IT24647

1. Solve each of the following sets of simultaneous congruences:

a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

Solution:

→ Total modulus $N = 3 \cdot 5 \cdot 7 = 105$

→ For modulus 3: $N_1 = 105/3 = 35$. Inverse of 35 mod 3 is $35 \equiv 2 \pmod 3$, and $2^{-1} \equiv 2 \pmod 3$ (since $2 \cdot 2 = 4 \equiv 1$).

contribution : $1 \cdot 35 \cdot 2 = 70$

→ For modulus 5: $N_2 = 105/5 = 21$. Inverse of 21 mod 5 is $21 \equiv 1 \pmod 5$, inverse $= 1$.

contribution : $2 \cdot 21 \cdot 1 = 42$

→ For modulus 7: $N_3 = 105/7 = 15$. Inverse of 15 mod 7 is $15 \equiv 1 \pmod 7$, inverse $= 1$.

Contribution : $3 \cdot 15 \cdot 1 = 45$

Sum $70 + 42 + 45 = 157$. Reduce mod 105: $157 \equiv 52 \bmod (105)$.

Answer : $x \equiv 52 \pmod{105}$. The smallest positive solution is $x = 52$.

IT24647

b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$

Solution:

→ $N = 11 \cdot 29 \cdot 31 = 9889$

→ $N_1 = 9889/11 = 899$. Inverse of 889 mod 11 : $899 \equiv 7 \pmod{11}$. Solve $7y \equiv 1 \pmod{11} \rightarrow y \equiv 8$

Contribution : $5 \cdot 899 \cdot 8 = 35960$

→ $N_2 = 9889/29 = 341$. $341 \equiv 20 \pmod{29}$. Inverse of 20 mod 29 is 3 because $20 \cdot 3 = 60 \equiv 2$ - not quite; correct inverse is 15 since $20 \cdot 15 = 300 \equiv 1 \pmod{29}$.

Contribution : $14 \cdot 341 \cdot 15 = 71610$

→ $N_3 = 9889/31 = 319$. $319 = 9 \pmod{31}$. Inverse of 9 mod 31 is 7 (since $9 \cdot 7 = 63 \equiv 1$).

contribution : 334495

Answer : $x \equiv 4944 \pmod{9889}$. The smallest positive solution is $x = 4944$.

IT24647

c) $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

Solution:

→ $N = 6 \cdot 11 \cdot 17 = 1122$

→ $N_1 = 1122/6 = 187$. Inverse of 187 mod 6 : $187 \equiv 1 \pmod 6$, inverse $= 1$.

Contribution: $5 \cdot 187 \cdot 1 = 935$

→ $N_2 = 1122/11 = 102$. $102 \equiv 3 \pmod{11}$. Inverse of 3 mod 11 is 4 (Since $3 \cdot 4 = 12 \equiv 1$).

Contribution : $4 \cdot 102 \cdot 4 = 1632$.

→ $N_3 = 1122/17 = 66$. $66 \equiv 15 \pmod{17}$. Inverse of 15 mod 17 is 8 (since $15 \cdot 8 = 120 \equiv 1$).

Contribution: $3 \cdot 66 \cdot 8 = 1584$

Sum $935 + 1632 + 1584 = 4151$. Reduce mod 1122: $4151 \bmod 1122 = 785$.

Answer: $x \equiv 785 \pmod{1122}$. The smallest positive solution is $x = 785$.