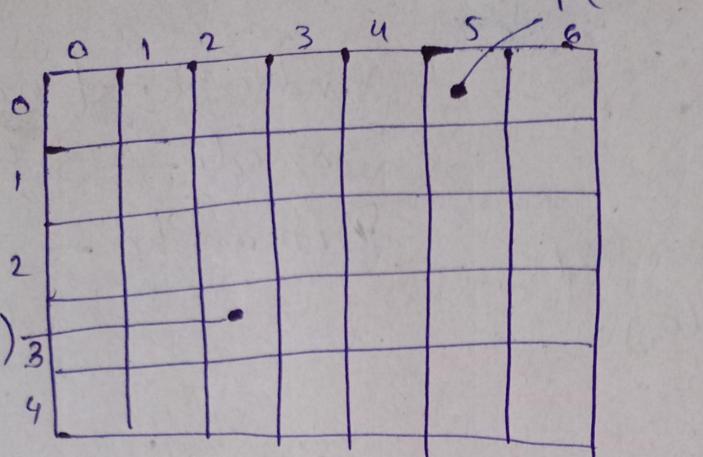


15/1/2025



$$D_e(P, Q) = \sqrt{(x-s)^2 + (y-t)^2}$$

$$= \sqrt{(0-3)^2 + (5-2)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ unit (Ans)}$$

This is Eucleidian distance.

