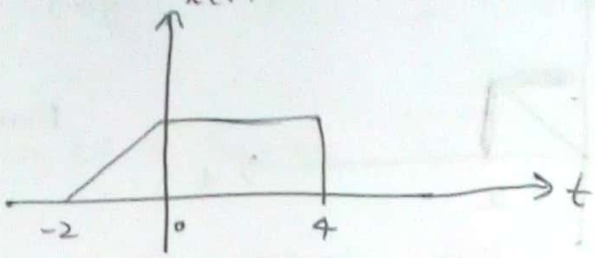
1) i)

Find a) $x\left(\frac{t}{2}-4\right)$

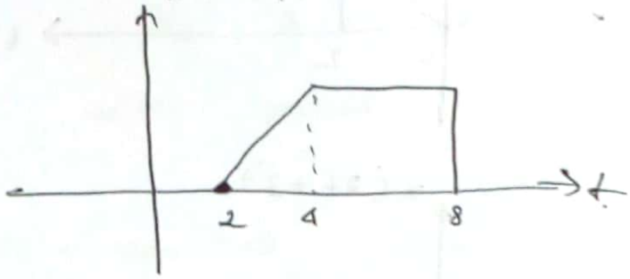
b) $x(-2t+2)$

$x(t)$

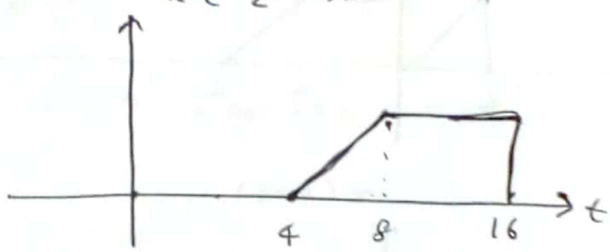


a)

$x(t-4)$

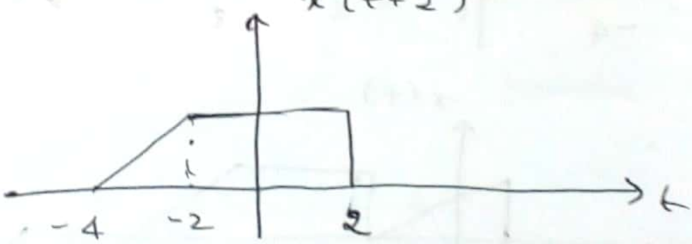


$x(t/2-4)$

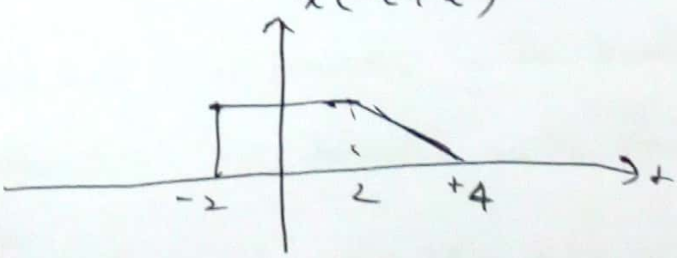


b)

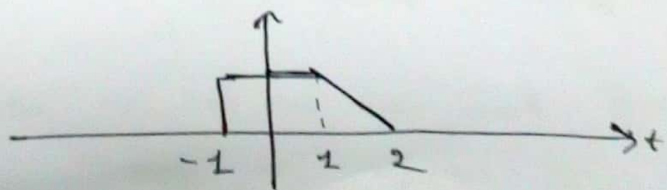
$x(t+2)$



$x(-t+2)$



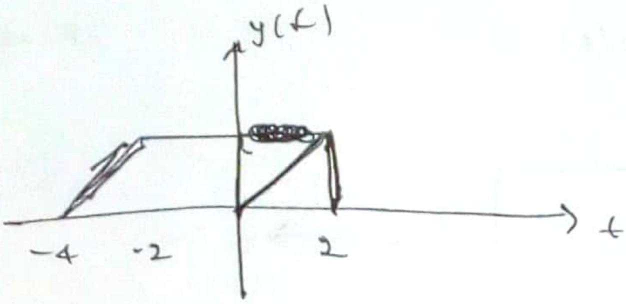
$x(-2t+2)$

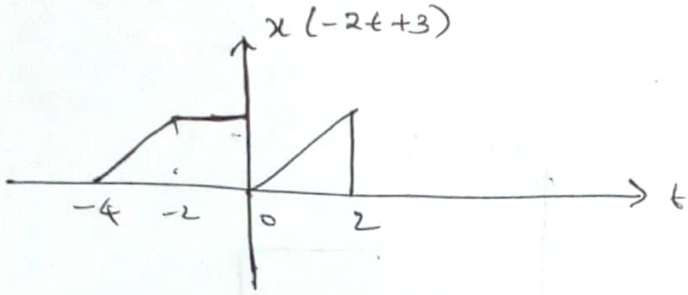


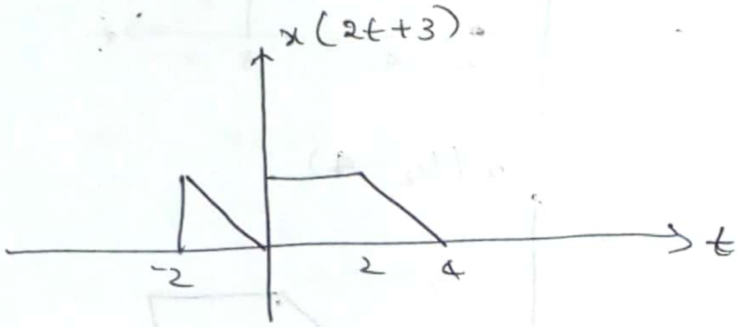
① ii)

