# Exercises 3

1. You are given the following points from a two dimensional distribution:
   $p_1 = (-1, 0)$, $p_2 = (1, 0)$, $p_3 = (1, 2)$, $p_4 = (3, 2)$, $p_5 = (4, 3)$, $p_6 = (5, 3)$
   Plot these points and notice that one direction seems more significant. Using PCA, determine this direction.

2. A ball is moving with constant velocity straight towards a camera along the optical axis. At time $t_0 = 0$ it covers 500 pixels, and at time $t_1 = 3$ it covers 750 pixels. At what time does it cover 1000 pixels? (The camera is assumed to be of pinhole type.)

3. A robot is trying to gather 3D information from an object. Since it has only a single camera, it rotates around the object to obtain multiple views from it, as seen in the figure below.
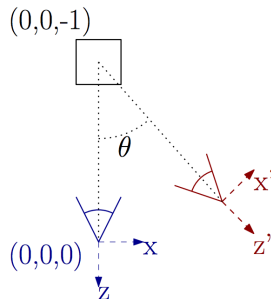
a) What is the relation between points P in 3D space and their image projections p on the image camera at $(0, 0)$? Consider unit focal length, $f = 1$, and centered image origin, with x increasing to the right and y increasing up.

b) What is the relation between 3D points $P = (x, y, z)$ in the original coordinate frame and $P' = (x', y', z')$ in the new coordinate frame after rotating θ radians?

c) What is the relation between image points p and p'?

d) At some point the encoders of the motors fail and the robot doesn't know how large θ is. Estimate θ given point correspondences between the two images. How many point do you need?

4. You are given the following binary image:

$$
\begin{array}{cccccc}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
\end{array}
$$

Compute the following:

- Moments: m_{00}, m_{10}, m_{01} and m_{20}
- Centers of gravity: x_0 and y_0
- Central moments: μ_{00}, μ_{01} and μ_{02}