

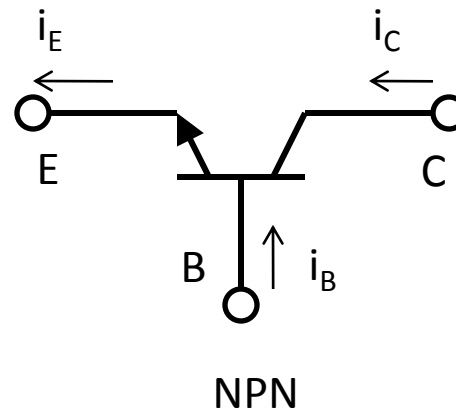
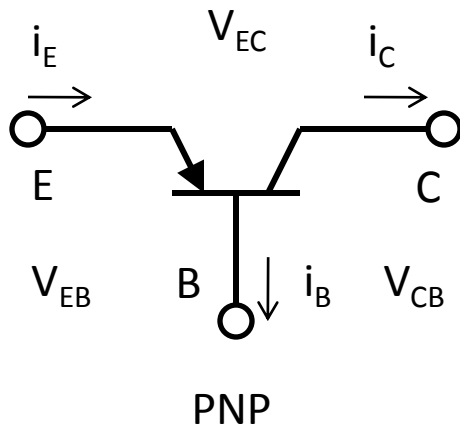
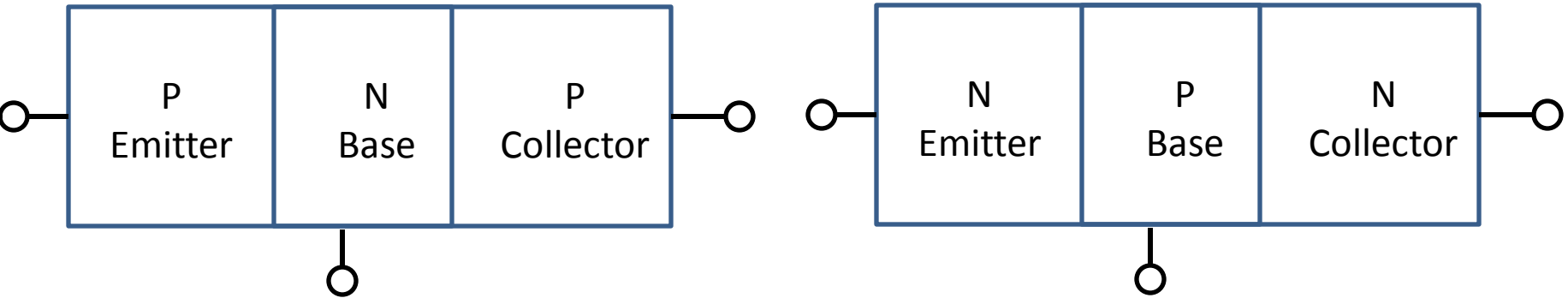
BJT and BJT Digital Circuits

EE 101

S. Lodha

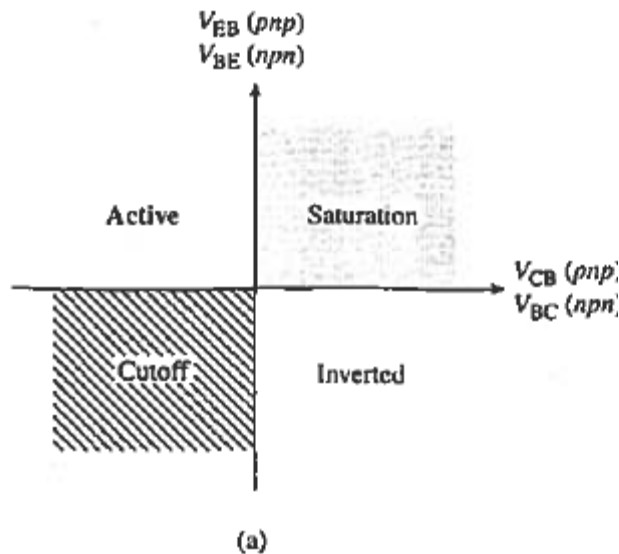
Reference material: L. Bobrow's Book

PNP and NPN

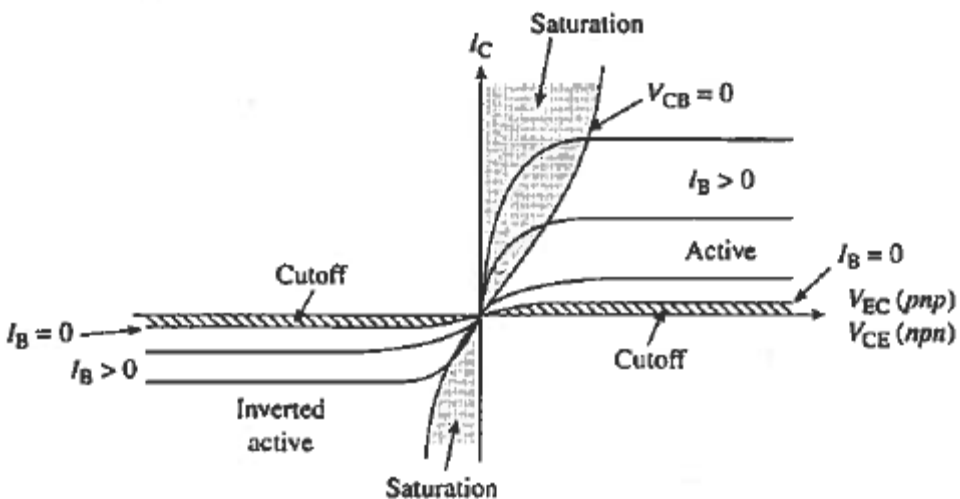


- $i_E = i_B + i_C$
- Terminal voltages also follow KVL

Biasing Modes

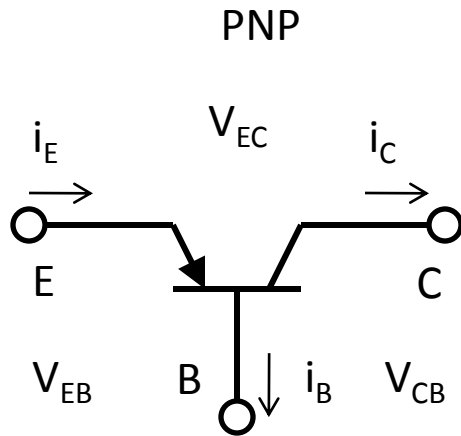


Biasing Mode	Biasing Polarity E-B Junction	Biasing Polarity C-B Junction
Saturation	Forward	Forward
Active	Forward	Reverse
Inverted	Reverse	Forward
Cutoff	Reverse	Reverse



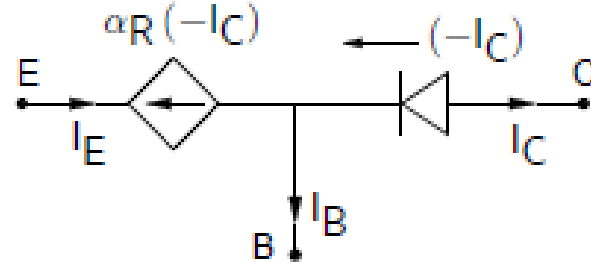
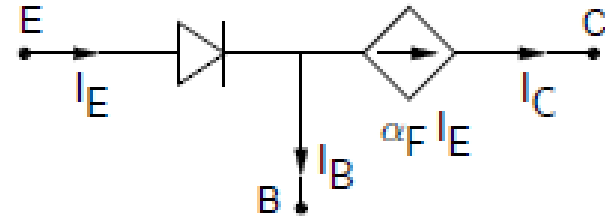
- Amplifier in active mode
 - Analog circuits
- Switch in cut-off and saturation regimes
 - Digital circuits

Biasing modes



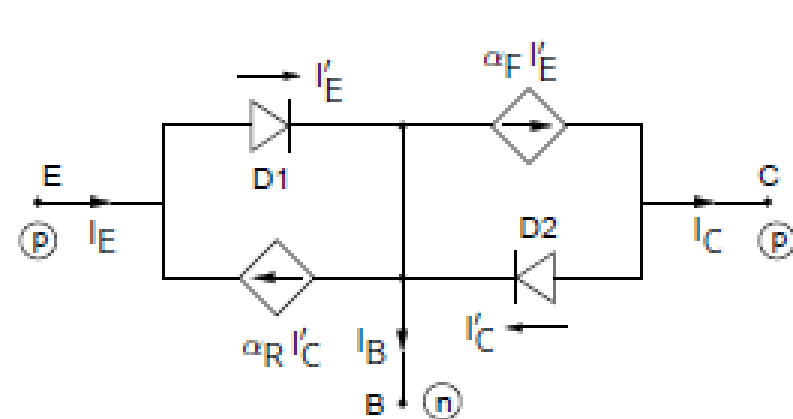
B-E F.B.
C-B R.B.

B-E R.B.
C-B F.B.



- Emitter and collector roles are reversed in reverse-active mode
- Forward and reverse $\alpha \rightarrow \alpha_F (>0.98)$ and $\alpha_R (<0.5)$ are quite different
 - Structure is not symmetric \rightarrow P+ / N / P- or N+ / P / N-
- Consequently β_F and β_R are very different too

Ebers-Moll (DC) Model



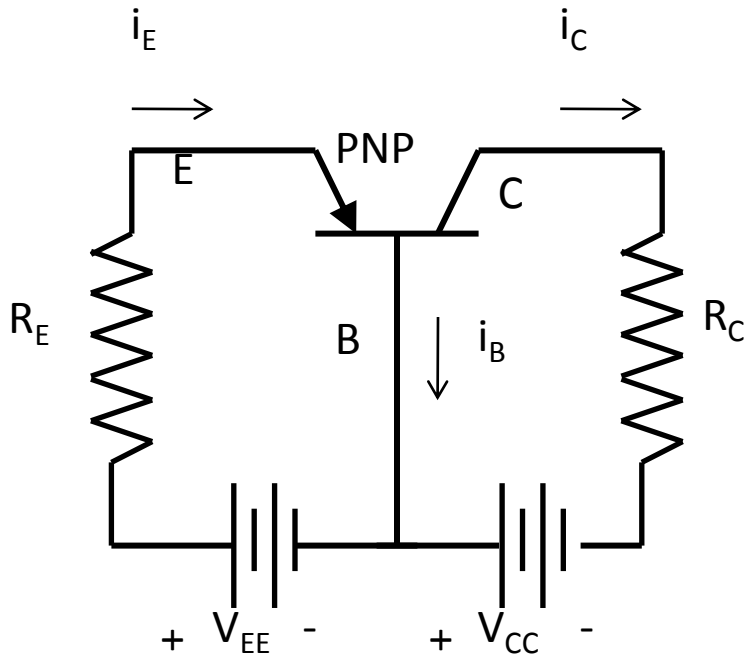
$$I'_E = I_{ES} \left[\exp \left(\frac{V_{EB}}{V_T} \right) - 1 \right], \quad I'_C = I_{CS} \left[\exp \left(\frac{V_{CB}}{V_T} \right) - 1 \right]$$

$$I_C = \alpha_F I_{ES} \left[e^{\left(\frac{V_{EB}}{V_T} \right)} - 1 \right] - I_{CS} \left[e^{\left(\frac{V_{CB}}{V_T} \right)} - 1 \right]$$

$$I_E = I_{ES} \left[e^{\left(\frac{V_{EB}}{V_T} \right)} - 1 \right] - \alpha_R I_{CS} \left[e^{\left(\frac{V_{CB}}{V_T} \right)} - 1 \right]$$

- Similar model for npn
- In saturation, for example, even though D_2 is F.B., $\alpha_F I'_E \gg I'_C$ (emitter is heavily doped) and the collector current flows out of collector terminal

