Assignment 1

Robotics

1) Explain SCARA robot configuration with a neat figure.

Answer:

SCARA (Selective Compliance Assembly Robot Arm) is a type of robotic arm that is widely used in manufacturing and assembly tasks due to its unique combination of speed, precision and flexibility.

Configuration:

(i) Horizontal Arm Movement —

The SCARA robot consists of two rotary joints (R-joints) that allow movement in the X-Y plane. This enables the arm to pivot and reach different positions with high precesion.

(ii) Vertical Linear Movement —

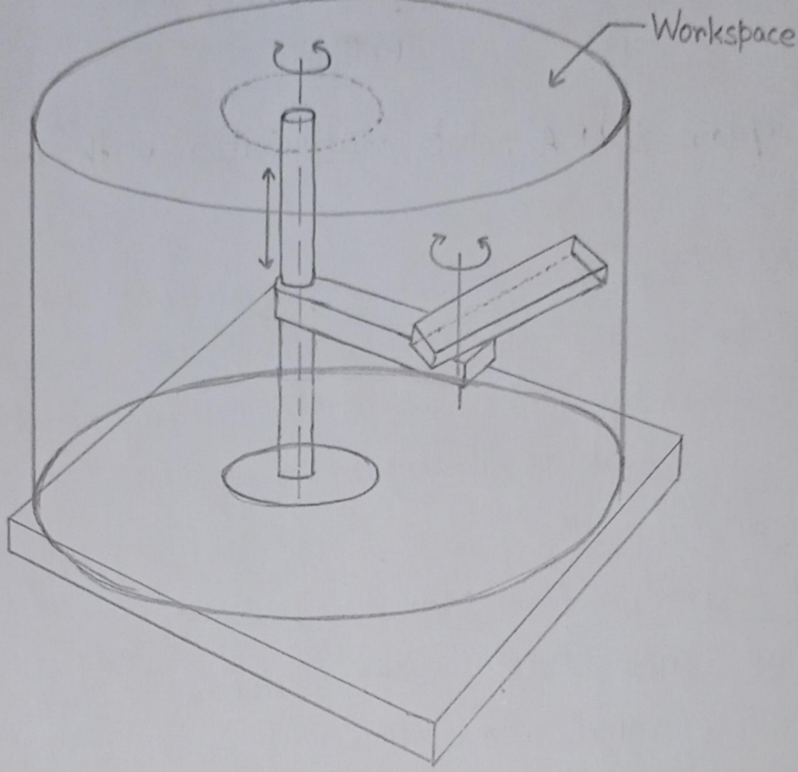
The arm incorporates a vertical linear actuator to provide movement along the Z-axis. This allows tasks like inserting components or applying vertical forces during assembly.

(iii) End-Effector —

The end of the arm is equipped with an end-effector, which could be a gripper, vaccum, or tool, depending on the specific application.

(iv) Base —

The base is stationary and houses the motors and controllers for the robot's movement.



The SCARA Configuration

2) Explain the classification of Robots as per Robotics Institute of America.

Answer :

According to the Robotics Institute of America, the following is the classification of Robots.

- Variable-Sequence Robot : A device that performs the successive stages of a task according to a predetermined method easy to modify.
- Playback Robot : A human operator performs the task manually by leading the Robot.

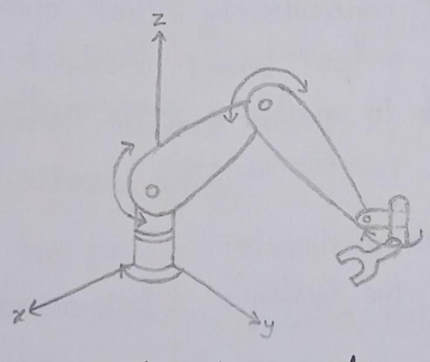
- Numerical Control Robot: The operator supplies the movement program rather than teaching it the task manually.
- Intelligent Robot: A robot with the means to understand its environment and the ability to successfully complete a task despite changes to the environment.

3) Explain any two types of robot reference frames.

Answer:

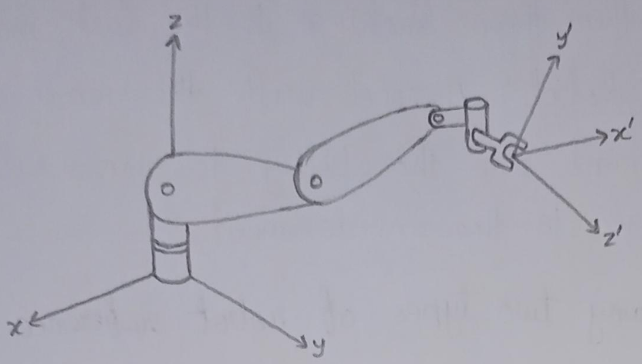
The explanation of two types of robot reference frames is below.

(i) Joint Reference Frame which is used to specify movements of each individual joint of the robot. In this case each joint may be accessed individually and thus only one joint moves at a time.



Joint reference frame

(ii) Tool Reference Frame which specifies the movements of the robots hand relative to the frame attached to the hand. The x', y' and z' axes attached to the hand define the motions of the hand relative to this local frame. All joints of the robot move simultaneously to create coordinated motions about the Tool frame.



Tool reference frame

4) Differentiate between Servo and Non-servo control system.

Answer:

Aspect	Servo Control System	Non-Servo Control System
Definition	A closed-loop control system that continuously adjusts its output based on feedback to maintain the desired performance.	An open-loop control system that operates without feedback and does not correct deviations.
Feedback Mechanism	Uses feedback to monitor and correct the system's output.	Does not use feedback for correction.
Accuracy	High accuracy due to real-time adjustments.	Lower accuracy as there is no feedback for correction.
Complexity	More complex due to sensors, feedback devices and controllers.	Simpler design, as it lacks feedback and complex controllers.
Cost	Relatively expensive due to added components.	Less expensive because of simpler mechanisms.

