Numerical Methods Mini Project

NUMERICALLY SOLVING TEMPERATURE DISTRIBUTION INSIDE FLOW THROUGH PIPE

WITH CONSTANT WALL TEMPERATURE

BY FINITE DIFFERENCE METHOD

BY

KONDAPALLY SAIKIRAN REDDY(2021PTH1060)

IMPORTANT ASSUMPTIONS

- Flow is incompressible
- In a fully developed flow field inside a pipe Velocity profile is azimuthally do not change and also constant along Z_cordinate.
- Temperature field is azimuthally symmetric and varies with respect to radius and along z_cordinate.
- Velocity profile is function of r_cordinate and Temperature profile is function of both Z and Radial cordinate.
- Also Velocity Profile is initially known.
- Therefore The convection-diffusion equation is a Two dimensional parabolic partial differential equation
- Energy equation is solved assuming all variables(Tmperature,velocity) are axisymmetric

Type of partial differential equation:

Energy equation is solved with radial coordinate system.

The axial diffusion term is neglected$(\frac{\partial^2 T}{\partial z^2})$

The Energy equation after reduction of terms:

$$V_z = 2V\_avg(1 - \frac{r^2}{R^2})$$

$$Vz\ \frac{\partial T}{\partial z} = \frac{1}{r}\ \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}$$

The above Energy equation is parabolic in nature

To solve the above equation we have chosen finite difference scheme consisting of combination of implicit and explicit methods.[**Crank Nicolson Method**]

Crank Nicolson method is widely used to solve Heat equations and its an second accurate method.

In our project,this method is implicit in z_cordinate

$\therefore$

j$\rightarrow radial(r)$

$i \rightarrow along\ pipe's\ length\ (z)$

$$\frac{\partial T}{\partial z}_{i+0.5} = \frac{T[i+1,j]-T[i,j]}{\Delta z}$$

$$\frac{\partial T}{\partial r}_{i+0.5} = \frac{T[i+1,j+1]-T[i+1,j-1]}{2*\Delta r} + \frac{T[i,j+1]-T[i,j-1]}{2*\Delta r}$$

$$\frac{\partial^2 T}{\partial r^2}_{i+0.5} = \frac{T[i+1,j+1]-2T[i+1,j]+T[i+1,j-1]}{\Delta r^2} + \frac{T[i,j+1]-2T[i,j]+T[i,j-1]}{\Delta r^2}$$

Booundary Conditions:

BC1 At r=0:

j=0 , for all i

$$\frac{\partial T}{\partial r}_{r=0} = 0 \;\; ; \quad T[i,j+1]=T[i,j-1]$$

BC2 At r=R

$j_{max}=\frac{R}{\Delta r}$, for all i

T[i, j_{max}]= T_w=wall temperature

BC3 At z=0

i=0 , for all j

T[0,j]=T_in=inlet temperature of fluid

<u>**Finite Difference Equations:**</u>

At r=0; j=0;

$$\lim_{r \to 0} \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\partial^2 T}{\partial r^2}_{r=0}$$

The energy equation reduces to:

$$(Vz\ \frac{\partial T}{\partial z})_{r=0} = 2 * \frac{\partial^2 T}{\partial r^2}_{r=0}$$

∴ **The finite difference equation is** (at j=0,for all i)

T[i+1,j+1]*(-2*K)+ **T[i+1,j]***(1+2*K)= **T[i,j]***(1-2*K)+ **T[i,j+1]***(2*K)

K=$\alpha\Delta z/2V\_avg\Delta r^2$

For r>0,j>0;for all i

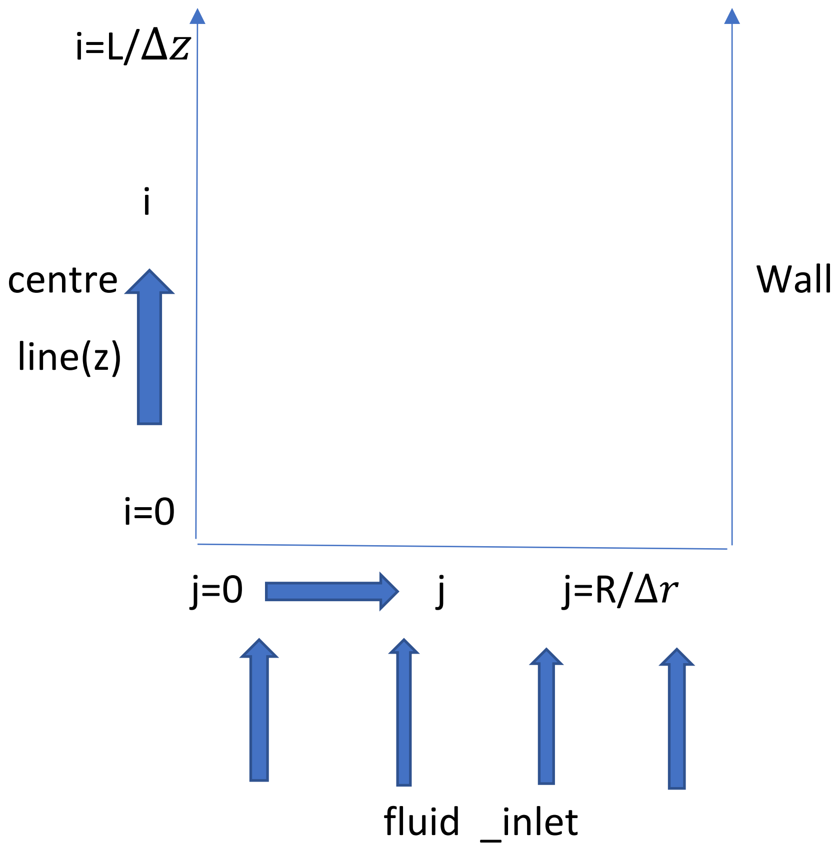
The Finite difference equation becomes:

K1(j)= $\dfrac{2*\text{v\_avg}*(1-\frac{(j*\Delta r)^2}{R^2})}{\Delta z}$

K2(j)= $\dfrac{\alpha}{2*j*\Delta r^2}$

K3(j)= $\dfrac{\alpha}{2*\Delta r^2}$

T[i+1,j+1]*(-K2(j)-K3(j))+**T[i+1,j]***(K1(j))+2*K3(j))+**T[i+1,j-1]***(K2(j)-K3(j)) **=** **T[i,j+1]***(K2(j)+K3(j))+**T[i,j]***(K1(j))-2*K3(j))+**T[i,j-1]***(K3(j)-K2(j))



INITIALIZATION:

Thermal Diffusivity of water is assumed to constant since temperature change is moderate, incompressible flow.

Thermal Diffusivity(α)=0.168*10^{-6}

Radius of Pipe(R)=0.5m

Length of pipe(Z)=500.2m

Average_velocity of fluid(V_avg)=0.001 m/s

Δr =0.01 (radial stepsize)

Δz=0.2m (axial stepsize)

T_W=$\mathbf{50^0 C}$ (Wall Temperature)

T_in=$\mathbf{20^0}C$ (Fluid inlet Temperature)

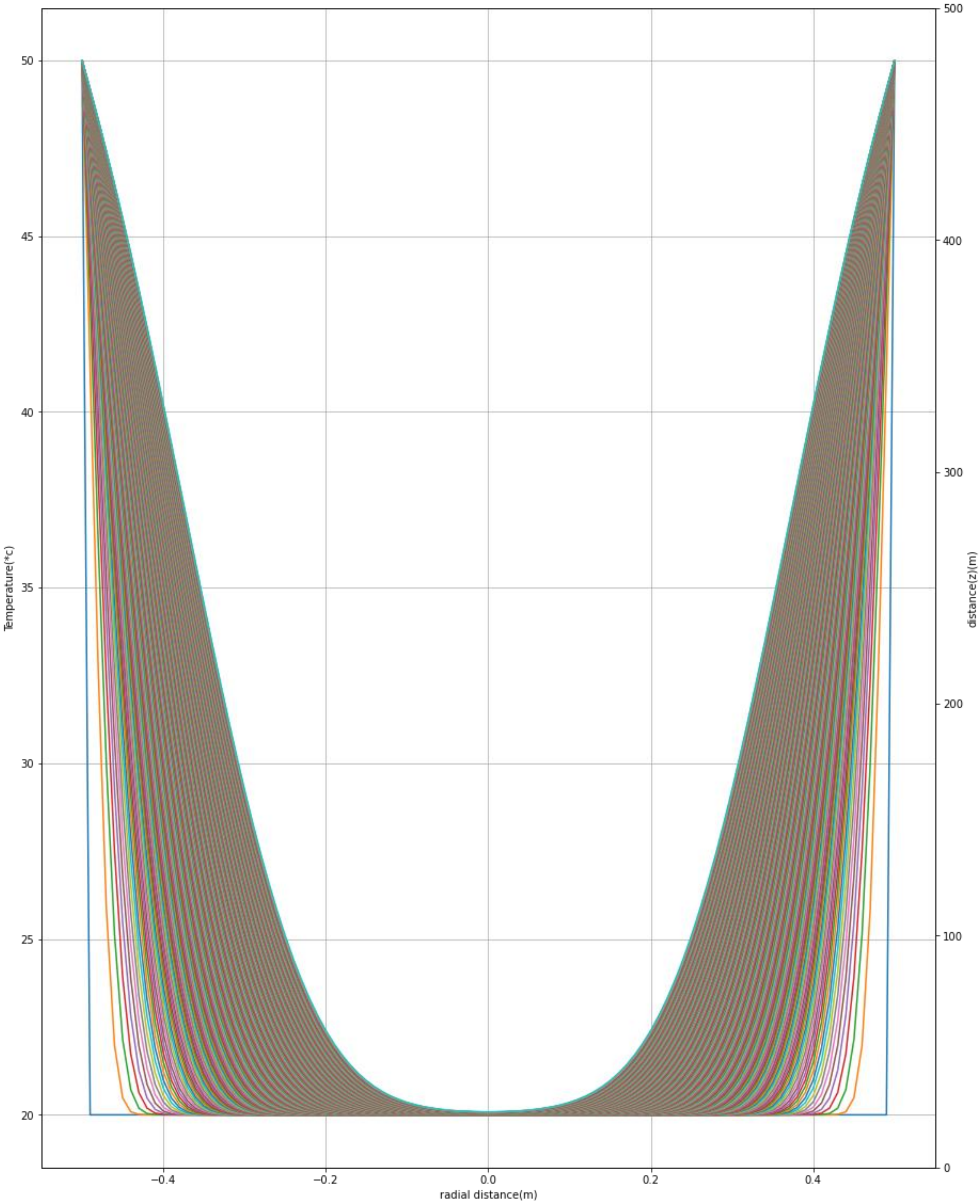
<u>RESULTS:</u>

From the below **Temperature contours** assume that the fluid is coming at the top of figure and Leaving at bottom of it