|| D_4 distance

$$D_u(P, Q) = |x-s| + |y-t|$$

$$= |0-3| + |5-2|$$

$$= 3+3 = 6 \text{ unit (Ans)}$$

|| Chessboard distance

$$D_B(P, Q) = \max(|x-s|, |y-t|)$$

$$= \max(3, 3) = 3 \text{ unit (Ans)}$$

Histogram Specification / or Histogram mapping

10 marks

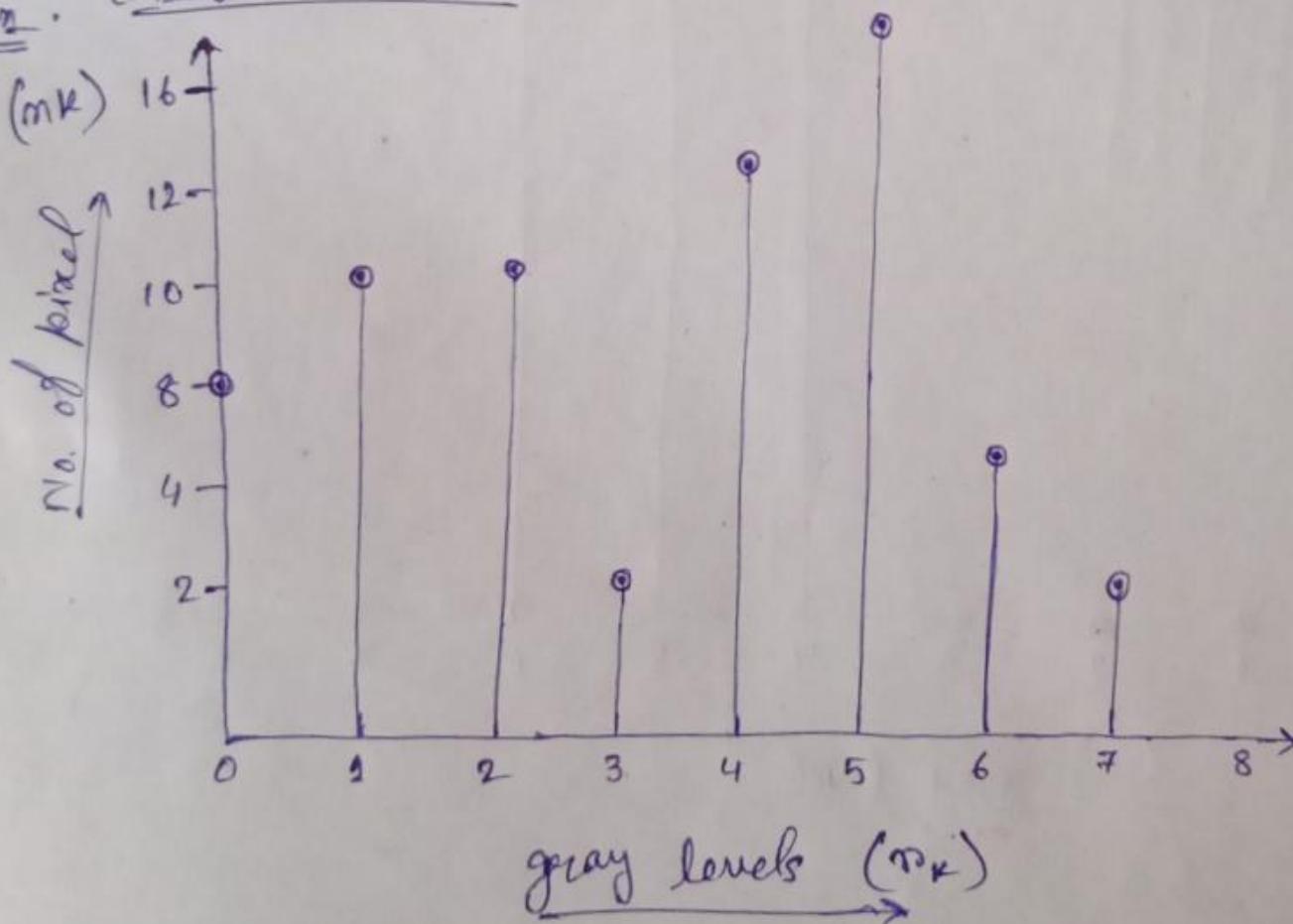
Original Image

gray level	0	1	2	3	4	5	6	7
No of pixel	8	10	10	2	12	16	4	2

Desired Image / Target Image

gray level	0	1	2	3	4	5	6	7
No of pixel	0	0	0	0	20	20	16	8

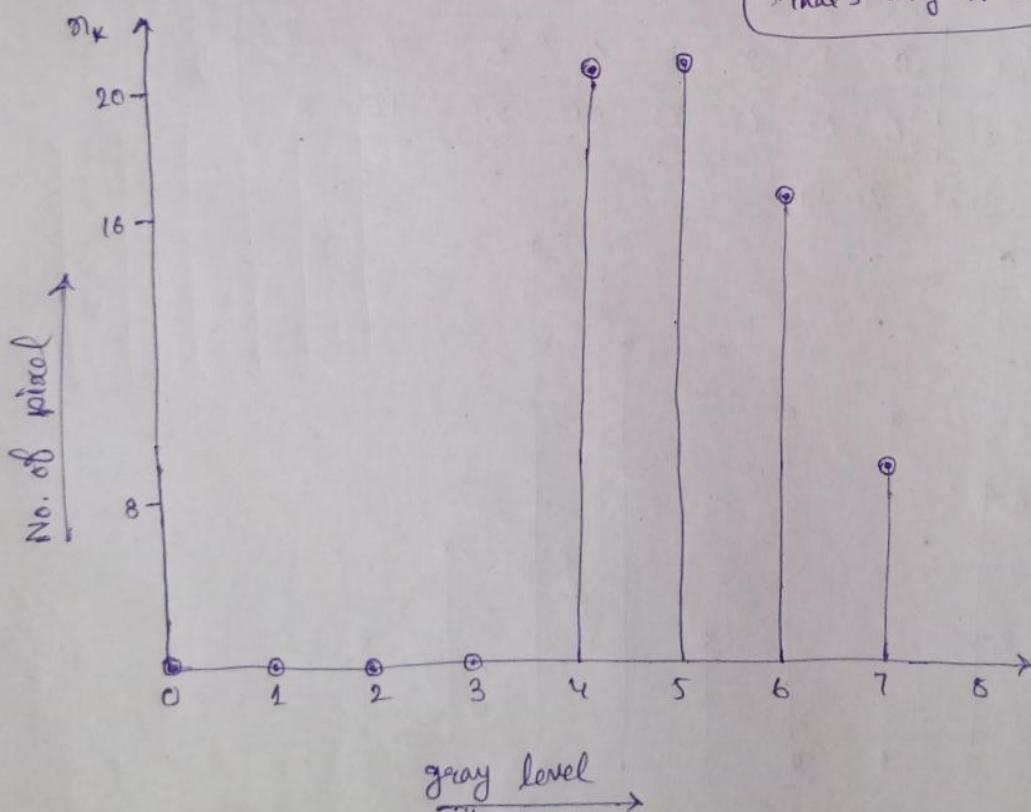
Soln: Original Image:



Histogram of original image

Gray levels γ_k	No. of pixels n_k	$P(\gamma_k) = n_k/n$ (PDF)	S_k CDF	$S_k \times 7$	Histogram equilized level
0	8	0.125	0.125	0.875	1
1	10	0.156 0.156	0.281	1.967	2
2	10	0.156 0.156	0.437	3.059	3
3	2	0.031 0.031	0.468	3.276	3
4	12	0.188 0.188	0.656	4.592	5
5	16	0.25 0.25	0.906	6.342	6
6	4	0.063 0.063	0.969	6.783	7
7	2	0.031	1	7	7
$n=64$					

Desired Image



Why $S_k \times 7$? → Here we are assuming 8 bits (0-7)
7 is 3 bits digits. $7 \rightarrow 111$. So,
 $2^3 = 8$. Let, $8 = L$. But in
formula we have to use $L-1$.
That's why it is $S_k \times (L-1)$