Aspect	Servo Control System	Non-Servo Control System
Response to Disturbances	Can compensate for disturbances and maintain desired performance.	Unable to adjust to disturbances; output may deviate from desired performance.
Energy Efficiency	May consume more energy due to continuous adjustments and monitoring.	Typically more energy-efficient as there is no continuous correction.
Control Mechanism	Uses a combination of controllers like PID or advanced control algorithms.	Operates based on predefined commands without dynamic adjustments.
Example	Servo motors, robotic arms and automatic doors.	Simple electric motors, traditional household appliances, and basic water pumps.

5) Derive 3D rotation matrix along X, Y, and Z-axis using direction cosine representation.

Answer:

The rotation matrix describes how a point or a vector rotates in 3D space about one of the coordinate axes. Using direction cosine representation, the rotation matrices for the X, Y, and Z axes can be derived as follows:

(i) Rotation about the X-axis:

Consider a point $P(x, y, z)$ rotated by an angle θ about the X-axis. The X-axis remains unchanged, and only the Y and Z coordinates are affected:

$x' = x$ (no change)
$y' = y\cos\theta - z\sin\theta$
$z' = y\sin\theta + z\cos\theta$

The rotation matrix for this transformation is:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

(ii) Rotation about the Y-axis:

For a rotation by an angle θ about the Y-axis, the Y-axis remains unchanged, and the X and Z coordinates are affected:

$x' = x\cos\theta + z\sin\theta$
$y' = y$ (no change)
$z' = -x\sin\theta + z\cos\theta$

The rotation matrix for this transformation is:

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

(iii) Rotation about the Z-axis:

For a rotation by an angle θ about the Z-axis, the Z-axis remains unchanged, and the X and Y coordinates are affected:

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$ (no change)

The rotation matrix for this transformation is:

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6) The coordinates point P in a reference frame are $[8, 6, 4]^T$. If the reference frame is rotated at an angle 45° along Z-axis, find the coordinate of point P in rotated frame.

<u>Answer:</u>

The rotation matrix for a rotation by $\theta = 45°$ about the Z-axis is:

$$R_z(45°) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

substitute $\cos(45°) = \sin(45°) = \frac{\sqrt{2}}{2}$

$$R_z(45°) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The initial coordinates of P are given as:

$$P = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix}$$

To find the rotated coordinates, multiply $R_z(45°)$ with P:

$P' = R_z(45°) \cdot P$

$$P' = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix}$$

$$P_x' = \frac{\sqrt{2}}{2}\cdot 8 - \frac{\sqrt{2}}{2}\cdot 6 = \frac{\sqrt{2}}{2}(8-6) = \sqrt{2}$$

$$P_y' = \frac{\sqrt{2}}{2}\cdot 8 + \frac{\sqrt{2}}{2}\cdot 6 = \frac{\sqrt{2}}{2}(8+6) = \frac{\sqrt{2}}{2}\times 14 = 7\sqrt{2}$$

$$P_z' = 4$$

The rotated coordinates of P are:

$$P' = \begin{bmatrix} \sqrt{2} \\ 7\sqrt{2} \\ 4 \end{bmatrix}$$

7) The coordinates point P in a reference frame are $[15, 3, 8]^T$. If the reference frame is rotated at an angle 60° along Y-axis, find the coordinate of point P in rotated frame.

Answer:

The rotation matrix for a rotation by $\theta = 60^\circ$ about the Y-axis is:

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Substitute $\cos(60^\circ) = \frac{1}{2}$ and $\sin(60^\circ) = \frac{\sqrt{3}}{2}$:

$$R_y(60^\circ) = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

The initial coordinates of P are:

$$P = \begin{bmatrix} 15 \\ 3 \\ 8 \end{bmatrix}$$

To find the rotated coordinates, multiply Ry (60°) with P:

$P' = R_y(60^\circ) \cdot P$

$$P' = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} 15 \\ 3 \\ 8 \end{bmatrix}$$

$$P'_x = \frac{1}{2} \times 15 + 0 \times 3 + \frac{\sqrt{3}}{2} \times 8 = \frac{15}{2} + \frac{8\sqrt{3}}{2} = \frac{15 + 8\sqrt{3}}{2}$$

$$P'_y = 0 \times 15 + 1 \times 3 + 0 \times 8 = 3$$

$$P'_z = -\frac{\sqrt{3}}{2} \times 15 + 0 \times 3 + \frac{1}{2} \times 8 = -\frac{15\sqrt{3}}{2} + \frac{8}{2} = \frac{-15\sqrt{3} + 8}{2}$$

The rotated coordinates of P are:

$$P' = \begin{bmatrix} \frac{15 + 8\sqrt{3}}{2} \\ 3 \\ \frac{-15\sqrt{3} + 8}{2} \end{bmatrix}$$

$\sqrt{3} \approx 1.732$

$$P'_x \approx \frac{15 + 8 \times 1.732}{2} = \frac{15 + 13.856}{2} = \frac{28.856}{2} \approx 14.428$$

$$P'_y = 3$$

$$P'_z \approx \frac{-15 \times 1.732 + 8}{2} = \frac{-25.98 + 8}{2} = \frac{-17.98}{2} \approx -8.99$$

Thus, the rotated coordinates are approximately:

$$P' \approx \begin{bmatrix} 14.428 \\ 3 \\ -8.99 \end{bmatrix}$$

8) Write a MATLAB program to animate one-link robot.