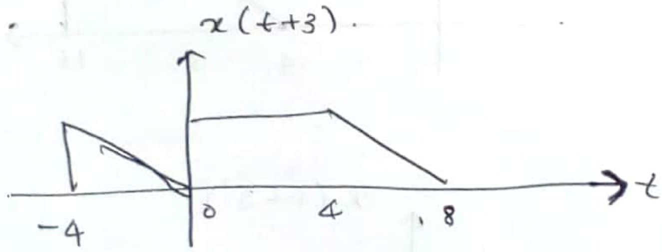
$y(t) = x(-2t+3)$

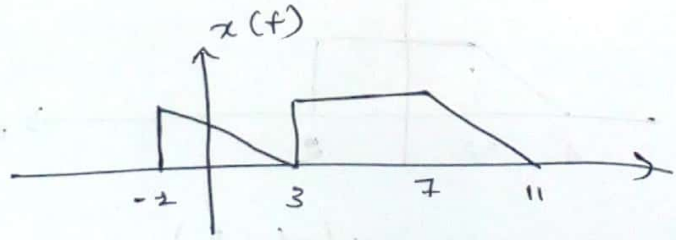
Find $x(t)$











(2) Check whether following signals are periodic or not

i) $\sin 2t + \cos 3t$

$T_1 = \sin 2t$ periodic with period π

$T_2 = \cos 3t$ periodic with period $\frac{2\pi}{3}$

$\frac{T_1}{T_2} = 3/2$ $\therefore$ $\sin 2t + \cos 3t$ is periodic

with period $3T_2 = 2T_1 = 2\pi$

ii) $\sin 7t + \cos 2\pi t$

$T_1 = \frac{2\pi}{7}$ $T_2 = 1$ $\frac{T_1}{T_2} = \frac{2\pi}{7}$ not rational

$\therefore$ $\sin 7t + \cos 2\pi t$ not periodic

iii) $\sin 8\pi t + \sin 3\pi t$

$T_1 = \frac{1}{4}$ $T_2 = \frac{2}{3}$ $\frac{T_1}{T_2} = \frac{3}{8}$

$\therefore$ $\sin 8\pi t + \sin 3\pi t$ periodic with period

$T = 8T_1 = 3T_2 = 2$

iv) $\sin 2\pi n + \cos 3\pi n$.

$N_1 = \sin 2\pi n$ is periodic with period 1

$N_2 = \cos 3\pi n$ is periodic with period 2

$\frac{N_1}{N_2} = \frac{1}{2}$ $\therefore$ $\sin 2\pi n + \cos 3\pi n$ is periodic with $N = 2$

v) $\sin 2n$ & $\cos 3n$ are not periodic for any integer N. Hence their sum is not periodic

(3) i) If $x(t)$ is odd P.T $\int_{-\infty}^{\infty} x(t)\,dt = 0$

$$S = \int_{-\infty}^{\infty} x(t)\,dt$$

Put $m = -t$

$$S = \int_{\infty}^{-\infty} x(-m)\,(-dm)$$

$$= \int_{-\infty}^{\infty} x(-m)\,dm$$

As $x(m)$ is odd $x(-m) = -x(m)$

$$S = -\int_{-\infty}^{\infty} x(m)\,dm$$

Substitute m with t

$$S = -\int_{-\infty}^{\infty} x(t)\,dt$$

$\Rightarrow$ $S = -S$ $\Rightarrow$ $2S = 0$ $\Rightarrow$ $\boxed{S = 0}$

$$\therefore \int_{-\infty}^{\infty} x(t)\,dt = 0$$

---

(3) ii) $y(t) = x_1(t) + x_2(t)$

Energy of $y(t) = \int_{-\infty}^{\infty} y^2(t)\,dt$

$$= \int_{-\infty}^{\infty} [x_1(t) + x_2(t)]^2\,dt$$

$$= \int_{-\infty}^{\infty} x_1^2(t)\,dt + \int_{-\infty}^{\infty} x_2^2(t)\,dt + 2\int_{-\infty}^{\infty} x_1(t)\,x_2(t)\,dt$$

$$= \text{Energy } x_1(t) + \text{Energy } x_2(t)$$

$$+ 2\int_{-\infty}^{\infty} x_1(t)\,x_2(t)\,dt.$$

$\therefore$ Energy of $y(t)$ = Energy of $x_1(t)$ + Energy of $x_2(t)$

If & only if $\int_{-\infty}^{\infty} x_1(t)\,x_2(t)\,dt = 0$

This can occur if $x_1(t) \cdot x_2(t)$ is odd function or

if $x_1(t) \cdot x_2(t) = 0$ for all t

and also in many other cases.

~~[illegible]~~

From this result we can know that

Energy of a signal = Energy of Even part of signal + Energy of odd part of signal

(4) i) A linear time invariant ~~signal~~ system is ~~gi~~ shown as

$$x(t) \longrightarrow y(t)$$

P.T

$$\frac{d(x(t))}{dt} \longrightarrow \frac{d[y(t)]}{dt}$$

Sol: $x(t) \longrightarrow y(t)$

$\Rightarrow$ $x(t+h) \longrightarrow y(t+h)$ as system is time invariant

$\therefore$ $(1)\,x(t+h) + (-1)\,x(t) \longrightarrow (1)\,y(t+h) + (-1)\,y(t)$

From Linearity

$\Rightarrow$ $x(t+h) - x(t) \longrightarrow y(t+h) - y(t)$

$$\therefore \frac{x(t+h) - x(t)}{h} \longrightarrow \frac{y(t+h) - y(t)}{h}$$

By linearity [Scaling: $h \neq 0$]

If we put limit $h \to 0$, it is nothing but

$$\frac{dx(t)}{dt} \longrightarrow \frac{dy(t)}{dt}$$

4) ii) $x(t) \longrightarrow e^{-t}x(t)$

Find impulse response of the system ?

$$u(t) \longrightarrow e^{-t}u(t)$$

$$\therefore \frac{d[u(t)]}{dt} \longrightarrow \frac{d[e^{-t}u(t)]}{dt}$$

$$\delta(t) \longrightarrow e^{-t}\delta(t) + u(t)(-1)e^{-t}$$

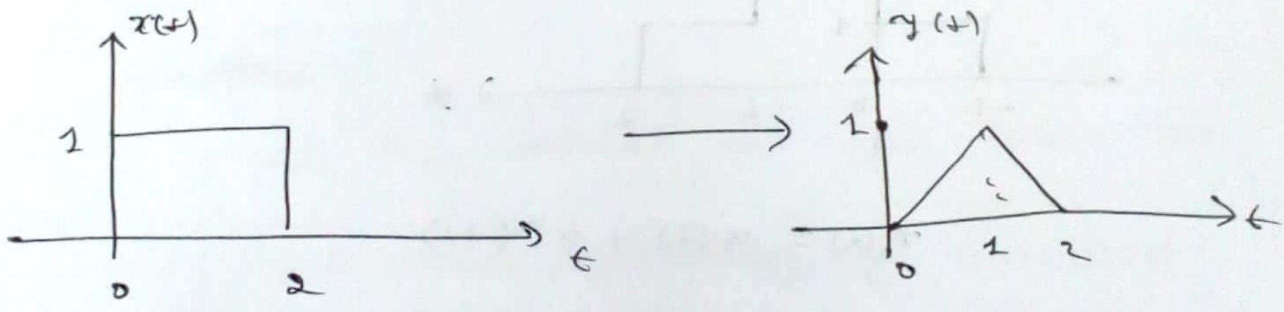
$$\therefore \delta(t) \longrightarrow e^{-t}\delta(t) - e^{-t}u(t)$$

$\therefore$ Impulse response

is $\boxed{h(t) = \delta(t) - e^{-t}u(t)}$

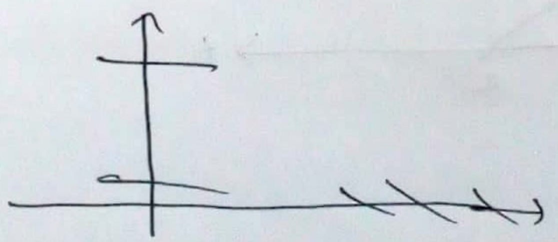
---

5) I/p & o/p of a LTI system is shown

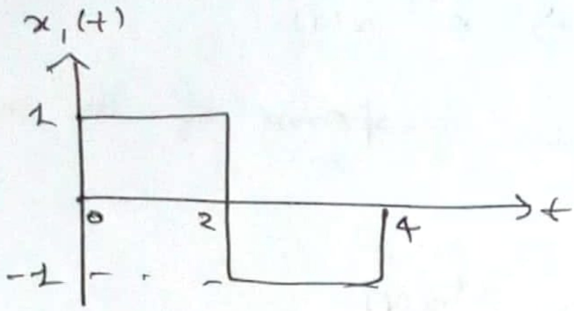


Find output for given i/p.

