K L E F

DEPARTMENT OF MATHEMATICS

II B. TECH CSE Regular

Course title: Probability, Statistics and Queuing theory

Course code: 21MT2103RB

Course Coordinator: Dr. K. Rajyalakshmi

Test 2 Scheme of Valuation

Date of Exam: 04/11/2022 Max. Marks: 50

1. Define the terms

Null hypothesis	1.5M
Alternative hypothesis	1.5M
Critical region	1.5M

2. Assume that the helium porosity of coal samples taken from any particular seam is normally distributed with true standard deviation 0.75.

i) Compute a 95% Confidence interval for the true average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85.

ii) Compute a 99% confidence interval for the true average porosity of another seam based on 16 specimens with a sample average porosity of 4.56.

Solution:

Given $\sigma=0.75$ 0.5M

i) 95% confidence interval for the population mean μ are:

n=20, $\bar{x} = 4.85$

$\bar{x}\pm1.96(\sigma/\sqrt{n})=4.85\pm1.96(0.75/\sqrt{20})=4.85\pm0.3287$, i.e., 4.5213 and 5.178

ii) 99% confidence interval for the population mean μ are:

n=16, $\bar{x} = 4.56$

$\bar{x}\pm2.58(\sigma/\sqrt{n}) =4.56 \pm2.58(0.75/\sqrt{16}) =4.56\pm0.4838$, i.e., 4.0762 and 5.0438

3. A manufacture claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of 6.28 kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of 5.61 kilograms. Test the manufacturer's claim at $\alpha=0.05$.

Solution:

Given $n_1=50$, $n_2=50$, $\bar{x}_1 = 86.7, \bar{x}_2 = 77.8$, $s_1=6.28=\hat{\sigma}_1$, $s_2=5.61=\hat{\sigma}_2$ 2M

Step 1: Set Null hypothesis 1M

H_0: μ_1-μ_2=12,

Step 2: Alternative hypothesis: H_1: $\mu_1>\mu_2$ (one-tailed) 1M

H_1: μ_1- μ_2>12

Step: 3 Choose the level of significance α=5% and 1% 1M

Test statistic: Under H_0, the test statistics is :

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})}} \cong \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})}} \sim N(0,1) \quad \text{(since samples are large).}$$

Where Standard error(S.E.)= $\text{S.E.}\,|\bar{x}_1 - \bar{x}_2| = \sqrt{(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})} \cong \sqrt{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})}$ 2M

$$Z = \frac{(86.7 - 77.8) - 12}{\sqrt{(\frac{6.28\text{^}2}{50} + \frac{5.61\text{^}2}{50})}} \sim N(0,1)$$

$$Z = -2.603$$

Conclusion: $|z| = 2.603 > 1.96$, the null hypothesis is rejected and we conclude that there is significant difference between the sample means. 1M

4. The following table gives the number of accidents that work place in an industry surveying varies days of the week. Test if the accidents are uniformly distributed over the week

Days	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	18	12	11	15	14

Solution:

Solution: To examine whether accidents are uniformly distributed over the week or not

Step 1: Set up the null hypothesis: Accidents are uniformly distributed over the week. 1M

Step: 2: Set alternative hypothesis: Accidents are uniformly distributed over the week. 1M

Step:3 Choose level of significance 5% 1M

Step: 4 Test statistic

Now, under the null hypothesis H_0, the test statistic is

$\chi^2 = \sum(\frac{(oi-ei)^2}{ei})$ 1M

Where o_i= observed frequencies

e_i = expected frequencies which is given by

e_i = (Total number of accidents) /No. of days=84/6=

Observed frequency (o_i)	Expected frequency (e_i)	o_i-e_i	$(\mathrm{oi}-\mathrm{ei})^2$	$\frac{(\mathrm{oi}-\mathrm{ei})^2}{ei}$
14	14	0	0	0
18	14	4	16	1.14
12	14	-2	4	0.2857
11	14	-3	9	0.6428
15	14	1	1	0.0714
14	14	0	0	0
N=84	N=84			2.14

Now, under the null hypothesis H_0, the test statistic is

$\chi^2 = \sum(\frac{(oi-ei)^2}{ei})$=2.14 3M

The table value of χ^2 at (n-1) = 6-1 =5d.f. is 11.07 and at 5% level of significance is.

χ^2 calculated value is less than χ^2 table value, so we accept the null hypothesis H_0.

