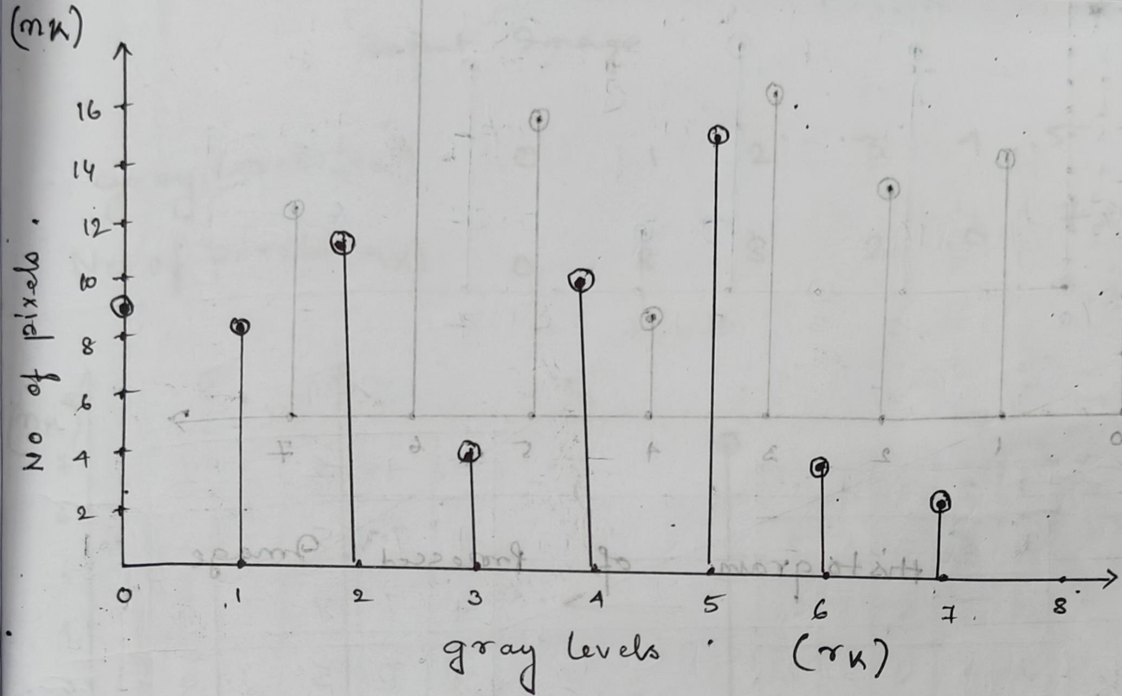
1. Histogram Equilization

Perform Histogram equalization for an 8x8 image shown below.

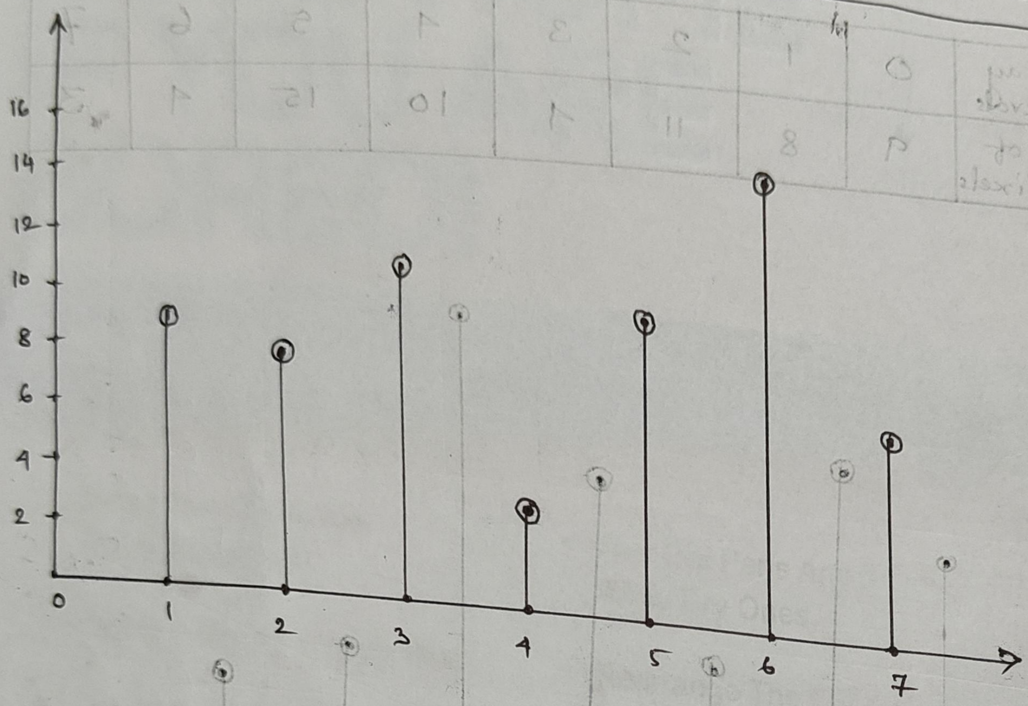
gray levels	0	1	2	3	4	5	6	7
No of Pixels	9	8	11	4	10	15	4	3



Histogram of Input Image.