Answer:

```
% MATLAB Program to Animate a One-Link Robot Arm
clc;
clear;
close all;
% Parameters
L=5;
theta_start = 0;
theta_end = 360;
theta_step = 5;
pause_time = 0.05;
% Create a figure for the animation
figure;
axis equal;
xlim([-L-1, L+1]);
ylim([-L-1, L+1]);
grid on;
title('One-Link Robot Animation');
xlabel('x-axis');
ylabel('y-axis');
% Base of the robot
hold on;
plot(0, 0, 'ko', 'MarkerSize', 10, 'MarkerFaceColor', 'k');
% Loop through each angle and animate
for theta = theta_start : theta_step : theta_end
    % Convert angle to radians
    theta_rad = deg2rad(theta);
```

```
    % Calculate the end coordinates of the robot arm
    x_end = L * cos(theta_rad);
    y_end = L * sin(theta_rad);
    % Plot the robot arm
    arm = plot([0, x_end], [0, y_end], 'r-', 'LineWidth',
          2);
    joint = plot(x_end, y_end, 'bo', 'MarkerSize', 8,
          'MarkerFaceColor', 'b');
    % Pause for animation
    pause(pause_time);
    % Delete the previous arm and joint to create motion
      effect
    if theta ~= theta_end
         delete(arm);
         delete(joint);
    end
end
% Hold the final position
hold off;
```

9) Write a MATLAB program to animate two-link robot.

Answer:

```
% MATLAB Program to Animate a Two-Link Robot Arm
clc;
clear;
close all;
% Robot parameters
L1= 5;
L2= 3;
theta1_start = 0;
theta1_end =90;
theta2_start = 0;
theta2_end = 90;
theta_step = 10;
pause_time =0.02;
% Create a figure for the animation
figure;
axis equal;
xlim([-L1-L2-1, L1+L2+1]);
ylim ([-L1-L2-1, L1+L2+1]);
grid on;
title ('Two-Link Robot Animation');
xlabel ('X-axis');
ylabel ('Y-axis');
% Base of the robot
hold on ;
plot (0, 0, 'ko', 'MarkerSize', 10, 'MarkerFaceColor', 'k');
```

```
% Animation loop
for theta1 = theta1_start : theta_step : theta1_end
    for theta2 = theta2_start : theta_step : theta2_end
        % Convert angles to radians
        theta1_rad = deg2rad(theta1);
        theta2_rad = deg2rad(theta2);
        % Calculate the joint and end-effector positions
        x_joint = L1 * cos(theta1_rad);
        y_joint = L1 * sin(theta1_rad);
        x_end = x_joint + L2 * cos(theta1_rad + theta2_rad);
        y_end = y_joint + L2 * sin(theta1_rad + theta2_rad);
        % Plot the robot arm
        link1 = plot([0, x_joint], [0, y_joint], 'r-', 'LineWidth',
                    2);
        link2 = plot([x_joint, x_end], [y_joint, y_end], 'b-',
            'LineWidth', 2);
        joint1 = plot(x_joint, y_joint, 'go', 'MarkerSize', 8,
            'MarkerFaceColor', 'g');
        end_effector = plot(x_end, y_end, 'mo', 'MarkerSize', 8,
            'MarkerFaceColor', 'm');
    % Pause for animation
    pause(pause_time);
    % Delete previous links and points to create motion
      effect
    delete(link1);
    delete(link2);
```

```
        delete(joint1);
        delete(end_effector);
    end
end
% Hold the final position
hold off;
```

10) Define Singularity and Write the properties of rotation matrix.

Answer:

Singularity refers to a condition where a robotic manipulator loses its ability to move in certain directions or loses one or more degrees of freedom.

// Properties of a Rotation Matrix

A rotation matrix is used to represent a rigid body rotation in 2D or 3D space. It has the following properties:

(i) Orthogonality: The rows and columns of a rotation matrix are orthogonal to each other:

$$R^T R = I$$

where R^T is the transpose of the rotation matrix, and I is the identity matrix.

(ii) Determinant: The determinant of a rotation matrix is always 1:

$$\det(R) = 1$$

(iii) Inverse Equals Transpose: For a rotation matrix, the inverse is equal to its transpose:

$$R^{-1} = R^T$$

(iv) Preservation of Vector Norm: A rotation matrix preserves the length (norm) of a vector during transformation

$$\|Rv\| = \|v\|, \quad \forall v \in \mathbb{R}^n$$

(v) Orthogonal Columns and Rows: Each column or row of the matrix has a unit length, and they are mutually orthogonal.

(vi) Eigenvalues: The eigenvalues of a 3D rotation matrix lie on the unit circle in the complex plane.

11) Explain various types of joints used in robots.

Answer:

The various types of robotic joints are below:

(i) Prismatic Joint

- Motion: Linear motion along a single axis.
- Characteristics:
  → Moves the connected links closer or farther apart.
- Applications:
  → Robotic arms in assembly lines for linear positioning tasks.

(ii) Revolute Joint

- Motion: Circular motion about a single fixed axis.
- Characteristics:
  → Provides one degree of rotational freedom.
- Applications:
  → Robotic manipulators, pick-and-place robots, and welding robots.

(iii) Cylindrical Joint

- Motion: Combination of translational and rotational motion along and about a single axis.
- Characteristics:
  → Enables both linear and rotational movement between two connected links.
- Applications:
  → Robots used for machining or pick-and-place operations where combined motion is required.

(iv) Spherical Joint

- Motion: Allows three degrees of rotational freedom.
- Characteristics:
  → Enables high flexibility and wide range of motion.
- Applications:
  → Humanoid robots and robots with dexterous manipulators.

(v) Cartesian Joint

- Motion: Provides translational motion along the X, Y, and Z axes.
- Characteristics:
  → Highly accurate and straightforward motion control.
- Applications:
  → Cartessian robots used in 3D printers, CNC machines, and pick-and-place systems.

(vi) Planer Joint

- Motion: Allows relative sliding motion between two connected links in a plane.
- Characteristics:
  → Movement includes translational motion in two directions and rotational motion about an axis normal to the plane.
- Applications:
  → Specific robotics research applications.