~~i) $x_1(t)$~~



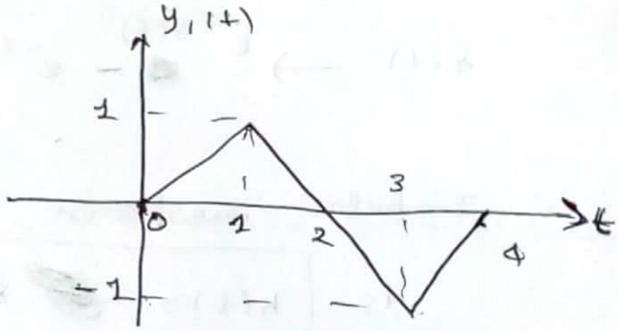
(5) i)



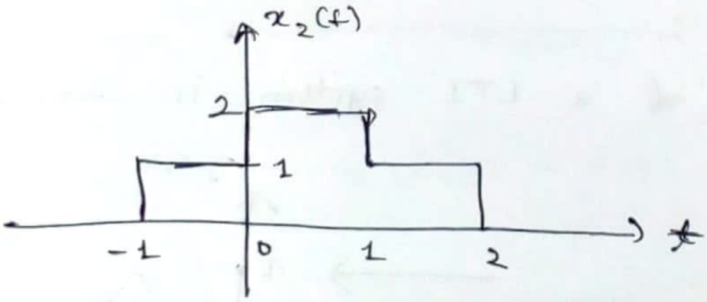
Soln: It can be seen that

$$x_1(t) = x(t) - x(t-2)$$

$\therefore$ o/p $y_1(t) = y(t) - y(t-2)$

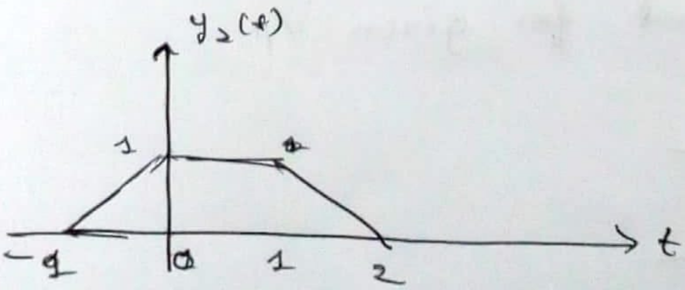


ii)

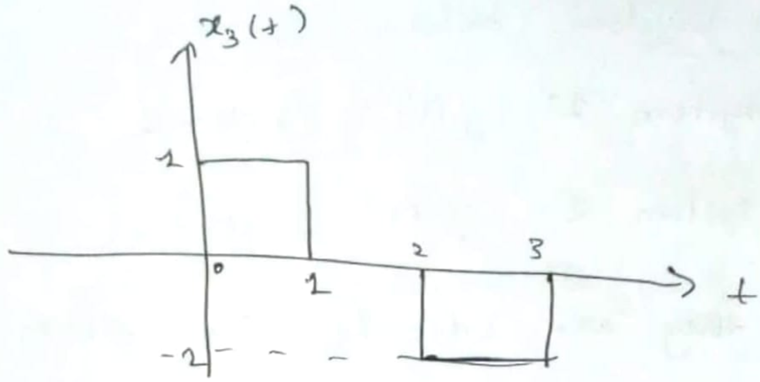


$$x_2(t) = x(t) + x(t+1)$$

$$\therefore \; y_2(t) = y(t) + y(t+1)$$

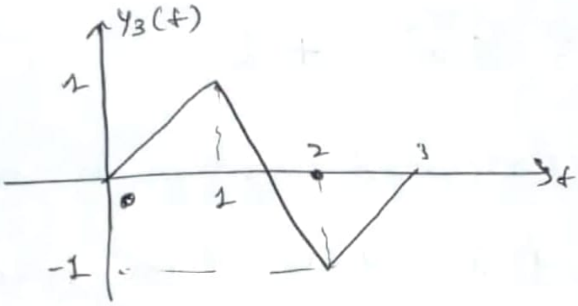


5 iii)

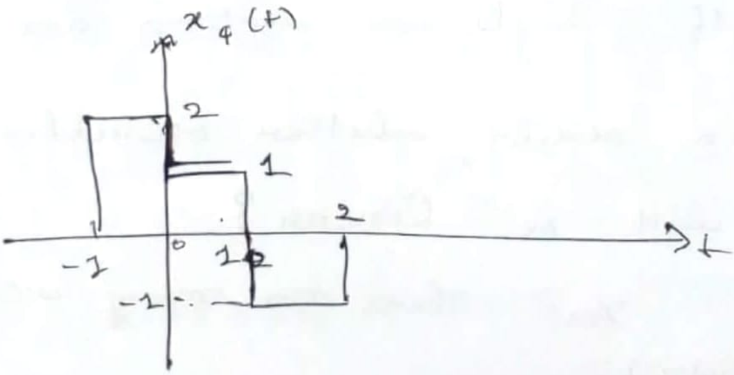


$$x_3(t) = x(t) - x(t-1)$$

$$y_3(t) = y(t) - y(t-1)$$

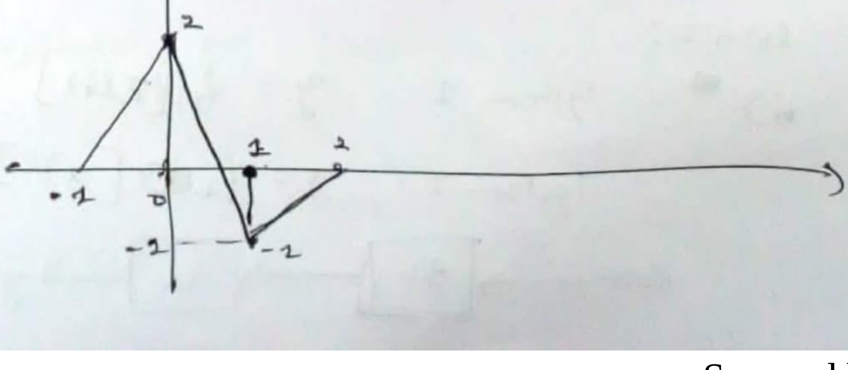


iv)



$$x_4(t) = 2x(t+1) - x(t)$$

$$y_4(t) = 2y(t+1) - y(t)$$



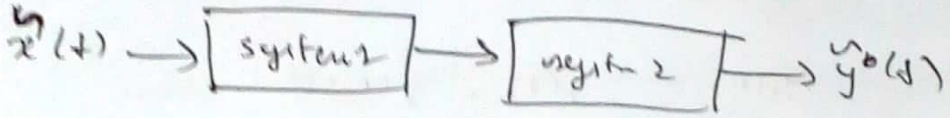
6) i) Two system behave as

system 1: $y(t) = 2x(t) + 2$

system 2: $y(t) = 2x(t) - 2$

If they are cascaded in series will the resulting system be LTI?