Gray levels r_k	No of Pixels n_k	$P(r_k) = n_k/n$ (PDF)	S_k CDF	$S_k \times 7$	Histogram equalized level
0	9	0.141	0.141	0.987	1
1	8	0.125	0.266	1.862	2
2	11	0.172	0.438	3.066	3
3	4	0.0625	0.5005	3.5035	4
4	10	0.156	0.6565	4.5955	5
5	15	0.234	0.8905	6.2335	6
6	4	0.0625	0.953	6.671	7
7	3	0.047	1	1	7
	$n = 64$				

Gray levels	1	2	3	4	5	6	7
No of pixels	9	8	11	4	16	15	7



Histogram of Processed Image

② Perform Histogram equalization for the following image.

$f(x,y)$

1	2	1	1	1
2	5	3	5	2
2	5	5	5	2
2	5	3	5	2
1	1	1	2	1

Input Image

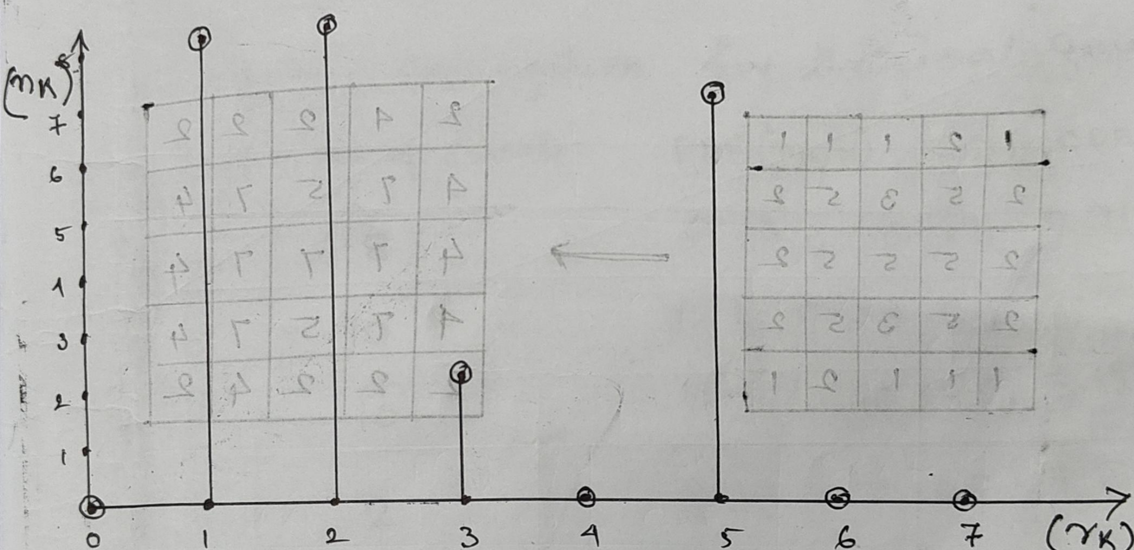
Max value = 5

5 is of 3bit

$\therefore 2^3 = 8$

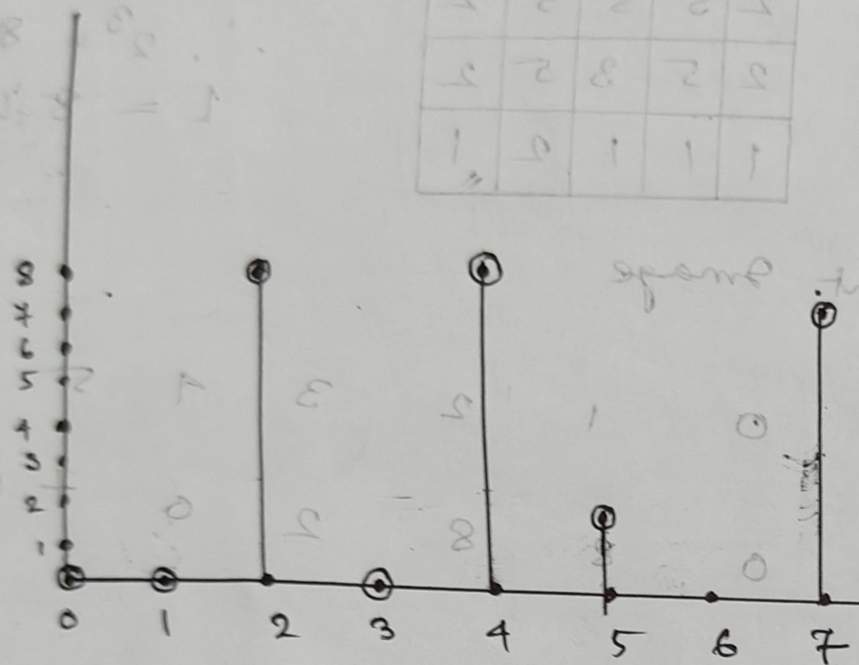
$L = 8 \quad L-1=7$

gray levels (r_K)	0	1	2	3	4	5	6	7
No of pixels (n_K)	0	8	8	2	0	7	0	0



graylevels r_K	No of pixels (n_K)	$P(K) = n_K/n$ (PDF)	SK CDF	SK×7	Histogram Equlized level
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1	7	7
6	0	0	1	7	7
7	0 $n=25$	0	1	7	7

gray levels	0	2	4	5
No of pixels	0	8	8	2



1	2	1	1	1
2	5	3	5	2
2	5	5	5	2
2	5	3	5	2
1	1	1	2	1

$\longrightarrow$

2	4	2	2	2
4	7	5	7	4
4	7	7	7	4
4	7	5	7	4
2	2	2	4	2

Histogram Specification / Matching.