Common base PNP in active mode



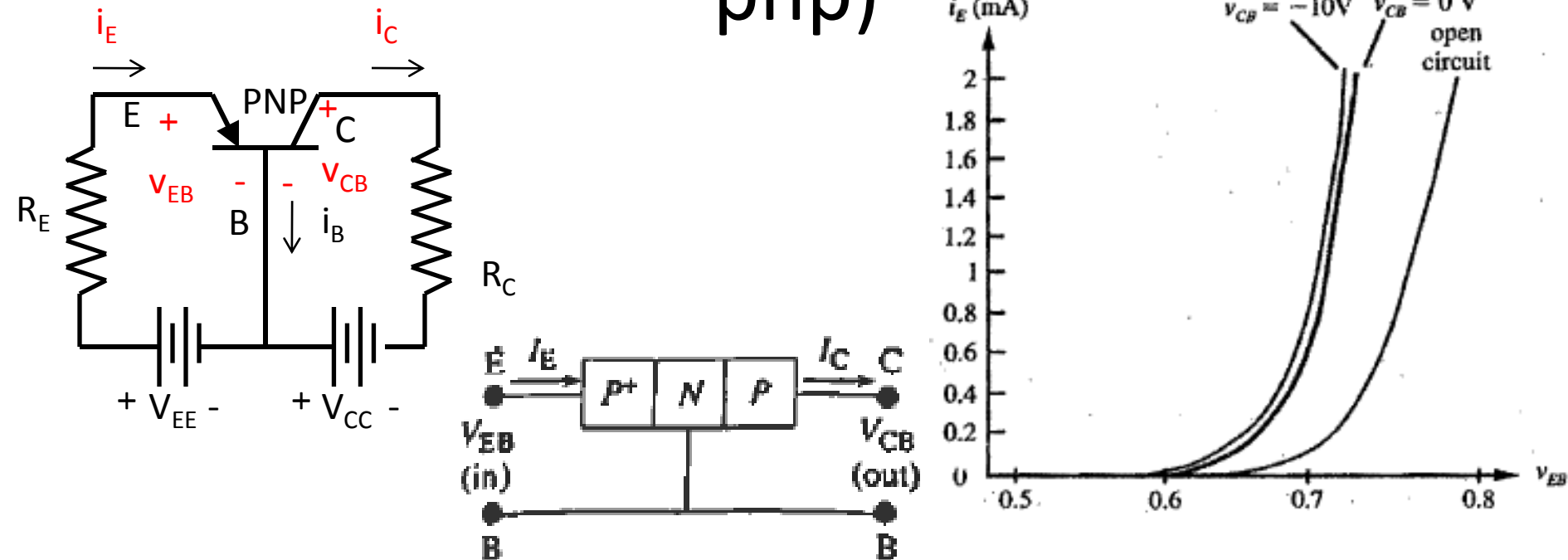
$$i_E = i_{EP} + i_{EN} \approx i_{EP}$$

$$i_C = \alpha i_{EP} + I_{CO} \approx \alpha i_E + I_{CO}$$

$$\alpha = \frac{i_C - I_{CO}}{i_E} \approx \frac{i_C}{i_E}$$

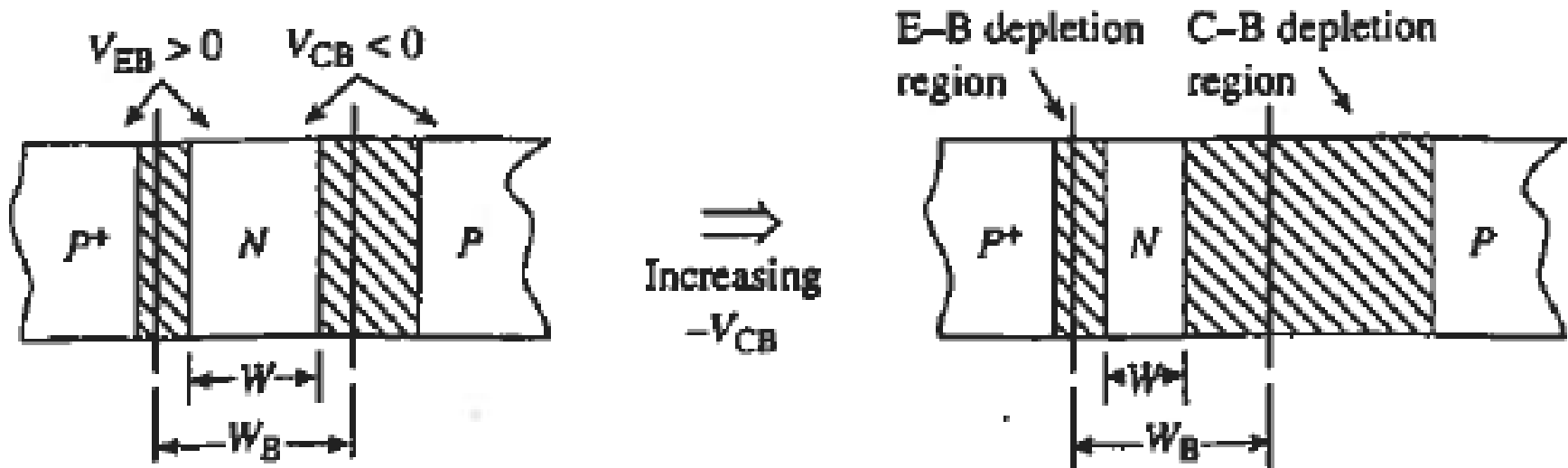
- Emitter current is mainly due to hole diffusion
- Collector current is due to holes that make it + reverse current
- Ratio of emitter/collector is the “large signal current gain” α
- α is typically 0.95-0.99

Input Characteristics (Common base pnp)



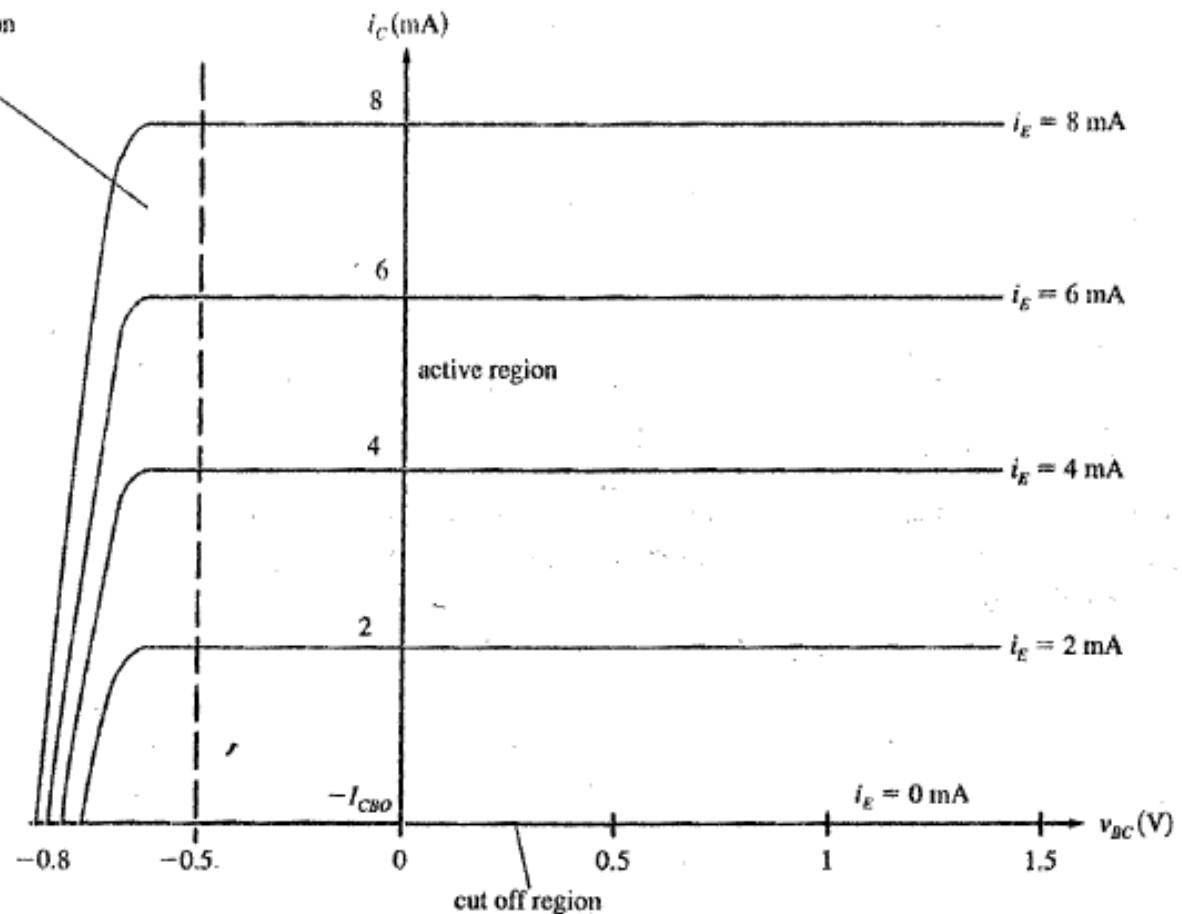
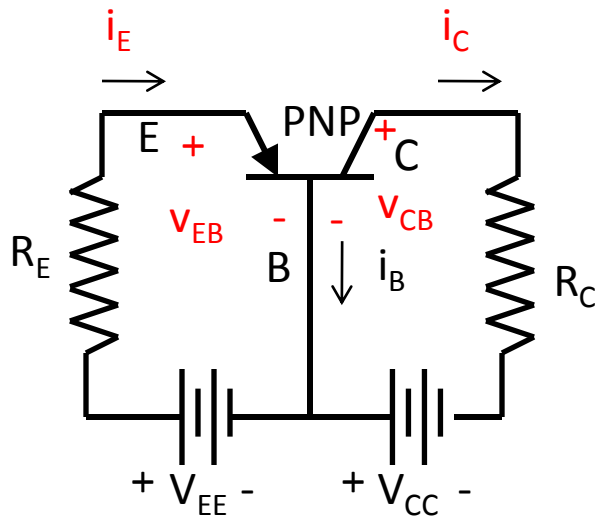
- Input characteristics i_E vs v_{EB} (varying v_{CB})
- As v_{CB} increases, base W decreases, dp/dx increases, i_E and i_C increase for the same v_{EB}
- This effect is also called “base width modulation”

Base width modulation



- Intuitively, as v_{CB} increases (reverse bias)
 - i_E increases (dp/dx increases)
 - i_B decreases (recombination in base decreases)
 - α increases (less recombination in base)
- Also called the “Early effect” → after J. Early

Output Characteristic: (Common base pnp)

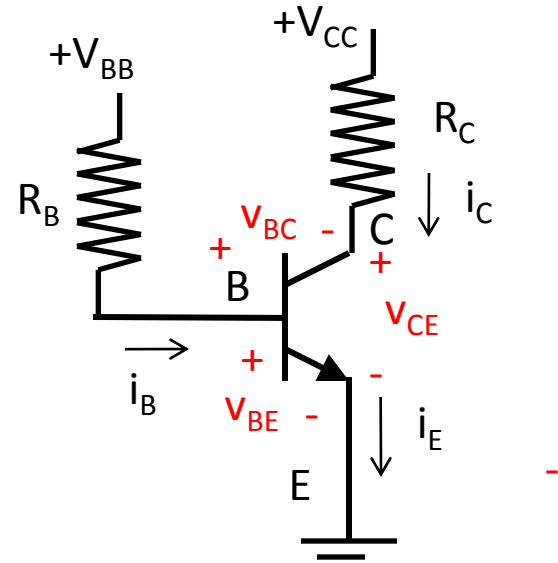
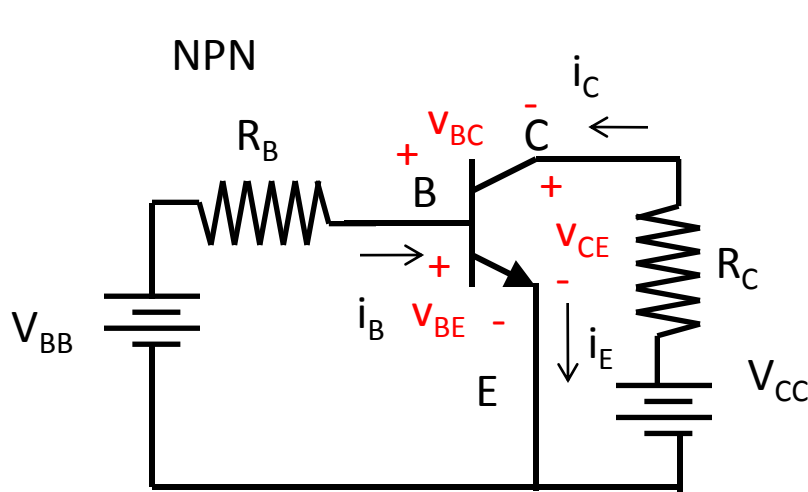


- Output characteristics i_C vs v_{BC} (varying i_E)
- $i_E=0$, $I_{CBO} \rightarrow$ CB current with emitter open, typically nA (~ 0)

