

Assignment 1

Probability and Random Processes

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EE22BTECH11038

Question 1.4.2

Find the intersection **O** of the perpendicular bisectors of **AB** and **AC**.

Solution:

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Vector equation of perpendicular bisector of **AB** is

$$(\mathbf{A} - \mathbf{B})^\top \left(\mathbf{x} - \frac{\mathbf{A} + \mathbf{B}}{2} \right) = 0 \quad (1)$$

where,

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad (3)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (5)$$

$$\Rightarrow (\mathbf{A} - \mathbf{B})^\top = (5 \quad -7) \quad (6)$$

\therefore The equation of perpendicular bisector of **AB** is

$$(5 \quad -7) \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -3 \\ 5 \end{pmatrix} \right) = 0 \quad (7)$$

$$\Rightarrow (5 \quad -7) \mathbf{x} = \frac{1}{2} (5 \quad -7) \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad (8)$$

Performing matrix multiplication yields

$$(5 \quad -7) \mathbf{x} = -25 \quad (9)$$

Vector equation of perpendicular bisector of **AC** is

$$(\mathbf{A} - \mathbf{C})^\top \left(\mathbf{x} - \frac{\mathbf{A} + \mathbf{C}}{2} \right) = 0 \quad (10)$$

where,

$$\mathbf{A} + \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} \quad (12)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (13)$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (14)$$

$$\Rightarrow (\mathbf{A} - \mathbf{C})^\top = (4 \quad 4) \quad (15)$$

\therefore The equation of perpendicular bisector of **AC** is

$$(4 \quad 4) \left(\mathbf{x} - \frac{1}{2} \begin{pmatrix} -2 \\ -6 \end{pmatrix} \right) = 0 \quad (16)$$

$$\Rightarrow (4 \quad 4) \mathbf{x} = \frac{1}{2} (4 \quad 4) \begin{pmatrix} -2 \\ -6 \end{pmatrix} \quad (17)$$

Performing matrix multiplication yields

$$(4 \quad 4) \mathbf{x} = -16 \quad (18)$$

$$\Rightarrow (1 \quad 1) \mathbf{x} = -4 \quad (19)$$

Solving both line equations by row reduction method:

$$\text{Augmented matrix} = \begin{pmatrix} 5 & -7 & -25 \\ 1 & 1 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -7 & -25 \\ 1 & 1 & -4 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow 5R_2 - R_1} \begin{pmatrix} 5 & -7 & -25 \\ 0 & 12 & 5 \end{pmatrix} \quad (20)$$

$$\begin{pmatrix} 5 & -7 & -25 \\ 0 & 12 & 5 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow \frac{12}{7}R_1 + R_2} \begin{pmatrix} \frac{60}{7} & 0 & \frac{-265}{7} \\ 0 & 12 & 5 \end{pmatrix} \quad (21)$$

$$\begin{pmatrix} \frac{60}{7} & 0 & \frac{-265}{7} \\ 0 & 12 & 5 \end{pmatrix} \xrightarrow{\begin{matrix} R_2 \leftrightarrow \frac{1}{12}R_2 \\ R_1 \leftrightarrow \frac{7}{60}R_1 \end{matrix}} \begin{pmatrix} 1 & 0 & \frac{-53}{12} \\ 0 & 1 & \frac{5}{12} \end{pmatrix} \quad (22)$$

$$\therefore \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \quad (23)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \quad (24)$$

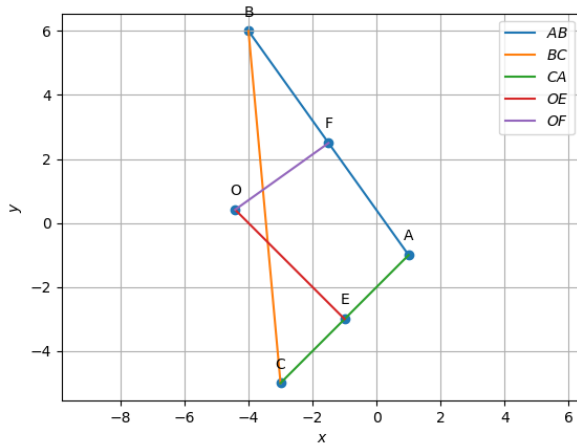


Fig. 0. O is the point of intersection of perpendicular bisectors of AB and AC

Therefore, the point of intersection of perpendicular bisectors of **AB** and **AC** is $\mathbf{O} = \frac{1}{12} \begin{pmatrix} -53 \\ 5 \end{pmatrix}$