Perform the Histogram Specification

Original Image

gray level	0	1	2	3	4	5	6	7
No of Pixel	8	10	10	2	12	16	4	2

Desired Image.

gray level	0	1	2	3	4	5	6	7
No of Pixel	0	0	0	0	20	20	16	8

Histogram equilization for Original Image

gray level	No of Pixels	PDF (n_k/N)	CDF	CDF×L-1	R
0	8	0.13	0.13	0.91	1
1	10	0.16	0.29	2.03	2
2	10	0.16	0.45	3.15	3
3	2	0.03	0.48	3.36	3
4	12	0.18	0.66	4.62	5
5	16	0.25	0.91	6.37	6
6	4	0.06	0.97	6.79	7
7	2	0.03	1.0	7	7

Histogram equilization for Target Image

gray level	No of Pixel	PDF (nk/N)	CDF	CDF×L-1	Sk	Round off
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	20	0.31	0.31	2.17	2	2
5	20	0.31	0.62	4.34	4	4
6	16	0.25	0.87	6.09	6	6
7	8	0.13	1.0	7	7	7

Modified Image

graylevel	0	1	2	3	4	5	6	7
No of Pixel	0	0	0	0	18	12	28	6

Mapping

gray scale	H	S	Map
0	1	0	4
1	2	0	4
2	3	0	5
3	3	0	5
4	5	2	6
5	6	4	6
6	7	6	7
7	7	7	7

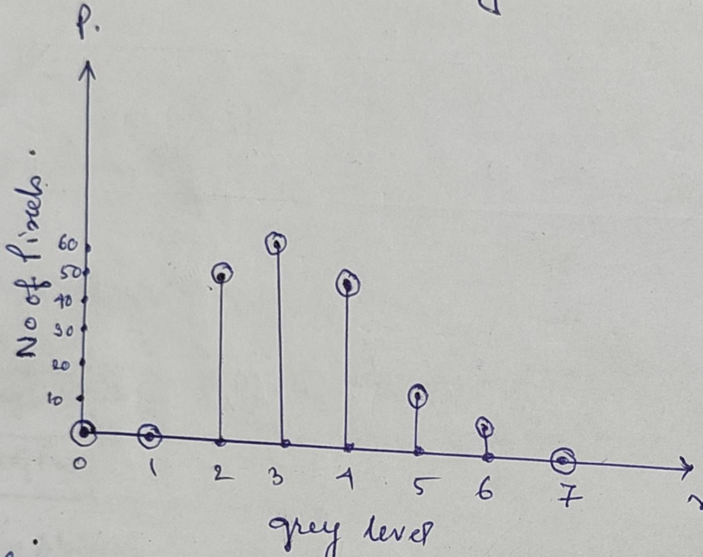
Modified Image

Gray scale	0	1	2	3	4	5	6	7
No of Pixel	0	0	0	0	18	12	28	6

Histogram Streching / contrast streching
(To make high constrast).

Grey Level	0	1	2	3	4	5	6	7
No of Pixels	0	0	50	60	50	20	10	0

Perform Histogram streching such that the new image has dynamic range [0-7]



Slope:

$$S = \left(\frac{S_{max} - S_{min}}{r_{max} - r_{min}}\right)(r - r_{min}) + S_{min}$$

$r_{min} = 2$

$r_{max} = 6.$

$S_{min} = 0$

$S_{max} = 7.$

$$S = \frac{7-0}{6-2}(r-2)$$

$$S = \frac{7}{4}(r-2)$$

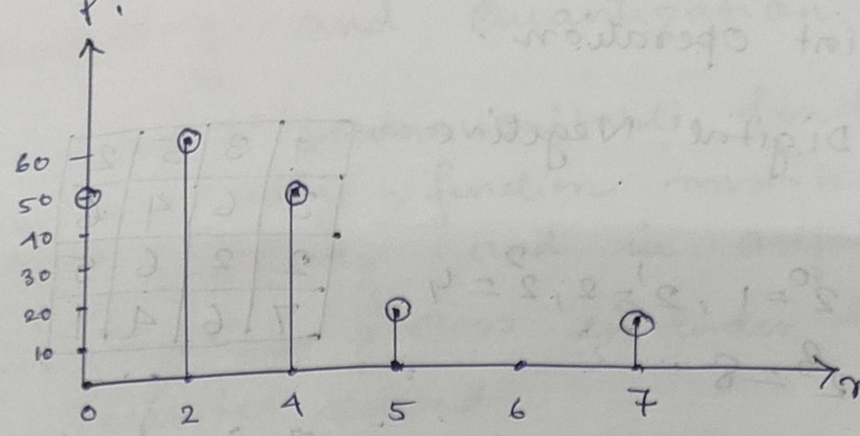
when $r = 2$ $S = 0.$

when $r = 3$ $S = 1.8 = 2$

when $r = 4$ $S = 3.5 = 4$

when $r = 5$ $S = 5.3 = 5$

when $r = 6$ $S = 7.$



② Apply contrast streching on 3bit grey level image of size 4×4.

2	1	2	1
4	5	5	6
3	2	1	4
6	2	1	6

Point operation.

1) Digital Negetive

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$2^0 = 1,\ 2^1 = 2,\ 2^2 = 4$

$2^3 = 8$.

