Recap

Covered → Fourier Series expansion & properties

Fourier Transform (Continuous time)

Key idea → for F.S → signal is periodic
↓ relax it to aperiodic signal

make $T \to \infty$
↓
$\omega_0 \to 0$

$(n\omega_0) \to$ continuous fn ω → $(-\infty, \infty)$

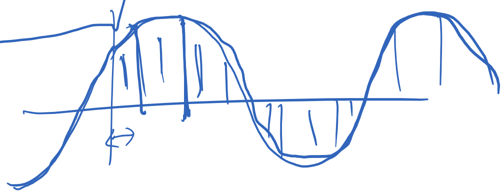
in F.S.

$$F(n) = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt.$$

↓ $F(n\omega_0)$ $\quad \omega_0 = \frac{2\pi}{T}$

Fourier Transform

$$\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$$



Exercise

Prove: $\int_{-\infty}^{\infty} e^{j\omega_1 t} e^{-j\omega_2 t}\, dt$

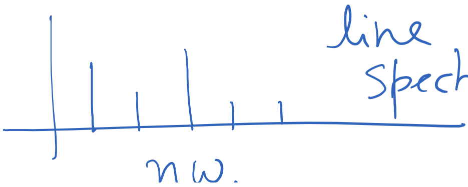
$= 0$ if $\omega_1 \neq \omega_2$

$\frac{1}{T}\sum |f(nT)|^2$
↓
$\int |f(t)|^2 dt$

$$\boxed{f(t) = \frac{1}{2\pi}\int F(\omega)\, e^{j\omega t}\, d\omega}$$ inverse FT

$F(\omega) \leftrightarrow f(t)$ pair FT

FS

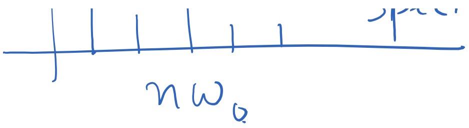


line spectrum

$n\omega_0$

FT

Cont.

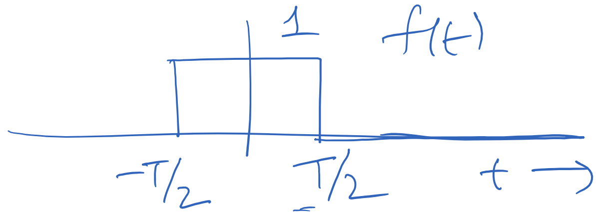


Claim → FS is a special case of FT

Can I represent FS using FT ?

SAn $\sum F_n \, \delta(\omega - n\omega_0)$

Ex

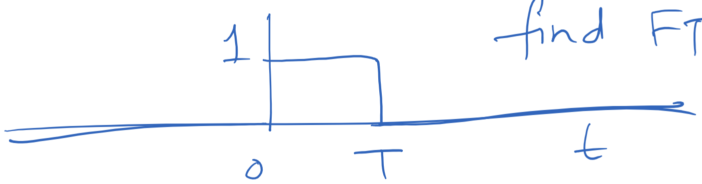


Find $F(\omega)$

$$F(\omega) = \int_{-T/2}^{T/2} 1 \cdot e^{-j\omega t} \, dt = \frac{-1}{j\omega} e^{-j\omega t} \Big|_{-T/2}^{T/2}$$

$$= T \operatorname{sinc} \frac{\omega T}{2}$$

A small variant



find FT

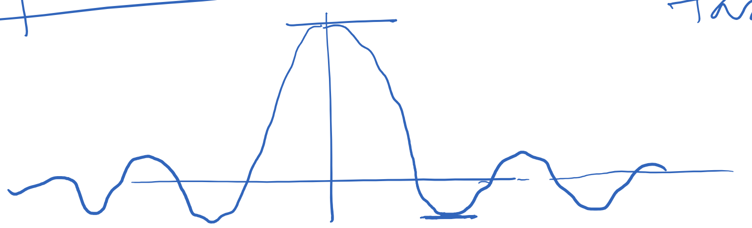
$\int_0^T$

Shifting property

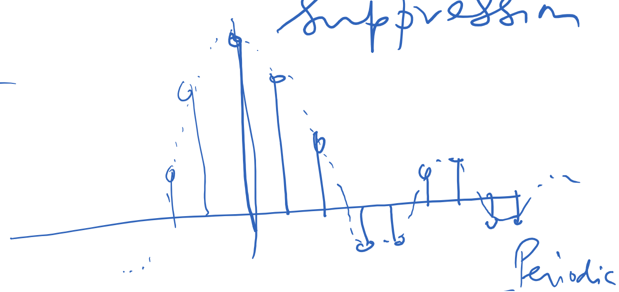
$$T \operatorname{sinc}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2}$$