Different biasing regions

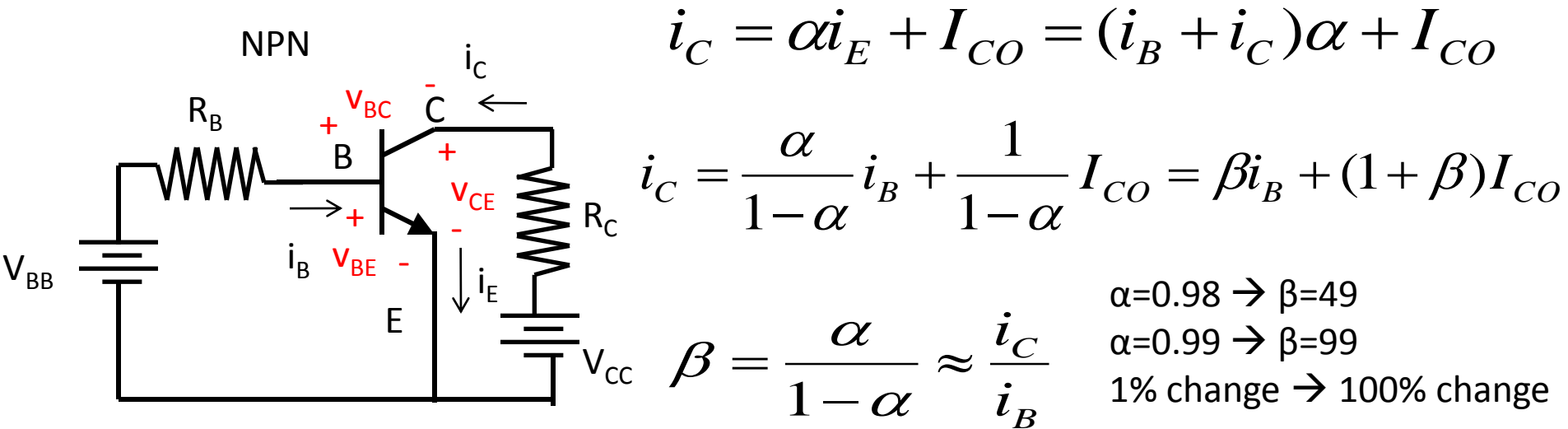
- Right of dashed line: $i_E > 0$ (FB), $v_{BC} > -0.5$ (V γ) (RB) \rightarrow Active region
- For $i_E > 0$, $-0.8 < v_{BC} < -0.5$, both FB \rightarrow saturation
- On or below $i_E=0 \rightarrow$ cutoff

Common emitter npn



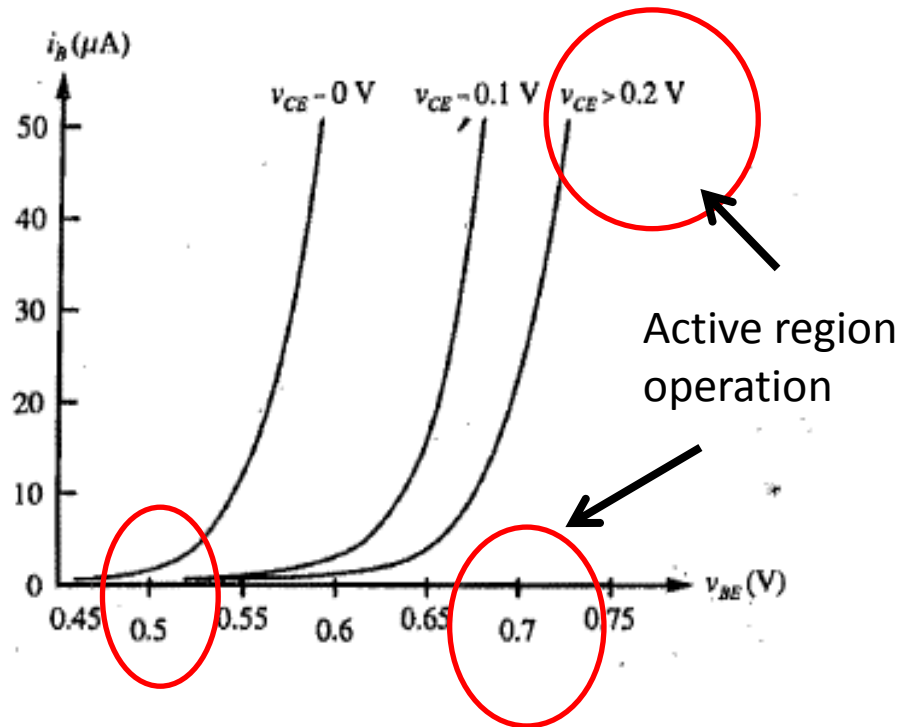
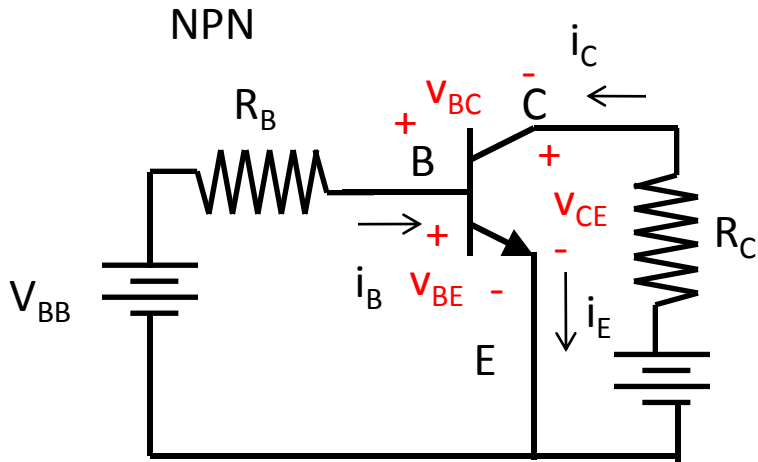
- Active region: Large $V_{BB} \rightarrow v_{BE} > 0 \rightarrow$ BE jn F. B.
- For CB R.B.: $v_{BC} = v_{BE} - v_{CE} < V_{\gamma}$ (V_{γ} cut-in voltage = 0.5 V)
– Therefore, $v_{CE} > v_{BE} - V_{\gamma} = 0.7 - 0.5 = 0.2$ V
- $v_{CE} > 0.2$ V ensures reverse bias CB junction

Common emitter npn



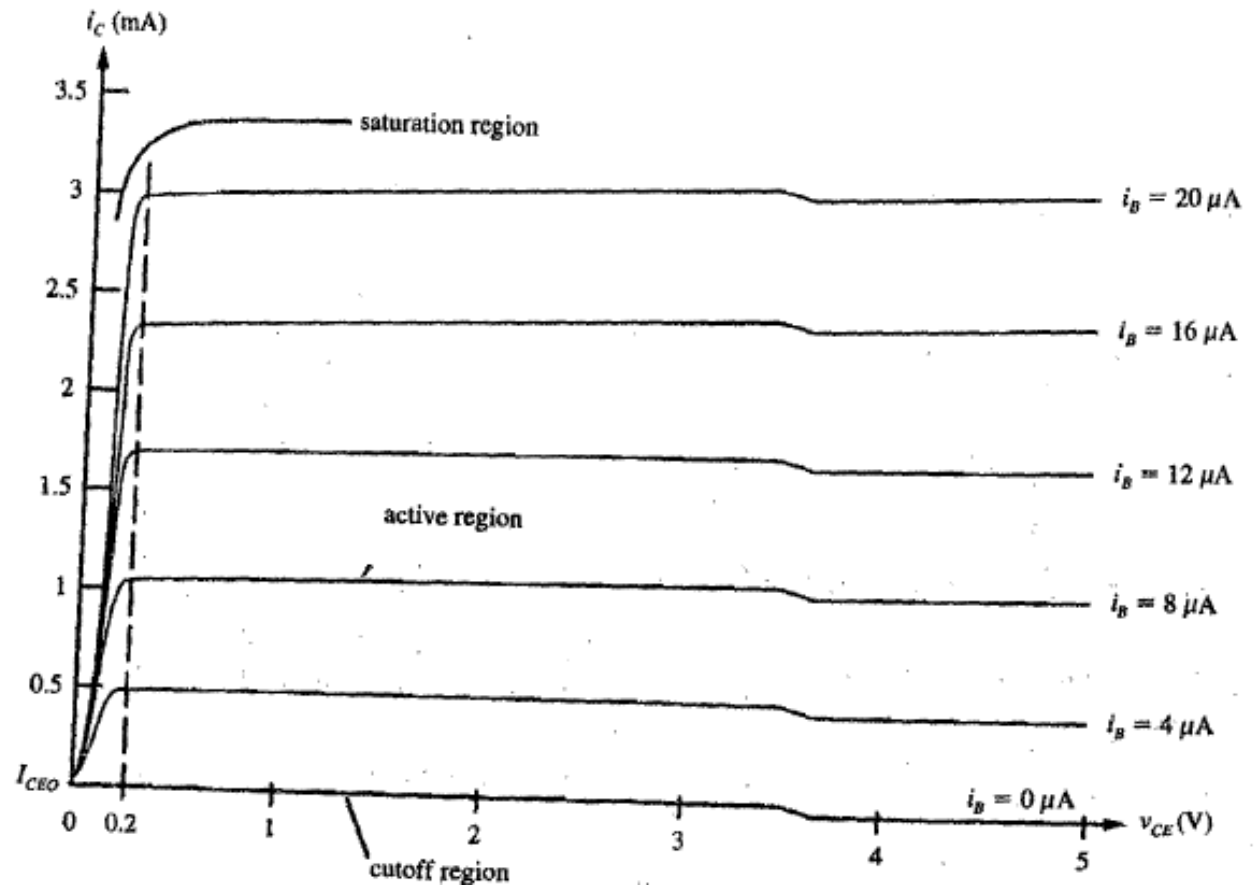
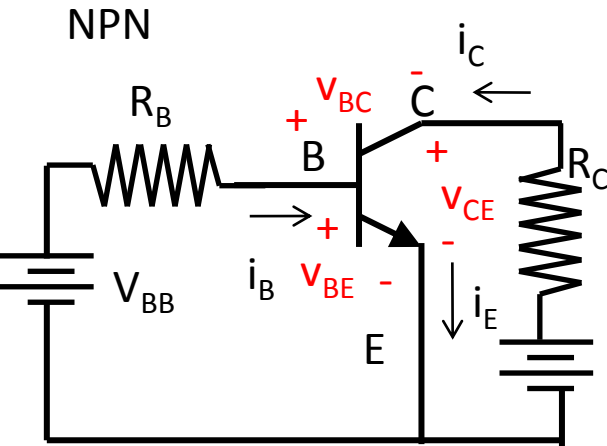
- β is the large signal common emitter gain, much larger than α

Input Characteristics (C.E. npn)



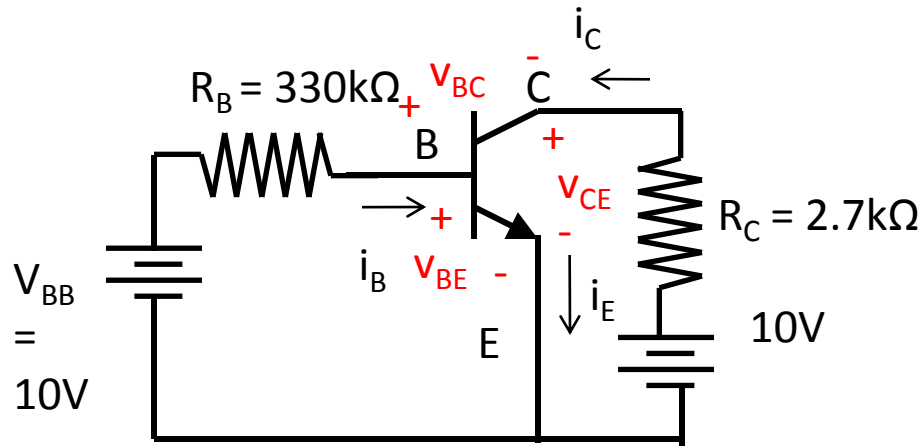
- As v_{CE} increases $\rightarrow v_{CB}$ ($v_{CB} = v_{CE} - v_{BE}$) increases $\rightarrow W_B$ decreases $\rightarrow i_B$ decreases (less recombination)
 - Therefore for same v_{BE} you get less i_B

Output Characteristics (C.E. npn)



- Active region: $i_B > 0$, $v_{CE} > 0.2$ V
- Saturation: $i_B > 0$, $v_{CE} < 0.2$ V
- Cutoff: $i_B \leq 0$. $I_{CEO} \rightarrow i_C (=i_E)$ when base is open ($i_B = 0$)

Example



Assuming active operation

$$v_{BE} = 0.7 \text{ V}, i_C = 100i_B$$

$$i_B = \frac{10 - v_{BE}}{330 \times 10^3} = \frac{10 - 0.7}{330 \times 10^3} = 28.2 \mu\text{A}$$