ans:



$\tilde{y}^{o}(t) = 4\tilde{x}(t) + 2$

Cascaded system is not linear.

But it is time invariant.

---

ii) If 2 linear systems are connected in series, whether resulting system will be linear?

Ans: Yes. There are many ways to prove.

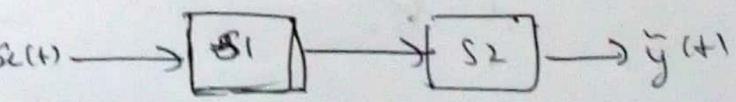
Soln 1:

Let two LTI system with impulse response $h_1(t)$ & $h_2(t)$, when they are cascaded resulting system will be a LTI system with impulse response $h_1(t) * h_2(t)$.

Soln 2:

ii) System 1: $y = f_1[x(t)]$

System 2: $y = f_2[x(t)]$



Let

$$\tilde{x}_1(t) \longrightarrow \tilde{y}_1(t) = f_2[f_1[\tilde{x}_1(t)]]$$

& $$\tilde{x}_2(t) \longrightarrow \tilde{y}_2(t) = f_2[f_1[\tilde{x}_2(t)]]$$

$$\therefore \quad a\tilde{x}_1(t) + b\tilde{x}_2(t) \longrightarrow f_2[f_1[a\tilde{x}_1(t) + b\tilde{x}_2(t)]]$$

$$f_2[f_1[a\tilde{x}_1(t) + b\tilde{x}_2(t)]] = f_2[a f_1[\tilde{x}_1(t)] + b f_1[\tilde{x}_2(t)]]$$

as f_1 is linear

$$\therefore \quad f_2[f_1(a\tilde{x}_1(t) + b\tilde{x}_2(t)]$$

$$= a f_2[f_1[\tilde{x}_1(t)]] + b f_2[f_1[\tilde{x}_2(t)]]$$

$$= a\tilde{y}_1(t) + b\tilde{y}_2(t)$$

$$\therefore \quad a\tilde{x}_1(t) + b\tilde{x}_2(t) \longrightarrow a\tilde{y}_1(t) + b\tilde{y}_2(t)$$

$\therefore$ cascaded system is linear.

---

5 iii) If a linear & a nonlinear systems are connected in series, whether the resulting system be linear.

Ans: No.

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6 iv) If two nonlinear systems are connected in series whether the resulting system be nonlinear?

Ans: Not always.

Ex: System 1: $2x(t) + 2 = y(t)$

System 2: $2x(t) - 4 = y(t)$

when they are cascaded resulted system will be

$$x(t) \longrightarrow \boxed{S1} \longrightarrow \boxed{S2} \longrightarrow y(t)$$

$$y(t) = 4x(t)$$

---

7 Compute the convolution

i) $x(n) = \alpha^{n} u(n)$ $\quad h(n) = \beta^{n} u(n-a)$

$|\alpha|, |\beta| < 1$

$|\alpha| \neq |\beta|$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \alpha^{k} u(k)\, \beta^{n-k} u(n-k-a)$$

$$= \beta^{n} \sum_{k=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^{k} u(n-k-a)$$

case i) If $n<a$

$$y(n) = 0$$

case ii) If $n>a$

$$y(n) = \beta^n \sum_{k=0}^{n-a} \left(\frac{\alpha}{\beta}\right)^k$$

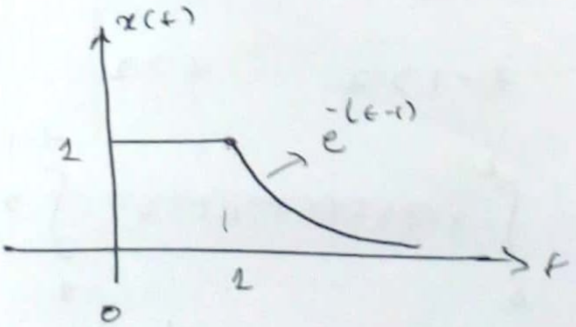
$$= \beta^n \left[\frac{1-\left(\frac{\alpha}{\beta}\right)^{n-a+1}}{1-\left(\frac{\alpha}{\beta}\right)}\right]$$

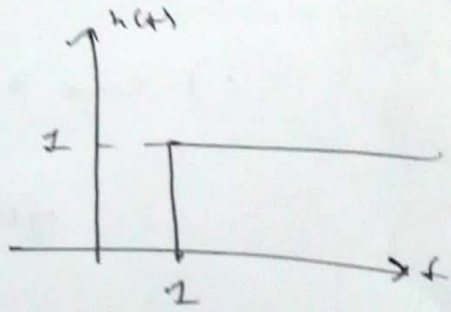
$$= \beta^n \left[\frac{\frac{\beta^{n-a+1}-\alpha^{n-a+1}}{\beta^{n-a+1}}}{\frac{\beta-\alpha}{\beta}}\right]$$

$$\boxed{y(n) = \beta^a \left[\frac{\beta^{n-a+1}-\alpha^{n-a+1}}{\beta-\alpha}\right]}$$

---

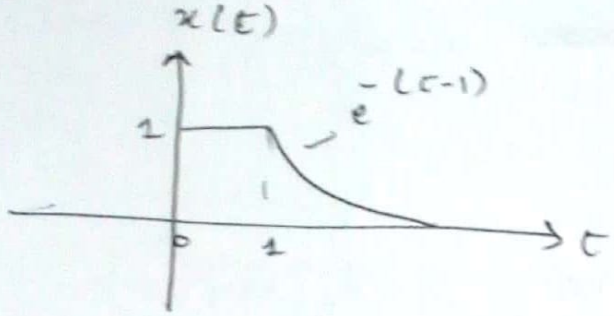
7 ii)