Histogram of desired image —

gray levels n_k	No. of pixels n_k	$P(n_k) = n_k/n$ (PDF)	S _k CDF	S _k × 7	Histogram equilized level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	20	0.313	0.313	2.191	2
5	20	0.313	0.626	4.403	4
6	16	0.25	0.876	6.132	6
7	8	0.125	1.001	7.007	7
$\sum n_k = 64$					

Mapping

Gray level	H(0)	H(D)	Mapping
0	1	0	4 ✓
1	2	0	4 ✓
2	3	0	5 ✓
3	3	0	5 ✓
4	5	2	6 ✓
5	6	4	6 ✓
6	7	6	7 ✓
7	7	7	7 ✓

Modified Image :

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	0	0	18	12	28	6

gray level slicing

① With Background

$$a \leq v \leq b$$

where $a=3$, $b=6$ (given)

Ex. 1
pixel

	0	1	2	3	4
0
1	.	.	.	253	.
2	.	.	250	253	.
3	.	2	.	.	.
4	0				

Ex. 2 $3 \leq v \leq 6 = 7$
otherwise 0

1	4	2	7	5
3	2	4	5	2
2	6	5	7	0
4	6	6	5	1
0	2	3	2	1

↓

1	7	2	7	7
7	2	7	7	2
2	7	7	7	0
7	7	7	7	1
0	2	7	2	1

② Without Background
 $3 \leq v \leq 6 = 7$
 otherwise 0

1	4	2	7	5
3	2	4	5	2
2	6	5	7	0
4	6	6	5	1
0	2	3	2	1

↓

0	7	0	7	7
7	0	7	7	0
0	7	7	7	0
7	7	7	7	0
0	0	7	0	0

Bit plane Slicing:

167	133	111
144	140	135
169	159	148

Step 1: Binary format for above image is

10100111	10000101	01101111
10010000	10001100	10000111
10011111	10011010	10010100

for 1st digit: Binary format of 167

1	0	1	0	0	1	1	1
MSB	7 th bit	6 th bit	5 th bit	4 th bit	3 rd bit	2 nd bit	LSB

Step 2: Bit plane slicing of above example.

