Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{1}$$

1 Vectors

Parameters	Values	Description
$\mathbf{m_1}$	$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} 3 \\ -9 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} -7 \\ 5 \end{pmatrix}$	CA
$\|\|A-B\|\|$	5.66	length of AB
$\|\|B-C\|\|$	9.49	length of BC
$\|\|C-A\|\|$	8.60	length of CA
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 4 \\ -4 \end{pmatrix}$	AB
c_1	-16	
$\mathbf{n_2}$	$\begin{pmatrix} -9 \\ -3 \end{pmatrix}$	BC
c_2	-12	
$\mathbf{n_3}$	$\begin{pmatrix} 5 \\ 7 \end{pmatrix}$	CA
c_3	-20	
Area	2	Area of Triangle
$\angle A$	80.54°	Angles
$\angle B$	63.43°	
$\angle C$	36.03°	

TABLE 1: Vectors.

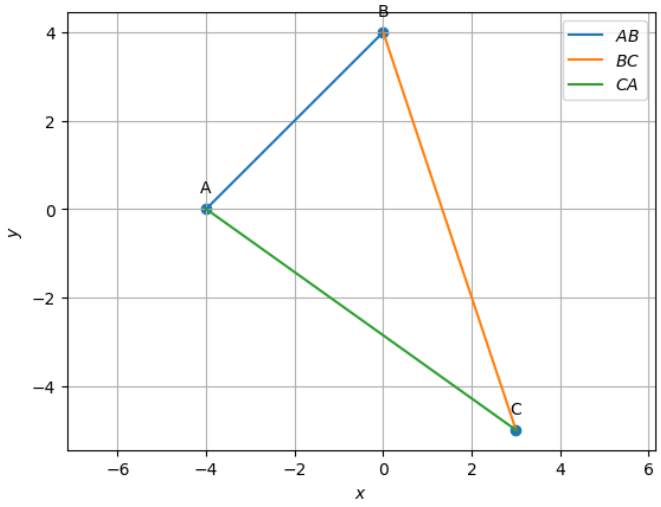

Fig. 1: triangle plotted using python

2 Median

Parameters	Value	Description
$\mathbf{D}$	$\begin{pmatrix} 1.5 \\ -0.5 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -0.5 \\ -2.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 5.5 \\ -0.5 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -0.5 \\ -5.5 \end{pmatrix}$	
c_4	2	
$\mathbf{m_5}$	$\begin{pmatrix} -0.5 \\ -6.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -6.5 \\ 0.5 \end{pmatrix}$	
c_5	2	
$\mathbf{m_6}$	$\begin{pmatrix} 5 \\ 7 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 7 \\ 5 \end{pmatrix}$	
c_6	-4	
$\mathbf{G}$	$\begin{pmatrix} -0.33 \\ -0.33 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

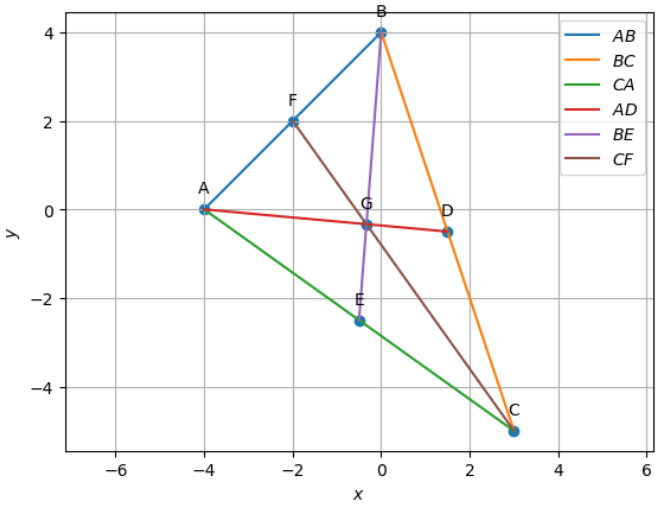

Fig. 2: medians plotted using python

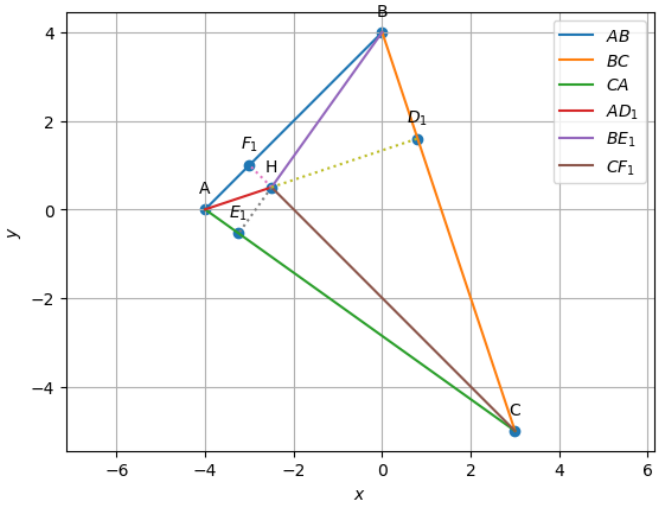

Fig. 3: altitudes plotted using python

3 Altitude

Parameters	Value	Description
$\mathbf{D_1}$	$\begin{pmatrix}0.8\\1.6\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-3.24\\-0.54\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-3\\1\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}-4.8\\-1.6\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-1.6\\4.8\end{pmatrix}$	
c_7	6.4	
$\mathbf{m_8}$	$\begin{pmatrix}3.24\\4.54\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}4.54\\-3.24\end{pmatrix}$	
c_8	-12.97	
$\mathbf{m_9}$	$\begin{pmatrix}6\\-6\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}-6\\-6\end{pmatrix}$	
c_9	12	
H	$\begin{pmatrix}-2.5\\0.5\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

