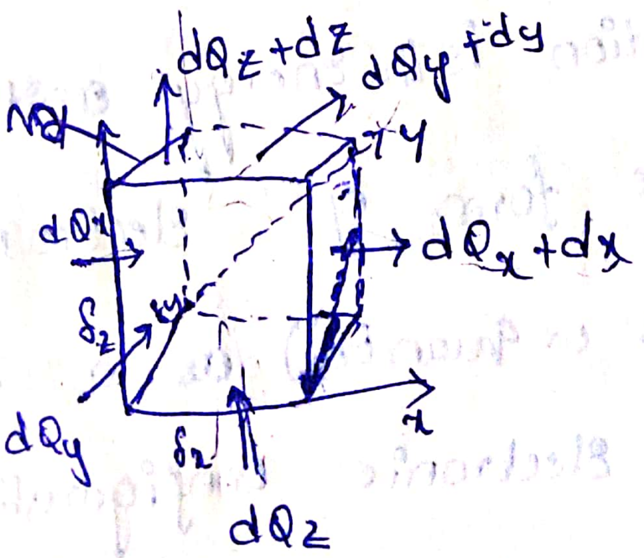
General heat conduction equation in Cartesian form:



→ let us consider that there is some heat source within the solid & heat is produce internally as a result of flow of electric current for nuclear or chemical reaction.

→ Let $q_G$ represent heat Generated internally per unit volume.

$$q_G \ (W/m^3).$$

Therefore, total rate of heat generation within the Element volume is given by $q_G \, \delta x \cdot \delta y \, \delta z$.

The Net heat flow due to conduction & Internal heat generated together where increase the internal energy of volume Element.

∴ The rate of accumulation of Internal energy with in the control (or) Elementary Volume.

$$\rho \, \delta x \, \delta y \, \delta z \, c \frac{\partial T}{\partial t}$$

Energy Balance on Volume Element:

Rate of energy stored within the solid = (Rate of heat Influx) - (Rate of heat outflux) + Rate of heat generation

$$\rho c\, \delta x\, \delta y\, \delta z \frac{\partial T}{\partial t} = (dQ_x + dQ_y + dQ_z) - (dQ_{x+dx} + dQ_{y+dy} + dQ_{z+dz}) + q_G\, \delta x\, \delta y\, \delta z .$$

$$\rho c\, \delta x \delta y \delta z \frac{\partial T}{\partial t} = k\, \delta x\, \delta y\, \delta z \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + q_G\, \delta x\, \delta y\, \delta z$$

$$\frac{\rho c}{k} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} .$$

Thermal diffusivity $\alpha = \frac{k}{\rho c} = \frac{\text{Heat conducted}}{\text{Heat stored}}$ , $m^2/s$.

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}}$$