Advancing Math Reasoning Capabilities in Small Language Models

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Motivation

LLMs excel in various NLP tasks but struggle with **mathematical reasoning** and are very resource intensive. Small Language Models offer a lightweight alternative but also perform poorly on reasoning tasks.

This project explores methods to enhance reasoning capabilities in SLMs, specifically focusing on the T5-small Transformer model.

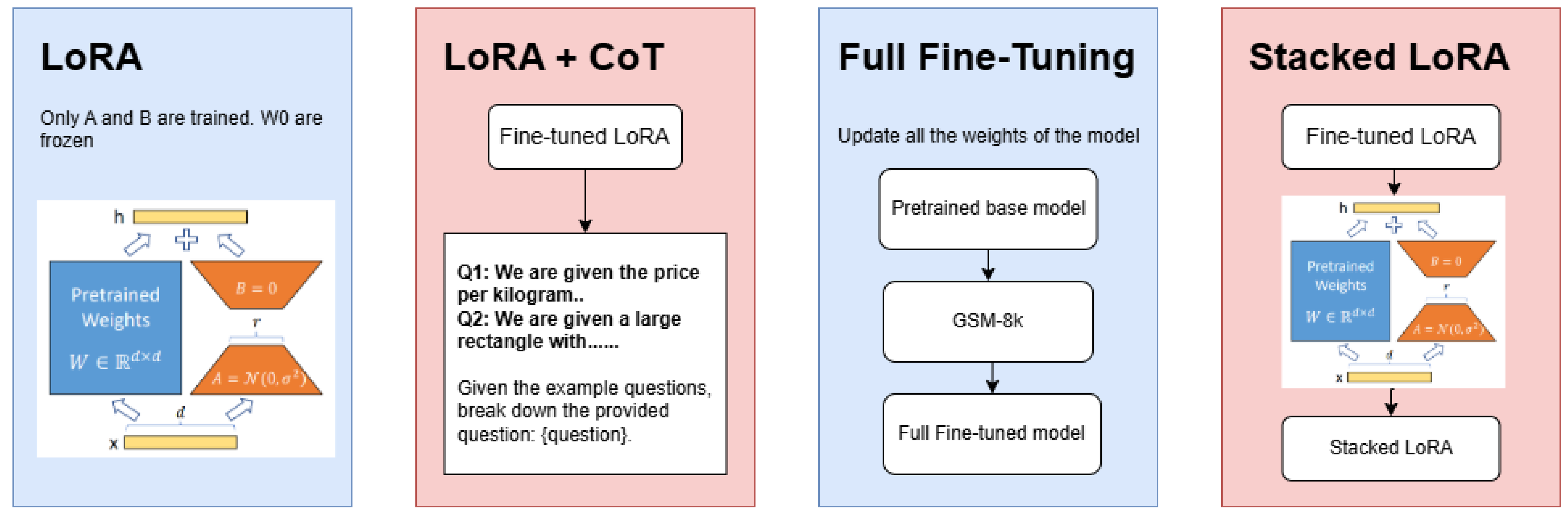


Figure 1: Overview of Fine-Tuning strategies

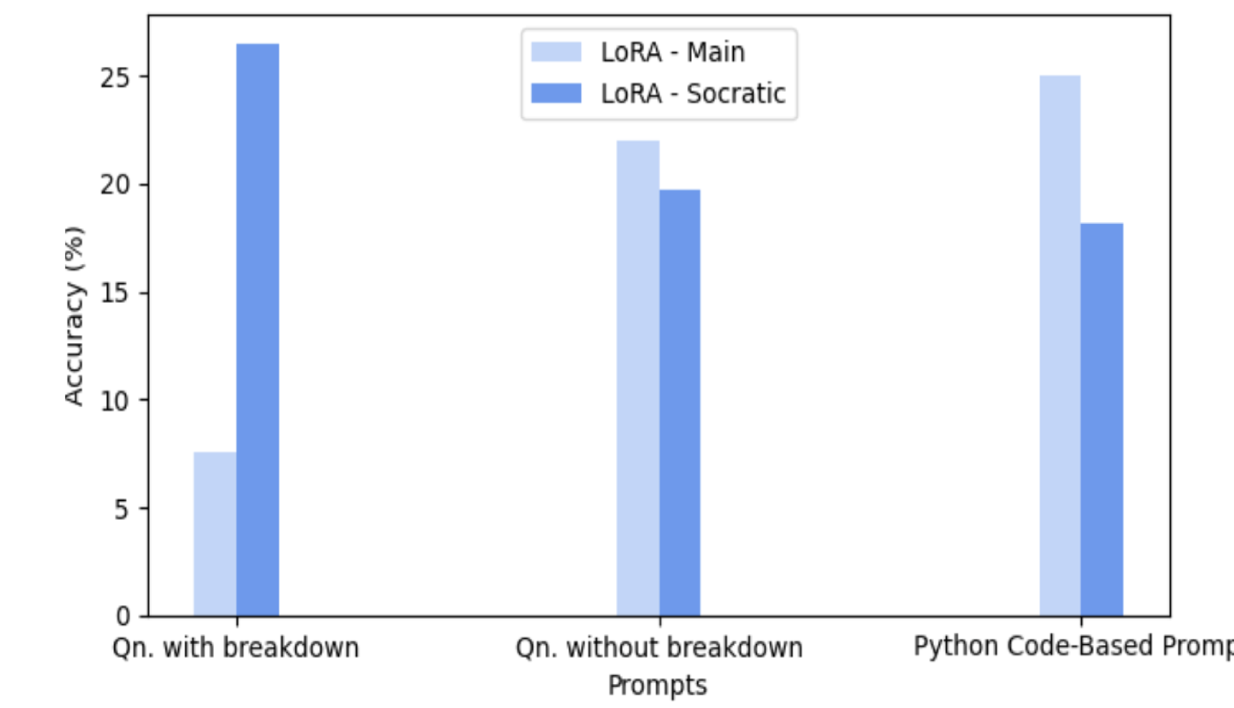


Figure 3: Chain-Of-Thought for LoRA

Methodology

Dataset: GSM8k (Grade School Math 8K) a dataset for mathematical reasoning tasks consisting of 8000 problems.

Base Model: T5 Small Transformer architecture with 60M parameters. Chosen for its efficiency and lightweight architecture.

Fine-Tuning Approaches:

- **Low-Rank Adaptation (LoRA)**
  - Re-train only low-rank matrices A and B, keeping pretrained model weights frozen, reducing memory requirements and computation costs. (Hu et al., 2021)
- **LoRA with Chain-of-Thought (CoT) Prompting**
  - The model is guided to reason through problems step-by-step using structured prompts. Fine-tuned LoRA model is used to generate intermediate reasoning steps.
- **Full Fine-Tuning**
  - All model parameters are updated during training resulting in a fully adapted version. Highest task performance but computationally expensive.
- **Quantized LoRA**
  - An efficient variant of LoRA applied to T5-Base (220M parameters) using 8-bit quantization.

Stacked LoRA (Our approach)

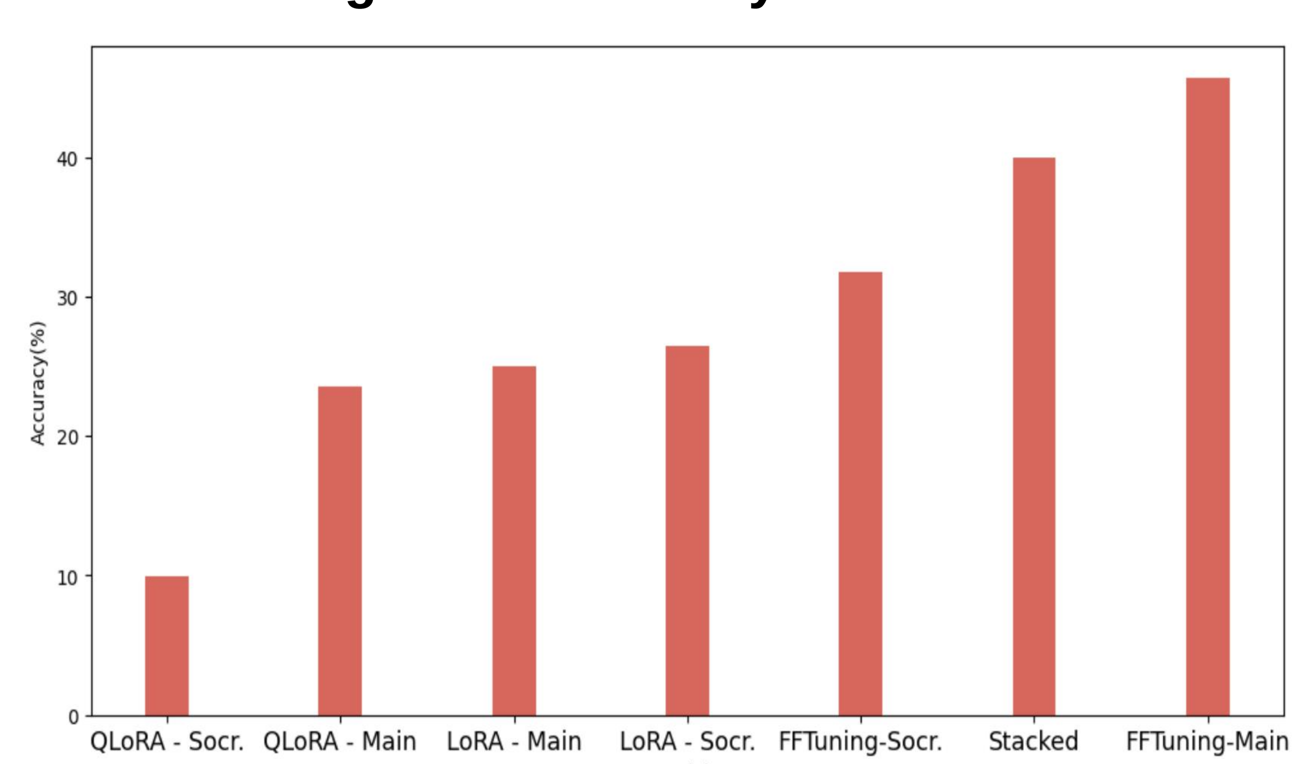
Stacked LoRA introduces additional low-rank layers to fine-tune the model further on a new dataset or task allowing knowledge transfer while training prior task-specific adaptations.