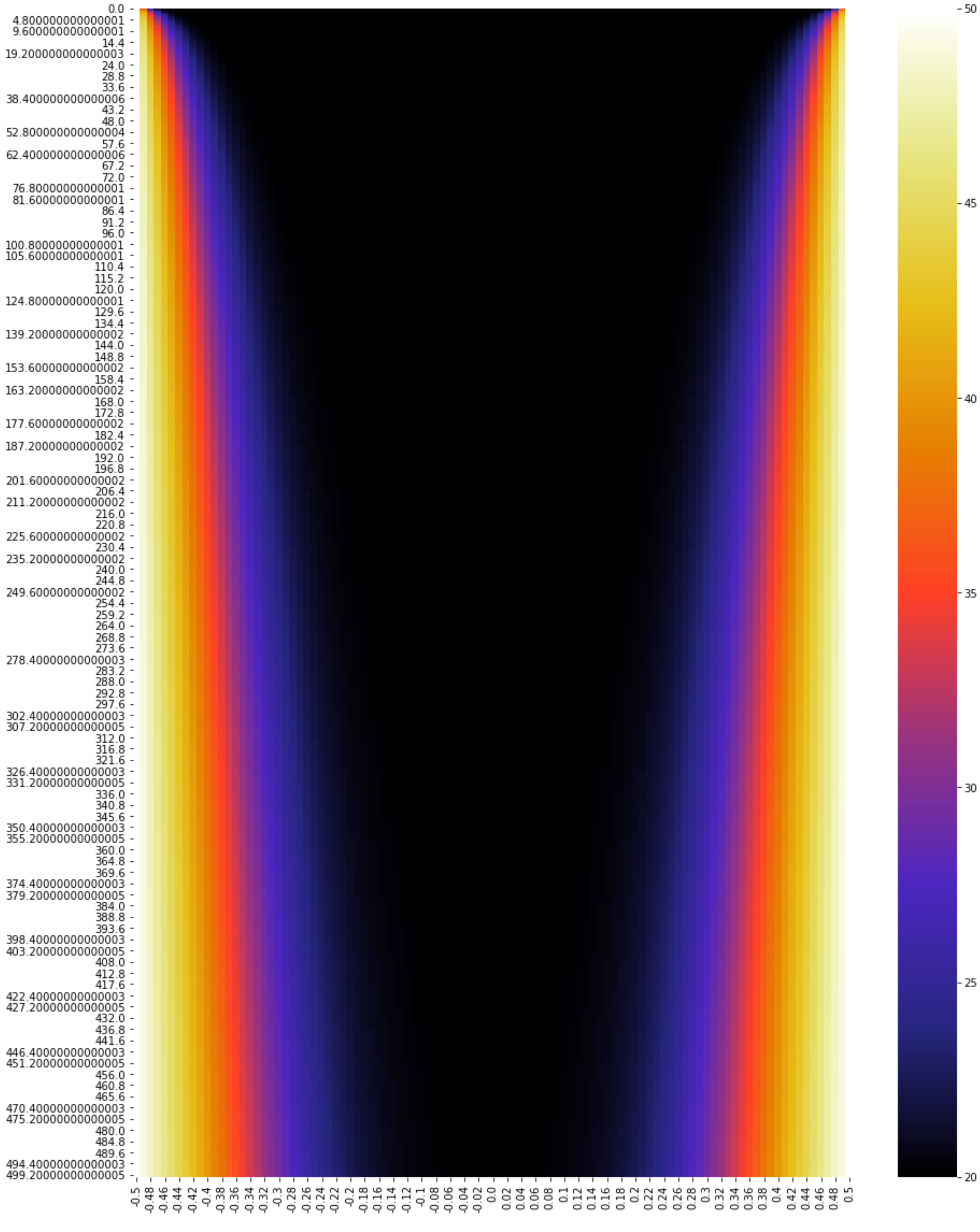
For the **Temperature profile figures** assume that fluid is coming at the bottom of figure and Leaving at Top of it

The results are obtained for various values of average velocity of fluid.

