WIDTH-1][TILE_SIZE +  
MAX_MASK_WIDTH-1]  
  
if((row_i >= 0) && (row_i < height) &&  
    (col_i >= 0) && (col_i < width) ) {  
    N_ds[ty][tx] = data[row_i*pitch + col_i];  
}  
else{  
    N_ds[ty][tx] = 0.0f;  
}
```



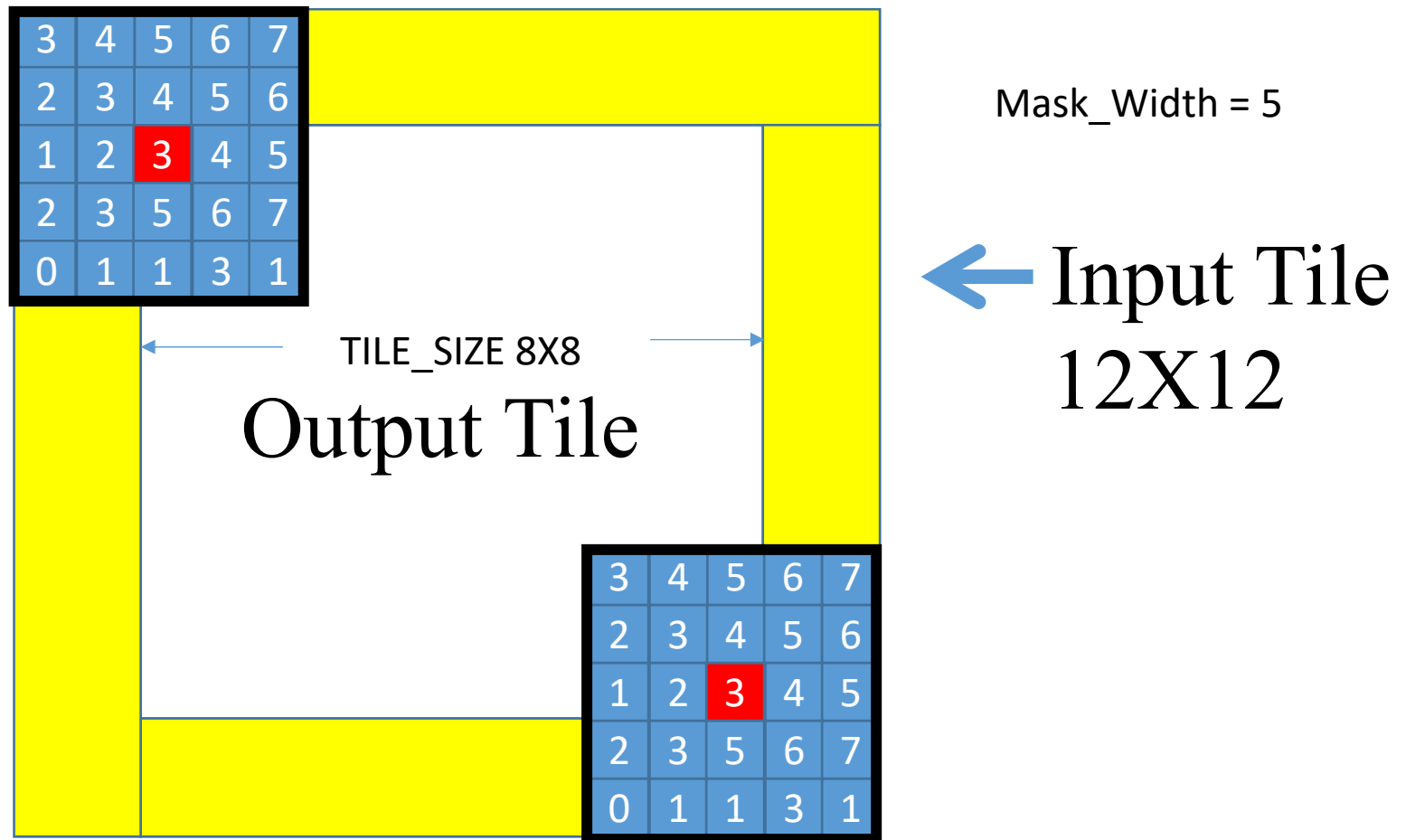
Threads that loads halos outside N should return 0.0

Some threads do not participate in calculating output.

```
float output = 0;
if(ty < O_TILE_SIZE && tx < O_TILE_SIZE)
{
    for(i = 0; i < MASK_WIDTH; i++) {
        for(j = 0; j < MASK_WIDTH; j++) {
            output += M[i][j] * N_ds[i+ty][j+tx];
        }
    }
    if(row_o < height && col_o < width)
        data[row_o * width + col_o] = output;
}
```



# Analysis 2D Tile convolution



# A Simple Analysis

- for a small 8X8 output tile example (Mask\_Width =5)
  - $12 \times 12 = 144$  N elements need to be loaded into shared memory
  - The calculation of each P element needs to access 25 N elements
  - $8 \times 8 \times 25 = 1,600$  global memory accesses are converted into shared memory accesses
- A reduction of  $1,600/144 = 11X$

# In General

- Tiled :
  - $(O\_TILE\_SIZE + Mask\_Width - 1)^2$  N elements need to be loaded into shared memory
- Basic:
  - The calculation of each P element needs to access  $Mask\_Width^2$  N elements
  - $O\_TILE\_SIZE^2 * Mask\_Width^2$  global memory accesses are converted into shared memory accesses
- The reduction is

$$O\_TILE\_SIZE^2 * Mask\_Width^2 / (O\_TILE\_SIZE + Mask\_Width - 1)^2$$

# Bandwidth Reduction for 2D

- The reduction is

$$O\_TILE\_SIZE^2 * Mask\_Width^2 / (O\_TILE\_SIZE + Mask\_Width - 1)^2$$

TILE_SIZE	8	16	32	64
Reduction Mask_Width = 5	11.1	16	19.7	22.1
Reduction Mask_Width = 9	20.3	36	51.8	64

$$9 \times 9 = 81$$

Input size:  $64 + 8 = 72$ ;  
Tile size:  $72 \times 72 = 5184$  or 20,736 bytes,  
larger than shared memory available.

# Summary

- Taking advance of
  - Constant memory and caching
- Tiled convolution algorithms
  - Addressing memory bandwidth issue in accessing  $N$