Hence, we conclude that the accidents are uniformly distributed over the week. 1M

5a. Five measurements of tar content of a certain kind of cigarette yielded 14.5, 14.2, 14.4, 14.3 and 14.6 mg per cigarette. Show that the difference between the mean of this sample, $\bar{x}$ = 14.4 and the average tar claimed by the manufacturer μ = 14.0, is significant at α = 0.05. Assume normality.

Solution:

Solution: Null Hypothesis H_0: the average tar claimed by manufacturer is 14 i.e., H_0: μ=14.

Alternative Hypothesis: H_1: μ≠14.

Choose level of significance 0.05 2.5M

Test statistic: Under H_0, the test statistic is :

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

Where $\bar{x}$ and

S^2 are to be computed from the sample values of I. Q.'s.

Calculations for Sample Mean and Standard deviation:

x	$(x-\bar{x})$	$(x-\bar{x})^2$
14.5	0.1	0.01
14.2	-0.2	0.04
14.4	0	0
14.3	-0.1	0.01
14.6	0.4	0.16
Total=72		0.22

Here n=5, $\bar{x}$=72/5=14.4 and S^2=0.22/5=0.044 2M

$|t| = \frac{0.4}{0.2097/\sqrt{4}} = 3.8132$ 1M

t-table value at 5% LOS for 4 degrees of freedom for two-tailed test is 2.776**.** 1M

5b) Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150^0F, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above 150^0, 50 water samples will be taken at randomly selected items and the temperature of each sample recorded. In the context of this situation, describe type I and type II errors. Which type of error would you consider more serious? Explain.

Solution:

Type I error: In reality, the plant is incompliance with regulations but announced it a not following the regulations.

It indicates that the mean temperature of the discharged water from the power plant has no negative effect on the river's eco system. But concluded as it has effect on eco system 2M

Type II error: In reality, the plant is not incompliance with regulations but announced it is following the regulations.

It indicates that the mean temperature of the discharged water from the power plant has negative effect on the river's eco system. But concluded as it has no effect on eco system 2M

Therefore, type II error is more dangerous 2M

6 The data in the following table represent the number of hours of relief provided by 5 different brands of headache tablets administered to 25 subjects experiencing fevers of 38^0C or more. Perform the analysis of variance and test the hypothesis at the 0.05 level of significance that the mean number of hours of relief provided by the tablets is the same for all 5 brands. Discuss the results.

Tablet				
A	B	C	D	E
5.2	9.1	3.2	2.4	7.1
4.7	7.1	5.8	3.4	6.6
8.1	8.2	2.2	4.1	9.3
6.2	6.0	3.1	1.0	4.2
3.0	9.1	7.2	4.0	7.6

Solution: Here we analyse the mean number of hours of relief provided by the tablets regardless of the five different brands.

This can be solved by using the ANOVA one-way classification.
Step 1: We set up the null hypothesis.
H_0: There is no significant difference between the mean number of hours of relief provided by the tablets regardless of the five different brands
Step 2: H_1: There is a significant difference between the mean number of hours of relief provided by the tablets regardless of the five different brands.

2M

Tablet				
A	B	C	D	E
5.2	9.1	3.2	2.4	7.1
4.7	7.1	5.8	3.4	6.6
8.1	8.2	2.2	4.1	9.3
6.2	6.0	3.1	1.0	4.2
3.0	9.1	7.2	4.0	7.6

Step 3: Calculate the correction factor (c.f.)

c.f. = $(137.9)^2/25=760.6564$ 1M

Step 4: We calculate the raw sum of squares

Raw sum of squares =898.61

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Step 5: Calculate the total sum of squares

Total sum of squares (S. S. T) = 898.61-760.66=137.9536 2M

Step 6: Calculate the treatment sum of square (S. S.A.) or between the groups

S. S. A. =78.4216

Step 7: Calculate the Error sum of squares

E. S. S. = S.S.T. – S.S.A.=59.532 1M

Step 8: Calculate the degrees of freedom

Degrees of freedom for S.S.A = k-1(where k is the number of groups)=5-1=4

Degrees of freedom for Error S. S. E. =N-k (where N is the total no. in the group) =20

Degrees of freedom for total S. S. T= N-1=25-1=24

Step 9: Find the value of Mean sum of squares of two variances as

Mean sum of square between the group M.S.S.A = $\frac{\text{S.S.A}}{\text{k}-1}$=19.6054

Mean sum of square within the group M.S.S.E = $\frac{\text{S.S.E}}{\text{N}-\text{k}}$=5982/10=2.9766 1M

Step 10: Variance ratio

F_1= M.S.S.A/ M.S.S.T.=6.5865.