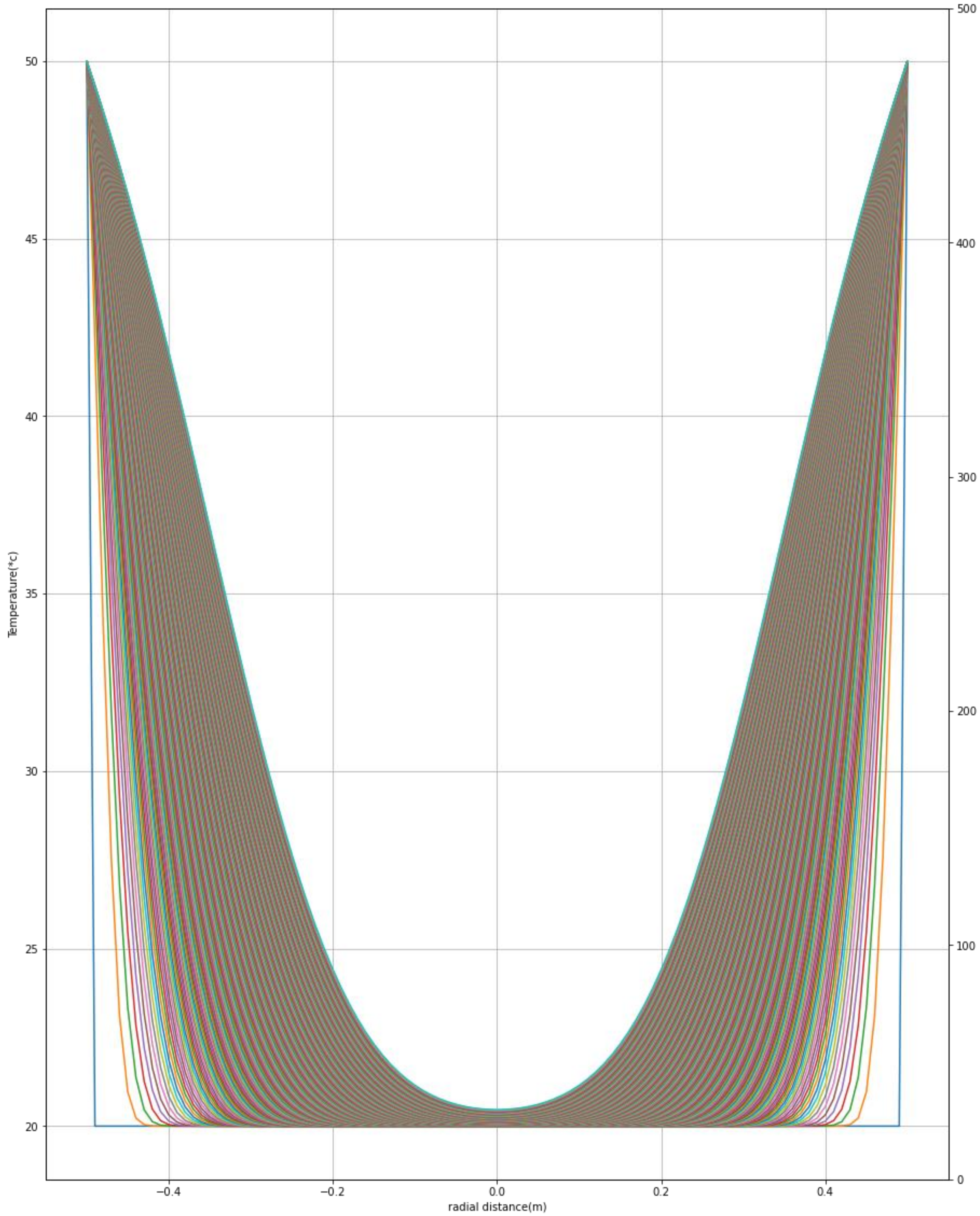
Temperature Profile for V_avg=0.08m/s



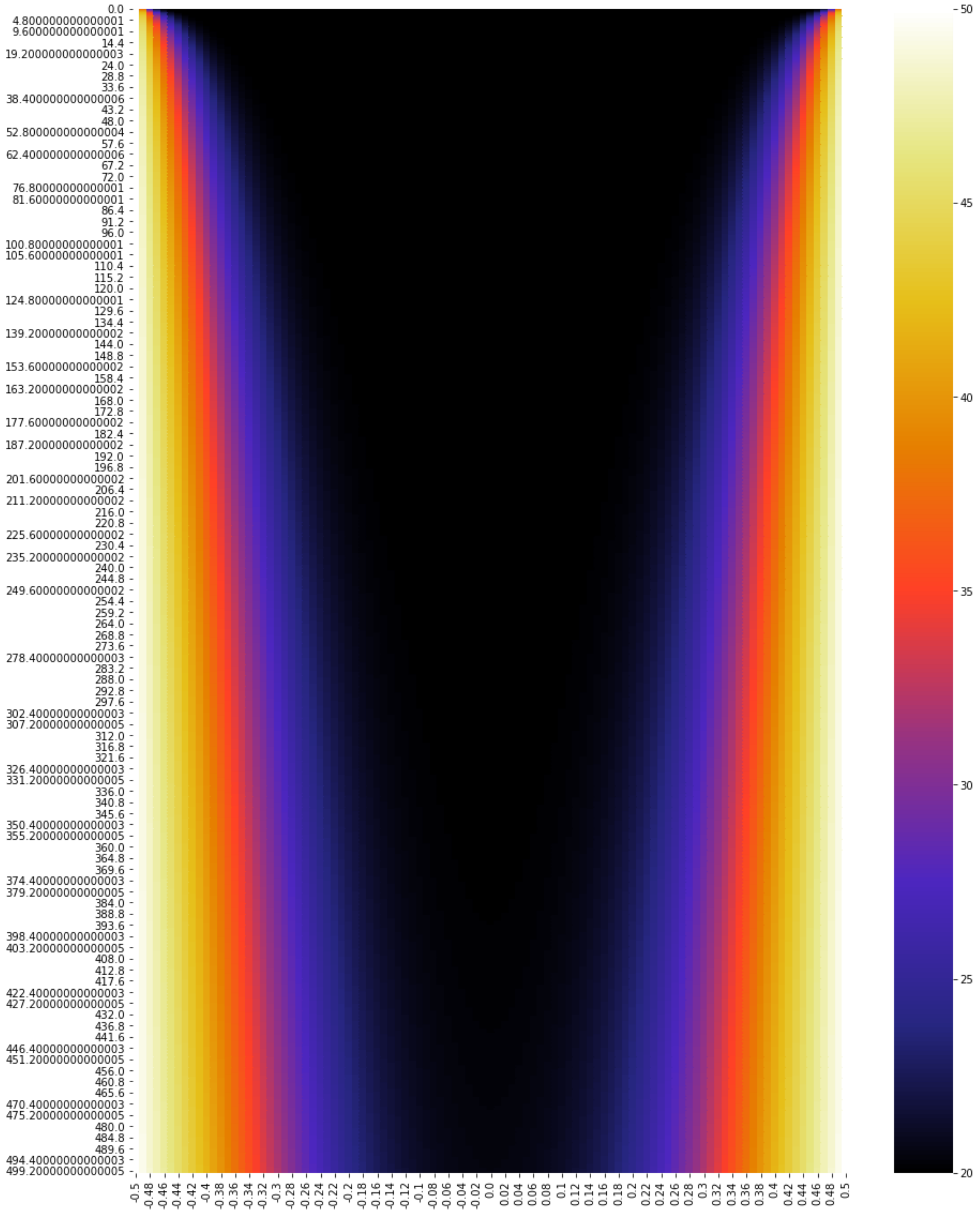