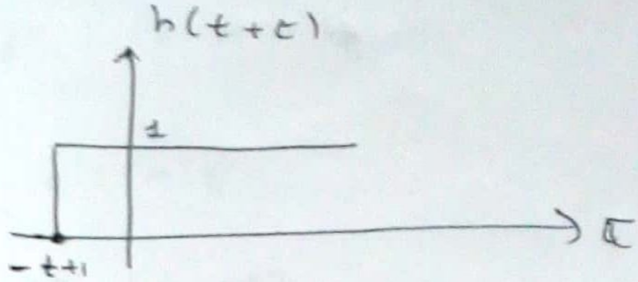


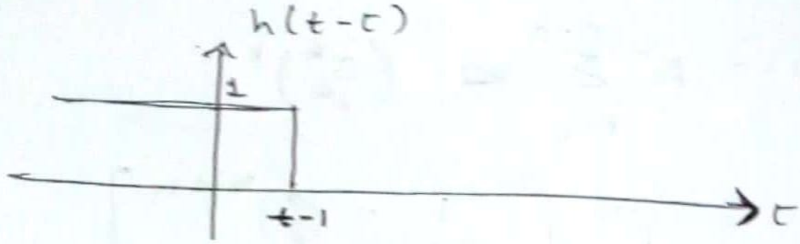


$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$







i) case 1: $t-1 \leq 0$ or $t \leq 1$

$$y(t) = 0$$

ii) case 2: $1 \geq t-1 > 0$ or $2 \geq t > 1$

$$y(t) = \int_0^{t-1} x(\tau)\, h(t-\tau)\, d\tau = \int_0^{t-1} 1.1\, d\tau$$

$$\boxed{y(t) = t-1}$$

iii) case 3: $t-1 > 2$ $t > 2$

$$y(t) = \int_0^1 x(\tau)\, h(t-\tau)\, d\tau + \int_1^{t-1} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_0^1 1\, d\tau + \int_1^{t-1} e^{-(\tau-1)}\, 1.\, d\tau$$

$$= \cancel{1 +} \cancel{\left[\frac{e^{-(\tau-1)}}{-1} \right]_1^{t-1}}$$

$$y(t) = 1 + e\left[\left[\frac{e^{-\tau}}{-1}\right]_1^{t-1}\right]$$

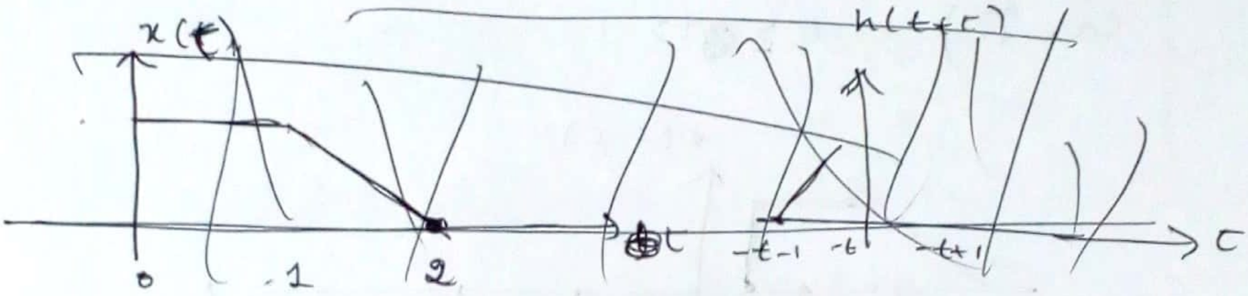
$$= 1 + e\left[e^{-1} - e^{-(t-1)}\right]$$

$$= 1 + \left[1 - e^{-(t-2)}\right]$$

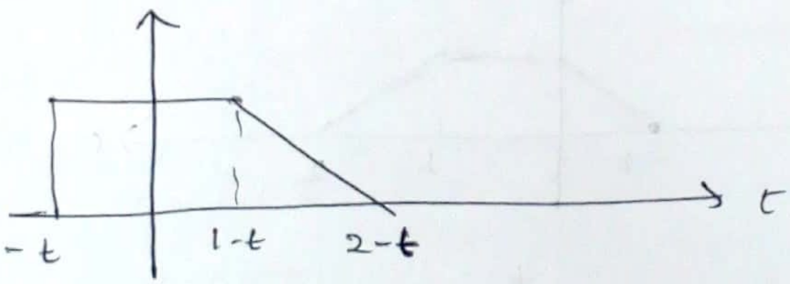
$$\boxed{y(t) = 2 - e^{-(t-2)}}$$

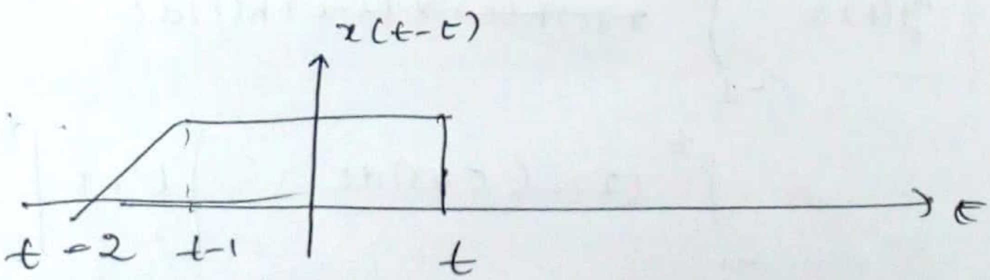
---

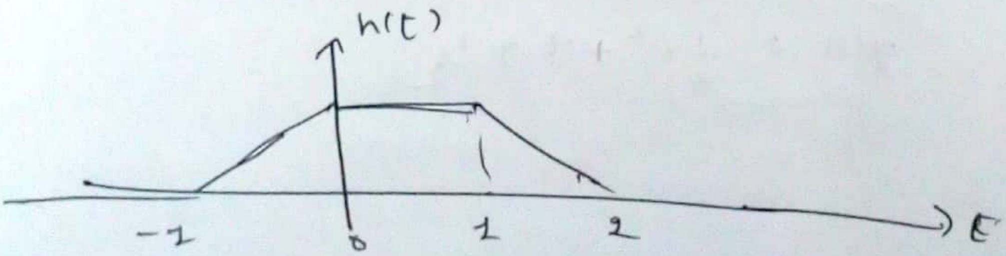
7 iii)