Parameters	Value	Description
$\mathbf{m_{10}}$	$\begin{pmatrix}-9\\-3\end{pmatrix}$	AD
$\mathbf{n_{10}}$	$\begin{pmatrix}3\\-9\end{pmatrix}$	
c_{10}	9	
$\mathbf{m_{11}}$	$\begin{pmatrix}5\\7\end{pmatrix}$	BE
$\mathbf{n_{11}}$	$\begin{pmatrix}-7\\5\end{pmatrix}$	
c_{11}	-9	
$\mathbf{m_{12}}$	$\begin{pmatrix}4\\-4\end{pmatrix}$	CF
$\mathbf{n_{12}}$	$\begin{pmatrix}4\\4\end{pmatrix}$	
c_{12}	0	
O	$\begin{pmatrix}0.75\\-0.75\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	4.81	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	161.08°	$\angle BOC = 2\angle BAC$
$\angle BAC$	80.54°	
$\angle AOC$	126.87°	$\angle AOC = 2\angle ABC$
$\angle ABC$	63.43°	
$\angle AOB$	72.05°	$\angle AOB = 2\angle BCA$
$\angle BCA$	36.03°	

TABLE 4: Perpendicular Bisector.

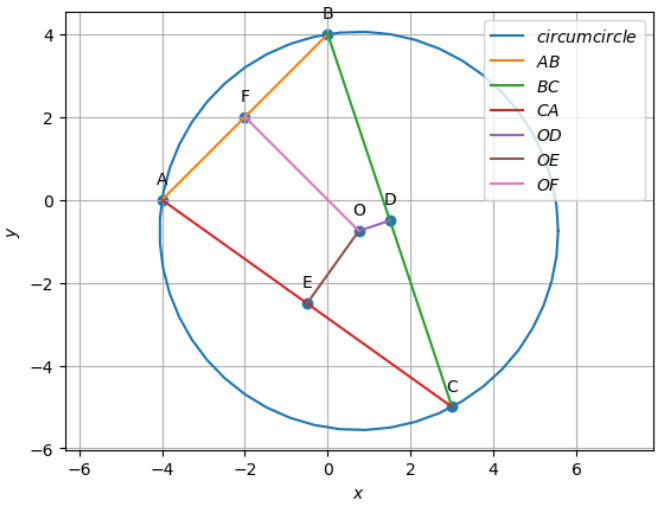

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

Parameters	Value	Description
$\mathbf{m_{13}}$	$\begin{pmatrix}3.12\\0.26\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}0.26\\-3.12\end{pmatrix}$	
c_{13}	-1.03	
$\mathbf{m_{14}}$	$\begin{pmatrix}-0.88\\-3.74\end{pmatrix}$	BI
$\mathbf{n_{14}}$	$\begin{pmatrix}-3.74\\0.88\end{pmatrix}$	
c_{14}	3.53	
$\mathbf{m_{15}}$	$\begin{pmatrix}-3.88\\5.26\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}5.26\\3.88\end{pmatrix}$	
c_{15}	-3.64	
$\mathbf{I}$	$\begin{pmatrix}-0.88\\0.26\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}1.03\\0.89\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-2.06\\-1.38\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}2.31\\1.68\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	2.02	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	40.27°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	31.72°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	18.01°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

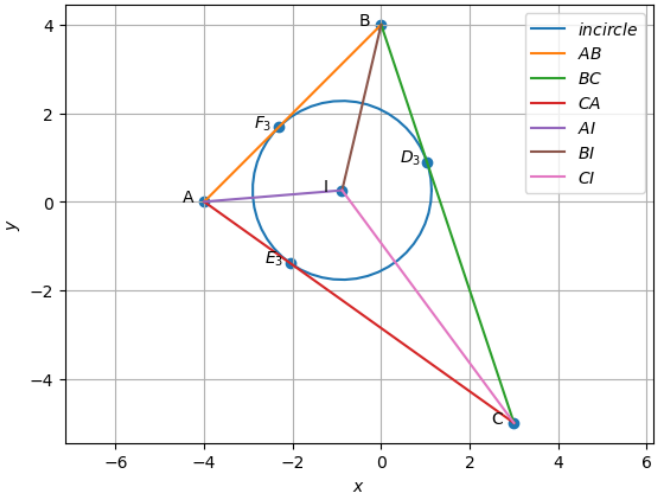

Fig. 5: Angle bisectors plotted using python