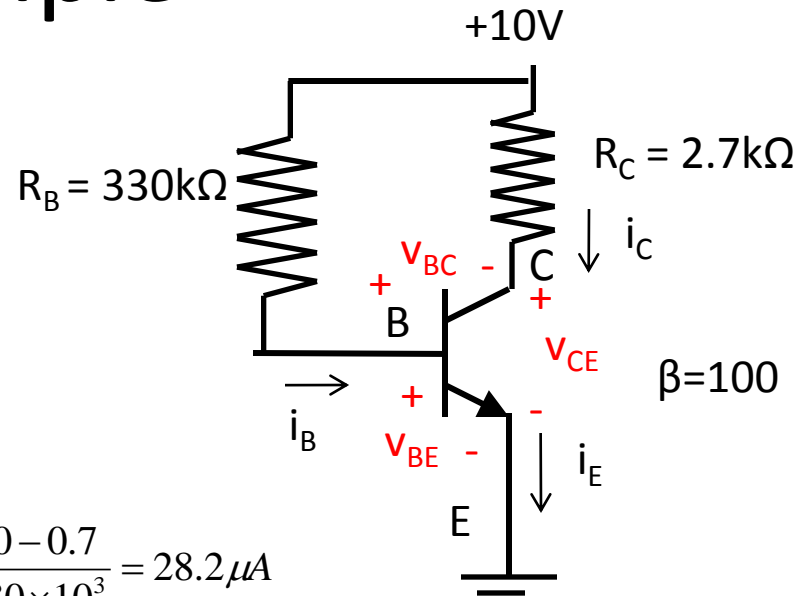
$$i_C = \beta i_B = 100(28.2 \times 10^{-5}) = 2.82 \text{ mA}$$

$i_B > 0$ indicates that BE junction is forward-biased,
for CB junction

$$v_{CE} = 10 - (2.7 \times 10^3)i_C = 2.39 \text{ V}$$

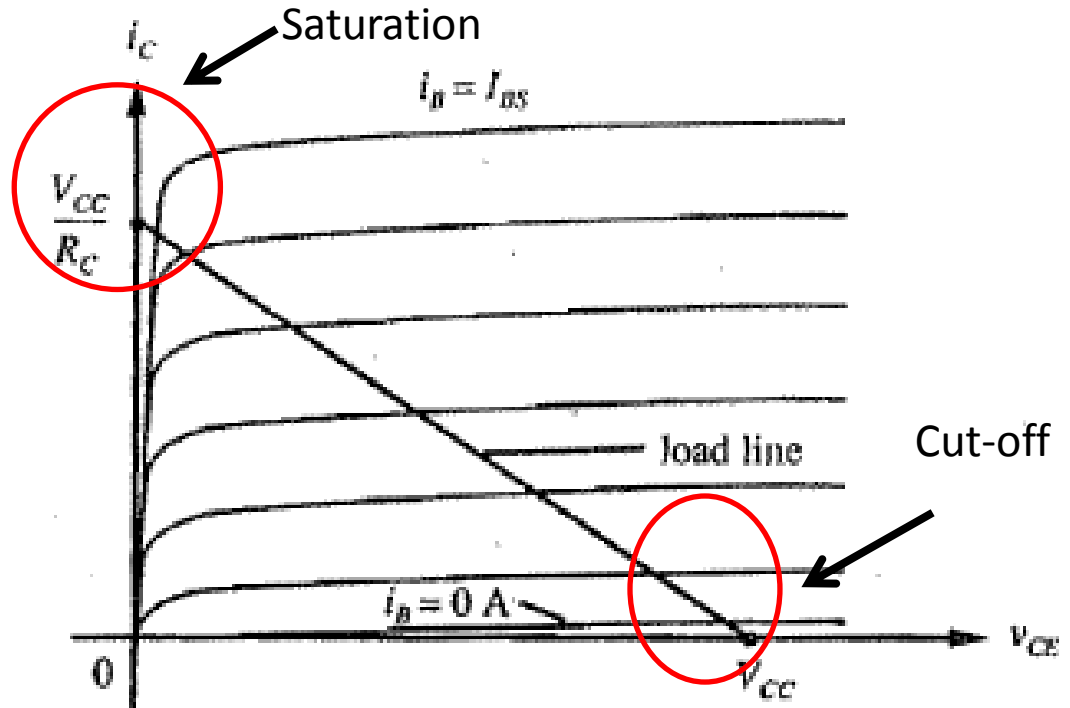
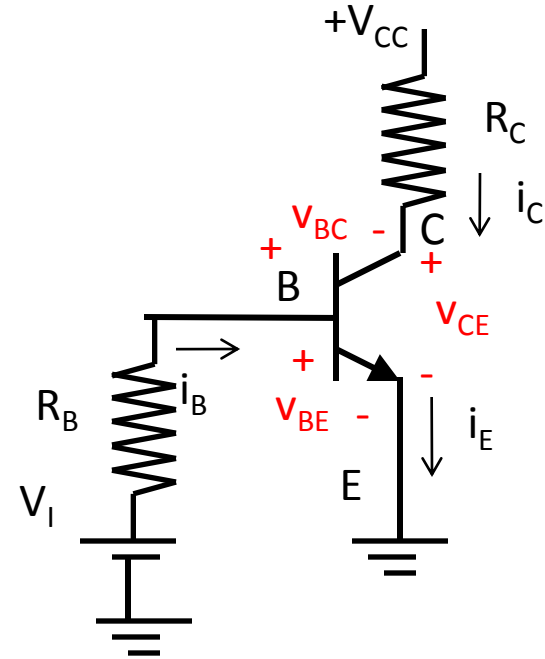
$$v_{BC} = v_{BE} - v_{CE} = 0.7 - 2.39 = -1.69 \text{ V} < V_\gamma = 0.5 \text{ V}$$

Hence the transistor is indeed operating in the active region.



Cut-off and saturation

$$i_C = \frac{V_{CC} - v_{CE}}{R_C} = -\frac{1}{R_C} v_{CE} + \frac{V_{CC}}{R_C}$$



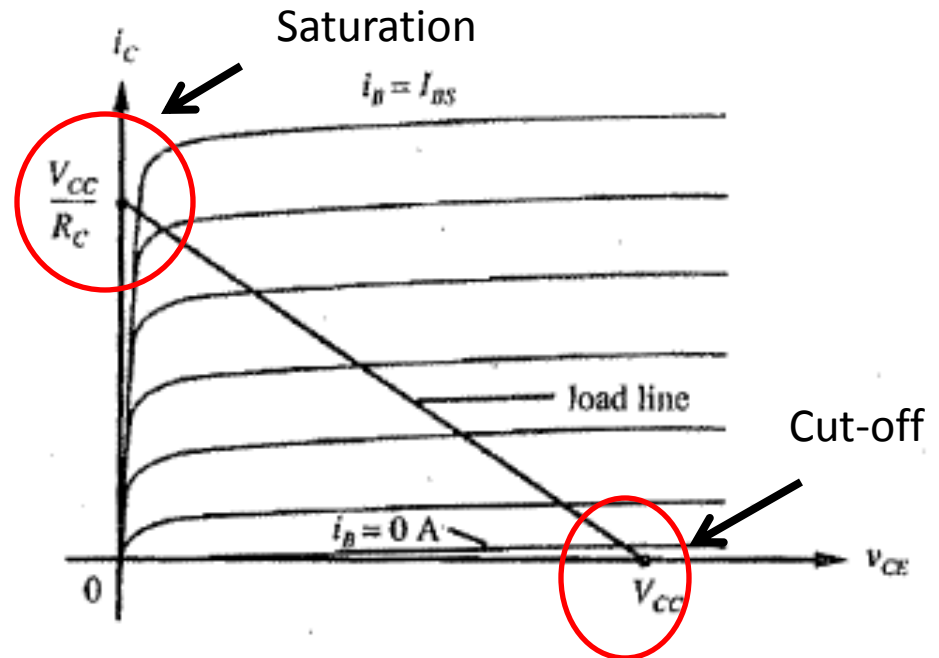
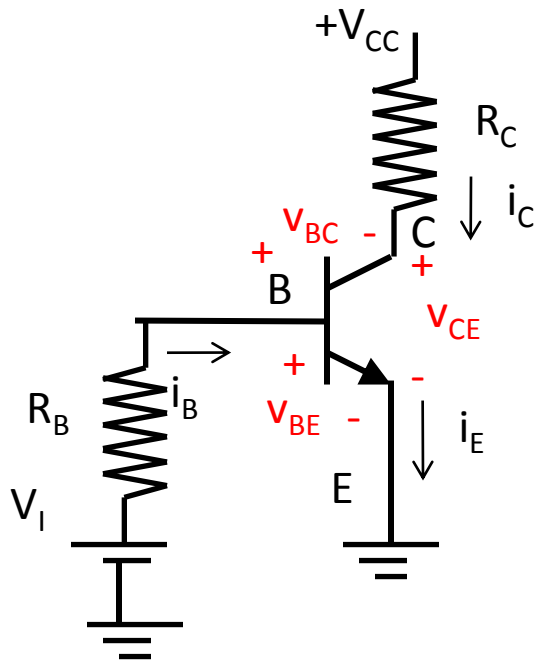
- Cut-off

- $i_B = 0 \rightarrow v_{BE} < 0.5 \text{ V} \rightarrow \text{R.B.}$
- $i_C = i_E = 0, v_{CE} = V_{CC}, v_{BC} = v_{BE} - v_{CE} < 0.5 - V_{CC} \rightarrow \text{R.B.}$

- Saturation

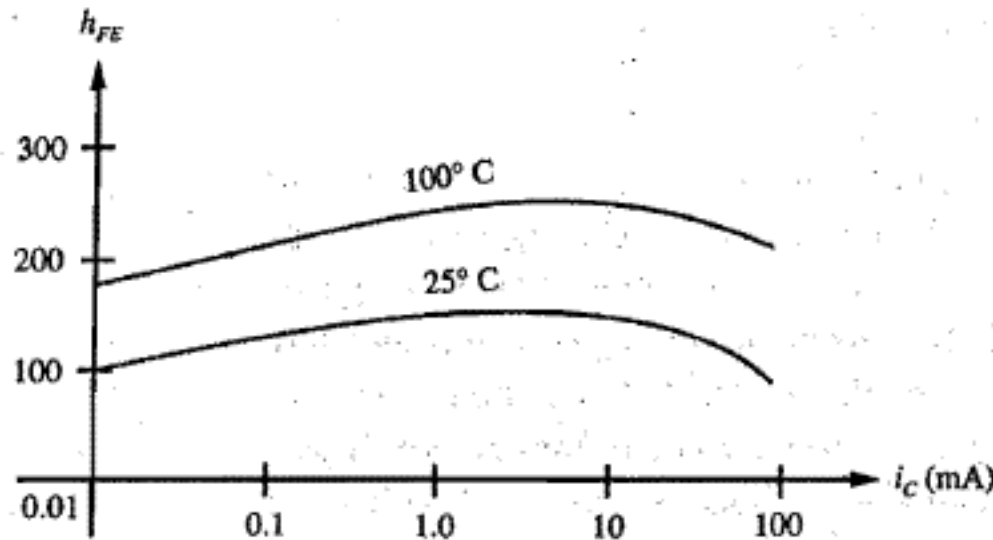
- $i_B > I_{BS}, i_C(\text{sat}) = (V_{CC} - v_{CE}(\text{sat})) / R_C \sim V_{CC} / R_C \rightarrow$ collector current is saturated
- $v_{CE} = v_{CE}(\text{sat}) \sim 0.2 \text{ V}, v_{BE}(\text{sat}) \sim 0.8 \text{ V}$
- $v_{BC} = v_{BE}(\text{sat}) - v_{CE}(\text{sat}) = 0.8 - 0.2 = 0.6 \text{ V} > 0.5 \text{ V} (V_{\gamma}) \rightarrow \text{BC is forward biased}$

Cut-off and saturation



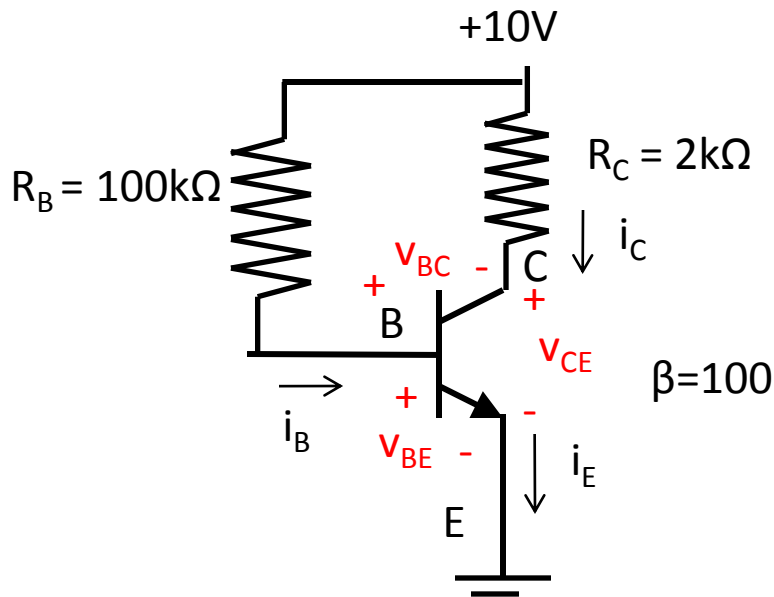
- ON (SAT), $v_{CE}=0.2\text{V}$, $i_C=V_{CC}/R_C \rightarrow$ short circuit
- OFF (CUTOFF), $v_{CE}=V_{CC}$, $i_C=0 \rightarrow$ open circuit
- Transistor behaves as a switch