(vii) Screw Joint

- Motion: Combination of rotational and translational motion along the same axis.
- Characteristics:
  → Motion resembles the action of a screw being turned.
- Applications:
  → Specialized robotic manipulators and some machining robots.

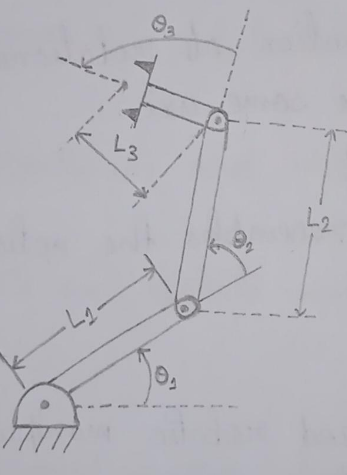
(viii) Parallel Joint

- Motion: Provides a constrained relative motion between two links using parallel linkages.
- Characteristics:
  → Ensures precise and stable movement.
- Applications:
  → Hexapods and Stewart platforms for flight simulators or high-precision machining.

(ix) Universal Joint

- Motion: Provides rotational motion around two perpendicular axes.
- Characteristics:
  - → Offers flexibility similar to a human wrist.
  - → Limited range of motion compared to spherical joints.
- Applications:
  - → Robotic wrists, drive shafts in robotic vehicles.

12) Find the DH-parameters for the following 3-link robot.



Answer:

To find the Denavit-Hartenberg (DH) parameters for the given 3-link robot –

(i) Identify the joint axes and assign the frames according to DH convention.

(ii) Calculate the following parameters for each joint:

- a_i : Link Length (distance along x_i between z_i and z_{i+1}).
- α_i : Link twist (angle between z_i and z_{i+1}, measured about

x_i).

- d_i : Link offset (distance along z_i between the common normal and x_i)
- θ_i : Joint angle (angle between x_i and x_{i+1}, measured about z_i).

From the image:

(i) Each joint appears to be revolute (denoted by θ_i)

(ii) Links L_1, L_2 and L_3 are the physical lengths of the robot's arms.

For Joint 1:

→ The base joint connects to L_1 and rotates about the z_1-axis with angle θ_1.

→ There is no offset or twist here, so $a_1 = 0$, $\alpha_1 = 0$, and $d_1 = 0$.

For Joint 2:

→ The second joint connects L_1 to L_2 and rotates about the z_2-axis with angle θ_2.

→ The link length is L_1, so $a_2 = L_1$, while $\alpha_2 = 0$ and $d_2 = 0$

For Joint 3:

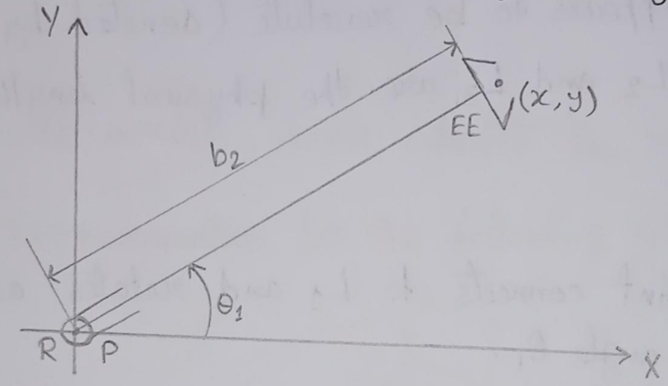
→ The third joint connects L_2 to L_3 and rotates about the z_3-axis with angle θ_3.

→ The link length is L_2, so $a_3 = L_2$, while $\alpha_3 = 0$ and $d_3 = 0$

DH Parameter Table

Joint i	a_i (Link Length)	α_i (Twist Angle)	d_i (Link Offset)	θ_i (Joint Angle)
1	0	0	0	θ_1
2	L_1	0	0	θ_2
3	L_2	0	0	θ_3

13) Find the DH-parameters for the following RP robot.



Answer:

(i) For Joint 1 (Revolute Joint):

- The base joint rotates about the Z_0-axis, and the frame does not translate along X_0.
- No link length ($a_1 = 0$) or offset ($d_1 = 0$)
- Twist angle ($\alpha_1 = 0$) because there is no angular displacement between Z_0 and Z_1.

(ii) Prismatic Joint (Joint 2):

- The prismatic joint extends along the link length ($a_2 = b_2$).
- Joint displacement (d_2) varies as the prismatic link extends or retracts.
- There is no angular rotation ($\theta_2 = 0$) or twist ($\alpha_2 = 0$).

DH Parameter Table.

Joint i	a_i (Link Length)	α_i (Twist Angle)	d_i (Link Offset)	θ_i (Joint Angle)
1	0	0	0	θ_1
2	b_2	0	d_2	0

14) Derive the position and velocity of the two-link robot. Also, derive its Jacobian Matrix.

Answer:

(i) Position of the End-Effector

The position of the end-effector (x, y) in terms of the joint angles θ_1 and θ_2 is derived as follows:

- The position of the end-effector is the sum of contributions from both links:

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Thus, the end-effector position is:

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$

(ii) Velocity of the End-Effector

The velocity of the end-effector is obtained by differentiating the position with respect to time:

$$\dot{P} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$

Differentiating x and y:

$$\dot{x} = -L_1 \sin\theta_1 \dot{\theta}_1 - L_2 \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\dot{y} = L_1 \cos\theta_1 \dot{\theta}_1 + L_2 \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

Thus, the velocity can be written in matrix form as:

$$\dot{p} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) & L_2 \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

(iii) <u>Jacobian Matrix</u>

The Jacobian matrix relates the joint velocities $(\dot{\theta}_1, \dot{\theta}_2)$ to the end-effector linear velocities $(\dot{x}, \dot{y})$:

$$\dot{p} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

From the velocity equations:

$$J = \begin{bmatrix} \frac{dx}{d\theta_1} & \frac{dx}{d\theta_2} \\ \frac{dy}{d\theta_1} & \frac{dy}{d\theta_2} \end{bmatrix}$$

Partial derivatives:

<u>For x</u> –

$$\frac{dx}{d\theta_1} = -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2),$$

$$\frac{dx}{d\theta_2} = -L_2 \sin(\theta_1+\theta_2)$$

<u>For y</u> –

$$\frac{dy}{d\theta_1} = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2)$$

$$\frac{dy}{d\theta_2} = L_2 \cos(\theta_1+\theta_2)$$

Thus, the Jacobian matrix is:

$$J = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) & L_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

15) Define Jacobian and derive the position and velocity of the one-link robot.

