Motion and optical flow

DD2423 Image Analysis and Computer Vision

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What about motion?

- Measuring sideways motion is very much like stereo.
- A single camera at two different instances in time can be seen as two cameras at two different locations.

Motion is more complex

However

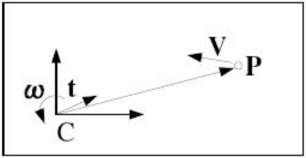
- Motion can be in any direction, not just along “epipolar lines”.
- One cannot tell how large the image motion is. For disparities one can have an idea of maximum and minimum values.
- The image motion arises from both
    - the motion of the camera (ego-motion), and
    - the motion of things in the scene (independent motion).

- Consider an observer moving with an angular velocity ω and translational velocity T in a static environment.
- In relation to the observer, a 3D point $P = (X, Y, Z)^\top$ moves as

$$\dot{P} = -T - \omega \times P \qquad (1)$$

or explicitly

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = -\begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} - \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



Motion field due to ego-motion

- The projection in the image is

$$\begin{cases} x = f\frac{X}{Z} \\ y = f\frac{Y}{Z} \end{cases} \Rightarrow \begin{cases} \dot{x} = f\frac{Z\dot{X}-X\dot{Z}}{Z^2} \\ \dot{y} = f\frac{Z\dot{Y}-Y\dot{Z}}{Z^2} \end{cases}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \frac{f}{Z}\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} - \frac{f}{Z}\begin{pmatrix} X \\ Y \end{pmatrix}\frac{\dot{Z}}{Z} \qquad (2)$$

Motion field due to ego-motion

- Combine the two equations (1) and (2)

$$\begin{cases} \dot{X} = -(T_x + \omega_y Z - \omega_z Y) \\ \dot{Y} = -(T_y + \omega_z X - \omega_x Z) \\ \dot{Z} = -(T_z + \omega_x Y - \omega_y X) \end{cases}$$

$$\begin{aligned} \frac{f}{Z}\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} &= -\frac{f}{Z}\begin{pmatrix} T_x + \omega_y Z - \omega_z Y \\ T_y + \omega_z X - \omega_x Z \end{pmatrix} \\ &= -\frac{f}{Z}\begin{pmatrix} T_x \\ T_y \end{pmatrix} - \begin{pmatrix} \omega_y f - \omega_z y \\ -\omega_x f + \omega_z x \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \frac{f}{Z}\begin{pmatrix} X \\ Y \end{pmatrix}\frac{\dot{Z}}{Z} &= -\begin{pmatrix} x \\ y \end{pmatrix}\frac{1}{Z}(T_z + \omega_x Y - \omega_y X) \\ &= -\begin{pmatrix} x \\ y \end{pmatrix}\left(\frac{T_z}{Z} + \omega_x \frac{y}{f} - \omega_y \frac{x}{f}\right) \\ &= -\begin{pmatrix} x \\ y \end{pmatrix}\left(\frac{T_z}{Z}\right) - \begin{pmatrix} x \\ y \end{pmatrix}\left(\omega_x \frac{y}{f} - \omega_y \frac{x}{f}\right) \end{aligned}$$

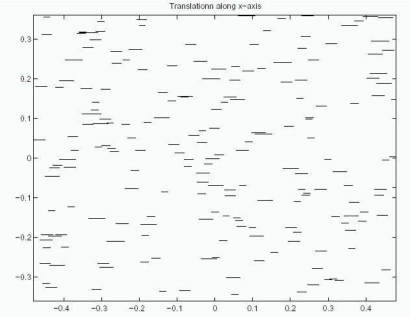
Motion field due to ego-motion

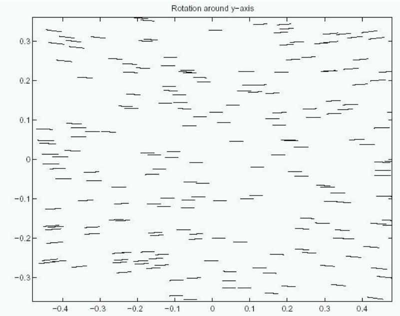
Add these together ⇒

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\frac{f}{Z} \begin{pmatrix} -T_x + \frac{x}{f} T_z \\ -T_y + \frac{y}{f} T_z \end{pmatrix}}_{\text{translation, scaled by 1/Z}} + \underbrace{\begin{pmatrix} \omega_x \frac{xy}{f} - \omega_y (f + \frac{x^2}{f}) + \omega_z y \\ \omega_x (f + \frac{y^2}{f}) - \omega_y \frac{xy}{f} + \omega_z x \end{pmatrix}}_{\text{rotation, independent of depth}}$$

- Translational component depends inversely on depth, scaling ambiguity: T and Z can be recovered only up to a scale.
- Rotational component does not depend on depth – impossible to estimate depth without translation.