Temperature contour for V_avg=0.08m/s



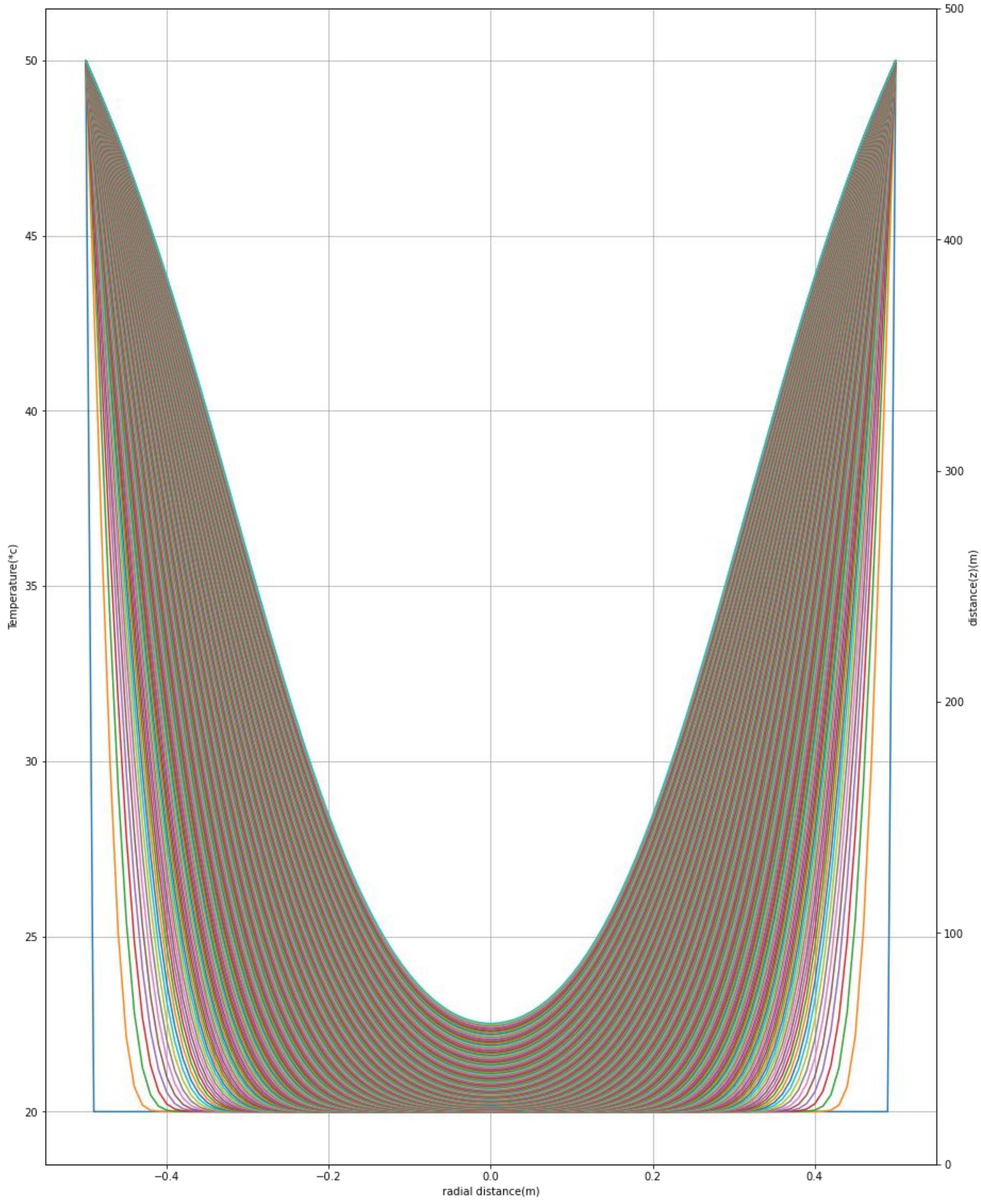
Temperature Profile for V_avg=0.06m/s



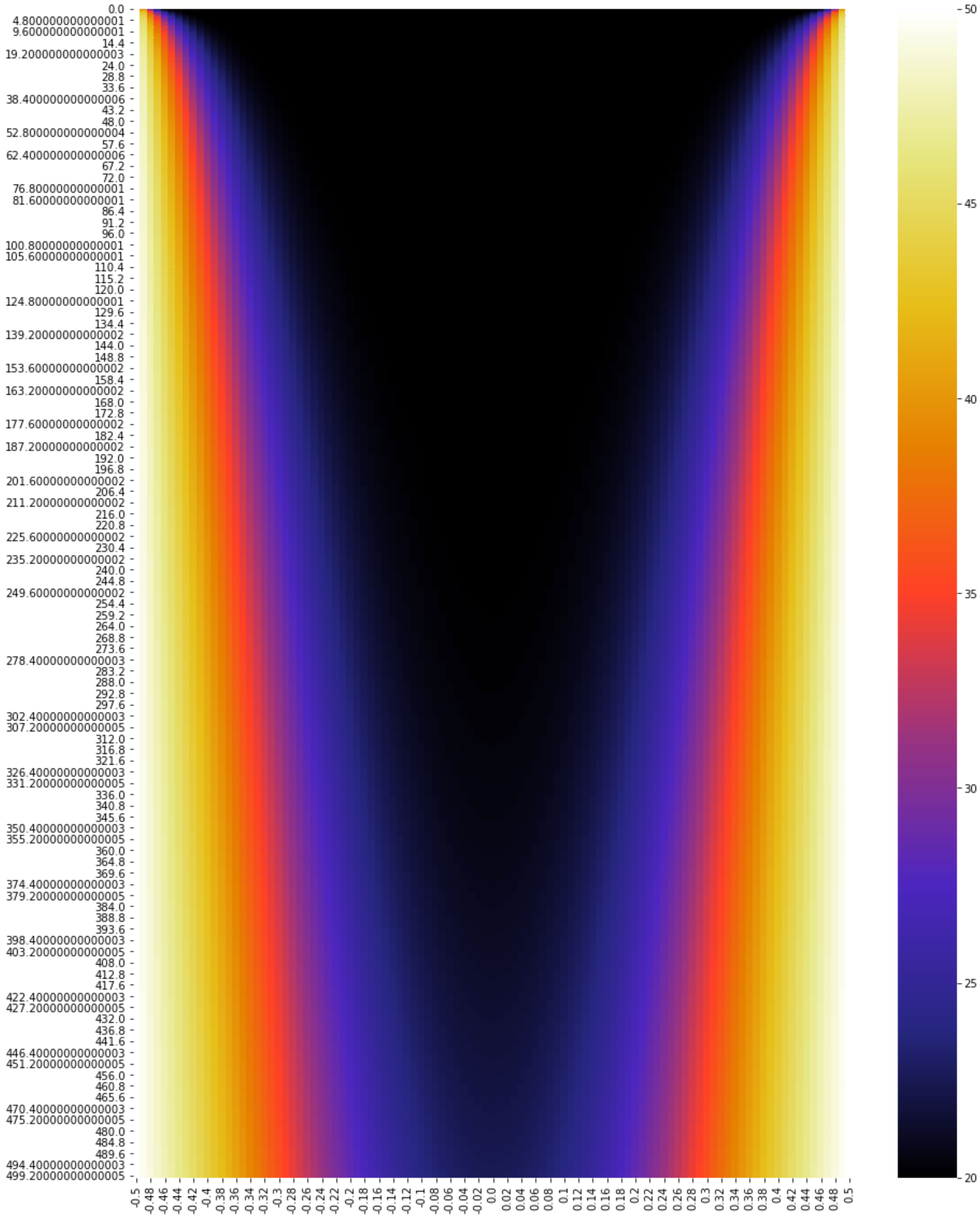