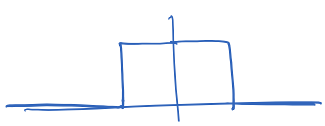
$$f(t - t_0) \longleftrightarrow F(\omega) e^{-j\omega T_0}$$

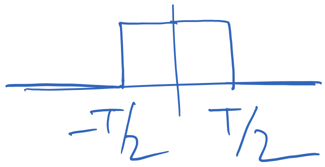
Spectrum

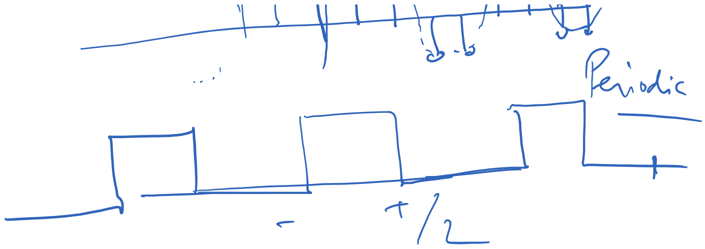


task → find the side lobe suppression



Signal 

Signal 



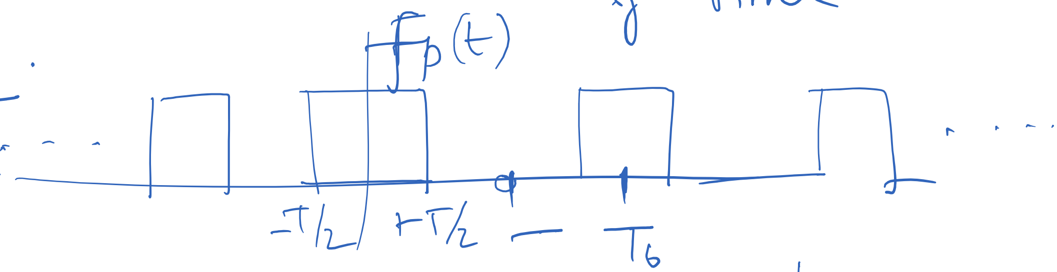
Existence condition

Dirichlet conditions

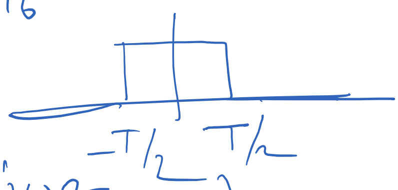
1. $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$

2/3 Finite no. of extrema/discontinuities over any finite duration of time

Exercise.

Periodic . . .  . . . .

Linearity property.

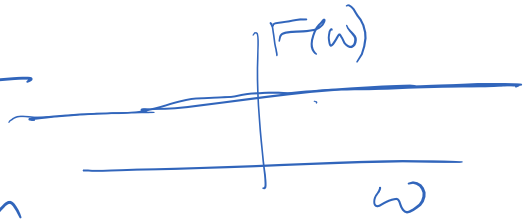


$$F_p(\omega) = F(\omega)\left[1 + e^{-j\omega T_0} + e^{-j\omega 2T_0} + \cdots\right]$$

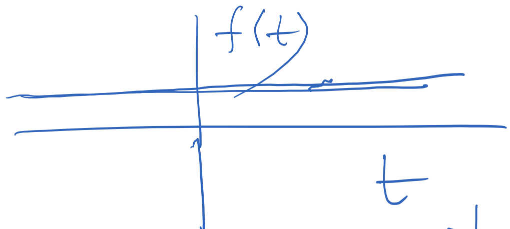
$\to F(\omega)$

Be careful of $\delta(t)$, $u(t)$ functions.

$$\mathcal{F}[\delta(t)] = 1$$



↘ dual of the problem



$\xrightarrow{FT}$ $2\pi\,\delta(\omega)$

Assume that it exists.

$F(\omega) =$

Use frequency shifting property.