Max pixel value is 7 ie 3 bit

$\therefore\ 2^3 = 8$ levels $\therefore\ n = 3$

$L = 2^3 = 8$

$s = (L-1) - r$

$= 8 - 1 - r$

$= 7 - r$.

If $r = 0$ $\quad s = 7 - 0 = 7$

$r = 1$. $\quad s = 7 - 1 = 6$.

$r = 2$ $\quad s = 7 - 2 = 5$.

$\vdots \qquad \vdots$

$r = 7$ $\quad s = 7 - 7 = 0$

2) Thresholding with $T = 4$.

$L = 8$

$L - 1 = 7$.

$$s = \begin{cases} L-1 = 7 & ;\ r \geq 4 \\ 0 & ;\ r < 4 \end{cases}$$

If $r = 0, 1, 2, 3 \rightarrow s = 0$

$r = 4, 5, 6, 7 \rightarrow s = 7$.

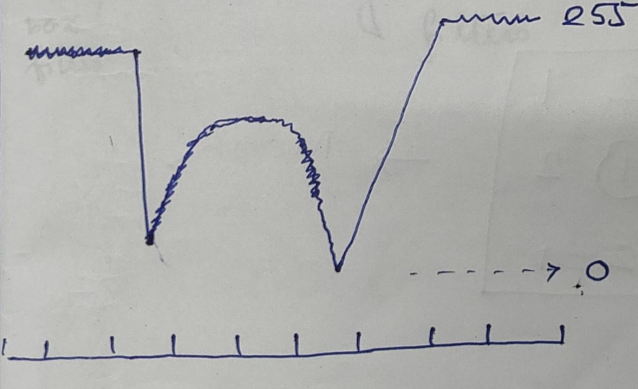
Sampling and Quantization.

In order to become suitable for digital processing an image function must be digitized both spatially and in amplitude.
This digitization process includes two main processes called

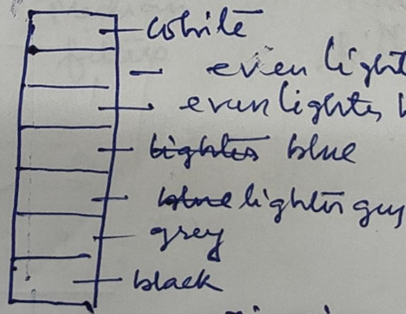
1) Sampling :- Digitizing the coordinate value is called sampling.

2) Quantization :- Digitizing the amplitude value is called as quantization.

0 black - low Intensity
grey - slightly higher
blue - Medium intensity
light blue - High Intensity
255 white :- very high Intensity.



Sampling



Quantization.

Smoothing Spatial filters

Smoothing spatial filters are used for blurring and noise reduction.

Blurring is used in preprocessing task such as removal of small details from an imag prior to (Large) object extraction.

Noise reduction can be accomplished by blurring with a linear filter and also by non linear filter.

Smoothing Spatial filters

- Linear filter
  - Mean/box filters
  - Weighted Avg filter
  - gausian filters
- Non-linear filters (Order-statistic filters)
  - Median filters
  - Max filters
  - Min filters

Smoothing linear filters:

They are also known as averaging filters (or) low pass filters as they are simply the average of the pixels contained in the neighbourshood of the filter mask.

The process results in an image with reduced sharp transitions in intensity which ultimately leads to noise reduction.

1. Box filter — all the coefficient are equal

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \text{Mask.}$$

3×3

2. Weighted average — give more (less) weight to pixel near (away from) the output location.

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \rightarrow \text{Mask.}$$

3. Gaussian filter — The weights are samples of 2D Gaussian filter.

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\frac{1}{16} \times \boxed{\begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{matrix}}_{3\times 3} \rightarrow \text{Mask}$$

- used to blur edges and reduce contrast
- similar to median filter but is faster.

Non linear filters (Order statistical filters).

Their response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the central pixel with the value determined by the ranking result.

1. Median filter :- find the median of the pixel value.

2. Min filter :- find the minimum of all the pixel value.

3) Max filter :- find the maximum of all the pixel value.

Consider the Image below and calculate the output of the pixel (2,2) if Smoothing is done using 3×3 neighbourhood using all the filters below.

a) Box / Mean filter
b) Weighted Avg filter
c) Median filter
d) Min filter
e) Max filter

1	8	8	0	7
4	7	9	5	7
5	4	6	8	6
4	2	0	1	5
0	1	0	2	0

a) Box filter

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{9} \times [7+9+5+4+6+8+2+0+1]$$

$$= \frac{1}{9} \times 42 = 4.66 \simeq 5$$

b) Weighted Average filter:

$$\text{Mask} \quad \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \frac{1}{16}\left[7\times1 + 9\times2 + 5\times1 + 4\times2 + 4\times2 + 8\times2 + 2\times1 + 0\times2 + 1\times1 \right]$$

$$= \frac{1}{16}[81] = 5.06 \simeq 5$$

c) Median filter
Write all the pixel in ascending order: $\cancel{0}, \cancel{1}, \cancel{2}, \cancel{4}, 5, \cancel{6}, \cancel{7}, \cancel{8}, 9$.

Median = 5

$\therefore$ 5 will be replaced with 6.

d) Min filter = 0.

e) Max filter = 9.

Fundemental of Spatial filtering.

The name filter is borrowed from frequency domain processing. It bassically refers to accepting (pasing) or rejecting certain frequency components.

