107 - 8F5AE23USVVI55C

FÖRSÄTTSBLAD TENTAMEN/ EXAMINATION COVER

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IFYLLES AV STUDENT OCH TENTAMENSVAKT/ TO BE FILLED IN BY THE STUDENT AND THE INVIGILATOR:

KURSKOD / COURSE CODE: DD2423	EFTERNAMN / FAMILY NAME: FILOTHEOU
KURSNAMN / COURSE NAME: Bildbehandling och datorseende	FÖRNAMN / FIRST NAME: ALEXANDROS
PROVKOD / TEST CODE: TEN1	NAMNTECKNING / YOUR SIGNATURE: [signature]
TENTAMENSDATUM / EXAMINATION DATE (Y/Y/Y/Y M/M D/D): 2016-01-15	PERSONNUMMER / PERSONAL NUMBER (Y/Y/M/M/D/D): 871108-5590

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SCR	1807	[signature]	27

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x	x	x	x	–	x														

IFYLLES AV INSTITUTIONEN / TO BE FILLED IN BY THE DEPARTMENT:

BEDÖMNING / ASSESSMENT

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A ≥12 ≥12	4	6	4.5	4	–	5	+5	=28.5												

BONUSPOÄNG/ BONUS POINTS: 5,0

SLUTSUMMA / FINAL POINTS: 28,5

BETYG/ GRADE: C

0406186500

Godkänns av examinator / approved by Examiner: Mårten Björkman

KTH
VETENSKAP
OCH KONST

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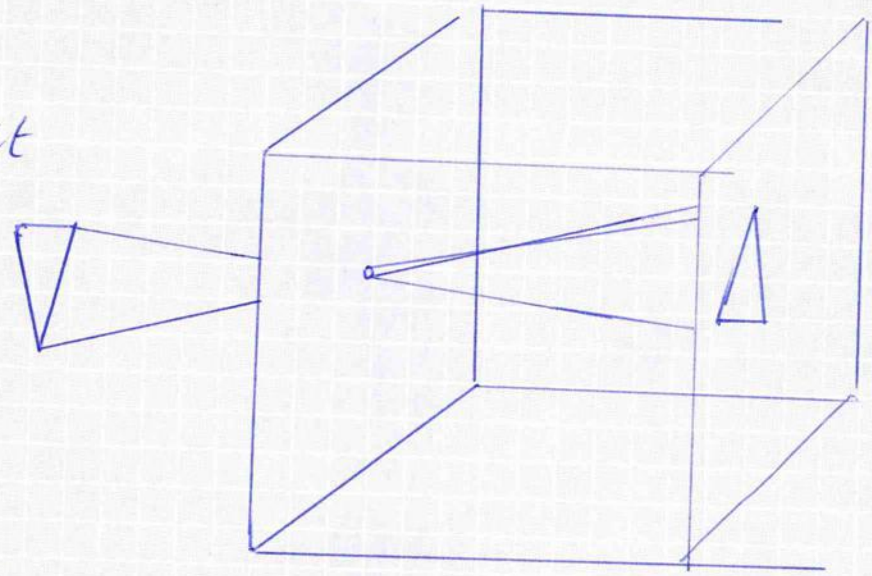
1)

object



Room with no light coming in except from the pinhole

Light comes in through an infinitesimally small hole in one wall. The object is projected to the opposite wall, scaled and inverted both horizontally and vertically, due to the intersection of the light rays with it.

It is not possible for a real-pinhole camera to be manufactured due to the infinitesimality of the hole.

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2)ii) Edge detection can help shape features, or produce, or verify segmentation methods. ~~that~~. Also, they denote ~~a~~ the boundaries of objects, hence they are valuable ~~for semantic~~ if we want to produce semantically rich results through computer vision or ~~…~~ image analysis.

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3) We can represent points located at infinity.

If (KX, KY, KZ, K) are the homogeneous coordinates of a point, then if $K=0$, the point is considered to be at infinity. ((X,Y,Z) are its coordinates in the Cartesian system).