$$e^{j\omega_0 t} \xleftarrow{\text{IFT}} 2\pi\,\delta(\omega-\omega_0)$$

$\rightarrow$ sinusoid at frequency ω_0

$$\cancel{\mathcal{F}[\delta(t)] = \int}$$

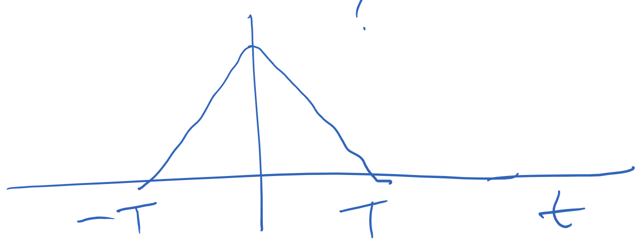
Other properties

Convolution

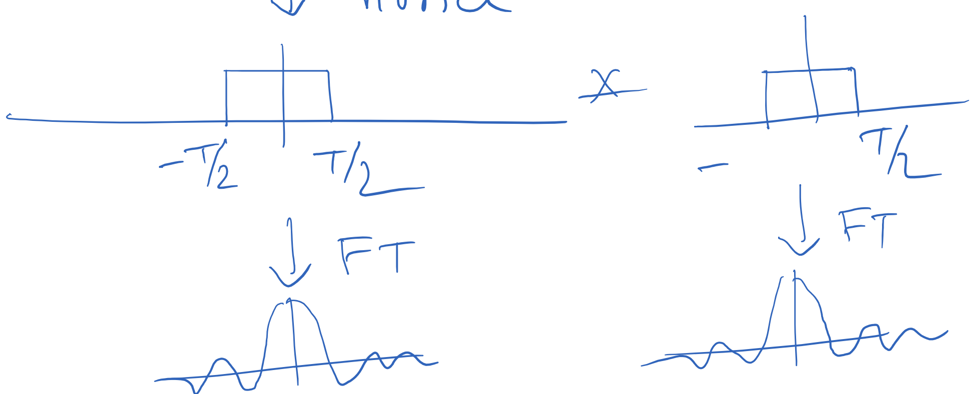
$$\mathcal{F}[f(t) * g(t)] = 2\pi\, F(\omega)\, G(\omega)$$

$$\mathcal{F}[f(t)\cdot g(t)] = \underbrace{\frac{1}{2\pi}}_{2\,?} F(\omega) * G(\omega)$$

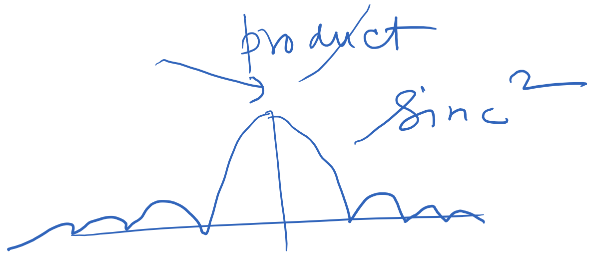
Example.



↓ notice



product



Ex → $\text{sinc}(\omega_1 t) * \text{sinc}(\omega_2 t)$

Scaling property

$$\mathcal{F}[f(at)] = \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

$$\mathcal{F}[f'(t)] = \int_{-\infty}^{\infty} f'(t)\, e^{-j\omega t}\, dt.$$

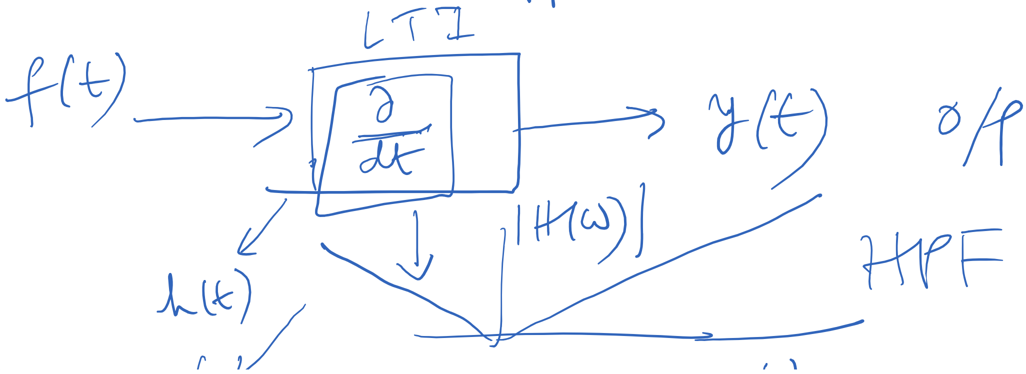
$$\cancel{\frac{d}{dt}\mathcal{F}[f(t)] = \frac{d}{dt}\int_{-\infty}^{\infty}} \quad \times$$

$$\mathcal{F}[f'(t)] = j\omega F(\omega)$$

$$\mathcal{F}\left[\int_{-\infty}^{t} f(\tau)\, d\tau\right] = \frac{1}{j\omega} F(\omega) + 2\pi\delta(\omega).$$

$$\mathcal{F}[f^{(n)}(t)] = (j\omega)^n F(\omega)$$

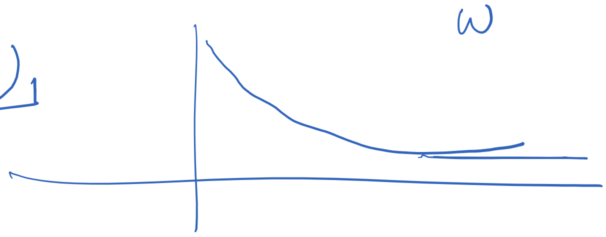
what happens at $\omega = 0$



$h(t)$

$F(\omega) \longrightarrow$ o/p

i/p $\quad \underbrace{j\omega \cdot F(\omega)}_{\mathcal{F}[h(t)]}$



LPF

Mid Sem Syllabus