1	1	0
1	1	1
1	1	1

8 bit
(MSB bit)
Plane

0	0	1
0	0	0
0	0	0

7 bit

1	0	1
0	0	0
0	0	0

6 bit

1	0	1
0	0	1
1	1	0

2 bit

0	0	0
1	0	0
1	1	1

5 bit

0	0	1
0	1	0
1	1	0

4 bit

1	1	1
0	1	1
1	0	1

3 bit

1	1	1
0	0	1
1	0	0

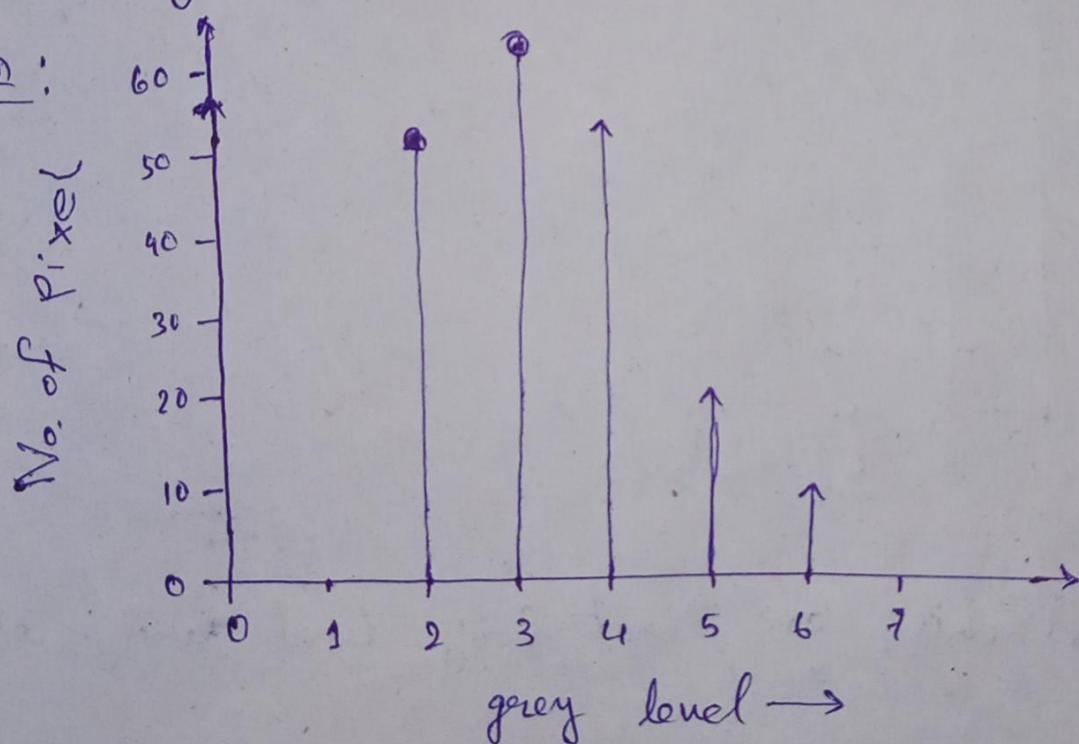
2 bit

LSB plane

DIPHistogram stretching / Contrast stretching

grey level	0	1	2	3	4	5	6	7
No. of pixels	0	0	50	60	50	20	10	0

Perform histogram stretching such that the new image has dynamic range $[0 \rightarrow 7]$.

Sol:

$$s_{\min} = 0$$

$$s_{\max} = 7$$

$$r_{\min} = 2$$

$$r_{\max} = 6$$

$$\text{Slope, } S = \left(\frac{S_{\max} - S_{\min}}{r_{\max} - r_{\min}} \right) \times (r - r_{\min}) + S_{\min}$$

when, $r=2$

$$S_2 = \left(\frac{7-0}{6-2} \right) \times (2-2) + 0 = \frac{7}{4} \times 0 + 0 = 0$$

when, $r=3$

$$S_3 = \left(\frac{7-0}{6-2} \right) \times (3-2) + 0 = \frac{7}{4} \times 1 + 0 = 1.75 \approx 2$$

when, $r=4$

$$S_4 = \left(\frac{7-0}{6-2} \right) \times (4-2) + 0 = \frac{7}{4} \times 2 + 0 = 3.5 \approx 4$$

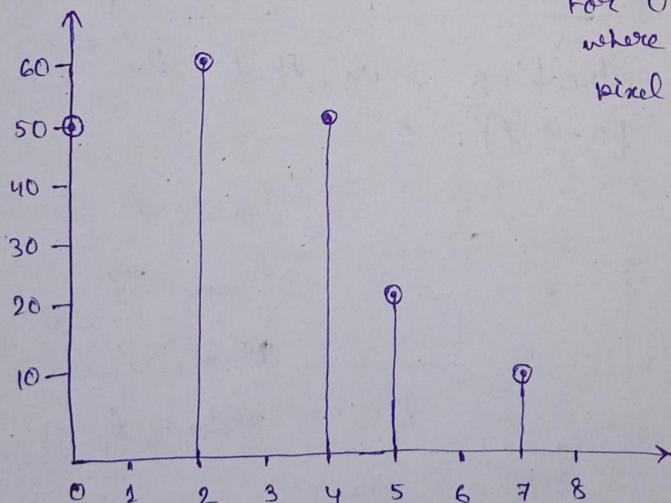
when, $r=5$

$$S_5 = \left(\frac{7-0}{6-2} \right) \times (5-2) + 0 = \frac{7}{4} \times 3 = 5.25 \approx 5$$

when, $r=6$

$$S_6 = \left(\frac{7-0}{6-2} \right) \times (6-2) + 0 = \frac{7}{4} \times 4 + 0 = 7$$

For 0, $r=2$ and in graph where grey level=2 there no. of pixel = 50.



Qn: Apply contrast stretching on 3-bit grey level image of size 4×4 .

Solu:

$f(x, y)$

2	1	2	1
4	5	5	6
3	2	1	4
6	2	1	6

DIP

$$f(x,y) =$$

6	7	7	7	3
5	1	1	2	2
5	0	5	0	3
0	0	3	3	3
5	1	1	4	4

- 1) Apply grey scale slicing for the image where $3 \leq v \leq 5$
- 2) Construct Image Histogram Equilization
- 3) Find Image Negative
- 4) Distance from P to Q (CITY Block)

3) Apply Grey Scale Slicing

with Background $a \leq v \leq b$ where, $a=3$
 $b=5$

6	7	7	7	3
5	1	1	2	2
5	0	5	0	3
0	0	3	3	3
5	1	1	4	4

$$3 \leq v \leq 5 = 7$$

otherwise v as it is

6	7	7	7	7
7	1	1	2	2
7	0	7	0	7
0	0	7	7	7
7	1	1	7	7

without
Background

0	0	0	0	7
7	0	0	0	0
7	0	7	0	7
0	0	7	7	7
7	0	0	7	7

Range
 $3 \leq v \leq 5 = 7$
without 0

3) Image Negative

Max Value = 7 ~ $(111)_2$

$$\begin{aligned} \text{Formula} \quad S &= (L-1) \cdot r^m \\ &= (8-1) \cdot 7^0 \\ &= 7 - 7^0 \end{aligned}$$