Common emitter dc gain



- $h_{FE} = i_C / i_B \sim \beta$
- For all practical purposes we will assume h_{FE} is constant

Example



Assuming active operation

$$v_{BE} = 0.7\text{ V}, i_C = 100i_B$$

$$i_B = \frac{10 - v_{BE}}{100 \times 10^3} = \frac{10 - 0.7}{100 \times 10^3} = 93\mu\text{A}$$

$$i_C = \beta i_B = 100(9.3 \times 10^{-5}) = 9.3\text{mA}$$

$$v_{CE} = -(2 \times 10^3)i_C + 10 = -8.6\text{V} < 0.2\text{V}$$

$$v_{BC} = v_{BE} - v_{CE} = 0.7 + 8.6 = 9.3 > V_\gamma = 0.5\text{V}$$

Therefore not in active mode

Assume saturation,

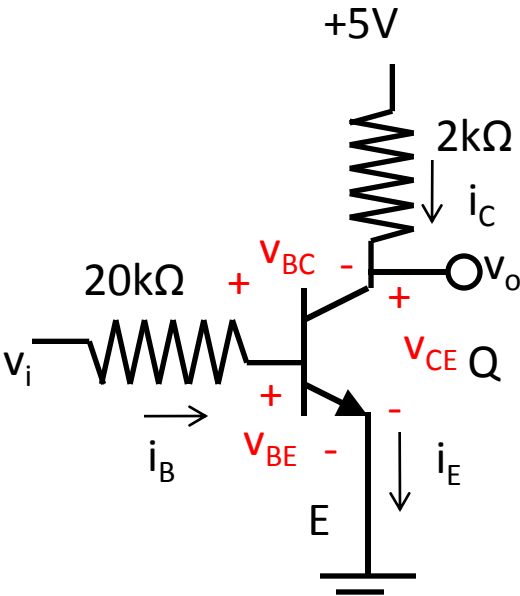
$$i_B = \frac{10 - v_{BE(sat)}}{100 \times 10^3} = \frac{10 - 0.8}{100 \times 10^3} = 92\mu\text{A}$$

$$i_C = \frac{10 - v_{CE(sat)}}{2 \times 10^3} = \frac{10 - 0.2}{2 \times 10^3} = 4.9\text{mA}$$

$$\text{In saturation, } i_B \geq \frac{i_C}{h_{FE}} = \frac{4.9 \times 10^{-3}}{100} = 49\mu\text{A}$$

Hence assumption of saturation is correct.

Example: BJT Inverter



low voltage=0.2 V (logic '0')

high voltage=5.0 V (logic '1')

For $v_i=0.2$ V, assume Q is OFF (cutoff)

$i_B=0$, $v_{BE}=v_i=0.2$ V < 0.5 V, indeed Q is off $\rightarrow i_C=0$ and $v_o=5$ V (logic '1')

For $v_i=5$ V, assume Q is ON (saturation)

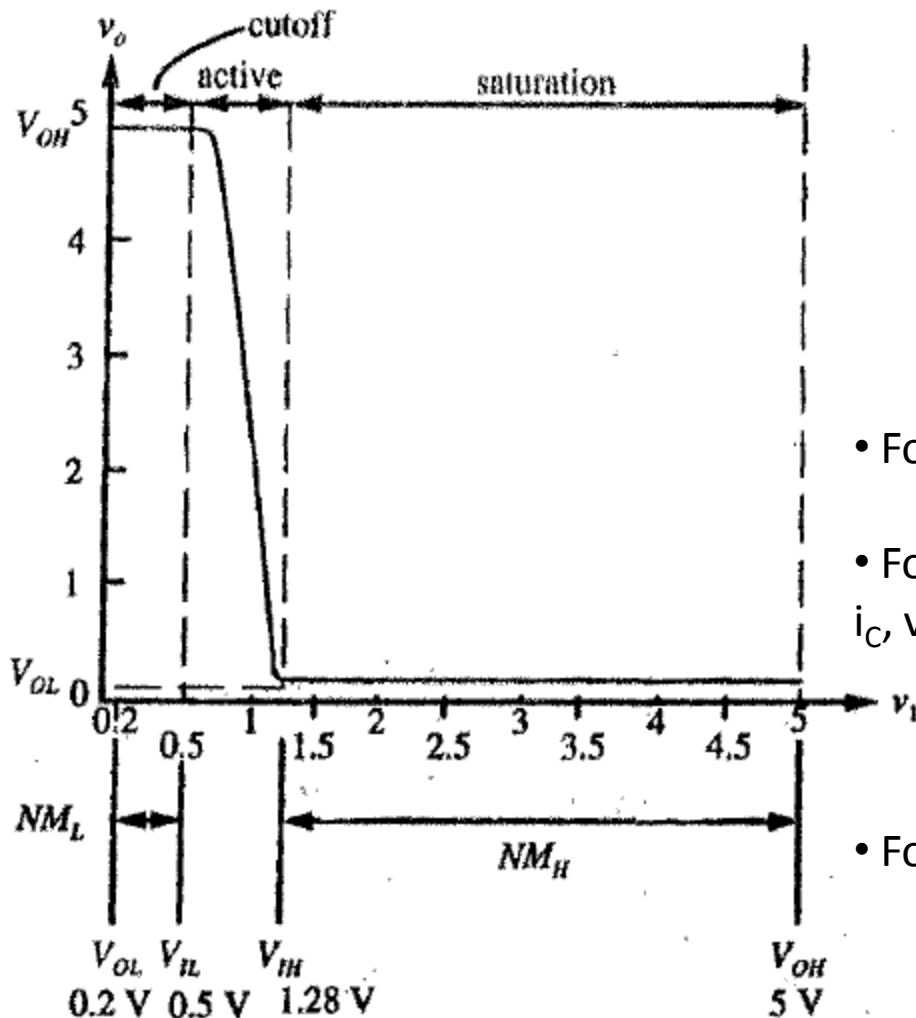
$i_B=0.21$ mA, $i_C=2.4$ mA and $v_i=5$ V (logic '1')

$$i_B \geq \frac{i_C}{h_{FE}} = \frac{2.4 \times 10^{-3}}{100} = 24 \mu A$$

Hence Q is indeed ON, and $v_o=0.2$ V (logic '0')

Hence this circuit acts as an “inverter”, $v_o = \bar{v}_{in}$

Transfer characteristic (v_o vs v_{in})



$V_{OH} \rightarrow$ Output Voltage high = 5 V

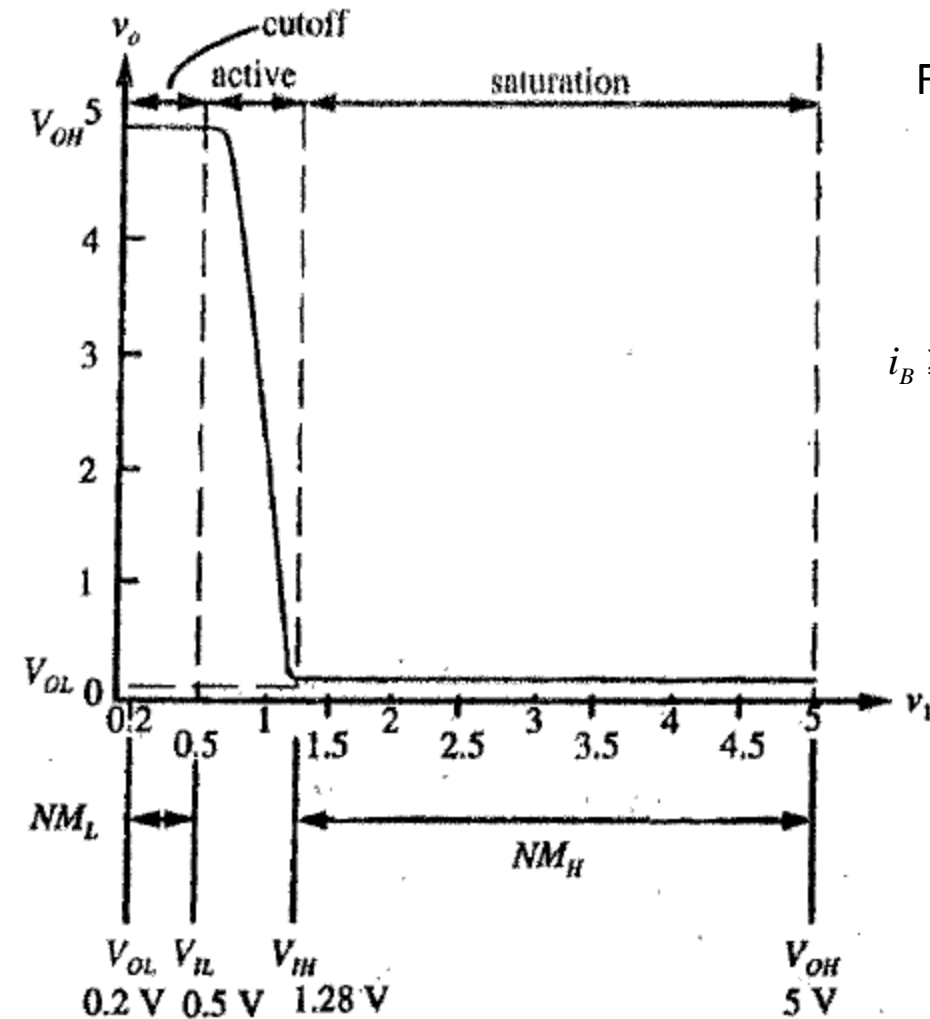
$V_{OL} \rightarrow$ Output Voltage low = 0.2 V

$$v_o = v_{CE} = 5 - (2 \times 10^3) i_C$$

$$v_{BC} = v_{BE} - v_{CE}$$

- For $0.2 < v_i < 0.5$ V, Q off
- For $v_i > 0.5$ V, Q enters active region for small i_B and i_C , v_o is ~ 5 V
 $\rightarrow 0.5$ V = V_{IL} (largest input voltage that still results in the same output as low input voltage)
- For increasing v_i , Q enters saturation and $v_o = 0.2$ V
 $\rightarrow V_{IH}$ = minimum input voltage that results in the same output as high input voltage

Finding V_{IH} and Noise Margins



For V_{IH} , and Q to be ON

$$i_B = \frac{v_i - 0.8}{20 \times 10^3}, i_C = \frac{5 - 0.2}{2 \times 10^3} = 2.4 \text{ mA}$$

$$i_B \geq \frac{i_C}{h_{FE}} \Rightarrow \frac{v_i - 0.8}{20 \times 10^3} \geq \frac{2.4 \times 10^{-3}}{100} \Rightarrow v_i \geq 1.28 \text{ V}$$

Hence, $V_{IH} = 1.28 \text{ V}$

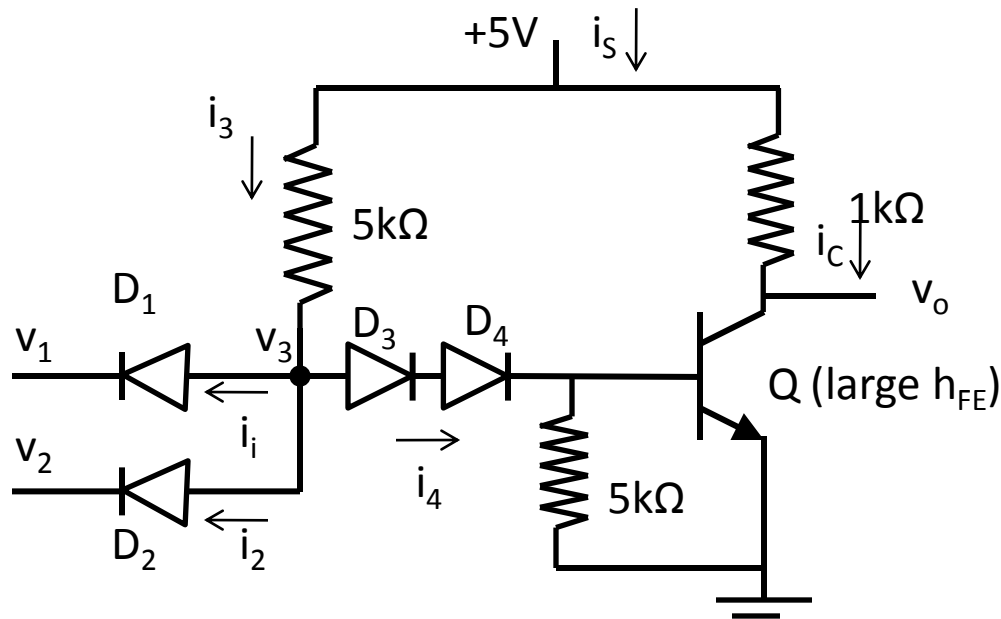
NOISE MARGINS

- minimum noise (unwanted) voltage at the input of a digital logic gate that causes output to deviate is called the noise margin of the gate

- $NM_L \rightarrow$ minimum voltage noise that can produce an output other than that from a low input = $V_{IL} - V_{OL}$

- $NM_H \rightarrow$ minimum voltage noise that can produce an output other than that from a high input = $V_{OH} - V_{IH}$

Diode-Transistor Logic (DTL)



v_1	v_2	v_o (NAND)
0	0	1
0	1	1
1	0	1
1	1	0

Case 1 ($v_1=v_2=0.2$ V)

Assume D_1 and D_2 are F.B.

$$\rightarrow v_3 = 0.7 + 0.2 = 0.9\text{V},$$

Assume D_3 and D_4 are off.

$$i_2 + i_1 = i_3 = \frac{5 - (0.9)}{5 \times 10^3} = 0.82\text{mA}, i_2 = i_3 = 0.41\text{n}$$

$$i_B = i_4 = 0, v_{BE} = 0\text{V}, v_3 - v_{BE} = 0.9 < 0.7 + 0.7 = 1.4\text{V}$$

Hence D_3 and D_4 are indeed off.

Q is off and v_o is 5V.

Case 2/3 ($v_1=0.2\text{V}, v_2=5\text{V} / v_1=5\text{V}, v_2=0.2\text{V}$)

Assume D_1 is on, D_2 is off $\rightarrow v_3 = 0.7 + 0.2 = 0.9\text{V}$,

$$v_3 - v_2 = 0.9 - 5 = -4.1 < 0.7\text{V} \rightarrow D_2 \text{ is off}$$

Assume D_3 and D_4 are off.

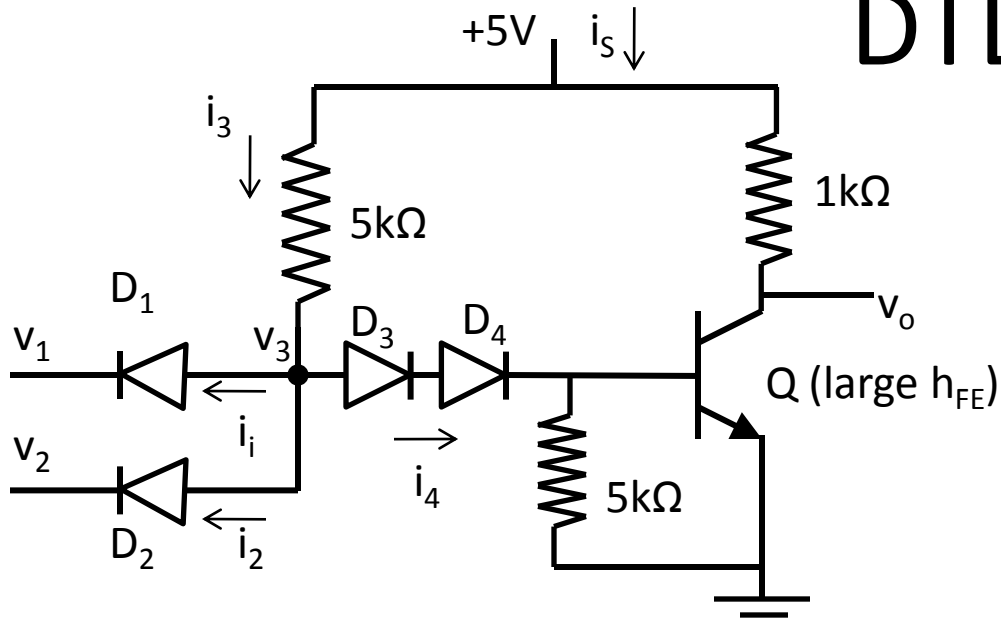
$$i_1 = i_3 = \frac{5 - (0.9)}{5 \times 10^3} = 0.82\text{mA} > 0$$

$$i_B = i_4 = 0, v_{BE} = 0\text{V}, v_3 - v_{BE} = 0.9 < 0.7 + 0.7 = 1.4\text{V}$$

Hence D_3 and D_4 are indeed off.

Q is off and v_o is 5V.

DTL



v_1	v_2	v_o (NAND)
0	0	1
0	1	1
1	0	1
1	1	0

Case 4 ($v_1=v_2=5\text{ V}$)

Assume D_1 and D_2 are off $\rightarrow i_1=i_2=0$.

Assume D_3 , D_4 , Q are on.

$$v_3 = 0.7 + 0.7 + 0.8 = 2.2\text{ V}$$

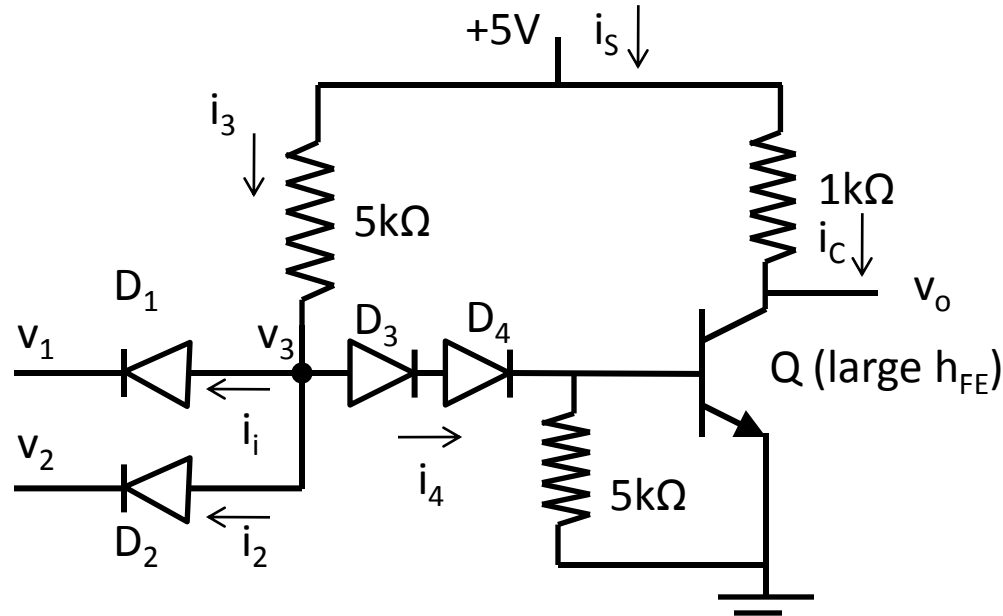
$5 - v_3 = -2.8\text{ V} \rightarrow D_1$ and D_2 are off

$$i_4 = i_3 = \frac{5 - 2.2}{5 \times 10^3} = 0.56\text{ mA} > 0$$

$$i_B = i_4 - \frac{0.8}{5 \times 10^3} = 0.4\text{ mA}, i_C = \frac{5 - 0.2}{1 \times 10^3} = 4.8\text{ mA}$$

$$h_{FE} \geq \frac{i_C}{i_B} = \frac{4.8}{0.4} = 12 \rightarrow Q \text{ is on, } v_o = 0.2\text{ V}$$

DTL: Power Dissipation



v_1	v_2	v_o (NAND)	
0	0	1	Q is off
0	1	1	
1	0	1	
1	1	0	Q is on

Case 1/2/3 ($v_1=0.2\text{V}$, $v_2=5\text{V}$ / $v_1=5\text{V}$, $v_2=0.2\text{V}$ / $v_1=v_2=0.2\text{V}$)

Only i_3 is flowing, hence power consumed

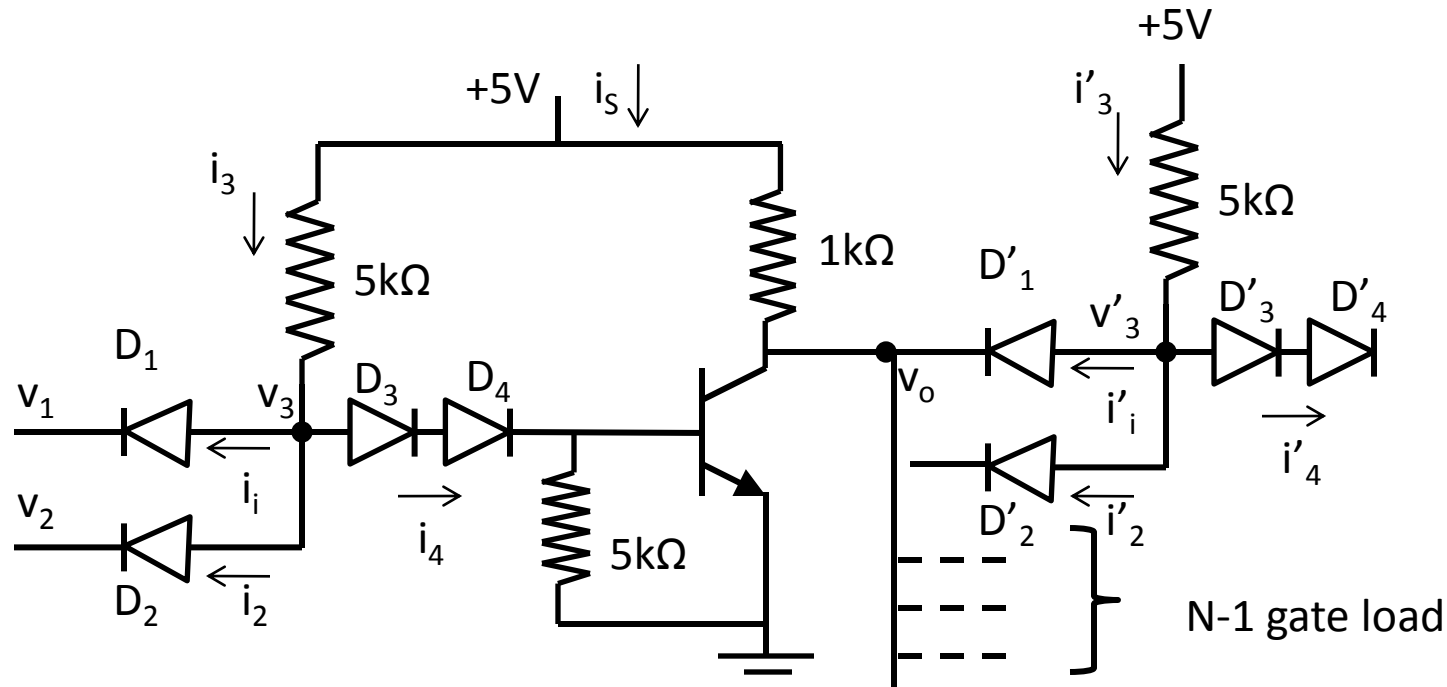
$$P_1 = 5i_s = 5i_3 = 5(0.82 \times 10^{-3}) = 4.1\text{mW}$$

Case 4 ($v_1=5\text{V}$, $v_2=5\text{V}$, $v_o=0.2\text{V}$)

$$P_0 = 5(i_3 + i_c) = 5(0.56 \times 10^{-3} + 4.8 \times 10^{-3}) = 26.8\text{mW}$$