- We apply stacked LoRA using a coding dataset to generalize to reasoning tasks.
- Weight update rule:

$$\boxed{W = W_0 + A_1 B_1 + f(A_2 B_2)}$$

Where W_0 are frozen pre-trained weights, A_1B_1 are LoRA weights trained on the first dataset, and A_2B_2 are newly introduced LoRA weights fine-tuned on the second dataset. $f(.)$ (e.g., ReLU)

Figure 2: Accuracy vs. Models



Results

- We assessed model performance using accuracy, perplexity, ROUGE-1 score and average semantic similarity to evaluate correctness, predictive quality, textual overlap and semantic alignment respectively.
- **Stacked LoRA** achieves an **accuracy** of 40%, demonstrating near-equivalent performance to full-fine tuning, with significantly reduced computational requirements.
- Full Fine-Tuning achieves the highest accuracy at 45% but compromises semantic similarity (62.81%).
- An alternative approach would be using larger quantized models like T5-base (220M parameterss) which has high semantic similarity while being computationally efficient through 8-bit quantization.

Table 1: Performance Evaluation

Model	ROUGE-1	Perplexity	Average Semantic Similarity
LORA - Main	0.5084	3.2633	79.70%
QLORA - Main	0.5156	2.4158	79.99%
LORA - Socratic	0.4678	3.2706	80.42%
QLORA - Socratic	0.5262	2.3754	75.96%
Full-Fine Tuning - Main	0.4136	2.2708	62.81%
Full-Fine Tuning - Socratic	0.4778	2.2353	62.50%
LoRA Stacked	0.3546	2.2587	61.38%

- This suggests that quantizing larger models may be more effective than fine-tuning smaller ones for mathematical reasoning tasks. (Wei et al., 2023)

We also evaluated the performance of three Chain-of-Thought(CoT) prompting strategies on LoRA with GSM8k main and Socratic datasets and observed their accuracy.

- **Question with breakdown**
- **Question without breakdown**
- **Python code-based prompt**

Python code-based prompt approach gave the highest accuracy, particularly effective on LoRA main (25%) ,but slightly lower performance on Socratic (18.2%).

Conclusion

- Stacked LoRA enhances T5-small's efficiency and reduces resource use but falls short of full fine-tuning in complex reasoning tasks. Further research is needed to improve its performance on challenging tasks.

References

Hu, E. J., Shen, Y., Wallis, P., Allen-Zhu, Z., Li, Y., Wang, S.,Wang, L., and Chen, W. Lora: Low-rank adaptation of large language models, 2021. URL https://arxiv.org/abs/2106.09685.

Wei, J., Wang, X., Schuurmans, D., Bosma, M., Ichter, B., Xia,F., Chi, E., Le, Q., and Zhou, D. Chain-of-thought prompting elicits reasoning in large language models, 2023. URL https://arxiv.org/abs/2201.11903.