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4) The magnitude holds the information that is comprehensible to humans via their vision system. The phase has no immediate visual value to humans, although I found a study where the researchers used the magnitude of one image ~~and com~~ in combination with the phase of another, and the combined "reconstructed" image was visually similar to the image from where the phase was taken.

The phase component holds the information from where each sinusoid that makes up the signal is bound to start.

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6D. ~~...~~ The camera sensor ~~must be~~ has to be somehow mounted to the inside of the camera and ~~aligned withe~~ its center aligned with the center of the camera's lens. It is ~~usual~~ expected that the sensor may ~~be not~~ not be perfectly ~~para~~ aligned with the plane glued to, ~~a~~ meaning it can be skewed ~~in either the~~ ~~Z~~ along either the Z-axis, rotated along the Z axis or its center may not be in line with the one of the lens. These types of imperfections ~~...~~ are tackled by the intrinsic camera parameters.

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8) ~~We assume that the image is one period of an infinitely repeated image~~

We assume that ~~an~~ image is the representation over one period of a periodic signal. This means that the periodic signal is the infinitely repeated x-wise and y-wise image.

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9) If W_{max} is the highest frequency present in an image, then aliasing occurs when we try to sample it at a sampling rate lower than $2 \cdot W_{max}$.

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10.) Gaussian filters are linear and shift invariant.

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11. ~~We compute the~~

First, we compute the x-wise and y-wise first order derivatives, f_x and f_y respectively. Then, the second moment matrix is formed by

$$S = \sum \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix}$$

that is, summing over all values in f_x squared, f_y squared and the product of the f_x and f_y for every corresponding image position.

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12) It contains information about ~~few features & such as corners~~ differences of Gaussians of features such as ~~corners~~ corners and combinations of features

-1p

Not, the feature descriptor.

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13) RANSAC works through these 5 steps:

1. First, pick a minimal set of points
2. Fit a ~~line~~ (line in this case) model to those points.
3. Count the number of points ~~that~~ whose distance is lower than a set threshold.
4. Repeat steps 1-3 for a predefined number of iterations.
5. Choose the model ~~to~~ with the highest number of points in step 3.

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14. Template-based methods are not robust because they look for a fixed template, or image inside another one and ~~that in the latter~~ in order for it to be recognised it has to be in exactly the same dimensions, orientation and brightness in it. There is little to no invariance to scale, point of view, occlusion, noise or illumination conditions with ~~these~~ this type of recognition method.

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15d) In binocular disparities it is easier because the cameras' ~~are~~ positions are fixed relatively to each other, and their baseline is known and fixed over time. However, in optical flow the structure of the scene might change along with the position of the camera, hence there are two types of motions going on: egomotion and scene motion. For depths to be computed, or disparities, the relative translation and rotation for each type of motion needs to be computed, and depths cannot be computed without translation.

-5.

-5p

-7

-5p

12/15

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1.i) (iii) For the first camera:

$$x_{1A} = P_A \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

$$x_{2A} = P_A \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 0,5 \\ 3 \\ 1 \end{bmatrix}$$

$$x_{3A} = P_A \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 1,5 \\ 3,5 \\ 1 \end{bmatrix}$$

$$x_{4A} = P_A \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

(iv) For the second camera:

$$x_{1B} = P_B \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

$$x_{2B} = P_B \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}$$

$$x_{3B} = P_B \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}$$

$$x_{4B} = P_B \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

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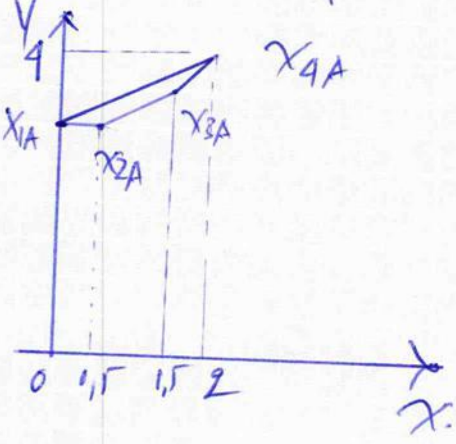
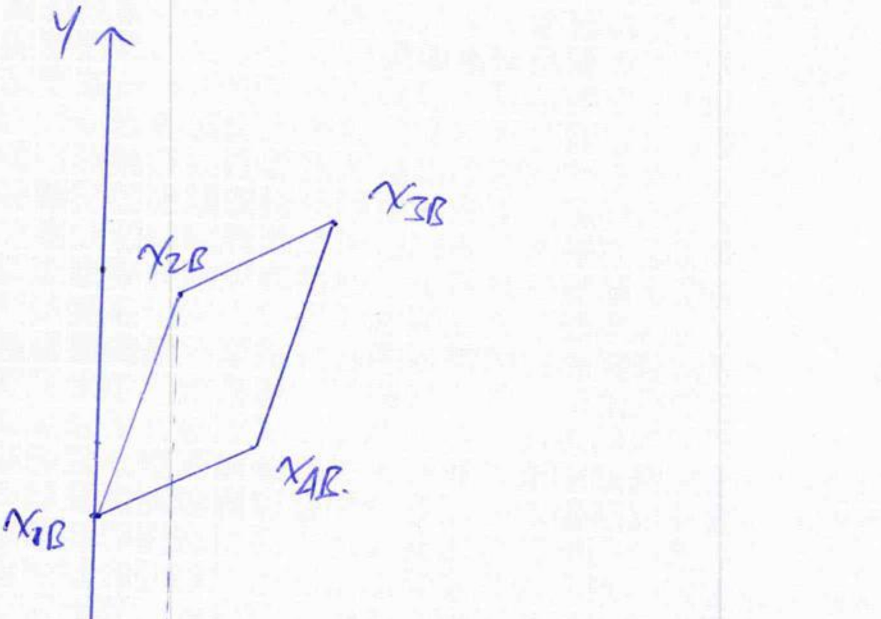
where we have assumed homogeneous coordinates. Then in 2D:

▶ For the first camera:



▶ For the second camera:



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The second camera is the affine one, since parallel lines in 3D are maintained in their projection in 2D. Hence the first one is the perspective one.

By looking at the projection matrices, we can tell that the second one is the affine one, since its last row is all zeros except for the last element, which is one.

4

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1) Fourier transform:

$$\hat{f}(w) = \int_{-\infty}^{+\infty} f(x) e^{-jwx} dx = \int_{-1/2}^{1/2} e^{-jwx} dx = \frac{-1}{jw}\left[e^{-jwx}\right]_{-1/2}^{1/2}$$

$$\hat{f}(w) = \frac{-1}{jw}\left(e^{-\frac{jw}{2}} - e^{\frac{jw}{2}}\right) = \frac{2}{w} \cdot \frac{e^{\frac{jw}{2}} - e^{-\frac{jw}{2}}}{2j}$$

$$\hat{f}(w) = \frac{2}{w} \cdot \sin\frac{w}{2} = \frac{\sin\frac{w}{2}}{\frac{w}{2}}$$

~~Unfortunately, a box filter introduce~~
Unfortunately, convolving an image with ~~its~~ a box filter introduces ringing effects, and, hence, it degrades the quality of the image. This is due to the sinc's structure.

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2). In 1D, a convolution of function f with function g is defined as:

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(a) g(x-a) \, da.$$

In our case:

$$f(x) * f(x) = \int_{-\infty}^{+\infty} f(a) \cancel{g} f(x-a) \, da$$

$$= \int_{-1/2}^{1/2} 1 \cdot f(x-a) \, da \quad (1)$$

$$f(a) = \begin{cases} 1, & -1/2 \leq a \leq 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$f(-a) = \begin{cases} 1, & -1/2 \leq -a \leq 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$f(-a) = \begin{cases} 1, & a \leq 1/2 \text{ and } a \geq -1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$f(x-a) = \begin{cases} 1, & x-1/2 \leq x-a \leq x+1/2 \\ 0, & \text{otherwise.} \end{cases}$$

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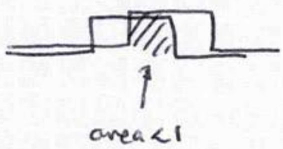
or, substitute $x-a=y$

then: $da=-dy$

$a=-1/2 \rightarrow y=x+1/2$

$a=1/2 \rightarrow y=x-1/2$



-1p   and ①:

$$f(x)*f(x)=\int_{x+1/2}^{x-1/2} -dy = \int_{x-1/2}^{x+1/2} dy = x+1/2-x+1/2$$

$$=1.$$

$$f(x)*f(x)=\begin{cases} 1-|x| & ; -1<x<1 \\ 0 & ; \text{otherwise} \end{cases}$$

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3) $$\hat{g}(w) = \int_{-\infty}^{+\infty} g(x) e^{-jwx} dx = \int_{-1}^{1} (1-|x|) e^{-jwx} dx$$

$$= \int_{-1}^{0} (1+x) e^{-jwx} dx + \int_{0}^{1} (1-x) e^{-jwx} dx$$

$$= \int_{-1}^{0} e^{-jwx} dx + \int_{0}^{1} e^{-jwx} dx + \int_{-1}^{0} x e^{-jwx} dx + \int_{0}^{1} x e^{-jwx} dx$$

$$= -\frac{1}{jw}\left[e^{-jwx}\right]_{-1}^{0} - \frac{1}{jw}\left[e^{-jwx}\right]_{0}^{1} + \underbrace{\int_{-1}^{0} x e^{-jwx} dx - \int_{0}^{1} x e^{-jwx} dx}_{I(x)}$$

$$= \frac{-1}{jw}\left(1 - e^{jw}\right) - \frac{1}{jw}\left(e^{-jw} - 1\right) + I(x)$$

-TP $$= \frac{e^{jw} - e^{-jw}}{jw} = \frac{2}{w} \cdot \sin w + I(x)$$

I don't have a mathematical handbook to solve $I(x)$. So far, the introduction of the sinc function means that the ringing effect is ~~..~~ here too I don't know however if $I(x)$ would cancel it.

6/8 You can use the product rule for integration. However, easier to use $f(x) * f(x) \to \hat{f}(w)\hat{f}(w)$ on 2.5 and 2.2.

$$I(x) = -\frac{2}{w}\sin w + \frac{\sin^2(w/2)}{(w/2)}$$

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1) $\int p(z)dz = \int (2z^4 - 3z^2 + \frac{8}{5})dz$

$= \frac{2}{5}z^5 - z^3 + \frac{8}{5}z + C$

ii) for $z = 0 \longrightarrow C = 1$

$z = 1 \longrightarrow$ ✓

So, $T(z) = \frac{2}{5}z^5 - z^3 + \frac{8}{5}z + 1.$

iii) For increasing the contrast:

$\frac{dT}{dz} \geq 1 \Leftrightarrow 2z^4 - 3z^2 + \frac{8}{5} \geq 1.$

$z^2(2z^2 - 3) \geq -\frac{3}{5}.$

and $z^2(2z^2 - 3) < -3/5$ for decreasing.

-0.5p

What is the range?

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3) (iii) Mean:

$$f_{mean}(x) = \left[\frac{7}{3}, \frac{11}{3}, 4, \frac{13}{3}, \frac{19}{3}, \frac{28}{3}, 11, \frac{37}{3}, \frac{35}{3}, 8\right].$$

(iv) Median:

$$f_{median}(x) = [2, 4, 4, 4, 6, 10, 11, 12, 11, 10].$$

-5p Binomial!

2.

-3p

4.5/9

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1) Moments:

$$M_{pq} = \sum_x \sum_y x^p y^q f(x,y)$$

$M_{00} = 12$

(2)

$M_{10} = 2\cdot 1 + 3\cdot 2 + 2\cdot 3 + 3\cdot 4 + 2\cdot 5 = 32$

$M_{01} = 1 + 2\cdot 2 + 4\cdot 3 + 3\cdot 4 + 2\cdot 5 = 39$

2)

$$\cancel{f_x = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \quad f_y = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}$$

(1)

$$f_x = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & -1 \end{bmatrix}, \quad f_y = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$C = \sum_x \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix} = \begin{bmatrix} 7/4 & 1 \\ 1 & \frac{5}{4} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 4 \\ 4 & 5 \end{bmatrix}$$

In order to find the orientation of the ellipse I would compute the eigenvalues of C first, then the eigenvector that corresponds to the lowest eigenvalue. ~~That gives us the~~

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That, plus the centers of gravity $\frac{m_{10}}{m_{00}}$, $\frac{m_{01}}{m_{00}}$ would give us the line in which the eclipse is oriented.

1

3) A morphological opening is an erosion followed by a dilation.

▶ Erosion.

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

X

▶ Dilation:

No point, there are no foreground pixels.

X

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2) Disparity: $d = \frac{f\cdot b}{Z} = \frac{10^3\cdot 10^{-1}}{Z} = \frac{100}{Z}$

$\cancel{Z = \frac{fb}{d} \rightsquigarrow \frac{dZ}{dd} = \frac{-fb}{d^2} = \frac{-Z}{d}}$

differentiating with respect to the depth

2

$\frac{dd}{dZ} = \frac{-fb}{Z^2} = \frac{-d}{Z} \Leftrightarrow dd = \frac{-d}{Z}\,dZ$

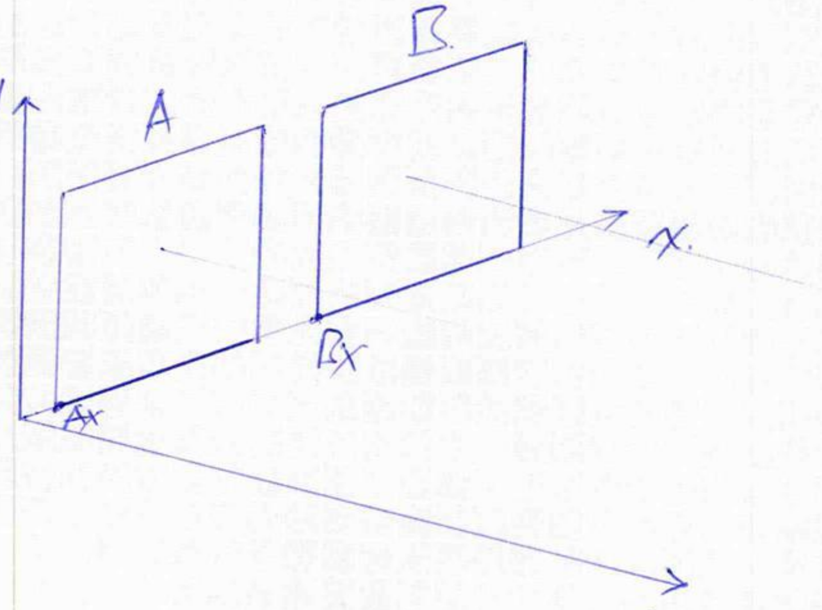
$dd = \frac{-1000\cdot 10^{-1}}{1^2}\cdot 10^{-3} = -10^{-1}$

0.1 pixel

hence we need an accuracy of 10 cm.

unit is wrong

3)



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① Camera A at $(A_x, 0, 0)$

① Camera B at $(B_x, 0, 0)$

① Rotation matrix:

$$R = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \theta = 2\pi$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

① Translation matrix:

$$T = \begin{bmatrix} B_x - A_x \\ 0 \\ 0 \end{bmatrix} \checkmark$$

① $P_B = R \cdot P_A + T = R \cdot P_A + R \cdot R^{-1} T$

$P_B = R(P_A + R^{-1} T)$

① $e = -R^{-1} T = \begin{bmatrix} A_x - B_x \\ 0 \\ 0 \end{bmatrix}$

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1 Essential matrix:

$$E = R \cdot [e]_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Ax+Bx \\ 0 & -Ax+Bx & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Ax+Bx \\ 0 & -Ax+Bx & 0 \end{bmatrix}$$

3

1) ?

0