Temperature contour for V_avg=0.06m/s



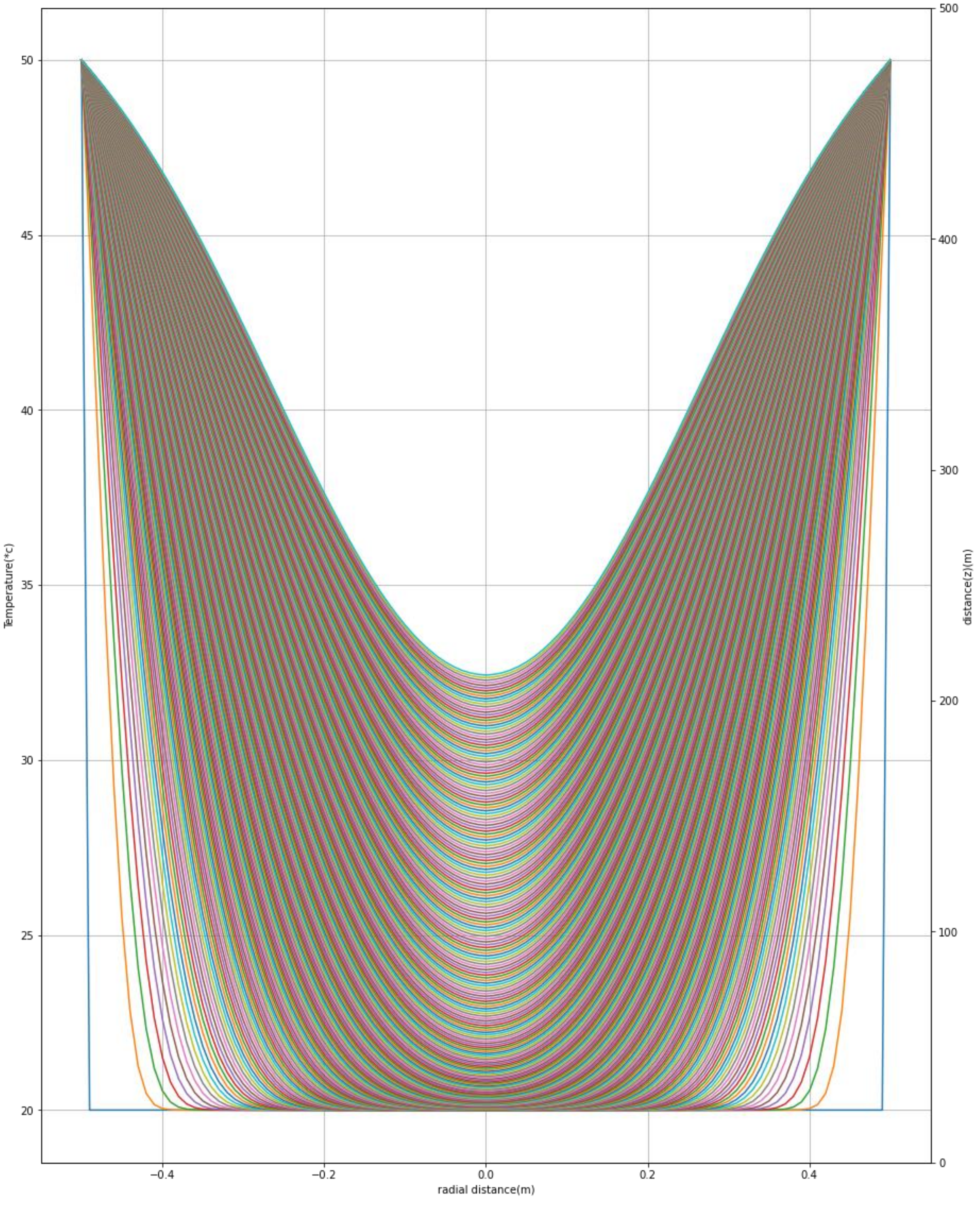
Temperature Profile for V_avg=0.04m/s



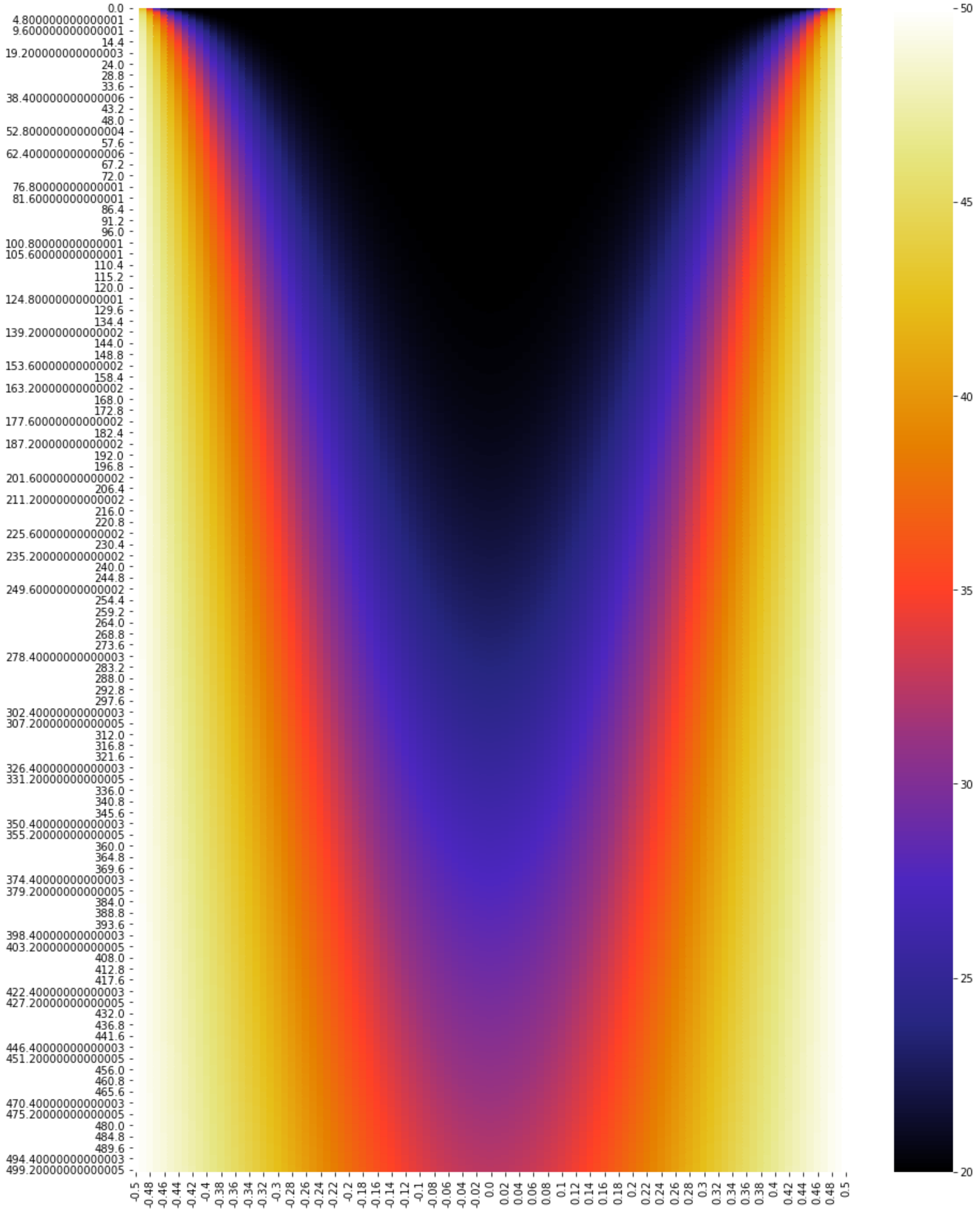
Temperature contour for V_avg=0.04m/s



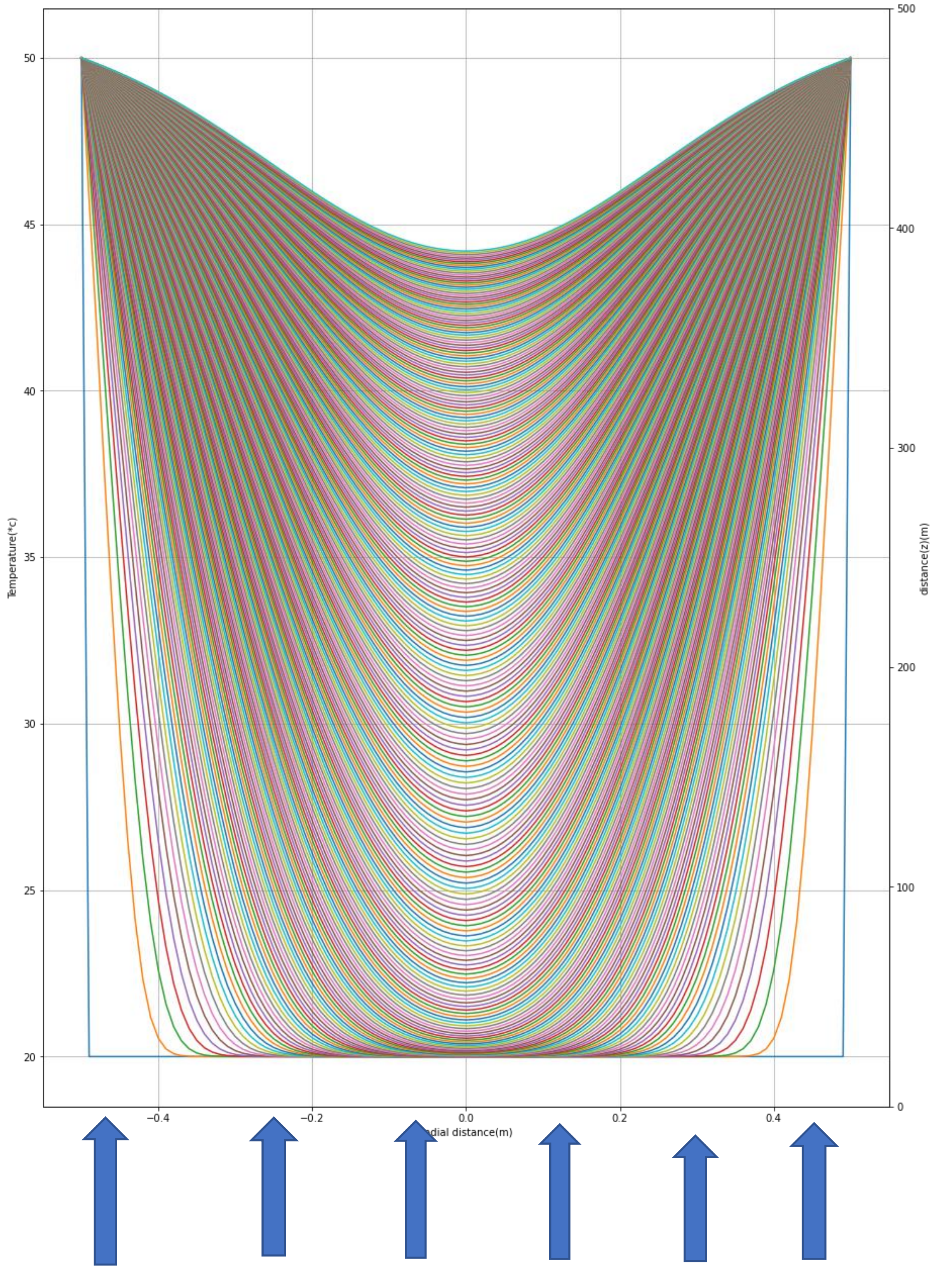
Temperature Profile for V_avg=0.02m/s



Temperature contour for V_avg=0.02m/s



Temperature Profile for V_avg=0.01m/s



Temperature contour for V_avg=0.01m/s