Answer:

The Jacobian matrix is a mathematical representation that relates the joint velocities of a robotic manipulator to the linear and angular velocities of its end-effector.

// Position of the End-Effector

The position of the end-effector (x, y) in the Cartesian space can be expressed as:

$$x = L_1 \cos\theta_1$$
$$y = L_1 \sin\theta_1$$

Thus, the position vector of the end-effector is:

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \end{bmatrix}$$

// Velocity of the End-Effector

The velocity of the end-effector $(\dot{x}, \dot{y})$ is obtained by differentiating the position vector P with respect to time:

$$\dot{x} = \frac{d}{dt}(L_1 \cos\theta_1) = -L_1 \sin\theta_1 \dot{\theta}_1$$

$$\dot{y} = \frac{d}{dt}(L_1 \sin\theta_1) = L_1 \cos\theta_1 \dot{\theta}_1$$

Thus, the velocity vector is:

$$\dot{P} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 \dot{\theta}_1 \\ L_1 \cos\theta_1 \dot{\theta}_1 \end{bmatrix}$$

16) Calculate the Jacobian of a two-link planer arm when $\theta_1 = 45°$ and $\theta_2 = 20°$

Answer:

For a two-link planar robot, the position of the end-effector in Cartesian space is derived based on the joint angles θ_1 and θ_2, and the lengths of the links L_1 and L_2.

Position Equations

The position of the end-effector (x, y) in terms of joint angles is:

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Jacobian Matrix

The Jacobian matrix (J) relates the joint velocities $[\dot{\theta}_1, \dot{\theta}_2]$ to the end-effector linear velocities $[\dot{x}, \dot{y}]$. It is given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Where the Jacobian matrix is:

$$J = \begin{bmatrix} \frac{dx}{d\theta_1} & \frac{dx}{d\theta_2} \\ \frac{dy}{d\theta_1} & \frac{dy}{d\theta_2} \end{bmatrix}$$

Partial Derivatives

(i) $\frac{dx}{d\theta_1} = -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2)$

(ii) $\frac{dx}{d\theta_2} = -L_2 \sin(\theta_1 + \theta_2)$

(iii) $\frac{dy}{d\theta_1} = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

(iv) $\frac{dy}{d\theta_2} = L_2 \cos(\theta_1 + \theta_2)$

Thus, the Jacobian matrix becomes:

$$J = \begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Given,

$\theta_1 = 45^\circ = \frac{\pi}{4}$ radians

$\theta_2 = 20^\circ = \frac{\pi}{9}$ radians

Link lengths: $L_1 = 1$, $L_2 = 0.5$

For $\theta_1 = 45^\circ$,

$\sin\theta_1 = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\cos\theta_1 = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

For $\theta_2 = 20^\circ$,

$\sin\theta_2 = \sin\left(\frac{\pi}{9}\right) \approx 0.342$, $\cos\theta_2 = \cos\left(\frac{\pi}{9}\right) \approx 0.940$

For $(\theta_1 + \theta_2) = 65^\circ$,

$\sin(\theta_1 + \theta_2) = \sin(65^\circ) \approx 0.906$

$\cos(\theta_1 + \theta_2) = \cos(65^\circ) \approx 0.423$

Now,

$$\frac{dx}{d\theta_1} = -L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2)$$
$$= -(1)(0.707) - (0.5)(0.906)$$
$$= -0.707 - 0.453 \quad = -1.16$$

$$\frac{dx}{d\theta_2} = -L_2 \sin(\theta_1 + \theta_2)$$
$$= -(0.5)(0.906) = -0.453$$

$$\frac{dy}{d\theta_1} = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$
$$= (1)(0.707) + (0.5)(0.423)$$
$$= 0.707 + 0.212 = 0.919$$

$$\frac{dy}{d\theta_2} = L_2\cos(\theta_1+\theta_2)$$
$$= (0.5)(0.423) = 0.212$$

Final Jacobian Matrix:

$$J = \begin{bmatrix} -1.16 & -0.453 \\ 0.919 & 0.212 \end{bmatrix}$$

17) Derive the inverse kinematic solution of a two-link manipulator.

Answer:

The two-link planar manipulator has two revolute joints.

L_1: Length of the first link

L_2: Length of the second link

(x, y): Position of the end-effector in Cartesian coordinates

Using the distance from the base to the end-effector:

$$r = \sqrt{x^2+y^2}$$

Apply the cosine rule in the triangle formed by L_1, L_2 and r:

$$r^2 = L_1^2 + L_2^2 - 2L_1L_2\cos\theta_2$$

Rearranging for $\cos\theta_2$:

$$\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$$

Thus, $$\theta_2 = \arccos\left(\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right)$$

Since arccos has two solutions, there are two configurations:

Elbow-Up : Positive θ_2

Elbow-Down : Negative θ_2

Compute θ_1

The angle ϕ between the x-axis and the line connecting the origin to the end-effector is:

$$\phi = \arctan\left(\frac{y}{x}\right)$$

compute the auxiliary angle ψ using the law of cosines:

$$\cos\psi = \frac{L_1^2 + r^2 - L_2^2}{2Lr}$$

$$\sin\psi = \sqrt{1-\cos^2\psi}$$

Thus ψ is:

$$\psi = \arctan\left(\frac{\sqrt{1-\cos^2\psi}}{\cos\psi}\right)$$

Finally, the shoulder angle θ_1 is:

$$\theta_1 = \phi - \psi$$

Substituting:

$$\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{L_2 \sin\theta_2}{L_1 + L_2\cos\theta_2}\right)$$

So,

Elbow Angle, $\theta_2 = \arccos\left(\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right)$

Shoulder Angle, $\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}\right)$

18) Derive the inverse kinematic solution of a three-link manipulator.