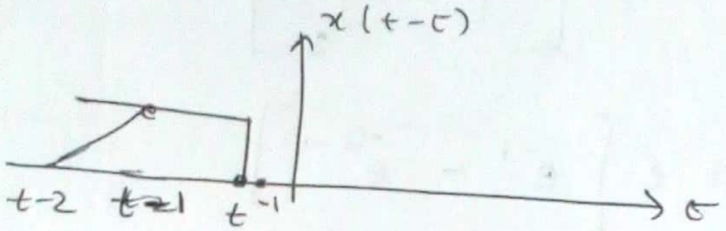
Soln: $x(t+\tau)$

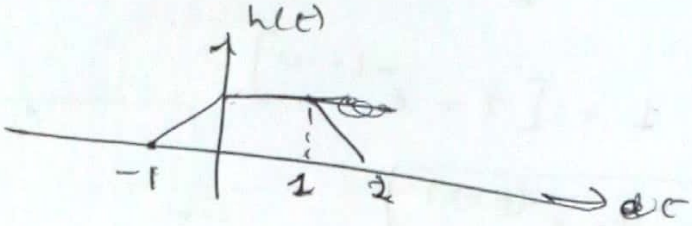






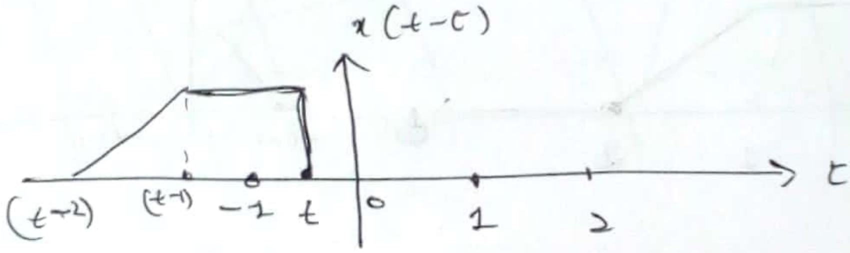
case i) $t < -1$

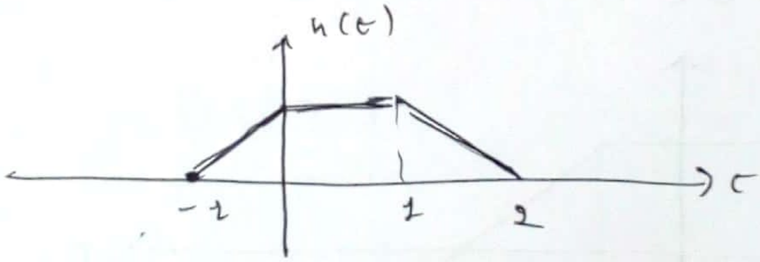




$$y(t) = 0$$

case ii) $0 \geq t > -1$





$$y(t) = \int_{-1}^{t} \cancel{x(\tau)h(\tau)}\, x(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_{-1}^{t} (1).(\tau+1)\, d\tau = \left[\frac{\tau^2}{2} + \tau\right]_{-1}^{t}$$

$$y(t) = \frac{1}{2}t^2 + t + 1/2$$

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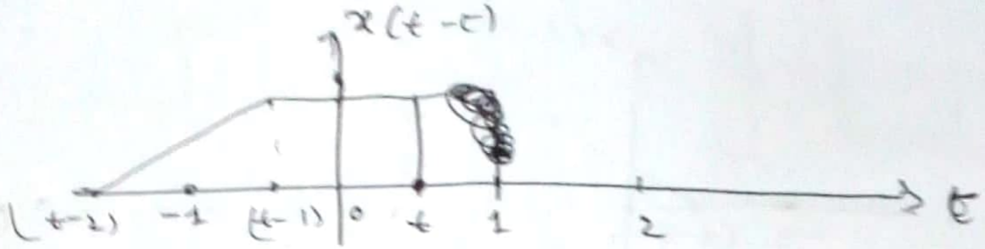
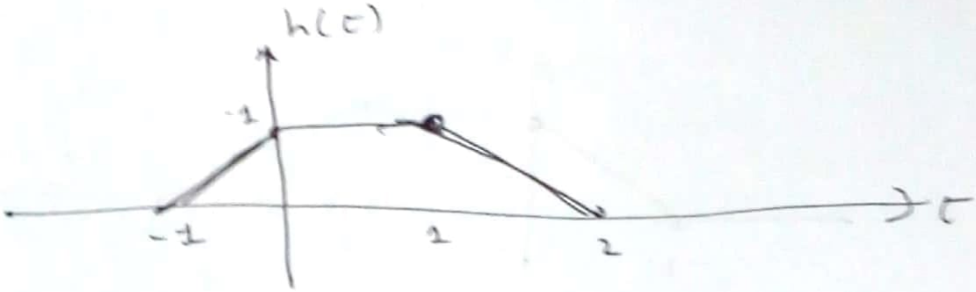
case iii) $1 \geq t > 0$





$$y(t) = \int_{-1}^{t-1} x(t-\tau)h(\tau)d\tau + \int_{t-1}^{0} x(t-\tau)h(\tau)d\tau$$

$$+ \int_{0}^{t} x(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_{-1}^{t-1} [\tau - t + 2](\tau+1)\, d\tau + \int_{t-1}^{0} (1)(\tau+1)\, d\tau$$

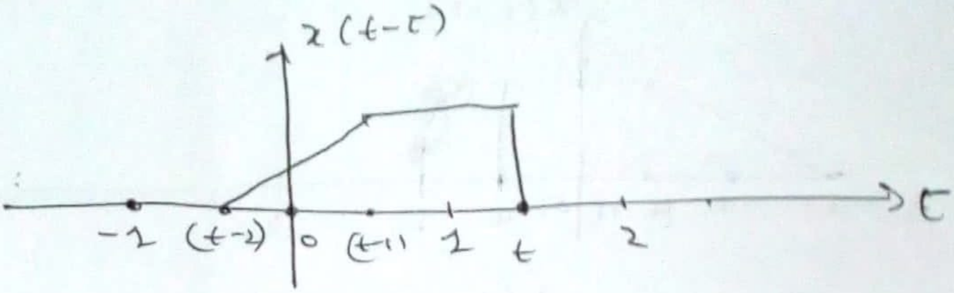
$$+ \int_{0}^{t} 1.1\, d\tau$$

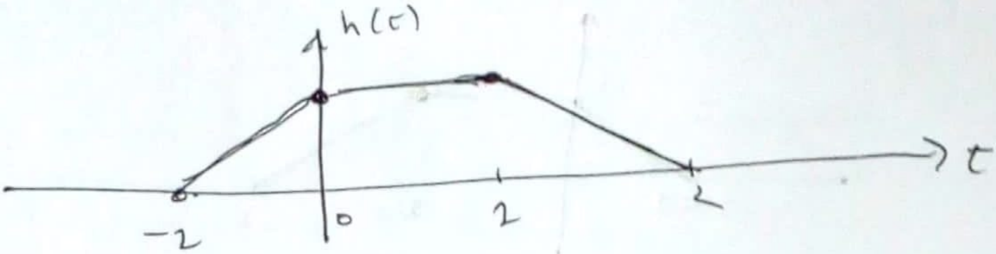
$$= \int_{-1}^{t-1} \left[\tau^2 + \tau(3-t) + (2-t)\right] d\tau$$

$$+ \int_{t-1}^{0} (\tau+1)\, d\tau + \int_{0}^{t} 1\, d\tau$$

$$\boxed{y(t) = \left[\frac{(t-1)^3+1}{3} + \frac{(t-1)^2-1}{2}(3-t) + (2-t)\right] + \left[1 - t - \frac{(t-1)^2}{2}\right] + t}$$

case iv) $2 \geq t > 1$





$$y(t) = \int_{(t-2)}^{0} (\tau - t + 2)(\tau + 1)\, d\tau + \int_{0}^{t-1} (\tau - t + 2)\, 1\, d\tau$$