Ex:- A filter that passes low frequeny is called as Low pass filter.

We can acomplish a similar smoothing directly on the image itself using spatial filters. also called (mask, templates and windows).

Convolution:

Let I = {0,0, 1, 0,0} be an image. Using the mask K = {3,2,8}, perform the convolution

I = {0,0,1,0,0}   K = {3,2,8}

1) zero padding process convolution

In convolution process we have to rotate the kernal by 180°.

8 2 3   8 2 3
0 0 0 0 1 0 0 0 0

ii) Initial position

Template

8 2 3

0 0 0 0 1 0 0 0 0

0

output is 0 located at centre pixel.

$8 \times 0 + 2 \times 0 + 3 \times 0 = 0$

iii) Position after one shift.

Template is shifted by one bit.

8 2 3

0 0 0 0 1 0 0 0 0

0 0

output is 0.

$8 \times 0 + 2 \times 0 + 3 \times 0 = 0$

iv) Position after 2 shifts.

8 2 3

0 0 0 0 1 0 0 0 0

0 0 3

Output produced is 3.

$(8 \times 0) + (2 \times 0) + (3 \times 1) = 3$

v) Position after 3 shifts

8 2 3

0 0 0 0 1 0 0 0 0

0 0 3 2

Output produced is 2.

$2 \times 1 + 3 \times 0 = 2$

vi) Position after 4 shifts

Template is shifted again

8 2 3
0 0 0 0 1 0 0 0 0 $8\times1+0+0 = 8$
0 0 3 2 8 0

The output produced is 8.

vii) Position after 5 shifts

Template is shifted again

8 2 3
0 0 0 0 1 0 0 0 0
0 0 3 2 8 0

o/p produced is 0.

viii) Correlation:

Let $I = \{0, 0, 1, 0, 0\}$ be an image using the mask $k = \{3, 2, 8\}$ perform correlation.

$I = \{0, 0, 1, 0, 0\}$ $k = \{3, 2, 8\}$

1) Zero padding process for correlation

3 2 8 3 2 8
0 0 0 0 1 0 0 0 0

ii) Initial Position Template.

3 2 8
0 0 0 0 1 0 0 0 0 0

0

Output produced is 0.

iii) Position after 1 shift.

3 2 8
0 0 0 0 1 0 0 0 0

0 0

O/P is 0.

iv) Position after 2 shifts

3 2 8
0 0 0 0 1 0 0 0 0

0 0 8

The O/P is = 8.

v) Position after 3 bits.

3 2 8
0 0 0 0 1 0 0 0 0

0 0 8 2

O/P produced is 2.

vi) Position after 4 shifts.

$$\begin{array}{cccccccccc} & & & & & 3 & 2 & 8 & & \\ & & & & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & & & & & & & \\ & 0 & 0 & 8 & 2 & 3 & & & & \end{array}$$

o/p produced is 3.

vii) Position after 5 shifts

$$\begin{array}{cccccccccc} & & & & & 3 & 2 & 8 & & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 8 & 2 & 3 & 0 & & & \end{array}$$

o/p produced is 0.

viii) final position template.

$$\begin{array}{cccccccccc} & & & & & & 3 & 2 & 8 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 8 & 2 & 3 & 0 & 0 & & \end{array}$$

o/p produced is 0. Further shifting exceeds the range.

So in the final position the o/p produceds is {0 0 8 2 3 0 0}

* Let $I = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ be an image

and $K = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ be the kernal.

Perform convolution and correlation.

1) For convolution rotate the kernal 180^0

$$\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$$
$\downarrow$
$$\begin{matrix} 3 & 4 \\ 1 & 2 \end{matrix}$$
$\downarrow$
$$\begin{matrix} 4 & 3 \\ 2 & 1 \end{matrix}$$

$\therefore K' = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

$$\begin{matrix} 0_4 & 0_3 & 0 & 0 \\ 0_2 & 3_1 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \qquad K' = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$\downarrow$

$4 \times 0 + 0 \times 3 + 0 \times 2 + 3 \times 1 = 3$.

$$\begin{matrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

b)

$$\begin{array}{cccc} 3 & 0_4 & 0_3 & 0 \\ 0 & 3_2 & 3_1 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

$4 \times 0 + 3 \times 0 + 2 \times 3 + 3 \times 1$

$6 + 3 = 9.$

$\Downarrow$

$$\begin{array}{cccc} 3 & 9 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

c)

$$\begin{array}{cccc} 3 & 9 & 0_4 & 0_3 \\ 0 & 3 & 3_2 & 0_1 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

Ans $= 0 + 0 + 0 + 6$

$\Downarrow$

$$\begin{array}{cccc} 3 & 9 & 6 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

d)

$3\times3 + 1\times3 = 12$

$$\begin{matrix} 3 & 4 & 6 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 3 & 4 & 6 & 0 \\ 12 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

e)

$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

f)

$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

g)

$$\begin{matrix} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 3 & 1 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

h)

$$\begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \Rightarrow \begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

i

$$\begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \rightarrow \boxed{\begin{array}{cccc} 3 & 9 & 6 & 0 \\ 12 & 30 & 138 & 0 \\ 9 & 21 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{array}}$$

For Correlation perform the same steps without rotating the Kernal by 180°.

Logarithmic transform and Power Law transform.

1. Logarithmic transformation

$$s = c \log (1+r)$$

where c is a constant and is assumed that $r \geq 0$

S represents the pixel values of the output image and r reprents the pixel values of input image and c is a constant and it is assumed that

$r \geq 0$.