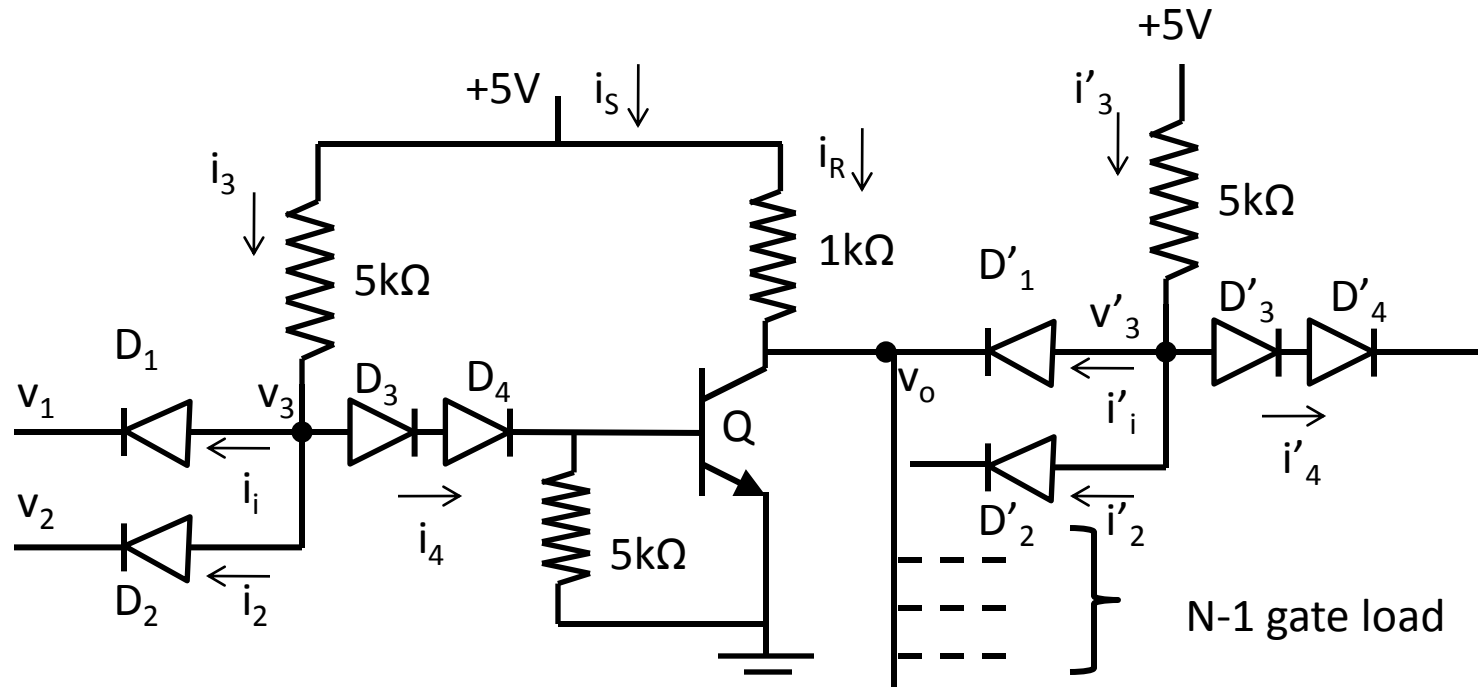
Average power (assuming the gate spends equal time in the 1 and 0 states) = $(P_0 + P_1)/2 = 15.45\text{ mW}$

DTL: Fan-out



- Fan-out= number of gates connected to the output of the logic gate

DTL Fan-out

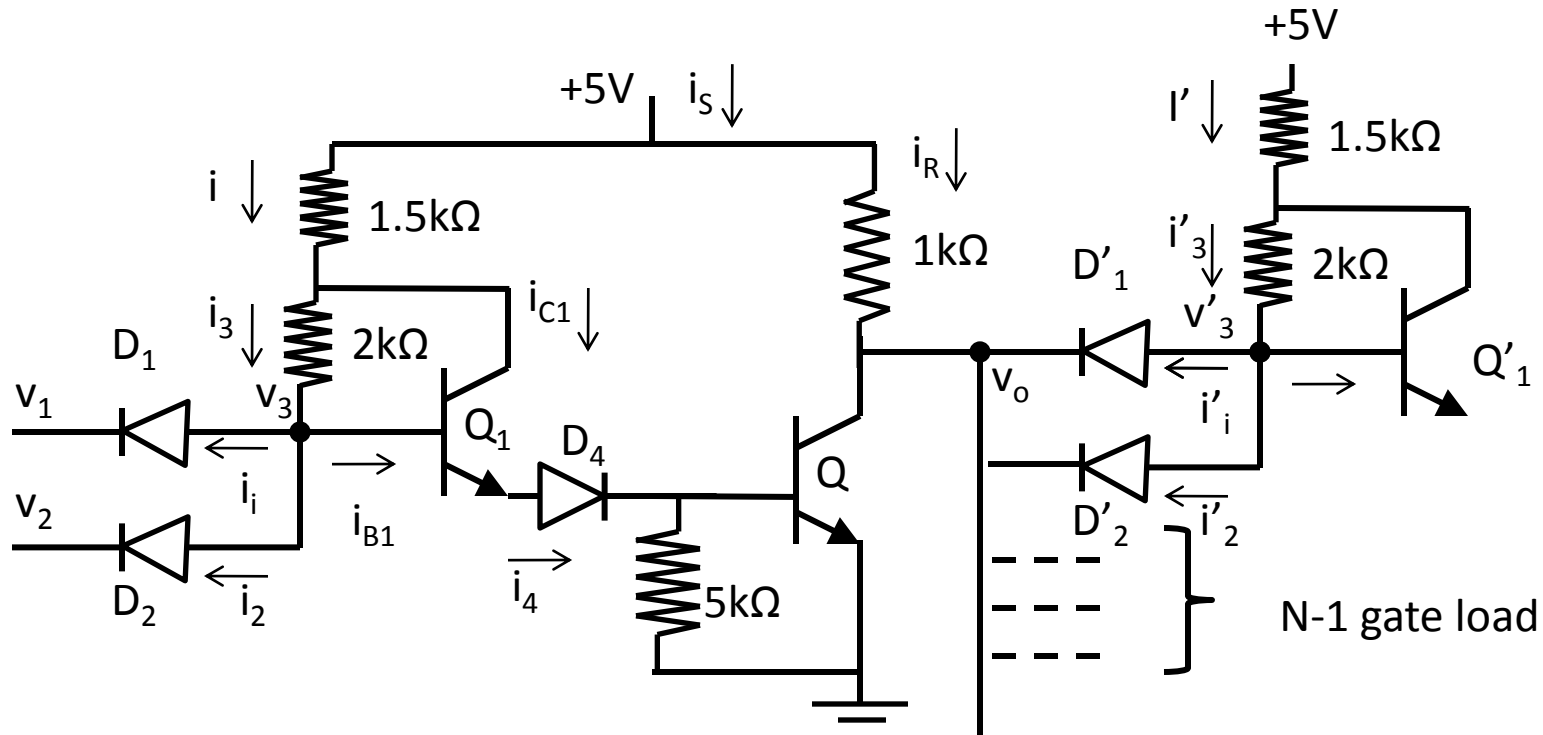


For $v_o = 5V$, D'_1 will be off, and similarly input diodes of other $N-1$ gates will be off.

For $v_o = 0.2V$, D'_1 will be on with $i'_3 \Rightarrow i_C = i_R + Ni'_1 = \frac{5-0.2}{1 \times 10^3} + N(0.82 \times 10^{-3}) = 4.8 \times 10^{-3} + N(0.82 \times 10^{-3})$

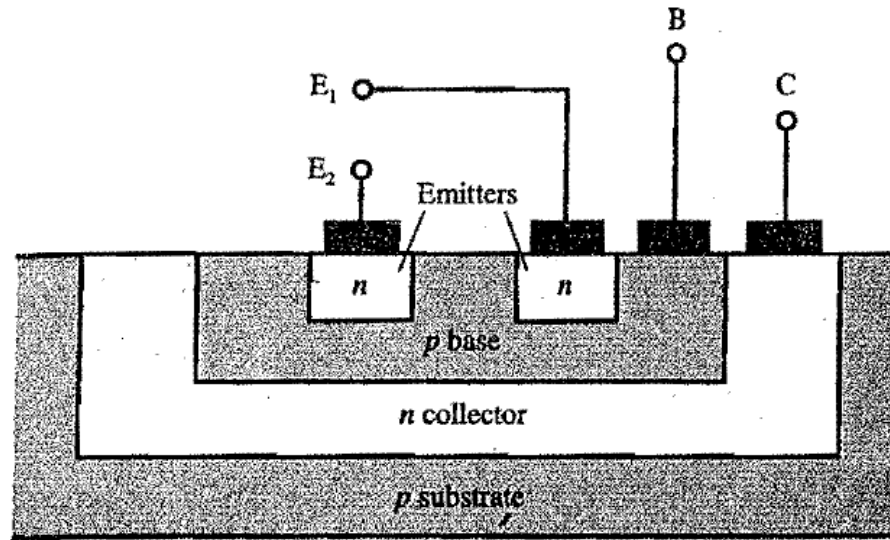
For Q to be on $h_{FE} i_B \geq i_C \Rightarrow 40(0.4 \times 10^{-3}) \geq 4.8 \times 10^{-3} + N(0.82 \times 10^{-3}) \Rightarrow N \leq 13.7$

DTL: Fan-out

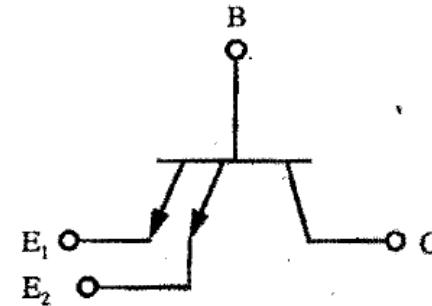


- Replacing D_3 by Q_1 can increase fan-out
 - Solve as HW and find out maximum N
 - Why?

TTL: Transistor-Transistor Logic



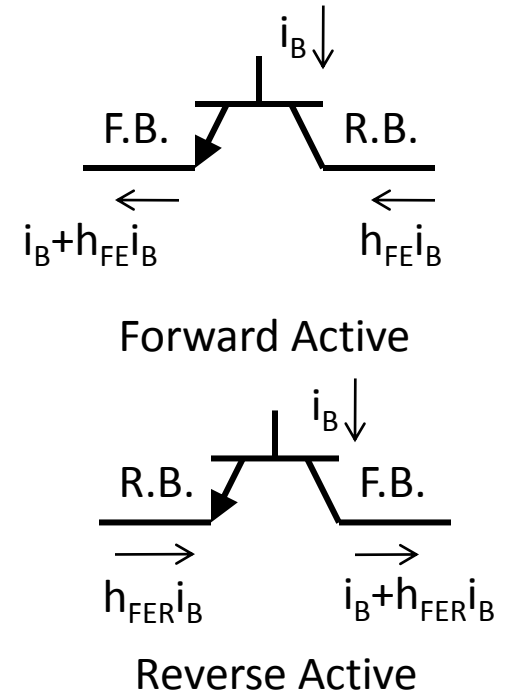
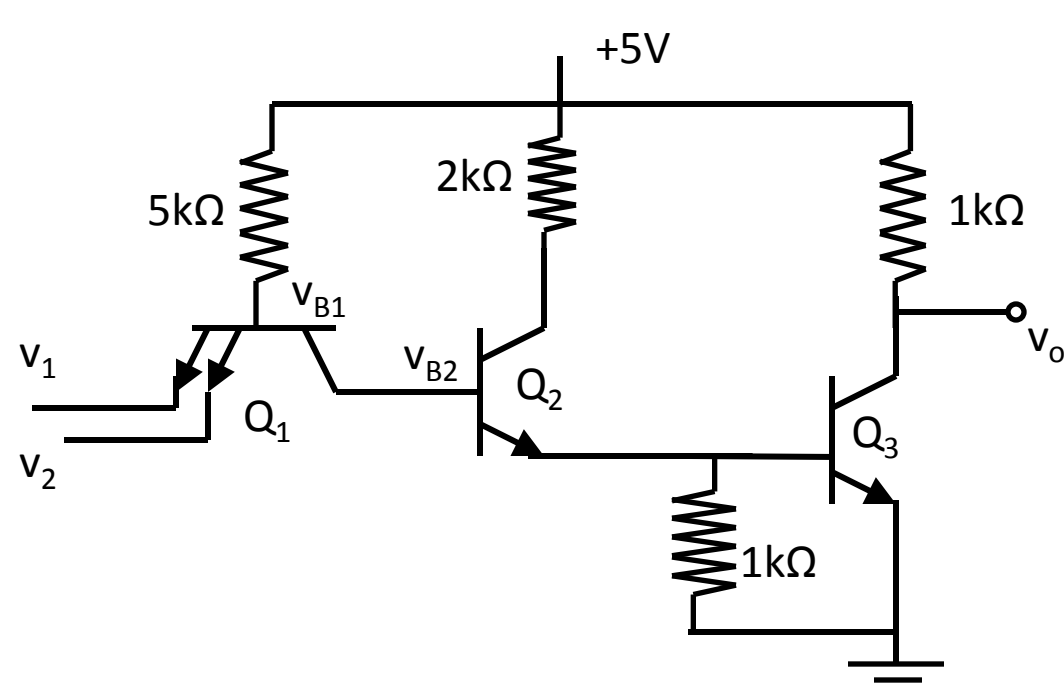
(a) two-emitter IC *n*pn transistor



(b) circuit symbol

- Multi-emitter BJT: Faster than DTL

TTL NAND Gate



Case 1/2/3 ($v_1=0.2V$, $v_2=5V$ / $v_1=5V$, $v_2=0.2V$ / $v_1=v_2=0.2V$)

