Advanced graph theory: Tutorial 1: CS60047 Autumn August 12, 2022

1. Show that a k-regular bipartite graph has a perfect matching.
2. Draw the 8-vertex 16-edge complement of the cube of 8 vertices of 12 edges.
3. Draw the Petersen graph P_{10} of $\binom{5}{2}$ vertices, where the vertices correspond to the 2-subsets of the first 5 natural numbers, and the edges correspond to disjoint pairs of such 2-subsets.
4. Find the size $\alpha(P_{10})$ of the largest independent set in the Petersen graph, and the smallest vertex cover of size $\beta(P_{10})$.
5. Show that a graph with girth 5 and minimum vertex degree $\delta \geq k$ has at least $k^2 + 1$ vertices.
6. For $k = 2$ and $k = 3$ find examples of graphs of girth 5 and minimum vertex degree k with exactly $k^2 + 1$ vertices.
7. Show that the Petersen graph is triangle-free.
8. Count the number of 5-cycles in the Petersen graph.
9. Show that all longest paths in a tree pass through a common vertex.
10. Show that this is not generally true for general graphs by giving an example.
11. Show that there are $n-k$ distinct paths of length k in a tree of diameter $2k - 3$.