Answer:

L_1: Length of the first link

L_2: Length of the second link

L_3: Length of the third link

(x, y): Position of the end-effector

ϕ: Orientation of the end-effector relative to the base frame

The end-effector orientation is defined by ϕ. First, determine the position of the wrist joint (x_w, y_w), which is the endpoint of the second link and the starting point of the third link:

$$x_w = x - L_3 \cos\phi$$

$$y_w = y - L_3 \sin\phi$$

The wrist joint (x_w, y_w) is effectively the endpoint of a two-link manipulator consisting of L_1 and L_2. The solution for θ_1 and θ_2 can be derived using the two-link manipulator inverse kinematics:

(i) Elbow angle (θ_2):

$$\cos\theta_2 = \frac{x_w^2 + y_w^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\theta_2 = \arccos\left(\frac{x_w^2 + y_w^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

(ii) Shoulder angle (θ_1):

$$\phi_1 = \arctan\left(\frac{y_w}{x_w}\right)$$

$$\cos\phi_2 = \frac{L_1^2 + x_w^2 + y_w^2 - L_2^2}{2L_1\sqrt{x_w^2 + y_w^2}}$$

$$\theta_2 = \arccos\left(\frac{L_1^2 + x_w^2 + y_w^2 - L_2^2}{2L_1\sqrt{x_w^2 + y_w^2}}\right)$$

So, $\theta_1 = \phi_1 - \phi_2$

Using the orientation of the end-effector ϕ and the previously calculated joint angles:

$$\theta_3 = \phi - (\theta_1 + \theta_2)$$

Wrist Position:

$$x_w = x - L_3 \cos\phi$$
$$y_w = y - L_3 \sin\phi$$

Joint Angles:

$$\theta_2 = \arccos\left(\frac{x_w^2 + y_w^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

$$\theta_1 = \arctan\left(\frac{y_w}{x_w}\right) - \arccos\left(\frac{L_1^2 + x_w^2 + y_w^2 - L_2^2}{2L_1\sqrt{x_w^2 + y_w^2}}\right)$$

$$\theta_3 = \phi - (\theta_1 + \theta_2)$$

19) Write a note on progressive advancements in robots.

Answer:

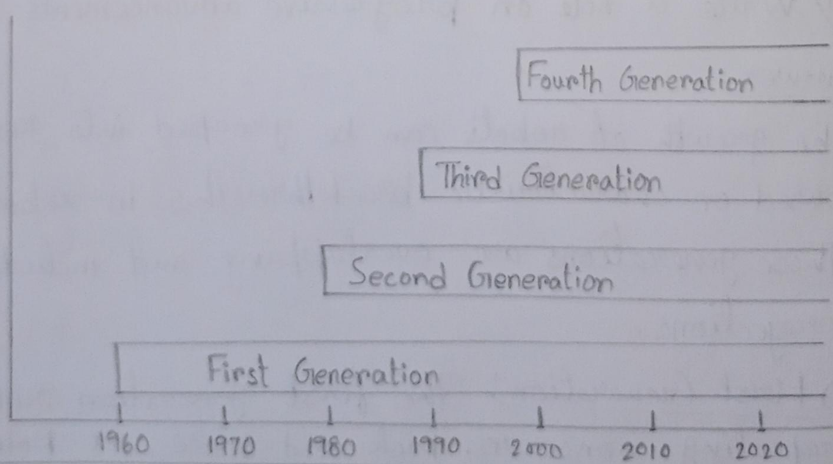
The growth of robots can be grouped into robot generations, based on characteristic breakthroughs in robot's capabilities. These generations are overlapping and include futuristic projections.

(i) First Generation: The first generation robots are repeating, nonservo, pick-and-place, or point-to-point kind.

The technology for these is fully developed and at present about 80% robots in use in the industry are of this kind. It is predicted that these will continue to be in use for a long time.

(ii) Second Generation: The addition of sensing devices and enabling the robot to alter its movements in response to sensuary feedback marked the beginning of second generation. These robots exhibit path-control capabilities. This technological breakthrough came around 1980s and is yet not mature.

(iii) Third Generation: The third generation is marked with robots having human-like intelligence. The growth in computers led to high-speed processing of information and, thus, robots also acquired artificial intelligence, self-learning and conclusion-drawing capabilities by past experiences. On-line computations and control, artificial vision, and active force/torque interaction with the environment are the significant characteristics of these robots. The technology is still in infancy and has to go a long way.



(iv) Fourth Generation: This is futuristic and may be a reality only during this millenium. Prediction about its features is difficult, if not impossible. It may be a true android or an artificial biological robot or a super humanoid capable of producing its own clones. This might provide for fifth and higher generation robots.

20) Write a note on Robot programming language.

Answer:

A robot programming language serves as a communication interface between a human programmer and a robotic manipulator, enabling the execution of textual commands to achieve desired robotic tasks. The process of robot programming can be categorized into two main types: on-line programming and off-line programming.

On-line Programming

- In on-line programming, the robotic manipulator executes commands immediately after they are entered by the programmer.
- This allows real-time verification of the robot's actions and immediate correction of any discrepancies.
- While effective for testing and debugging, this approach ties up the robot, preventing it from performing its regular tasks during programming.

Off-line Programming

- Off-line programming involves creating and testing the program in a simulated environment without direct interaction with the physical robot.

- This method eliminates production downtime, as the robot can contine performing its regular tasks while the program is being developed.
- Once the programmer is satisfied with the program's correctness, it is uploaded to the robot manipulator for execution in the real environment.

Key Features

(i) Structure: Robot Programming languages are modeled on conventional programming languages, incorporating their own vocabulary, grammar, and syntax.

(ii) Commands:

→ Definition of points, paths, and frames.

→ Control of joint motion, end-effectors

→ Interaction with external sensors and devices, along with environmental interactions.

(iii) Functionality:

→ Traditional programming functions such as arithmetic, logic, and trigonometric operations.

→ Condition testing, looping structures, debugging tools, and input-output operations.

→ Data management features like storage, retrieval, and program updates.

Execution Modes

Once a program is developed and uploaded, it is executed by the robot in a "run" mode, ensuring continuous operation without further manual intervention.

21) Explain different application fields of robots.

Answer:

Over the years, advancements in robotics have enabled their application across a diverse range of fields. Below are some major application areas of robots:

(i) Industrial Applications

Robots are extensively used in industries to improve efficiency, precision, and safety.

- Assembly: Performing repetitive tasks like assembling electronic devices, vehicles, and consumer goods.
- Welding: Robotic arms handle precision welding in automotive and aerospace manufacturing.

(ii) Medical and Healthcare

In the healthcare domain, robots assists in a variety of roles:

- Surgical Robots: Systems like the da Vinci Surgical System perform minimally invasive surgeries with precision.
- Pharmaceutical Robots: Used in drug discovery, manufacturing and packaging.

(iii) Defense and Security

Robots enhance operational safety and effectiveness in military and security contexts:

- Explosive Ordnance Disposal (EOD): Robots like bomb disposal robots are used to handle explosives safely.
- Combat Robots: Some robots are equipped for battlefield assistance and defense operations.

(iv) Exploration

Robots are deployed in environments that are too hazardous or inaccessible for humans:

- Space Exploration: Robots like the Mars rovers conduct research on other planets.
- Underwater Exploration: Submersible robots explore deep oceans, studying marine life and underwater ecosystems.
- Mining: Robots are used for drilling, extracting minerals, and ensuring safety in hazardous mining operations.

(v) Agriculture

Robotics plays a significant role in modern farming techniques:

- Planting and Harvesting: Robots assist in sowing seeds, watering crops and harvesting produce.
- Weeding and Pest Control: Autonomous systems like drones spray pesticides and remove weeds with precision.

(vi) Domestic Application

In homes, robots make life easier and more convenient:

- Cleaning Robots: Devices like vaccum-cleaning robots and window-cleaning robots.
- Smart Assistants: Devices with robotic integration like Amazon Alexa and Google Assistant with robotic arms.

(vii) Education and Research

Robots are widely used in academic and research institutions:

- AI and Robotics Research: Universities and tech companies use robots for advancing artificial intelligence, machine learning, and robotics technology.

(viii) Disaster Management

Robots play a critical role in disaster response scenarios:

- Search and Rescue: Robots equipped with sensors and cameras assists in locating and rescuing trapped individuals.
- Firefighting Robots: Specialized robots are designed to extinguish fires and operate in hazardous environments.

(ix) Construction

Robots are incresingly used in the construction industry:

- Demolition Robots: Safely demolish old structures.
- Inspection Robots: Ensure the structural integrity of buildings and bridges.

(x) Transportation and Logistics

Robots are transforming the logistics and transportation industry:

- Autonomous Vehicles: Self-driving cars, trucks, and drones revolutionize logistics and commuting.
- Warehouse Automation: Robots like those used by Amazon handle sorting, storing, and picking items in warehouses.

22) Define the following:

a. Workspace
b. Degrees of Freedom
c. Rover
d. Actuators
e. Robot controller
f. Point-to-point robot
g. Continuous path-controlled robot

Answer:

a) Workspace: Depending on the configuration and size of the links and wrist joints, robots can reach a collection of points called a workspace.
Alternately workspace may be found empirically, by moving each joint through its range of motions and combining all space it can reach and subtracting what space it cannot reach.

b) Degrees of Freedom (DoF): Degrees of freedom represent the number of independent movements or motions a robot or its manipulator can make. Each degree of freedom corresponds to a single axis of motion.

c) Rover: A rover is a mobile robot specifically designed for exploration tasks in remote or hazardous environments. Rovers are commonly used in space missions and are equipped with sensors, cameras, and actuators for navigation and data collection.

d) Actuators : Actuators are mechanical or electromechanical devices that convert energy into motion, enabling the robot's joints or end-effectors to move.

- Motors of electric actuators are powered by electricity.
- Hydraulic actuators use fluid pressure for motion.
- Pneumatic actuators use compressed air for movement.

e) Robot Controller : The robot controller is the brain of the robot. It is a computing unit responsible for executing the robot's program, processing input data, and controlling the movement of actuators and sensors in real time.

- Functions –

→ Motion planning
→ Sensor feedback processing
→ Communication with external devices
→ Coordination of robotic subsystems

f) Point-to-point robot : Only the end points are programmed, the path used to connect the end points are computed by the controller.
User can control velocity, and may permit linear or piece wise linear motion.
Feedback control is used during motion to ascertain that individual joints have achieved desired location.
Often used hydraulic drives, recent trend towards servomotors.
Loads up to 500lb and large reach
Applications –
→ pick and place type operation
→ machine loading

g) Continuous path-controlled robot: In addition to the control over the endpoints, the path taken by the end effector can be controlled.
Path is controlled by manipulating the joints throughout the entire motion, via closed loop control
Applications —
→ spray painting, polishing, grinding, arc welding

23) Write the classification of sensors.
Answer:
Sensors can be classified based on various criteria. Below are the classifications:
(i) Based on the types of Input Measured
- Position Sensors:
  → Encoders
  → Potentiometers
- Velocity Sensors:
  → Doppler radar
- Acceleration Sensors:
  → Accelerometers

(ii) Based on the Working Principle
- Optical Sensors:
  → Utilize light-based signals
- Magnetic Sensors:
  → Detect magnetic fields

(iii) Based on Power Requirement
- Active Sensors:
  → Require external power to operate and generate an output signal.

- Passive Sensors:
  → Do not require external power; they rely on detecting environmental changes.

(iv) Based on the Nature of Output

- Analog Sensors:
  → Provide continuous output signals proportional to the measured quantity.
- Digital Sensors:
  → Provide discrete digital signals

(v) Based on the Application Area

- Automotive Sensors:
  → Used in vehicles
- Robotics Sensors:
  → Used for navigation and interaction

(vi) Based on Contact

- Contact Sensors:
  → Require physical contact to detect the measured property.
- Non-Contact Sensors:
  → Detect changes without physical contact.

(vii) Based on Technology

- Bio-Sensors:
  → Detect biological changes
- IoT Sensors:
  → Smart sensors connected to IoT networks.

24) Define kinetic energy and derive the kinetic energy of two-link robotic arm.

Answer:

Kinetic energy is the energy possessed by a body due to its motion.

Kinetic energy, $K = \frac{1}{2} mv^2$

m = mass of the object

v = velocity of the object

Link 1: Length L_1, mass m_1, angular velocity $\dot{\theta}_1$

Link 2: Length L_2, mass m_2, angular velocity $\dot{\theta}_2$

a) Velocity of the Center of Mass of Link 1

The position of the center of mass of link 1 is:

$$x_{COM1} = \frac{L_1}{2} \cos\theta_1 \quad , \quad y_{COM1} = \frac{L_1}{2} \sin\theta_1$$

The velocity of the center of mass is:

$$v_{COM1} = \sqrt{\left(\frac{dx_{COM1}}{dt}\right)^2 + \left(\frac{dy_{COM1}}{dt}\right)^2}$$

Using angular velocity:

$$v_{COM1} = \frac{L_1}{2} \dot{\theta}_1$$

b) Velocity of the Center of Mass of Link 2

The position of the center of mass of link 2 is:

$$x_{COM2} = L_1 \cos\theta_1 + \frac{L_2}{2} \cos(\theta_1 + \theta_2)$$

$$y_{COM2} = L_1 \sin\theta_1 + \frac{L_2}{2} \sin(\theta_1 + \theta_2)$$

The velocity of the center of mass is:

$$v_{COM2} = \sqrt{\left(\frac{dx_{COM2}}{dt}\right)^2 + \left(\frac{dy_{COM2}}{dt}\right)^2}$$

Total Kinetic Energy,

$$K = \frac{1}{2} m_1 v_{CoM1}^2 + \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 v_{CoM2}^2 + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

where,

I_1 and I_2 are the moments of inertia of the links.

v_{CoM1} and v_{CoM2} are the velocities of the centers of mass of the links.

$$v_{CoM1} = \frac{L_1}{2} \dot{\theta}_1$$

$$v_{CoM2} = \sqrt{\left(-L_1 \sin\theta_1 \dot{\theta}_1 - \frac{L_2}{2} \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)\right)^2 + \left(L_1 \cos\theta_1 \dot{\theta}_1 + \frac{L_2}{2} \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)\right)^2}$$

Final Expression,

$$K = \frac{1}{2} m_1 \left(\frac{L_1}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 v_{CoM2}^2 + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

25) Derive the dynamic equation of motion of one-link robotic arm.

Answer:

Link length = L

Mass of the link = m

Moment of inertia about the joint axis = I

Join angle = $\theta(t)$

External torque applied at the joint = T

Gravitational acceleration = g

The Lagrangian ($\mathcal{L}$) is defined as:

$$\mathcal{L} = K - U$$

where, K = Kinetic energy

U = Potential energy

The velocity of the center of mass is,

$$v_{CoM} = \frac{L}{2}\dot{\theta}$$

translational kinetic energy,

$$K_t = \frac{1}{2}m\left(\frac{L}{2}\dot{\theta}\right)^2 = \frac{1}{8}mL^2\dot{\theta}^2$$

rotational kinetic energy,

$$K_r = \frac{1}{2}I\dot{\theta}^2$$

Total Kinetic Energy,

$$K = K_t + K_r = \frac{1}{8}mL^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

Potential energy, $U = mgh$

where,

$$h = \frac{L}{2}\sin\theta$$

Thus, $U = mg\frac{L}{2}\sin\theta$

now,

$$\mathcal{L} = K - U = \left(\frac{1}{8}mL^2 + \frac{1}{2}I\right)\dot{\theta}^2 - mg\frac{L}{2}\sin\theta$$

The equation of motion is derived using the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) - \frac{d\mathcal{L}}{d\theta} = \tau$$

$$\frac{d\mathcal{L}}{d\dot{\theta}} = \left(\frac{1}{8}mL^2 + \frac{1}{2}I\right)2\dot{\theta} = \left(\frac{1}{4}mL^2 + I\right)\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \left(\frac{1}{4}mL^2 + I\right)\ddot{\theta}$$

$$\frac{d\mathcal{L}}{d\theta} = -mg\frac{L}{2}\cos\theta$$

The dynamic equation of motion for the one-link robotic arm is —

$$\left(\frac{1}{4}mL^2 + I\right)\ddot{\theta} + mg\frac{L}{2}\cos\theta = \tau$$