Q_1 active $\rightarrow v_{B1}=0.9V \rightarrow i_{B1}=0.82mA$

Q_1 saturation $\rightarrow v_{B1}=1.0V \rightarrow i_{B1}=0.8mA$

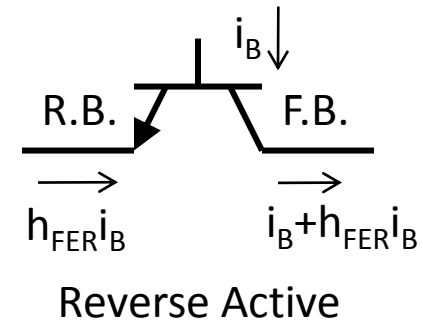
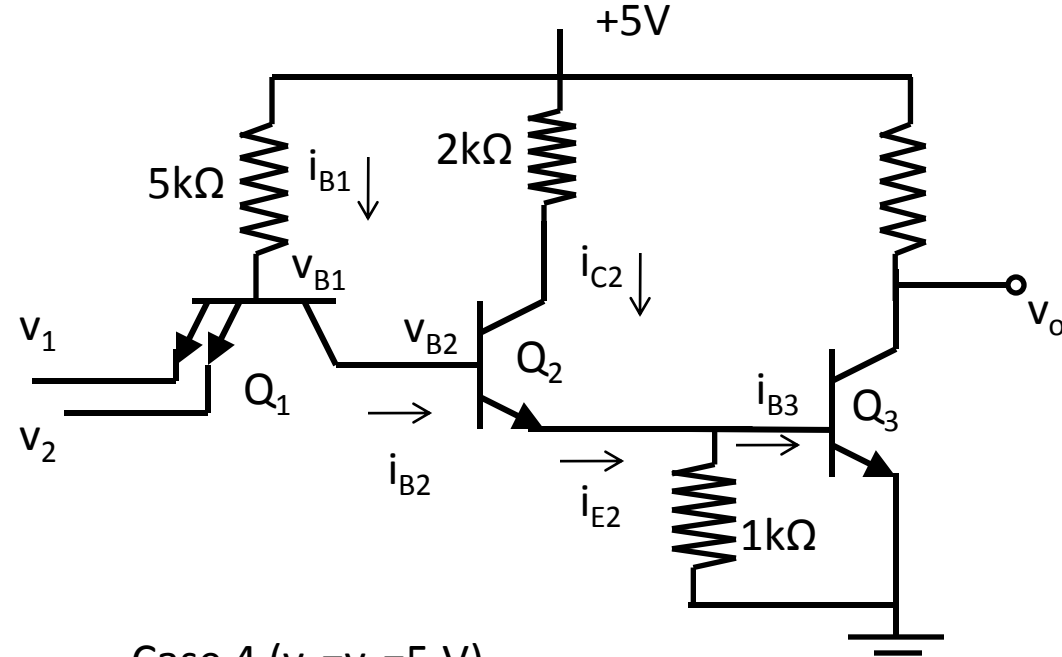
Assume Q_2 and Q_3 off.

$I_{CQ2} = -i_{B2} = i_{C1} \rightarrow nA$

$$h_{FE1} \geq \frac{i_{C1}}{i_{B1}} \Rightarrow v_{B2} = v_{CE1} + v_1 = 0.2 + 0.2 = 0.4V$$

Hence Q_2 and Q_3 are off and $v_o=5V$

TTL NAND Gate



Case 4 ($v_1=v_2=5\text{ V}$)

$Q_1 \rightarrow$ B.E. is R.B. and B.C. is forward-biased \rightarrow reverse active mode!

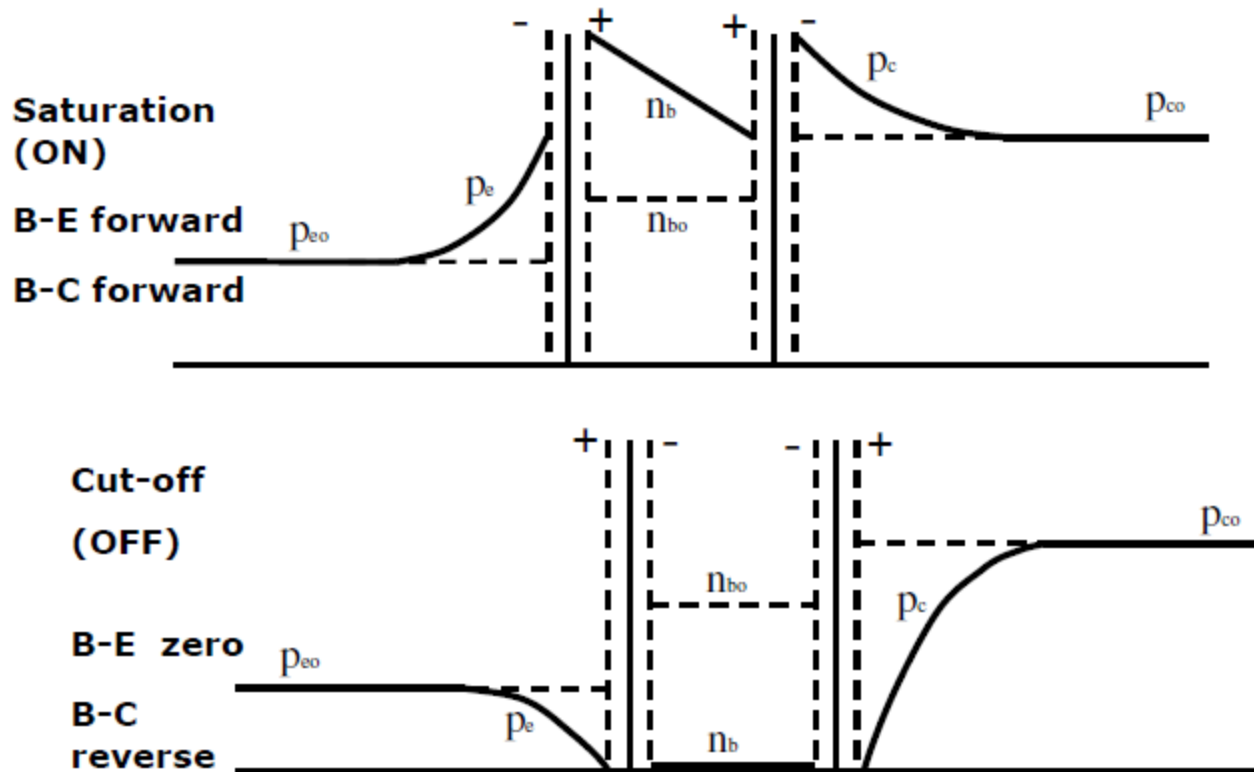
$i_{B2}=i_{B1}+h_{FER}i_{B1}$ turns Q_2 and Q_3 on. $v_{B1}=v_{BC1}+v_{BE1}+v_{BE2}=0.7+0.8+0.8=2.3\text{ V}$, $i_{B1}=0.54\text{mA}$

$i_{C2}=5-(0.2+0.8)/2000=2\text{mA}$, For Q_2 to be on, $h_{FE2} \geq \frac{i_{C2}}{i_{B2}} \Rightarrow h_{FE2} \geq \frac{2 \times 10^{-3}}{(1+h_{FER})0.54 \times 10^{-3}} = \frac{3.7}{1+h_{FER}}$

$i_{B3}=i_{B2}+i_{C2}-(0.8/1000)=(0.54+2-0.8) \times 10^{-3}=1.74\text{mA}$, $i_{C3}=(5-0.2)/1000=4.8\text{mA}$,

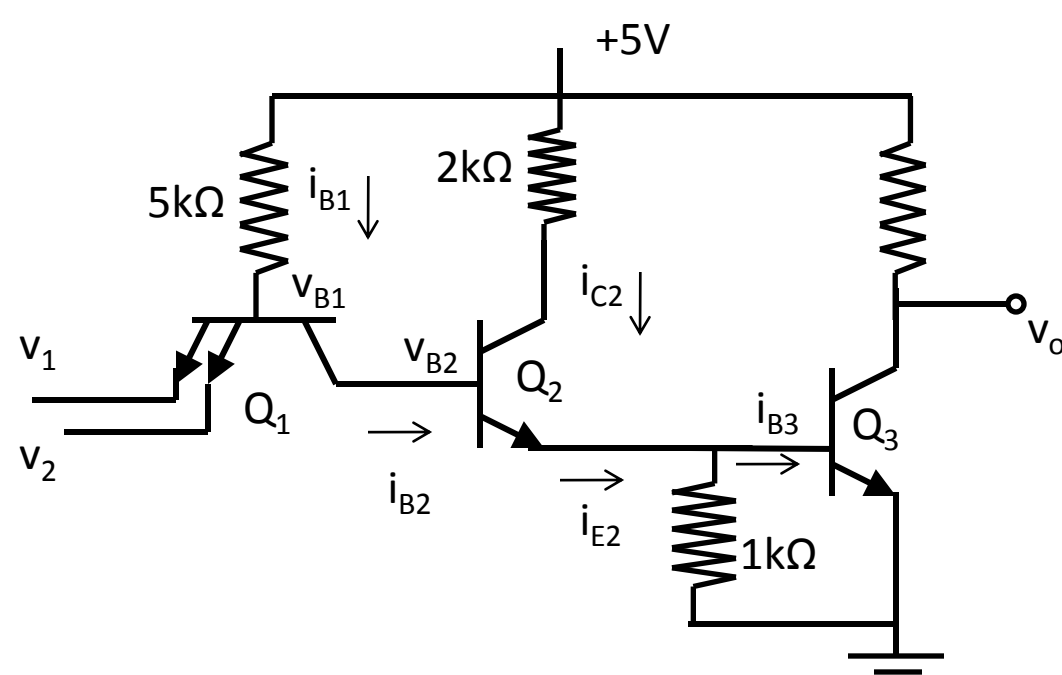
$h_{FE3} \geq \frac{i_{C3}}{i_{B3}} \Rightarrow h_{FE3} \geq \frac{4.8 \times 10^{-3}}{1.74 \times 10^{-3}} = 2.76$ Hence, Q_3 is also on and $v_o=0.2\text{V}$.

What determines switching speed?



- ON (SAT) \rightarrow OFF (CUT OFF) requires time to remove excess minority carriers in the base

Switching speed in TTL circuits



$v_1=v_2=5V \rightarrow Q_1$ is reverse active, Q_2 and Q_3 are ON

$$v_{B2}=v_{BE2}+v_{BE3}=0.8+0.8=1.6 \text{ V}$$

Say v_1 switches to 0.2 V

$$v_{B1}=0.9 \text{ V}$$

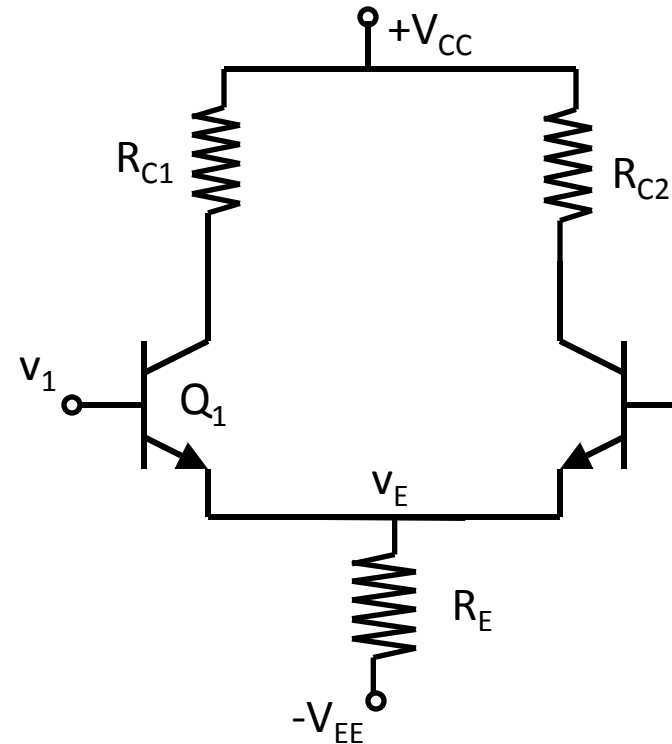
$$v_{B1}-v_{B2}=0.9-1.6=-0.7 \text{ V} \rightarrow \text{R.B.}$$

Hence Q_1 is in active mode

$$i_{C1}=h_{FE1}i_{B1}=0.82\text{mA} \times 50 = -41\text{mA} = i_{B2}$$

A large base current quickly removes stored charge from Q_2 and Q_3 . Subsequently Q_1 turns ON and Q_2 and Q_3 turn off.

Emitter-Coupled Logic (ECL)- “Non-saturating logic”



If $-V_{EE}+0.7 < v_1 < V_{CC}+0.5 \text{ V} \rightarrow Q_1$ is active

Also, $v_1 > v_R + 0.2 \text{ V} \rightarrow v_R - v_1 < -0.2 \text{ V}$

Then,

$$V_{BE2} = v_R - v_E = v_R - (v_1 - 0.7)$$

$$= 0.7 + v_R - v_1 < 0.7 - 0.2 = 0.5 \text{ V} \rightarrow Q_2 \text{ is cutoff}$$

- Fastest logic \rightarrow no saturation, only cut-off and active mode
- See example 7.18 in Bobrow's book for an ECL NOR/OR gate