

Question : Is set of odd numbers with binary operations (+), i.e. $\langle O, + \rangle$ an abelian group?

Solution :

$$O = \{ \dots -3, -1, 1, 3, 5, 7, 9, 11, 13, 15 \dots \}$$

1. Closure: if $a, b \in O$ then $a+b \in O$

$$\text{let, } a = 3$$

$$b = 5$$

$$\text{the } 3+5 = 8 \text{ not } \in O$$

So, condition fails this requirement.

2. Identify element : 0 is even number, so in $e \in G$ is work out.

3. Associative : if $\forall a, b, c \in O$ then $a+(b+c) = (a+b)+c \in O$

$$\text{Let, } a = 3,$$

$$b = 5$$

$$c = -5$$

$$\text{then, } 3+(5-5) = (3+5)-5 = 3 \in O$$

So, this condition is satisfying.

4. Inverse Element : if $a, a' \in O$ and $(a+a') = e \in O$

$$\text{Let, } a = 3$$

$$a' = -3$$

$$\text{then, } 3-3 = 0 \in O$$

So, the set is not satisfying the inverse property.

5. Commutative: if $\forall a, b \in O$, then $(a+b) = b+a \in O$

$$\text{let, } a = 5 \\ b = 7$$

then,

$$5+7 = 7+5 = 12 \text{ not } \in O$$

So, this condition is satisfying.

Here, given odd numbers set is not an abelian, because it is not passing the all condition of abelian.