Step 11: Prepare ANOVA table

Sources of variation	Degrees of freedom	Sum of squares	Mean sum of squares	Variance ratio	
				F-calculated value	F-tabulated value
Between the groups (columns) Within the groups (Error)	5-1=4 24-4= 20	78.4216 59.532	19.6054 2.9766	6.586	2.866
Total	25-1=24	137.9536	-	-	-

4.5M

Step 12: Conclusion F-Calculated value is >Table value. Therefore it is significant and we reject H_0 5% level of significance . Hence there is significant difference between the mean number of hours relief provided by five different brands. 1M

7. Mention the characteristics of queuing process

The best way to understand how a queue operates is to examine the characteristics of the basic queue elements. There are six basic characteristics of a queueing system.

a) **Arrival Process:** The arrival process describe the arrival patterns at the queue. Do customers arrive individually or do they arrive in groups? Do customers arrive at a fairly constant rate, or is there some pattern to their arrivals? Is arrival process predictable or random? 1M

b) **Service Process:** The service process represents the time taken to serve customers referred to as service time. Is the service time constant, or does it vary from customer to customers? Are customers served in bulk, as in an elevator? 1M

c) **Queue discipline:** The queue discipline specifies the order in which the customers in the queue are served. Are customers served in a first-come, first-served (FCFS) basis, or perhaps in last-come, first –served (LCFS) basis, or service in random order (SIRO)? 1M

d) **No. of servers:** Is there a single server or multiple servers. Is there a single queue that feeds all servers, separate queues at each server, or some variation of the two? 1M

e) **The calling population:** The population of potential customers, referred to as the calling population, may be assumed to be finite or infinite. 0.5M

8. If arrival rate is 3 customers/day and service rate is 5 customer/day for M/M/1 queuing system then the expected number of customers in the system at a certain day.

Solution:

Here we have λ= 3/day, μ=5/day 1M

and $\rho = \frac{\lambda}{\mu}$=3/5 1M

Expected number of customers in the system L_s= L_s=$\frac{\lambda}{\mu-\lambda}$ 2M

L_s=3/2. 0.5M

9. Customers arrive to a haircut salon according to a Poisson process with a mean arrival rate of 5/hr. Because of the reputation of the salon, customers were always willing to wait. Customer processing time was exponentially distributed with an average of 10 min. Answer the following questions
i) Average no. of customers in the shop and average no. of customers waiting for haircut.
ii) Average no. of waiting when there is at least one person waiting.
iii) Calculate percentage of time a customer can walk right in without having to wait at all?
iv) If the salon has only four seats, what is the probability that a customer, upon arrival, will not be able to find a seat and have to stand?
v) How much time customers spend waiting in the queue?

Solution: Here we have λ= 5/hour, μ=1/10 per minute=6 per hour and $\rho = \frac{\lambda}{\mu}$=5/6 1M

i) Average number of customers in the shop
=L_s=$\frac{\lambda}{\mu-\lambda} = 5$ 2M
Average number of customers waiting for hair cut
=L_q=$\frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{25}{6(6-5)} = 4\frac{1}{6}$

ii) Average number waiting where there is at least one person waiting
$P(n>1)=(P^2+P^3+\ldots)P_0=P^2$ 1M

$$=\frac{L_q}{P(n>1)}=\frac{\rho^2}{\frac{1-\rho}{\rho^2}}=\frac{1}{1-\rho}=\frac{\mu}{\mu-\lambda}=6$$

iii) Probability that a customer can walk right in =Probability that there is no unit in the system=$P_0 = 1-\frac{\lambda}{\mu}$=1/6➔% of customers can walk in =100P_0=100/6=16.7% 2M

iv) Probability that an arrival will not be able to find a seat=$P(n\geq 5)=P_0\rho 5$ 1M

v) Waiting time of the customer in the queue $W_q=\frac{L_q}{\lambda}=\frac{\lambda}{\mu(\mu-\lambda)}$ =5/6hour 1M

10. Patients arrive at a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.
    (i). Find the effective arrival rate at the clinic.
    (ii). Obtain the probability that an arriving patient will not wait? Will he find a vacant seat in the room.
    (i) Determine the expected waiting time until a patient is discharged from the clinic.