No. of bit
 $L = 3$
Formula - I
 $L = 2^m$
 $2^3 = 8$

Negative

1	0	0	0	4
2	6	6	5	5
2	7	2	7	4
7	7	4	4	4
2	6	6	3	3

④ *

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

 $f(x,y) = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 6 & 7 & 7 & 7 & 3 \\ 1 & 5 & 1 & 1 & 2 \\ 2 & 5 & 0 & 5 & 0 \\ 3 & 0 & 0 & 3 & 3 \\ 4 & 5 & 1 & 1 & 4 \end{matrix}$

P(x_1, y_1)
(1, 3)
Q(x_2, y_2)

$$\begin{aligned} d(P, Q) &= |x_1 - x_2| + |y_1 - y_2| \\ &\text{let } P(1, 3), Q(3, 1) \\ d(P, Q) &= |1 - 3| + |3 - 1| \\ &= 2 + 2 = 4 \end{aligned}$$

2)

Grey Level	0	1	2	3	4	5	6	7
No. of Pixel	4	4	2	5	2	4	1	3

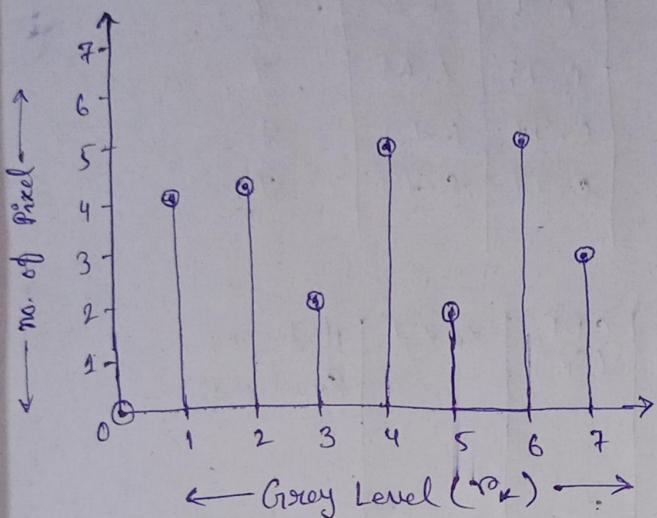
n = 25

Grey Level (r_K)	No. of Pixel (n_K)	$P(r_K) = \frac{n_K}{n}$ (PDF)	S_K (CDP)	$S_K \times 7$	Histogram Equalized
0	4	0.16	0.16	1.12	1
1	4	0.16	0.32	2.24	2
2	2	0.08	0.4	2.8	3
3	5	0.2	0.6	4.2	4
4	2	0.08	0.68	4.76	5
5	4	0.16	0.84	5.88	6
6	1	0.04	0.88	6.16	6
7	3	0.12	1	7	7
				$\overline{n = 25}$	

• New Image

6	7	7	7	4
6	2	2	3	3
6	1	6	1	4
1	1	4	4	4
6	2	2	5	5

Grey Level	0	1	2	3	4	5	6	7
No. of Pixel	0	4	4	2	5	2	5	3



Input Image :-

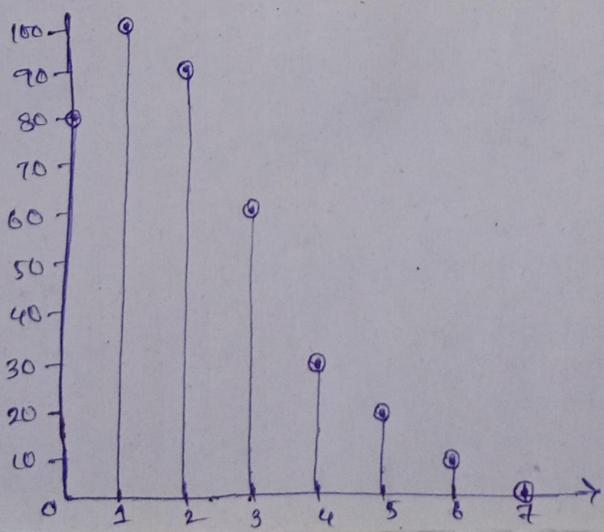
Qn)

Grey level	0	1	2	3	4	5	6	7
No. of pixel	80	100	90	60	30	20	10	0

Target / Desired Image :-

Grey level	0	1	2	3	4	5	6	7
No. of Pixel	0	0	0	60	80	100	80	70

Ans:



Grey Level (r_k)	No. of Pixel (n_k)	PDF $P(r_k) = \frac{n_k}{n}$	CDF (S_k)	$S_k \times 7$	Histogram Equilized
0	80	0.20	0.20	1.4	1
1	100	0.25	0.45	3.15	3
2	90	0.23	0.68	4.76	5
3	60	0.15	0.83	5.81	6
4	30	0.07	0.9	6.3	6
5	20	0.05	0.95	6.65	7
6	10	0.02	0.97	6.79	7
7	0	0	0.97	6.79	7
$\sum n_k = 390$					

Grey Level (r_k)	No. of Pixel (n_k)	PDF $P(r_k) = \frac{n_k}{n}$	CDF (S_k)	$S_k \times 7$	Histogram Equilized
0	0	0			
1	0	0			
2	0	0			
3	60	0.15			
4	80	0.20			
5	100	0.25			
6	80				
7	70				
$\sum n_k = 390$					

Logical Operation

① AND operation:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

		f(x,y)		
		1	1	1
0		1	0	0
0	1	0	0	0
1	0	0	1	0
1	1	1	0	1

		f ₁ (x,y)		
		1	1	1
0		0	0	1
0	1	0	0	0
1	0	0	0	1
1	1	0	1	1

		g(x,y)		
		1	1	1
0		0	0	0
0	1	0	0	0
1	0	0	0	0
1	1	0	0	0

② OR operation:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

		g(x,y)		
		1	1	1
0		0	1	1
0	1	1	1	1
1	0	1	1	1
1	1	1	1	1

③ NOT operation:

Y	Y
0	1
1	0

$$\Theta(x,y) \rightarrow \text{of}[f(x,y)]$$

0	0	0
1	0	1
1	0	1
1	0	1

Logical Image Arithmetic:

- ① Detect Differences: Image subtraction can be used to find difference between of the objects.
- ② De-noise: Adding successive image of the same scene can reduce random noise.
- ③ Collection of Illumination: Dividing two images can correct shadow and non-homogenous illumination.
- ④ Adjust Brightness: Multiplying pixel values by number > 1 , while dividing the factor > 1 can darken the image.
- ⑤ Blend Images: Varying the value ~~of~~ between 0 & 1. Then it make transition images.

MSE = Mean Square Error

SNR = Signal to Noise Ratio

PSNR = Peak Signal to Noise Ratio

These are used to measure the quality of images and sequence.

Qn What is MSE?

Ans) MSE is a error measurement of original image and distorted image.

• PSNR → Ratio of the max. possible signal power to the power of the noise that effects the signal.

Unit is DB [Decibal]

• Measure a quantity of contrast or reconstructed image. It indicates that higher PSR value indicates better quality.

- SNR → Measure quality of any signal.
- MSE → Avg. of the sequence errors between the original and distorted image. It compares the original image with distorted image. The lower noise MSE value indicates lesser.

Problems:

$$\textcircled{1} \quad f(x,y) = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 2 \end{bmatrix}, \quad \hat{f}(x,y) = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

MSE Formula:

$$\text{MSE} = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2$$

Sol:

$$\text{MSE} = \frac{1}{9} [(3-3)^2 + (2-2)^2 + (1-2)^2 + (1-1)^2 + (2-1)^2 + (1-2)^2 + (3-1)^2 + (2-1)^2 + (2-1)^2]$$

$$= \frac{1}{9} \times 9 = 1$$

Problem \textcircled{2}:

$$f(x,y) = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 2 \end{bmatrix}, \quad \hat{f}(x,y) = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

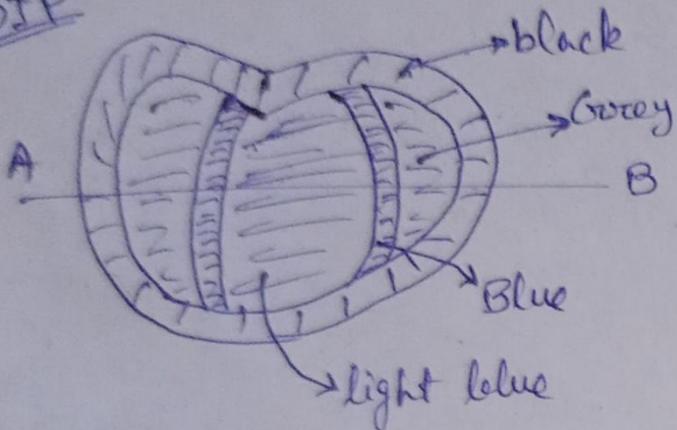
SNR Formula:

$$\text{SNR} = 20 \log_{10} \frac{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [f(x,y)]^2}{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2}$$

Sol:

$$\text{SNR} = 20 \log_{10} \frac{[9+4+1+1+4+1+9+4+4]}{9}$$

$$= 20 \log_{10} \frac{37}{9} = \underline{12.27}$$



darker $\rightarrow 0$

0 - black = low intensity

grey - slightly higher

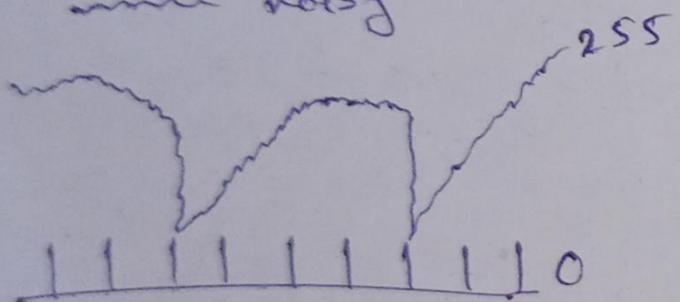
blue - Medium Intensity

light blue - High Intensity

white - very high intensity

Sampling & Quantization:

— clear
~~~~ noisy



## Image Negative & Thresholding:

### ① Image Negative:

|    |    |    |    |
|----|----|----|----|
| 12 | 68 | 64 | 32 |
| 8  | 12 | 15 | 32 |
| 8  | 10 | 16 | 28 |
| 36 | 40 | 52 | 65 |

Original

How much is  $L = ?$

$$65 = 1000100$$

$$L = 2^n = 2^7 = 128$$

$$S = (L-1) - R = 127 - 80$$

|     |     |     |    |
|-----|-----|-----|----|
| 115 | 59  | 63  | 95 |
| 119 | 115 | 112 | 95 |
| 119 | 117 | 111 | 99 |
| 91  | 87  | 75  | 62 |

Negative Image

Problem ① :

Thresholding with  $T = 4$ .

|   |   |   |   |
|---|---|---|---|
| 4 | 3 | 5 | 2 |
| 3 | 6 | 4 | 6 |
| 2 | 2 | 6 | 5 |
| 7 | 6 | 4 | 1 |

$$\text{If } R = 0, 1, 2, 3 \Rightarrow S = 0 \\ R = 4, 5, 6, 7 \Rightarrow S = 7$$

$$S = \begin{cases} L-1 = 7; & R \geq 4 \\ 0; & R < 4. \end{cases}$$

$$L = ?$$

7 is max. number

$$14 \rightarrow 3 \text{ bit}$$

$$2^3 = 8 \therefore L = 8, L-1 = 7$$

$$S = (L-1) - R$$

Ans:

|                   |                   |                   |   |
|-------------------|-------------------|-------------------|---|
| $\frac{7-4}{2^3}$ | 0                 | $\frac{7-5}{2^2}$ | 0 |
| 0                 | $\frac{7-6}{2^1}$ | $\frac{7-4}{2^3}$ |   |
| 0                 | 0                 |                   |   |
| $\frac{7-7}{2^0}$ | $\frac{7-6}{2^1}$ | $\frac{7-4}{2^3}$ | 0 |
| = 0               | 21                | = 3               | 0 |

Zooming:  $\xleftarrow{\text{Zoom in}} \xrightarrow{\text{(Doubling Row & Column)}}$

|   |   |
|---|---|
| 4 | 3 |
| 3 | 6 |

$2 \times 2$

|   |   |   |   |
|---|---|---|---|
| 4 | 4 | 3 | 3 |
| 4 | 4 | 3 | 3 |
| 3 | 3 | 6 | 6 |
| 3 | 3 | 6 | 6 |

point operation.

Shrinking:

## Sampling & Quantization :-

D - 31/11/25

These are the processes used to convert analog signal into digital signal.

1) What is sampling?

→ The process of collecting data from an analog source at regular intervals.

→ It determines the special resolution of an image.

→ Digitizing the coordinate value is called sampling.

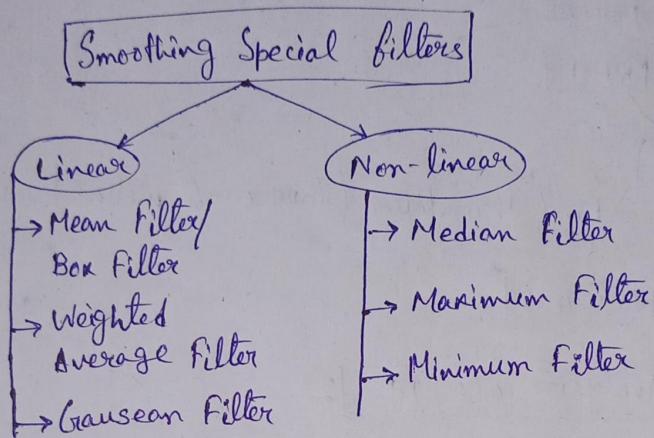
2) What is quantization?

→ The process of converting a continuous set of values into a discrete set of values.

→ Also digitizing the amplitude value is called quantization.

Smoothing Special filters are used for blurring and noise reduction.

- Bluring is used in Pre-processing task such as removal of small details from an image prior to (large) object extraction.
- Noise Reduction can be accomplished by blurring with a linear as well as non-linear filter.



Smoothing Linear Filters:- They are known as averaging or low-pass-filters as they are simply average of the pixels content in neighbourhood of the pixel.

The process in a result in an image with reduced sharp transition in intensity which ultimately leads to noise reduction.

(A) Mean Filter/Box Filter:- (B) Weighted Average Filter

$$\frac{1}{9} \alpha \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \left. \right\} \text{Mask}$$

- Gives more (less) weight to pixel near (away from) the output location.

(iii) Gaussian Filter:- Weights are samples of 2D gaussian filter.

$$g_{rs}(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Blur Edge Reduce contrast

It is used to Blur image & reduce its contrast.

- Non Linear Filter / Order Statistical Filter :-

These response is based on ordering (ranking). The pixels containing image area encompassed by the filter and then replacing value of the pixel by the value determined the ranking result

Median Filter : - Median of the pixel values

Maximum Filter : - To find the maximum of all pixel values

Minimum Filter : - To find the minimum of all pixel values

- Median of the Pixel Values:-

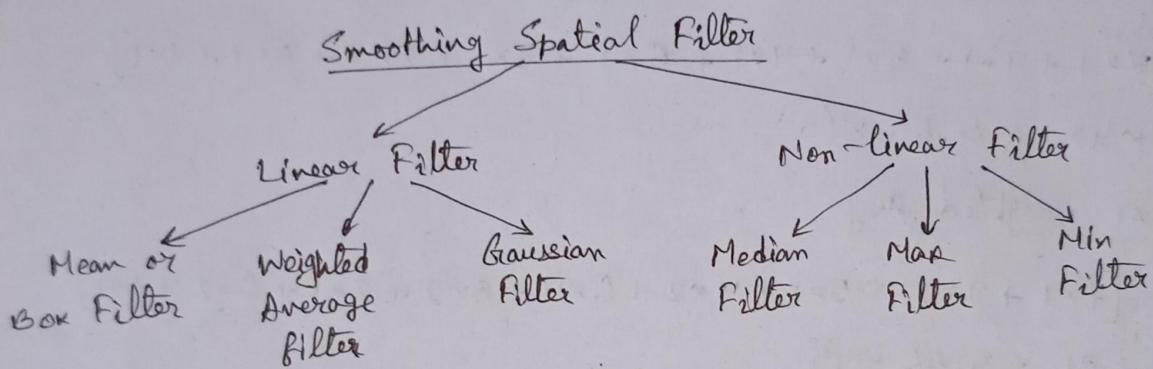
|   |   |   |
|---|---|---|
| 3 | 1 | 0 |
| 5 | 4 | 6 |
| 2 | 3 | 2 |

8, 7, 2, 1, ③, 8, 4, 5, 6  
↓  
Median

## Smoothing Spatial Filter:

Smoothing Spatial Filters are used for blurring and noise reduction.

- Blurring: Blurring is used in preprocessing task. Such as removal of small details from an image prior to object extraction.
- Noise reduction: It can be done by blurring with both linear and non-linear filters.



## Mask:

### ① Mean / Box filter

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad 3 \times 3$$

### ② Weighted Avg. filter

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

### ③ Gaussian Filter

$$G_G(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

For 2D,

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Qn: Consider the image and calculate the output of the pixel (2,2). If ~~the~~ smoothing is done using (3×3) neighbourhood. Mean Filters have it is used.

$$= \frac{1}{9} [8 \times 1 + 9 \times 1 + 5 \times 1 + 4 \times 1 + 6 \times 1 + 8 \times 1 + 2 \times 1 + 0 \times 1 + 1 \times 1] \\ = \frac{1}{9} \times 42 = 4.66 \approx 5$$

⑧ For weighted Avg.

$$= \frac{1}{16} [7 \times 1 + 9 \times 2 + 5 \times 1 + 4 \times 2 + 6 \times 4 + 8 \times 2 + 2 \times 1 + 0 \times 2 + 1 \times 1] \\ = \frac{1}{16} \times 81 = 5.06 \approx ⑤$$

## Non-Linear Filter

i) Median Filter, [Mask is empty]. | let,  
pos( $u_{i,j}$ )

2, 4, 4, 7, ⑥, 1, 8, 8, 9

let. pos = (2, 2)

$$\text{Ans} = 5$$

Mark replace with mark value

Min replace with min value

題 padding :

|             |   |   |   |   |   |
|-------------|---|---|---|---|---|
| <u>ext:</u> | 0 | 0 | 0 | 0 | 0 |
|             | 0 | 2 | 2 | 2 | 0 |
|             | 0 | 1 | 0 | 2 | 0 |
|             | 0 | 8 | 0 | 5 | 0 |
|             | 0 | 0 | 0 | 0 | 0 |

if n is odd  
mid val  
else  
mid 2 avg