110	120	90
91	94	98
90	91	99

i) $c = 1$

ii) $c = \frac{L}{\log_{10}(1+L)}$

$2^7 = 128$

$L = 128$

$n = 7$.

r	s
110	$s = 1 \cdot \log_{10}(1+110) = \log(111) = 2.04$
120	$s = 1 \cdot \log_{10}(1+120) = \log(121) = 2.08$
90	$s = 1 \cdot \log_{10}(1+90) = \log(91) = 1.95$
91	$s = 1 \cdot \log_{10}(1+91) = \log(92) = 1.96$
94	$s = 1 \cdot \log_{10}(1+94) = \log(95) = 1.97$
98	$s = 1 \cdot \log_{10}(1+98) = \log(99) = 1.99$
99	$s = 1 \cdot \log_{10}(1+99) = \log(100) = 2$

$$\begin{bmatrix} 110 & 120 & 90 \\ 91 & 94 & 98 \\ 90 & 91 & 99 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Input Image   o/p Image

ii) $$c = \frac{L}{\log_{10}(1+L)} = \frac{128}{\log_{10}(1+128)} = \frac{128}{\log_{10}(129)}$$

$$= 60.66$$

$$\Rightarrow 61$$

110	120	90
91	94	98
90	91	99

o/p image.

$\Rightarrow$

61×2.04	61×2.08	61×1.95
61×1.96	61×1.97	61×1.99
61×1.95	61×1.96	61×2

124.44	126.88	118.95
119.56	120.17	121.39
118.95	119.56	122

=

124	127	119
120	120	121
119	120	122

b) Power law transformation.

$$s = c r^{\gamma}$$

where c and γ are positive constant

Given $c = 1$ and $\gamma = 0.2$ Calculate for

110	120	90
91	94	98
90	91	99

Input image.

$$s = c r^{\gamma}$$

$$= 1 \cdot r^{0.2}$$

$$= r^{0.2}$$

r	s			
110	$s = 1\cdot(110)^{0.2}$	$= 2.56$	$\approx$	3
120	$s = 1\cdot(120)^{0.2}$	$= 2.60$	$\approx$	3
90	$s = (90)^{0.2}$	$= 2.45$	$\approx$	2
91	$s = (91)^{0.2}$	$= 2.46$	$\approx$	2
94	$s = (94)^{0.2}$	$= 2.48$	$\approx$	2
98	$s = (98)^{0.2}$	$= 2.50$	$\approx$	3
99	$s = (99)^{0.2}$	$= 2.50$	$\approx$	3

$$\begin{bmatrix} 3 & 3 & 2 \\ 2 & 2 & 3 \\ 2 & 2 & 3 \end{bmatrix}$$

O/P image.

sharpening Spatial filter

- The principal objective of sharpening is to highlight transitions in intensity. Applications include electronic printing, medical imaging, industrial inspection

Blurring - pixel averaging

reversing Sharpening - Spatial differentiation.

The strength of the response of a derivative operator is proportional to the degree of intensity discontinuity of the image at the point at which the operator is applied.

Therefore image differentiation enhance edges and other discontinuities (such as noise) and deemphasizes areas which have slowly varying intensities.

Foundation of sharpening filters

1) First order derivative filters

1) 1st order derivative of a one dimensional function $f(x)$

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

2) Second-order derivative of a one dimensional function $f(x)$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Laplacian filter :-

- It highlights gray level discontinuities in an image.
- It deemphasizes regions with slowly varying gray levels.

Formula

$$\boxed{\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y}}$$

where $\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y$$
$$- 4f(x, y).$$

Laplacian mask :

$$\begin{bmatrix} f(x-1,y-1) & f(x,y-1) & f(x+1,y-1) \\ f(x-1,y) & f(x,y) & f(x+1,y) \\ f(x-1,y+1) & f(x,y+1) & f(x+1,y+1) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Apply laplacian filter on the given image on the center pixel.

$$\begin{bmatrix} 8 & 5 & 4 \\ 0 & 6 & 2 \\ 1 & 3 & 7 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

I/P Image Mask

$= (8\times 0) + (5\times 1) + (4\times 0) + (0\times 1)$

$+ (6\times -4) + (2\times 1) + (1\times 0)$

$+ (3\times 1) + (7\times 0) = -14$

$= -14$

Apply laplacian filter on the given image by a 3×3 mask.

$$\begin{bmatrix} 10 & 20 & 20 & 10 & 10 \\ 20 & 20 & 20 & 10 & 10 \\ 10 & 10 & 10 & 20 & 20 \\ 10 & 10 & 20 & 50 & 20 \\ 20 & 20 & 10 & 10 & 10 \end{bmatrix} \text{ by } \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Image Segmentation.

1) Thresholding : It is carried out with the assumption that the range of intensity levels carried out by objects of interest is different from background.

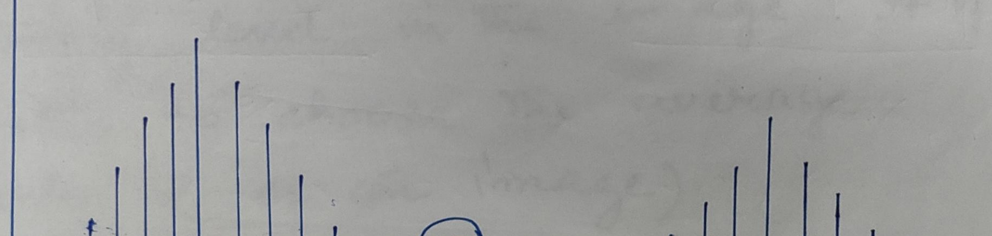
2) Steps :

a) A threshold T is selected

b) Any point (x,y) in the image at which $f(x,y) > T$ is called an object point.

c) The segmented image denoted by $g(x,y)$ given by

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \\ 0 & \text{if } f(x,y) \leq T. \end{cases}$$



single Threshold

Types of Thresholding

1) Global Thresholding

T is constant.

2) variable Thresholding

T changes over an image.

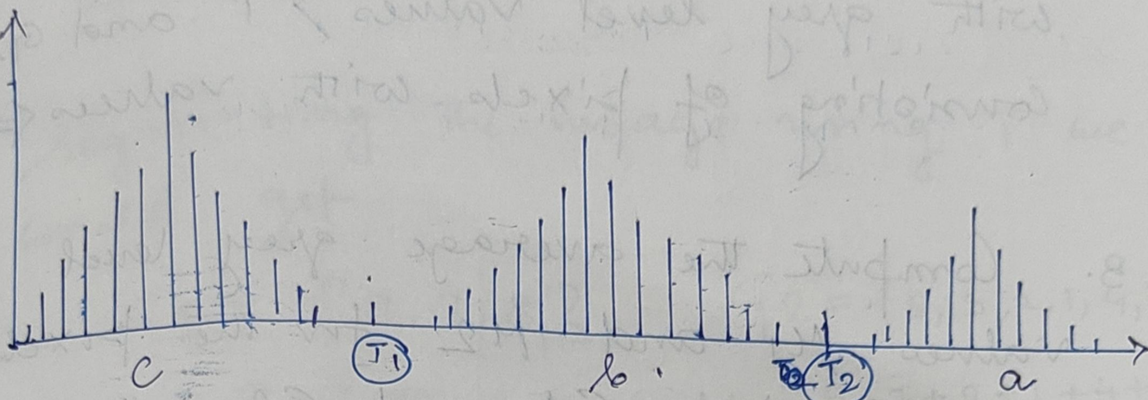
3) local or regional thresholding

In variable thresholding if the value of T at any point (x, y) in an image depends on the properties of a neighbourhood of (x, y).

4) Dynamic or adaptive Thresholding

In variable thresholding if the value of T depends on spatial coordinates (x, y)

A histogram with three dominant modes (two types of light objects on a dark background)



$$g(x,y) = \begin{cases} a & \text{if } f(x,y) > T_2 \\ b & \text{if } T_1 < f(x,y) \le T_2 \\ c & \text{if } f(x,y) \le T_1 \end{cases}$$

Procedure for global thresholding

1) Select an initial estimate for T (This value should be greater than the minimum and less than the maximum entensity level in the image. It is better to choose the average entensity of an image)

2. Segment the image using T this will produce 2 groups of pixels : G1 consisting of all pixels with grey level values > T and G2 consisting of pixels with values <=T

3. Compute the average grey level values μ_1 and μ_2 for the pixels in regions G1 and G2.

4. Compute a new threshold value

$$T = \frac{1}{2}(\mu_1 + \mu_2)$$

5. Repeat step 2 through 4 until the difference in T in successive iterations is smaller than a predefined parameter T_0

Eg:-

5	3	9
2	1	7
8	4	2

Let $T = \frac{5+3+9+2+1+7+8+4+2}{9}$

$T = \frac{41}{9} = 4.55 \simeq 5$

Segmenting the image using T we would get

$G1 = \{9, 7, 8\}$ $G2 = \{5, 3, 2, 1, 4, 2\}$

$M_1 = \frac{9+7+8}{3}$ $M_2 = \frac{5+3+2+1+4+2}{6}$

$= \frac{24}{3} = 8$ $= \frac{17}{6} = 2.83 \simeq 3$

$\therefore T = \frac{1}{2}(8+3) = \frac{1}{2} \times 11 = 5.5 \simeq 5$

Procedure for Adaptive thresholding

1. Convolve the image with a suitable statistical operator ie mean or median

2. Subtract the original from the convolved image

3. Threshold the difference image with C

4. Invert the threshold image.

Region Growing.

1. Apply region growing on the following image with initial point at (2,2) and threshold value as 2. by applying 4-connectivity.

	0	1	2	3
0	0	1	2	0
1	2	5	6	1
2	1	4	7	3
3	0	2	5	1

Solution:-

Condition

1. Absolute difference <= 2
2. 4 way connectivity.
3. Seed point = 7 ie position (2,2).

T = 2

	0	1	2	3
0	0	1	2	0
1	2	5^a	6^a	1
2	1	4	7^a	3
3	0	2	5^a	1

7 - 6 = 1 ✓

7 - 4 = 3 ✗

7 - 5 = 2 ✓

The segmented region is shown in the following figure.

	0	1	2	3
0	0	1	2	0
1	2	5^a	6^a	1
2	1	4	7^a	3
3	0	2	5^a	1

$\Downarrow$

Make the pixel marked as a by 1 otherwise 0.

0	0	0	0
0	1	1	0
0	0	1	0
0	0	1	0

(2) Apply region growing on the following image with seed point as and threshold value as 3.

5	6	6	7	6	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4

Seed point = 6. $T = 3$.

Absolute difference $<= 3$.

As no connectivity is mentioned then take 8 way connectivity.

5^a	6^a	6^a	7^a	6^a	7^a	6^a	6^a
6^a	7^a	6^a	7^a	5^a	5^a	4^a	7^a
6^a	6^a	4^a	4^a	3	2	5^a	6^a
5^a	4^a	5^a	4	2	3	4^a	6^a
0	3	2	3	3	2	4^a	7^a
0	0	0	0	2	2	5^a	6^a
1	1	0	1	0	3	4^a	4^a
1	0	1	0	2	3	5	4

Segmented Output Image:

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	0	0	1	1
1	1	1	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0

Region Split and Merge.

Apply split and merge on the following image with threshold value equal to 3.

5	6	6	6	7	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4

Ans. Condition : Absolute difference $<= 3$

Max value = 7 Min value = 0

$|7-0| = 7$ which is greater than 3

Therefore we will split the region into 4 sub regions. (splitting takes place when condition does not satisfy)

					b1		b2		
	5	6	6	6	7	7	6	6	
(a)	6	7	6	7	5	5	4	7	(b)
	6	6	4	4	3	2	5	6	b3
	5	4	5	4	2 (b4)	3	4	6	
	0	3	2	3	3	2	4	7	
	0	0	0	0	2 (d1)	2	5 (d2)	6	(d)
(c)	1	1	0	1	0	3	4	4	
	1	0	1	0	2 (d3)	3	5 (d4)	4	

In region (a)

Max = 7 Min = 4

$|7-4| = 3$ which is equal to threshold

Therefore, no need to split.

In Region (b) max value = 7 and min value = 2

$|7-2| = 5$ which is greater than 3. Therefore we will split the region into 4 equal sub parts.

In Region (c)

Max value = 3 min value = 0

$|3-0| = 3$ It satisfies the Condition so no need to split.

In Region (d)

Max value = 7 and min = 0

$|7-0| = 7$ which is greater than 3 threshold so The region will ~~split~~ split into 4 sub regions

~~In region (b)~~

~~Max value~~

Further we will check all the subregions since all of them are <= 3 no further spliting is required.

Merging

Check adjacent regions, if they fall within the threshold then merge.

Consider region a and b_1

Max value = 7, Min value = 4

$|7-4| = 3$ Then it less than or equal to threshold then merge.

Consider region ab_1 and b_2

Max = 7 Min = 4.

$|7-4| = 3$ ✓ Merge.

Consider the region $ab_1 b_2$ and b_4

Max = 7 Min = 4.

$|7-4| = 3$ ✓ Merge

Consider the region $ab_1 b_2 b_4$ and d_2

Max = 7 Min = 4.

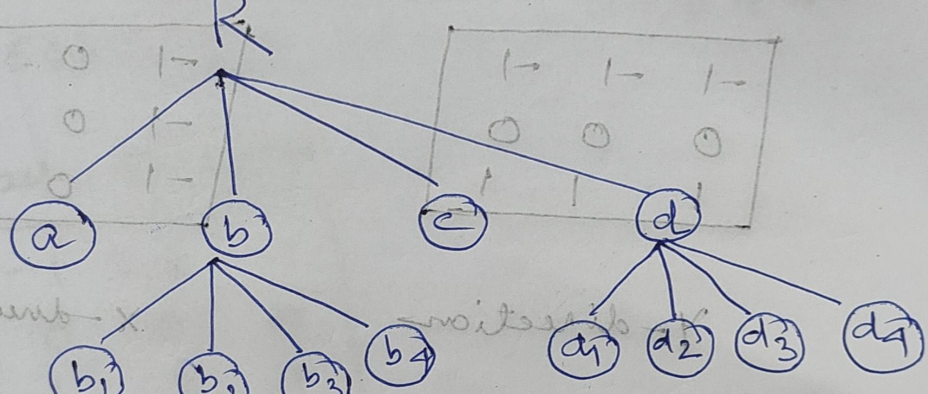
$|7-4| = 3$ ✓ Merge

Similarly merge $ab_1 b_2 b_4 d_2$ with d_4

Similarly merge c, d_1, b_3 and d_3

Final Segmentation

5	6	6	6	7	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4



Quad tree structure for splitting.

** Gradient Based Technique :-

(1) Roberts operator (Cross gradient)

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$G = \sqrt{G_x^2 + G_y^2}$$

(2) Sobel Operator

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Y-direction X-direction

(3) Prewitt Operators

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Y-direction X-direction

$$G = \sqrt{G_x^2 + G_y^2}$$

Apply Roberts, Sobel, Prewitt operators in the pixel at (1,1) in the following image :-

50	50	100	100
50	50	100	100
50	50	100	100
50	50	100	100

Input image.

1) Roberts operator (use any of the mask for Roberts operator

Mask for Roberts operator

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 50 \times (-1) + 100 \times 1 = -50 + 100 = 50$

2) Sobel operator

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$= 50 \times (-1) + 50(-2) + 100(-1) + 50 \times 1 + 50 \times 2 + 100 \times 1$

$= -\cancel{50} - \cancel{100} - \cancel{100} + \cancel{50} + \cancel{100} + \cancel{100} = 0.$

3) Prewitt operator

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$= -1 \times (50 + 50 + 100) + 1\,(50 + 50 + 100)$

$= 0.$