Solution:

$\lambda = 30/\text{hour},\ \mu = 20/\text{Hour}$, N=14

$$\rho=\frac{\lambda}{\mu}=1.5$$

i) Effective arrival rate $\lambda_{\text{eff}} = \lambda(1-P_N) = 30(1-P_{14}) =$

ii) P_0

P(n<14)

iii) W_s

$$P_0 \ and\ W_s=\frac{P_0}{\lambda}\ \sum_{n=0}^{N} n\,\rho^n$$

11. City hospital's eye clinic offers free vision tests every Wednesday evening. There are three ophthalmologists on duty. A test takes, on the average, 20 min. And the actual time is found to be approximately exponentially distributed around this average. Clients arrive according to a Poisson process with a mean of 6 /hr, and patients are taken on a first-come, first-served basis. The hospital planners are interested in knowing: (i) what is the average number of people waiting. (ii) the average amount of time a patient spends at the clinic and (iii) the average percentage ideal time of each of the doctors.

Solution: We have λ=6/hr, $\mu=\frac{1}{20}$/min=3/hr and s=3, $\rho=\frac{\lambda}{s\mu}=\frac{2}{3}$ 2M

$$P_0=\left[\sum_{n=0}^{s-1}\frac{(\rho s)^n}{n!}+\sum_{n=s}^{\infty}\frac{(\rho s)^n}{s^{n-s}s!}\right]^{-1}$$

$$=\left[1+2+\frac{2^2}{2\,!}+\frac{2^3}{3\,!}+\frac{2^4}{3\,.\,3\,!}+\frac{2^5}{3^2\;\;3\,!}+...\right]^{-1}$$

$$=\left[5+\frac{2^3}{3\,!}\;\frac{1}{1-\frac{2}{3}}\right]^{-1}=\frac{1}{9}$$ 3M

Avverage number of people waiting in the queue

$$L_q=\frac{\left(\frac{\lambda}{\mu}\right)^s\rho\,P_0}{s\,!\,(1-\rho)^2}=\frac{2^3\left(\frac{2}{3}\right)}{3\,!\left(1-\frac{2}{3}\right)^2}\left(\frac{1}{9}\right)=\frac{8}{9}$$

2M

Average amount of time a patient spends at the clinic

$$=W_s=\frac{L_q}{\lambda}+\frac{1}{\mu}=\frac{8/9}{6}+\frac{1}{3}=28.9\text{ min.}$$ 2M

Long term average fraction of idle time for any server in an M / M / s is equal to

$1-\rho = 1 - \frac{2}{3}=\frac{1}{3}.$ 1M

Therefore, each physician is idle $\frac{1}{3}$ of the time, given that 3 servers on duty, two of them will be busy at any time (on average), since $\frac{\lambda}{\mu}=2$. 1M

Furthermore, the fraction of time that there is at least one idle doctor can be computed as

$$P_0+P_1+P_2=P(W_q=0)=5/9.$$

i.e. 55.5% of time there is at least one idle doctor. 1.5M

12. A small mail order business has one telephone line and a facility for call waiting for two additional customers. Orders arrive at the rate of one per minute and each order requires 2 minutes and 30 seconds to take down the particulars. Model this system as an M/M/1/3 queue and answer the following questions and also compare the results with M/M/2/3 and M/M/2/4

i) Obtain the expected number of calls waiting in the queue.

ii) Obtain the mean wait in queue.

Solution:

We have $\lambda=1/\text{minute}$, $\mu=1/2.5\text{min}$, $\rho=\frac{\lambda}{s\mu}$ 3.5

Model 2: M/M/1/3

L_q

W_q 3M

Model 4: M/M/2/3

L_q

W_q

3M

Model 4: M/M/2/4

L_q

W_q

3M

All the faculty are involved in preparation of the key and scheme of the valuation and verified.

The following questions are allotted to the faculty to evaluate the answer scripts of Test 1 examination as per question wise:

Board 1		
S. No	**Name of the faculty**	**Q. No**
1	Dr K Rajyalakshmi (CC)	1, 2, 3 and 4
2	Dr V S Bhagavan	1, 2, 3 and 4
3	Dr N Srimannarayan	5a, 5b and 6a,6b
4	Dr N Vedavathi	7,8,9,10
5	Dr M Radha Madhavi	7,8,9,10
6	Dr W Sridhar	11a, 11b and 12a, 12b

10% of the total scripts are evaluated by Course Coordinator Dr. K. Rajyalakshmi.

Signature(s) of the faculty:

1. Dr. V. S. Bhagavan-
2. Dr N Srimannarayana
3. Dr N Vedavathi
4. Dr M Radha Madhavi
5. Dr W Sridhar
6. Dr K Rajyalakshmi

Signature of the Course Coordinator: