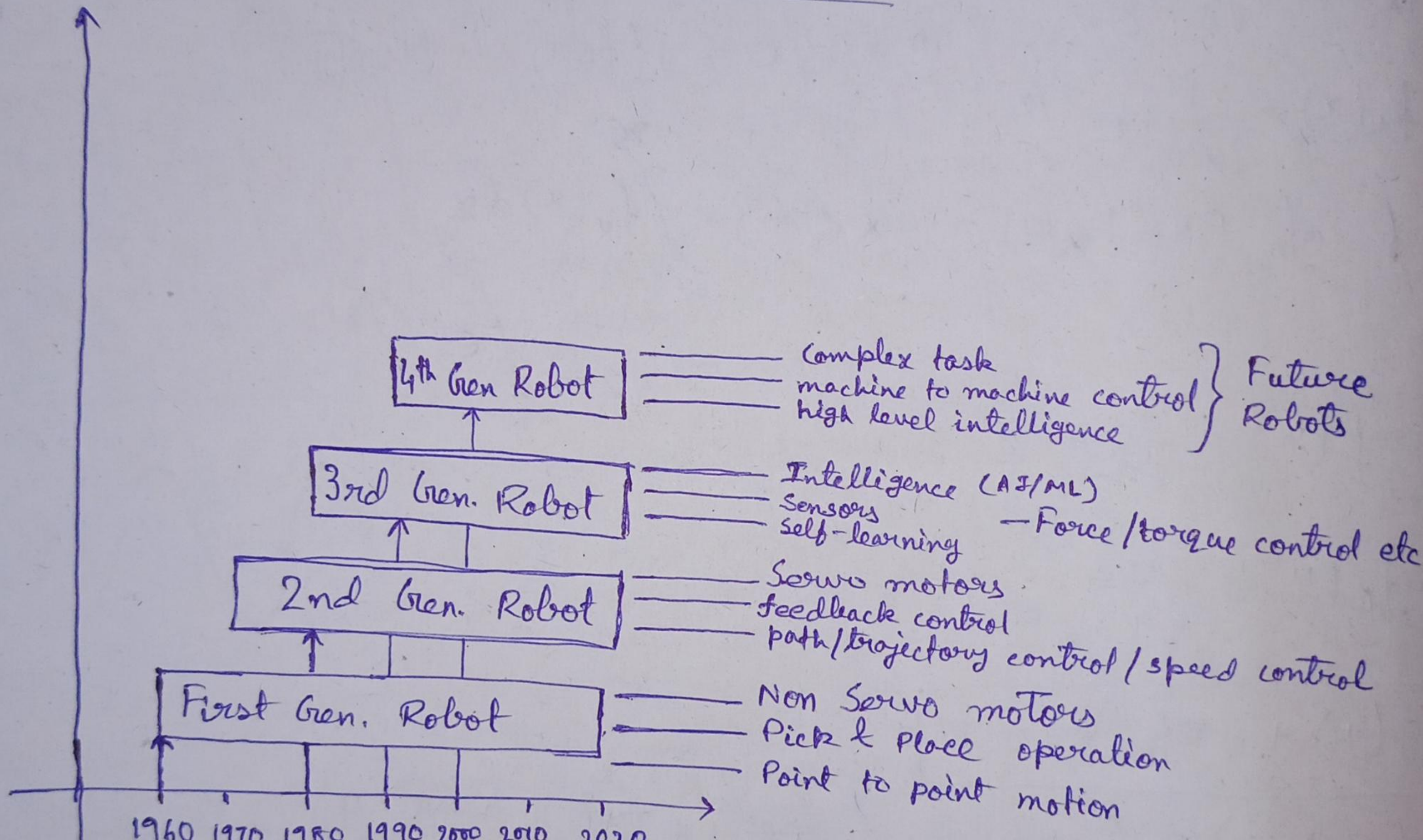
Robotics

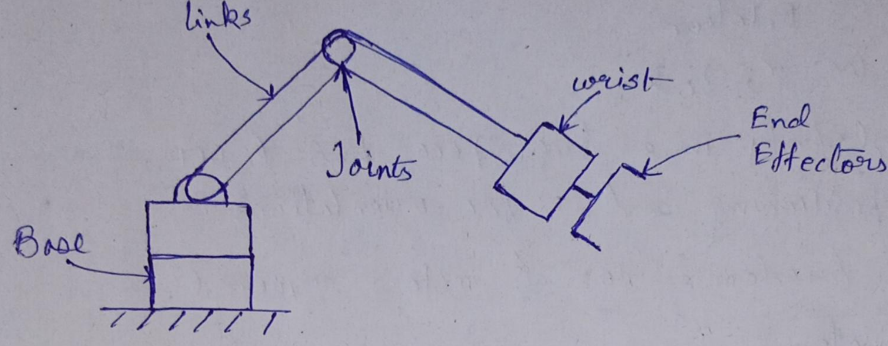
13/8/2024

Progressive Advancement of Robots :— (10 marks)



~~Robotics~~

Robot Anatomy



Base: Base are having two types in the robots. ~~the~~
e.g. (i) fixed base (ii) moveable base

(i) fixed ~~base~~: — The base of the robot which is fixed to the ground is called fixed base robot or manipulator.

(ii) moveable base: — If the base ~~are~~ of the robot is having legs or wheels attached to it is called moving based robot or rovers.

Links: The links are structural member after robot which with stands the forces ~~at~~ act in on it.

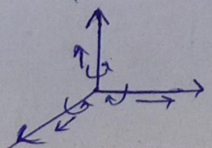
(*) Industrial robots has 6 links.

Joints: There are different types of joints are ~~are~~ used in robots —

(i) Rebolute Joint ~~,~~ (ii) Prismatic Joint, (iii) Cylindrical Joint, (iv) Spherical Joint, (v) Screw Joint, (vi) Universal Joint

Date: 21/8/2024

(*) The no. of movement that an object performs in a 3D space is called degrees of freedom.



6 = DOF ~~6D~~

6 = DOF

↳ 3 translation

@ x, y, z

↳ 3 Rotation

@ x, y, z

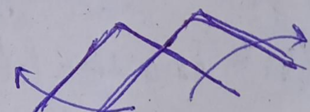
- The rigid body in a free space has 6 DOF –
- 3 for positioning and 3 for orientations.

Degree of freedom = no. of motors required.

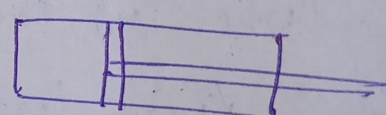
Robot Anatomy :

(A) Joints

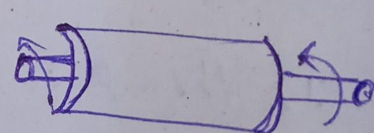
1. Revolute joint →



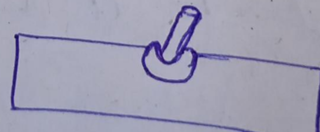
2. Personate joint →



3. Cylindrical joint →



4. Spherical joint



5. Screw joint
6. Universal joint

Robot Kinematics

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 5 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}_{3\times1} \qquad B^T = \begin{bmatrix} 4 & 3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3\times1}$$

$$A \cdot B = \begin{bmatrix} 2\times4 + 4\times3 + 6\times2 \\ 5\times4 + 4\times3 + 2\times2 \\ 3\times4 + 2\times3 + 1\times2 \end{bmatrix} = \begin{bmatrix} 32 \\ 36 \\ 20 \end{bmatrix}$$

$$|A| = 2(4\times1 - 2\times2) - 4(5\times1 - 3\times2) + 6(5\times2 - 3\times4)$$

$$= -8$$

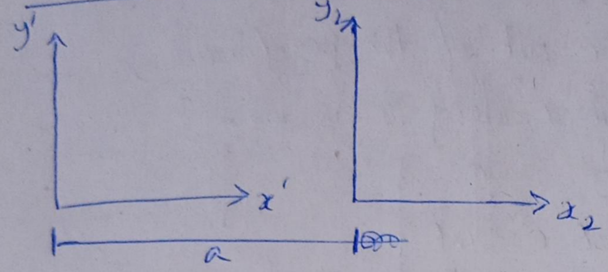
$$A^{-1} = \frac{1}{|A|}(\text{Adj } A)$$

$$B \times C = \begin{vmatrix} i & j & k \\ 4 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = (3\times1 - 2\times2)i - (4\times1 - 3\times2)j + (4\times2 - 3\times3)k$$

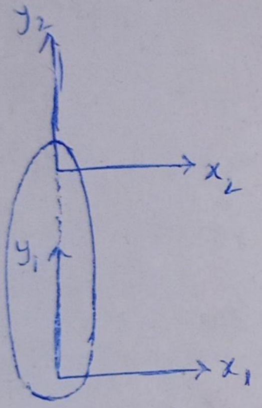
$$= -i + 2j - 1k$$

$$= \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

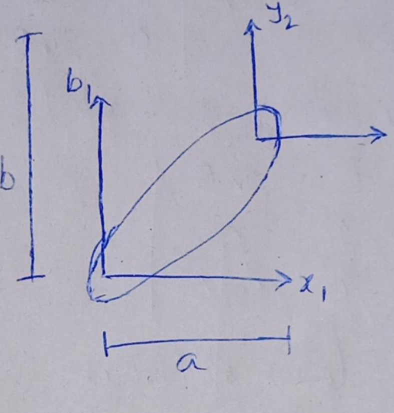
Translation of frames



$$\vec{a} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

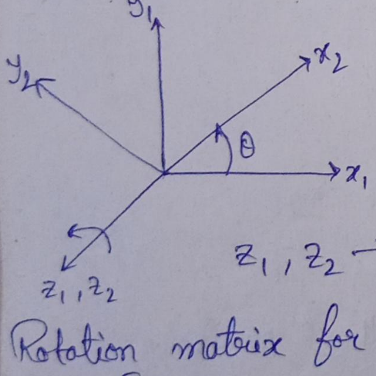


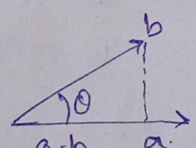
$$\vec{a} = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix}$$



$$\vec{a} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

Rotation of frames





$a \cdot b = |a||b| \cos\theta$

$a \cdot b = ab \cos\theta$

$z_1, z_2 \rightarrow$ out of the page (direction)

Rotation matrix for Rotation @ z at an angle θ

$$[R] = \begin{bmatrix} x_2 \cdot x_1 & y_2 x_1 & z_2 x_1 \\ x_2 \cdot y_1 & y_2 y_1 & z_2 y_1 \\ x_2 z_1 & y_2 z_1 & z_2 z_1 \end{bmatrix}$$

x_1, y_1, z_1 & x_2, y_2, z_2 are unit vectors

$x_2 \cdot x_1 = \cos\theta$

$x_2 \cdot y_1 = \cos(90^\circ - \theta) = \sin\theta$

$x_2 \cdot z_1 = \cos(90^\circ) = 0$

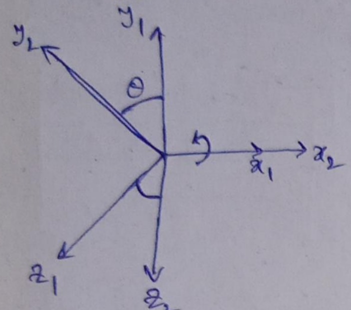
$y_2 x_1 = \cos(90^\circ + \theta) = -\sin\theta$

$y_2 y_1 = \cos\theta$ $\qquad$ $y_2 z_1 = \cos(90^\circ) = 0$

$z_2 x_1 = \cos(90^\circ) = 0$ $\qquad$ $z_2 y_1 = \cos(90^\circ) = 0$

$z_2 z_1 = \cos(0^\circ) = 1$

$$[R]_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



y_1 & x_2 are out of the page (direction)

Rotation matrix along x is what?

$$[R]_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation matrix along y is what?

$$[R_y] = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

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Properties of Rotation Matrix :

1. Rotation matrix belongs to 3-dimentional space $\rightarrow R \in SO(3)$.
2. Determinant of Rotation matrix is equal to 1.
   e.g. $|[R]_x| = \cos^2\theta + \sin^2\theta = 1$.
3. $R^{-1} = R^T$
4. Each row & each columns are mutually orthogonal.
5. Each rows & each columns are unit vectors

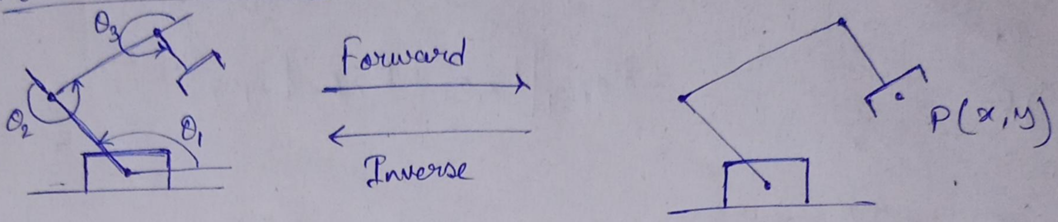
II **Kinematics**: It is the study of motion without considering the cause of the motion (i.e. Force)

- The kinametic quantities are Displacement or position, velocity and acceleration.

III **Dynamics**: It is the study of motion considering its costs (Force / Torque).

- Kinametics has 2 types —
  (i) Forward or direct Kinametics
  (ii) Inverse Kinematics

Forward Kinametics: → Calculating (x, y) position using θ angles.

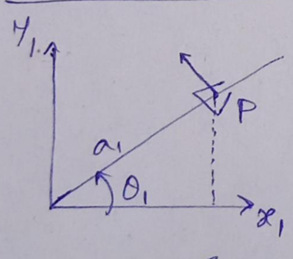


If the joint angles of each joint are known and the end effection positions are calculated, then the process is known as forward kinematics. The forward kinematics has an <u>unique solution</u>. Each to calculate

Inverse kinematics:

If the end effector position of the Robot known and joint angles of each joint are calculated, then this process is called Inverse kinematics. The inverse kinematics has multiple solutions difficult to calculate.

Kinametics of single / one link Robot



$$\vec{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix}$$

$$P = R_z \cdot t_x = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$$

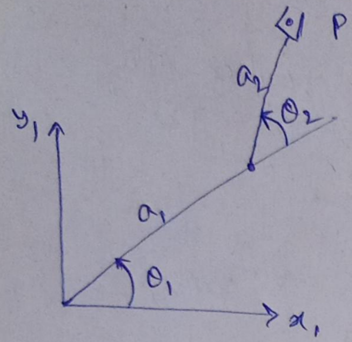
Rotation on z-axis.

Translation on x-axis.

$$= \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix}$$

$\therefore$ P is position of End Effector of 1 link manipulator / Robot.

Position of End Effector of 2-link Robot / Manipulator:



$$\vec{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = R_{z,\theta_1} \cdot t_{x,a_1} + R_{z,\theta_1} \cdot R_{z,\theta_2} \cdot t_{x,a_2}$$

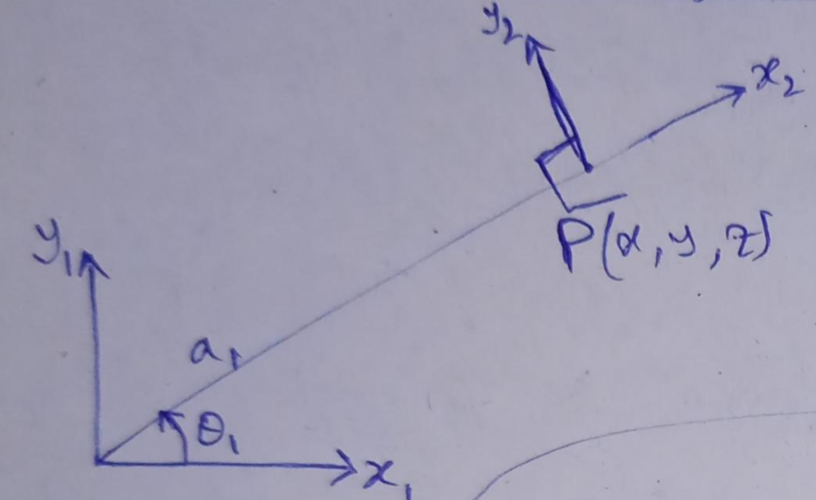
$$= R_{z,\theta_1}\left[t_{x,a_1} + R_{z,\theta_2}\, t_{x,a_2}\right]$$

$$\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) \\ a_1 s\theta_1 + a_2 s(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

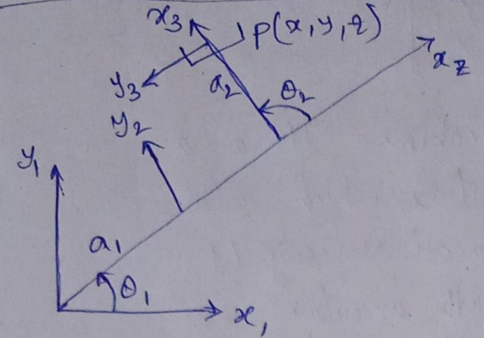
Forward Kinematics of single-link manipulation / Robot



$$P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = R_{z\theta}, t_{x,a} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix}$$

Forward Kinematics of 2-link manipulator



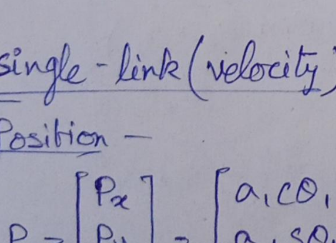
$$P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = R_{z_1,\theta_1} t_{x,a_1} + R_{z_1,\theta_2} R_{z,\theta_1} t_{x,a_2}$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} c\theta_1 c\theta_2 - s\theta_1 s\theta_2 & -(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) & 0 \\ c(\theta_1+\theta_2) & s(\theta_1+\theta_2) & 0 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) \\ a_1 s\theta_1 + a_2 s(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

3-link Robot



$$P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = R_{z,\theta_1} t_{x,a_1} + R_{z_1,\theta_1} R_{z,\theta_2} t_{x a_2} + R_{z_1,\theta_1} R_{z,\theta_2} R_{z_1,\theta_3} t_{x,a_3}$$

$$P = \begin{bmatrix} a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) + a_3 c(\theta_1+\theta_2+\theta_3) \\ a_1 s\theta_1 + a_2 s(\theta_1+\theta_2) + a_3 \cdot s(\theta_1+\theta_2+\theta_3) \\ 0 \end{bmatrix}$$

single-link (velocity)

Position — **Velocity**

$$P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ 0 \end{bmatrix} ; \quad \dot{P} = \frac{\partial P}{\partial t} = \underbrace{\frac{\partial P}{\partial \theta_1}}_{J} \times \frac{\partial \theta_1}{\partial t} = \begin{bmatrix} -a_1 s\theta_1 \\ a_1 c\theta_1 \\ 0 \end{bmatrix}$$

$$\dot{\theta}_1 = \begin{bmatrix} -a_1 s\theta_1 \dot{\theta}_1 \\ a_1 c\theta_1 \dot{\theta}_1 \\ 0 \end{bmatrix}$$

2nd - link velocity

$$P = \begin{bmatrix} a_1 c\theta_1 + a_2 c(\theta_1 + \theta_2) \\ a_1 s\theta_1 + a_2 s(\theta_1 + \theta_2) \\ 0 \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

Jacobian - It is a matrix which relates linear velocity (P) with angular velocity (θ).

Velocity

$$\dot{P} = \frac{\partial P}{\partial \theta_1} \times \frac{\partial \theta_1}{\partial t} + \frac{\partial P}{\partial \theta_2} \times \frac{\partial \theta_2}{\partial t}$$

$$= \underbrace{\begin{bmatrix} \frac{\partial P}{\partial \theta_1} & \frac{\partial P}{\partial \theta_2} \end{bmatrix}}_{} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J\theta$$

$J \rightarrow$ Jacobian

$$= \begin{bmatrix} -a_1 s\theta_1 - a_2 s(\theta_1 + \theta_2) & -a_2 s(\theta_1 + \theta_2) \\ a_1 c\theta_1 + a_2 (\theta_1 + \theta_2) & a_2 c(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 s\theta_1 \dot{\theta}_1 - a_2 s(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ a_1 c\theta_1 \dot{\theta}_1 + a_2 s(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix}$$

3-link velocity

$$\dot{P} = \begin{bmatrix} -a_1 s\theta_1 \dot{\theta}_1 - a_2 s(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) - a_3 s(\theta_1 + \theta_2 + \theta_3)(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \\ a_1 c\theta_1 \theta_1 + a_2 c(\theta_1 + \theta_2)(\dot{\theta}_1 + \theta_2) + a_3 c(\theta_1 + \theta_2 + \theta_3)(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$$

Acceleration $\rightarrow$ (single link)

$$\ddot{P} = \frac{\partial}{\partial t}(J\dot{\theta}) = J\ddot{\theta} + \dot{J}\dot{\theta}$$

$$= \begin{bmatrix} -a_1 s\theta_1 \\ a_1 c\theta_1 \\ 0 \end{bmatrix} \ddot{\theta}_1 + \begin{bmatrix} -a_1 c\theta_1 \dot{\theta}_1 \\ -a_1 s\theta_1 \dot{\theta}_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

2nd link

acceleration of 2-link –

$$\ddot{P} = J\ddot{\theta} + \dot{J}\dot{\theta} = \begin{bmatrix} -a_1 s\theta_1 - a_2 s(\theta_1+\theta_2) & -a_2 s(\theta_1+\theta_2) \\ a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) & a_2 c(\theta_1+\theta_2) \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -a_1 c\theta_1\dot{\theta}_1 - a_2 s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & -a_2 c(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) \\ -a_1 s\theta_1\dot{\theta}_1 - a_2 s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) & -a_2 s(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

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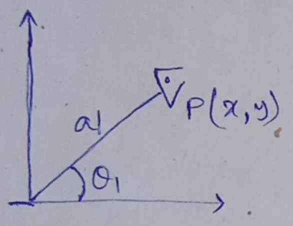
Inverse Kinematics

Single or 1-link Robot / Manipulator :



$P_x = a_1 c\theta_1 \longrightarrow (1)$

$P_y = a_1 s\theta_1 \longrightarrow (2)$

Known: a_1, P_x, P_y

To find: θ_1

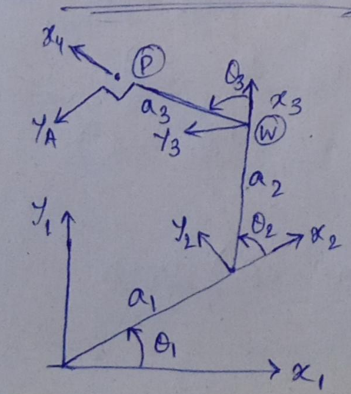
divide eqn (2) by (1)

$$\frac{a_1 s\theta_1}{a_1 c\theta_1} = \frac{P_y}{P_x}$$

$$\tan\theta_1 = \frac{P_y}{P_x} \Rightarrow \boxed{\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)}$$

$\boxed{\theta_1 = \text{atan2}(P_y, P_x)}$ for right quadrant

3-Link Robot :



$P_x = a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) + a_3 c(\theta_1+\theta_2+\theta_3)$

$P_y = a_1 s\theta_1 + a_2 s(\theta_1+\theta_2) + a_3 s(\theta_1+\theta_2+\theta_3)$

known $\rightarrow$ $a_1, a_2, a_3, P_x, P_y, \psi = (\theta_1+\theta_2+\theta_3)$

To find $\rightarrow$ $\theta_1, \theta_2, \theta_3$

Step 1: Convert 3-link robot to 2-link Robot for calculation.

$W_x = a_1 \cos\theta_1 + a_2 c(\theta_1 + \theta_2) \rightarrow (1)$

$W_y = a_1 s\theta_1 + a_2 s(\theta_1 + \theta_2) — (2)$

$W_x = P_x - a_3 c\psi — (3)$

$W_y = P_y - a_3 s\psi — (4)$

Elimination methods

(1) As θ_1 leaving, use,

Squaring & Adding (1) & (2)

$$W_x^2 + W_y^2 = a_1^2 c^2\theta_1 + a_2^2 c^2(\theta_1 + \theta_2) + 2a_1a_2 c\theta_1 c(\theta_1 + \theta_2) + a^2 s^2\theta_1 + a_2^2 s^2(\theta_1 + \theta_2) + 2a_1a_2 s\theta_1 s(\theta_1 + \theta_2)$$

$$W_x^2 + W_y^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos(\theta_1 - (\theta_1 + \theta_2))$$

$$= a_1^2 + a_2^2 + 2a_1a_2 c\theta_2$$

$$c\theta_2 = \frac{W_x^2 + W_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$s\theta_2 = \pm\sqrt{1 - c^2\theta_2} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{s\theta_2}{c\theta_2}\right)$$

$$\Rightarrow \theta_2 = \operatorname{atan2}(s\theta_2, c\theta_2)$$

Calculating θ_1,

$W_x = a_1 c\theta_1 + a_2 c\theta_1 - a_2 s(\theta_1) s(\theta_2)$

$W_y = a_2 s\theta_1 + a_2 s(\theta_1) c(\theta_2) + a_2 c(\theta_1) s(\theta_2)$

$W_x = (a_1 + a_2 c\theta_2) c\theta_1 - a_2 s\theta_2 s\theta_1 — (5)$

$W_y = (a_1 + a_2 c(\theta_2)) s\theta_1 + a_2 s\theta_2 c\theta_1 — (6)$

Multiplying eqn 5 by $a_2 s\theta_2$ & eqn (6) by $(a_1 + a_2(\theta_1 + \theta_2))$

$W_x a_2 s\theta_2 = (a_1 + a_2 c\theta_2) a_2 s\theta_2 c\theta_1 - a_2^2 s^2\theta_2 s\theta_1 — (7)$

$W_y (a_1 + a_2 c\theta_2) = (a_1 + a_2 c\theta_2)^2 s\theta_1 + (a_1 + a_2 c\theta_2) a_2 s\theta_2 c\theta_1 — (8)$

Subtract (7) from (8)

$$W_y(a_1 + a_2 c\theta_2) - W_x a_2 s\theta_2 = (a_1^2 + a_2^2 c^2\theta_2 + 2a_1a_2 c\theta_2) s\theta_1 + a_2^2 s^2\theta_2 s\theta_1$$

$$= (a_1^2 + a_2^2 + 2a_1a_2c\theta_2)s\theta_1$$

$$s\theta_1 = \frac{w_y(a_1 + a_2c\theta_2) - w_x a_2 s\theta_2}{\Delta}$$

where $\Delta = a_1^2 + a_2^2 + 2a_1a_2c\theta_2 = w_x^2 + w_y^2$

$$c\theta_1 = \pm\sqrt{1 - s^2\theta_1}$$

$$\tan\theta_1 = \left(\frac{s\theta_1}{c\theta_1}\right) \Rightarrow \theta_1 = \text{atan2}(s\theta_1, c\theta_1)$$

$$\theta_3 = \psi - \theta_1 - \theta_2$$

12/11/24

Denavit - Hartenberg (DH) Parameters :

The DH parameters shows / gives the current configuration of the robot. Current configuration means its position and orientation.

There are 4 parameters in DH parameters.

(i) Joint offset (b or d),
(ii) Joint Angle (θ)
(iii) Link Length (a),
(iv) Twist Angle (α).

To find DH Parameters

1st Stage: Assign Frames

(a) First assign z-axis along the direction of motion.
(b) Assign the x-axis $\perp$ to & passing through z_{i-1} & z_i (i = Link no.)
(c) Assign y-axis using Right hand rule

Stage 2: Write DH Parameters :

	x_i & x_{i+1} @ z_i		z_i & z_{i+1} @ x_{i+1}	
i	b	θ	a	α
1	0	θ(J.V)	a_1	0
2	0	θ_2(J.V)	a_2	0

b_1 – dist b/w x_1 & x_2 @ z_1

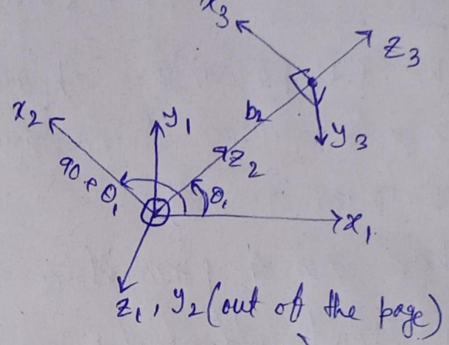
JV – joint variable

b_2 – dist. b/w x_2 & x_3 @ z_2

Revolute Prismatic

RP - manipulation

	x_i & x_{i+1} @ z_i		z_i & z_{i+1} @ x_{i+1}	
	b	θ	a	α
1	0	$90+\theta_1$(J.V)	0	90°
2	b_2(J.V)	0	0	0



RP manipulation:

	x_i & x_{i+1} @ z_i		z_i & z_{i+1} @ x_{i+1}	
	b	θ	a	α
1	0	θ_1(JV)	a_1	-90°
2	b_1(JV)	0°	0°	0°