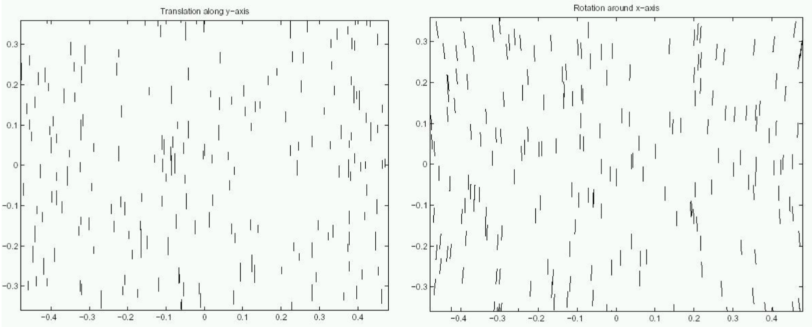
Motion flows





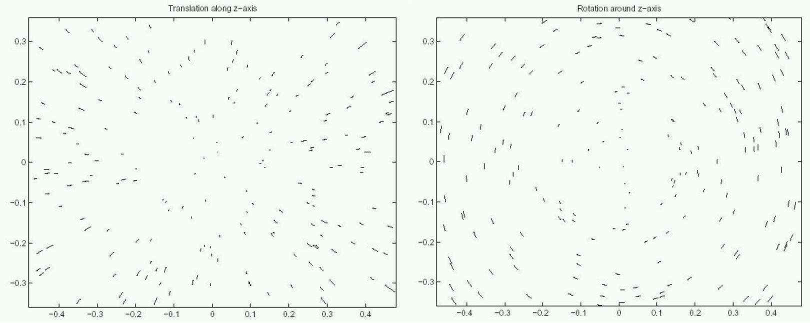
Translation T_x, Rotation ω_y

Translational and rotational flows are very similar.



Translation T_y, Rotation ω_x

Translational and rotational flows are very similar.



Translation T_z,   Rotation ω_z

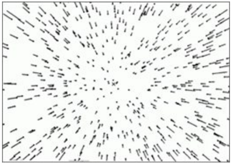
Except for fowards motion and rotation around optical axis.

$$\frac{1}{f}\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = -\frac{1}{Z}\begin{pmatrix}T_x\\ T_y\end{pmatrix} + \begin{pmatrix}x/f\\ y/f\end{pmatrix}\frac{T_z}{Z} = \frac{1}{Z}\begin{pmatrix}x/f\cdot T_z - T_x\\ y/f\cdot T_z - T_y\end{pmatrix}$$

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}x_{FOE}\\ y_{FOE}\end{pmatrix} = \frac{f}{T_z}\begin{pmatrix}T_x\\ T_y\end{pmatrix}$$

- The flow-field expands from a point, the Focus of Expansion.

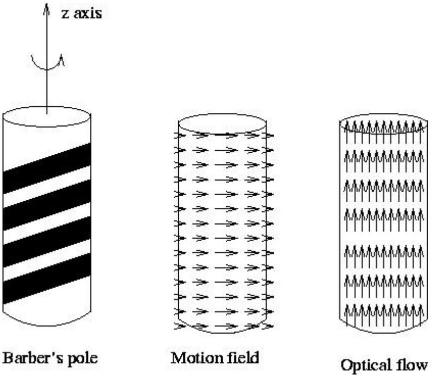
$$\begin{pmatrix}x_{FOE}\\ y_{FOE}\\ f\end{pmatrix} = \frac{f}{T_z}\begin{pmatrix}T_x\\ T_y\\ T_z\end{pmatrix}$$



- Conclusion: Translation direction can be seen directly in image.
- Comparison: Flow vectors in 3D are parallel and parallel lines "intersect" in a Vanishing Point.

Optical flow

- Optical flow is the apparent motion of brightness patterns.
- Generally, optical flow corresponds to motion field, but not always.
- For example, motion field and optical flow of a rotating barber's pole are different, as illustrated in the figure



Optical flow constraint equation

- Denote the intensity of a single scene point by $I(x(t), y(t), t)$.
- This is a function of three variables, as we now have spatio-temporal variation in our signal.
- To see how I changes, we differentiate with respect to time t:

$$\frac{dI}{dt} = I_x \frac{dx}{dt} + I_y \frac{dy}{dt} + I_t$$

Optical flow constraint equation

- If we assume that the image intensity of each visible scene point is constant over time (brightness constancy), we have

$$\frac{dI}{dt} = 0$$

which implies

$$I_x u + I_y v + I_t = 0$$

where the partial derivatives of I are denoted by subscripts, and u and v are the x and y components of the optical flow vector.

- This equation is called the *optical flow constraint equation*, since it expresses a constraint on the components the optical flow.

Optical flow constraint equation

- The optical flow constraint equation can be rewritten as

$$(I_x, I_y) \cdot (u, v) = -I_t$$

- Thus, the component of the image velocity in the direction of the image intensity gradient at the image of a scene point is

$$(u, v) = \frac{-I_t}{I_x^2 + I_y^2}(I_x, I_y)$$

- We cannot determine the component of the optical flow along an edge. This ambiguity is known as the *aperture problem*.

The aperture problem

(a) (b)

(a) Line feature observed through a small aperture at time t.

(b) At time $t+\delta t$ the feature has moved to a new position. It is not possible to determine exactly where each point has moved. From local image measurements only the flow component perpendicular to the line feature can be computed.

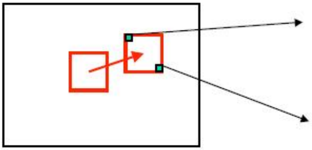
Normal flow: Component of flow perpendicular to line feature.

Local smoothness: Lucas, Kanade (1984)

- One pixel is not enough (one equation, two unknowns).

$$(I_x, I_y) \cdot (u, v) = -I_t$$

- Assume local smoothness (constancy) in a windows.



$$\begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_{t1} \\ I_{t2} \\ \vdots \end{bmatrix}$$

$$\boxed{A\vec{u} = b}$$

Local smoothness: Lucas, Kanade (1984)

Goal: Minimize $\left\| A\vec{u} - b \right\|^2$

Method: Least-Squares

$$A\vec{u} = b$$

$$\Downarrow$$

$$\underbrace{A^T A}_{2x2} \underbrace{\vec{u}}_{2x1} = \underbrace{A^T b}_{2x1}$$

$$\Downarrow$$

$$\boxed{\vec{u} = \left(A^T A\right)^{-1} A^T b}$$

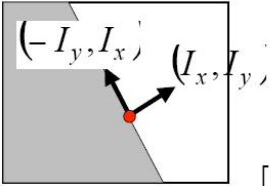
$$\bar{u} = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- $A^T A$ is the Second moment matrix used for corner detection.
- We need this matrix to be invertible $\Rightarrow$ No zero eigenvalues.

Behaviour due to Second moment matrix

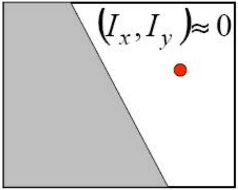
- Edge ➔ $A^T A$ becomes singular



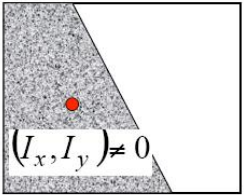
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} -I_y \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} -I_y \\ I_x \end{bmatrix}$ is eigenvector with eigenvalue 0

- Homogeneous ➔ $A^T A \approx 0$ ➔ 0 eigenvalues



- Textured regions ➔ two high eigenvalues



Some problems remain

- Lucas & Kanade (and similar) methods only handle flow smaller than the standard deviation of the Gaussian blurring filter.
- Possible solutions:
  - Iteratively shift windows for matching over time.
  - Search coarse-to-fine using Gaussian pyramids.

Summary of good questions

- Why is motion more complex than stereo?
- What is a Focus of Expansion?
- What is Motion field and what is Optical flow?
- How do you derive the optical flow constraint?
- What is a Second moment matrix?

Readings

- Szeliski: Chapters 8.1 – 8.2, 8.4

At the end .....

- Only one exercise session left on Thursday.
- Please fill in the course evaluation that will be added to web page!
- What about the exam?
  - 13 January, 14:00-19:00
  - Exam registration is needed. Tell me if you are not!
  - Allowed tools: calculator and mathematical handbook (e.g Beta)

Labs

- Reread what you did. What were you supposed to have learned?
- It will help you on both practical and theoretical parts of the exam.

Exercises

- Go through the problems!
- Likely that something similar is on the exam.
- AND, fill in “Course evaluation form”!