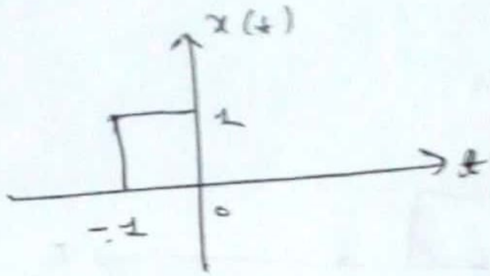
$$+ \int_{t-1}^{1} 1 \cdot 1\, d\tau + \int_{1}^{t} 1 \cdot (-\tau + 2)\, d\tau$$

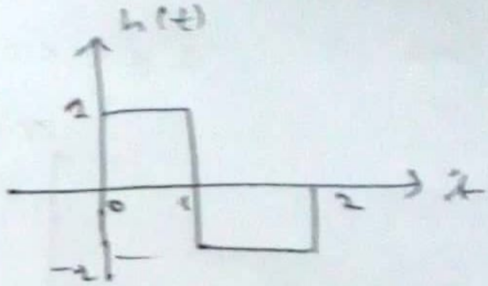
$$\boxed{y(t) = \left[(t-2)^2 - (3-t)\frac{(t-2)^2}{2} - \frac{(t-2)^3}{3} \right] + \left[\frac{(t-1)^2}{2} + (2-t)(t\ldots \right. \\ + [2-t] + \left[-\frac{t^2}{2} + 2t - \frac{3}{2} \right]}$$

Similarly for other cases it has to be foun[d]

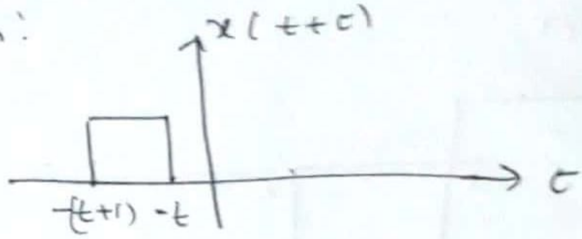
There will be 7 cases.

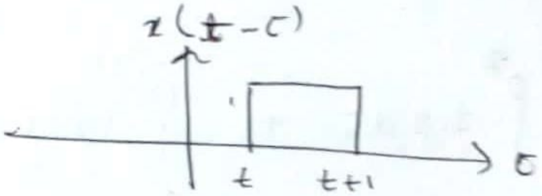
7 iv)

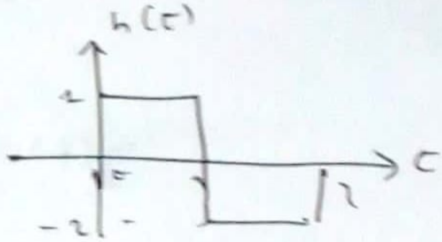




Soln:



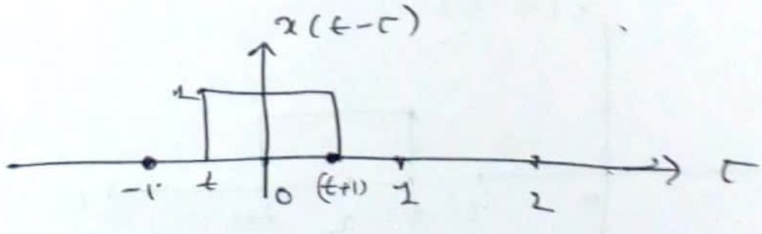


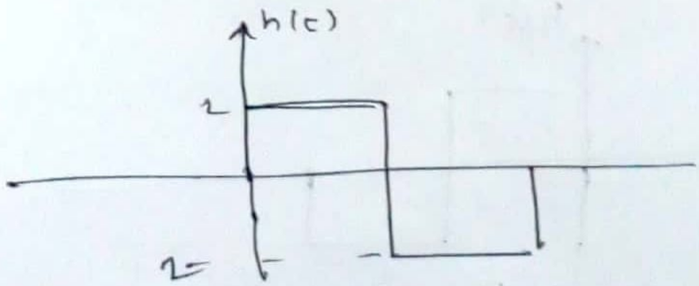


case 1: $t+1<0$ or $t<-1$

$$y(t)=0$$

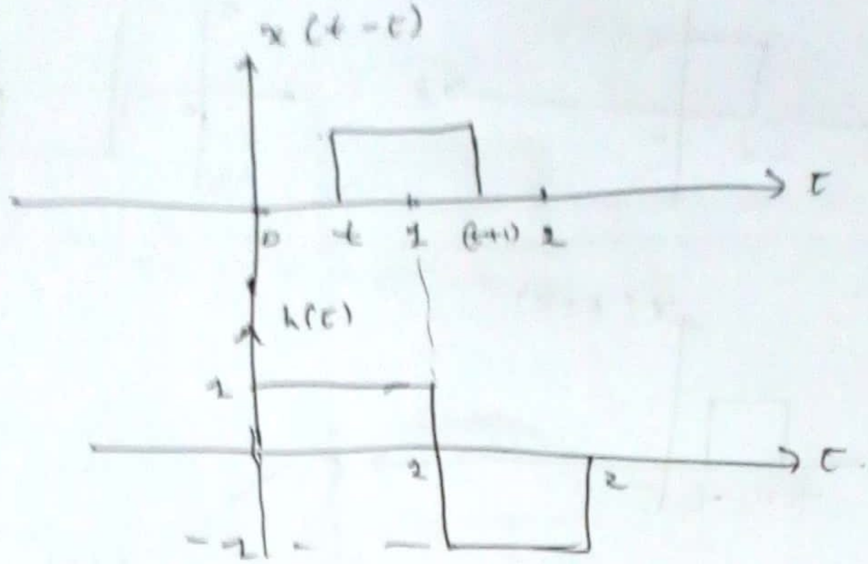
case 2: $0 \le t < -1$





$$y(t) = \int_{0}^{t+1} 1 \, d\tau = \underline{t+1}$$

Case 3) $1 \leq t < 0$

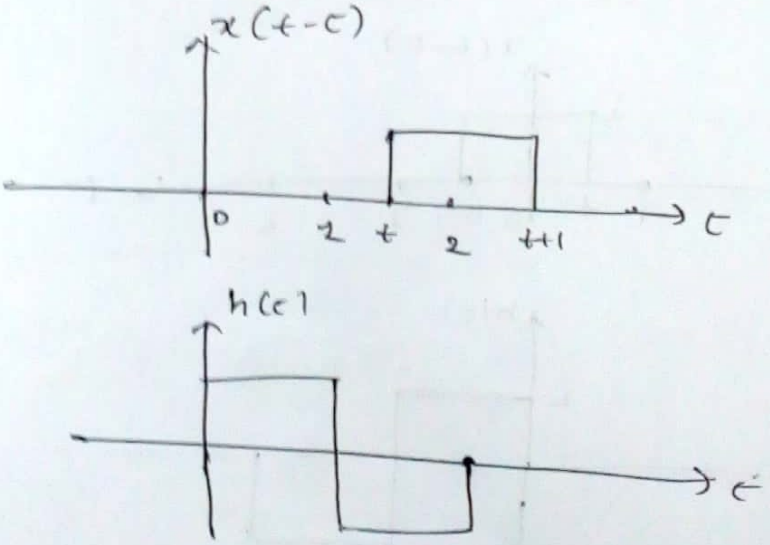


$$y(t) = \int_t^1 1.1 \, d\tau + \int_1^{t+1} (1)(-1) \, d\tau$$

$$= (1-t) - t$$

$$\boxed{y(t) = 1 - 2t}$$

Case 4: $t > 1$



$$y(t) = \int_t^2 1(-1) \, d\tau = \underline{\underline{t-2}}$$

$$\boxed{y(t) = t-2}$$

8) Determine whether the following system are causal, memoryless & BIBO stable.

i) $h(t) = e^{-2t} u(t)$

$h(t) = 0 \quad t < 0$ hence causal.

$h(t) \neq 0$ for all $t \neq 0$ hence not memoryless

$\int_{-\infty}^{\infty} h(t)\, dt = \int_{0}^{\infty} e^{-2t}\, dt$ is finite.

Hence BIBO stable.

ii) $h(t) = 2^t u(t-3)$

Causal

not memoryless

$\int_{-\infty}^{\infty} h(t)\, dt = \int_{3}^{\infty} 2^t\, dt$ is not finite

Hence not BIBO stable

iii) $h(t) = 3\delta(t)$

Causal

$h(t) = 0$ for all $t \neq 0$ Hence memoryless

BIBO stable

iv) $h(t) = e^{-t} u(t+\pi)$

$h(t) \neq 0 \;\; \forall\; t < 0$ Hence not causal.

not memoryless

8) v) $h(n) = 2^{-n} u(n)$

$h(n) = 0 \quad \forall \; n < 0$. $\therefore$ Causal

$h(n) \neq 0 \quad \forall \; n \neq 0$ $\therefore$ Hence not memoryless

$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} 2^{-n}$ is finite. Hence BIBO stable

vi) $h(n) = 2^{n} u(-n)$

$h(n) \neq 0 \quad \forall \; n < 0$ $\therefore$ Not causal,

Not memoryless

$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{0} 2^{n} = \sum_{n=0}^{\infty} 2^{-n}$ is finite,

Hence BIBO stable

vii) $h(n) = \left(\frac{1}{3}\right)^{n} u(-n)$

Not causal.

Not memoryless.

$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^{n} = \sum_{n=0}^{\infty} 3^{n}$ is not finite

Hence not BIBO stable

viii) $h(n) = \left(\frac{1}{3}\right)^{n} u(n+1)$

not causal

Not memoryless

BIBO stable.

9) Check whether following are true or not

i) $x(n) * [h(n)g(n)] = [x(n) * h(n)]g(n)$

Not true.

We will prove it by counter example

Let $g(n) = \delta(n)$

$\therefore$ LHS $= x(n) * [h(n).\delta(n)]$

$= x(n) * [h(0)\,\delta(n)]$

$= h(0)\, x(n)$

RHS $= [x(n) * h(n)].\delta(n)$

~~RHS will be~~

RHS $= 0 \quad \forall \quad n \neq 0$

But LHS $\neq 0 \quad \forall n \neq 0$.

ii) $y(t) = x(t) * h(t)$ Then, P.T.

$y(2t) = 2\, x(2t) * h(2t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\therefore \quad y(2t) = \int_{-\infty}^{\infty} x(\tau)\, h(2t-\tau)\, d\tau$$

Let $\tau' = \frac{\tau}{2}$

$$y(2t) = \int_{-\infty}^{\infty} x(2\tau') \, h(2t - 2\tau') \, (2 d\tau')$$

$$= 2 \int_{-\infty}^{\infty} x(2\tau) \, h[2(t - \tau')] \, d\tau'$$

$$= 2 \, x(2t) * h(2t)$$

iii) $x(t)$ & $h(t)$ are odd. Then

$y(t) = x(t) * h(t)$ is even.

Soln: $y(t) = \int_{-\infty}^{\infty} x(\tau) \, h(t - \tau) \, d\tau$

$$\therefore y(-t) = \int_{-\infty}^{\infty} x(\tau) \, h(-t - \tau) \, d\tau$$

$$\tau' = -\tau$$

$$y(-t) = \int_{\infty}^{-\infty} x(-\tau') \, h[-t + \tau'] \, (-d\tau')$$

$$= \int_{-\infty}^{\infty} x(-\tau') \, h[-(t - \tau')] \, d\tau'$$

as $x(t)$ & $h(t)$ are odd

$x(-\tau') = -x(\tau')$

& $h[-(t - \tau')] = h(t - \tau')$

$$\therefore y(-t) = \int_{-\infty}^{\infty} x(\tau') \, h[t - \tau') \, d\tau$$

$\boxed{\therefore y(-t) = y(t)} \Rightarrow y(t)$ is even

10) Check whether following systems are linear, Time invariant, causal.

i) $y(t) = \sin t \cdot x(t)$

It is linear.

$a x_1(t) + b x_2(t) \longrightarrow a \sin t \, x_1(t) + b \sin t \, x_2(t)$

Not time invariant

If $x(t)$ is delayed by T_0 o/p will be

$\sin t \cdot x(t - T_0) \neq y(t - T_0) = \sin(t - T_0) \, x(t - T_0)$

System is causal as o/p depends on present value of i/p

ii) $y(t) = x(at)$

a>1 a<0

It is linear, as

$a x_1(t) + b x_2(t) \longrightarrow a x_1(at) + b x_2(at)$

It is not Time invariant.

when i/p is delayed by T_0, o/p will be

$x[a(t - T_0)] = x[at - aT_0] \neq y(t - T_0)$
$= x[at - T_0]$

~~System is causal as o/p depends on future i/p~~

System is causal if $0 < a < 1$

iii) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

Linear.

Time invariant.

Not causal. as all i/p are required to find any point.

iv) $y(n) = \sum_{k=-\infty}^{n} x(k)$

Linear.

Time invariant.

Causal.

v) $y(t) = \frac{d\,x(t)}{dt}$

Linear

Time invariant.

vi) $y(n) = \max\{x(n), x(n-1), \ldots, x(\infty)\}$

Not linear.

Time invariant.

Causal.