NCERT 12.13.3.27

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EE22BTECH11038*

Question 12.13.3.27:A biased die is such that $\Pr(4) = \frac{1}{10}$ and other scores being equally likely. The die is tossed twice. If X is the 'number of fours seen', find the variance of the random variable X.

Solution: Since, X = number of fours seen on tossing a die twice, X={0, 1, 2}
Also,

$$\Pr(4) = \frac{1}{10} \tag{1}$$

$$\Pr(4') = \frac{9}{10} \tag{2}$$

So,

$$p_X(k) = {}^2C_k \Pr(4)^k \Pr(4')^{2-k} \tag{3}$$

$$= \begin{cases} {}^2C_0 \Pr(4')^2 = \frac{81}{100} & k = 0 \\ {}^2C_1 \Pr(4) \Pr(4') = \frac{18}{100} & k = 1 \\ {}^2C_2 \Pr(4)^2 = \frac{1}{100} & k = 2 \end{cases} \tag{4}$$

$$Var(X) = E(X^2) - (E(X))^2 \tag{5}$$

$$Var(X) = \sum k^2 p_X(k) - \left(\sum k p_X(k)\right)^2 \tag{6}$$

$$= \left(0 + \frac{18}{100} + \frac{4}{100}\right) - \left(0 + \frac{18}{100} + \frac{2}{100}\right)^2 \tag{7}$$

$$= \frac{22}{100} - \left(\frac{20}{100}\right)^2 \tag{8}$$

$$= \frac{11}{50} - \frac{1}{25} \tag{9}$$

$$= \frac{9}{50} \tag{10}$$

$$Var(X) = 0.18 \tag{11}$$

Parameter	Value	Description
$\Pr(4)$	$\frac{1}{10}$	Probability of dice showing 4
$\Pr(4')$	$\frac{9}{10}$	Probability of dice not showing 4
$p_X(0)$	$\frac{81}{100}$	0 fours appear when 2 dice are rolled
$p_X(1)$	$\frac{18}{100}$	1 four appears when 2 dice are rolled
$p_X(2)$	$\frac{1}{100}$	2 fours appear when 2 dice are rolled

TABLE 0: Random Variables

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