Gate EC 36.2023

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Question 36.2023:
A random variable X, distributed normally as N(0,1), undergoes the transformation Y=h(X), given in Fig. 0. The form of probability density function of Y is (In the options given below, a,b,c are non-zero constants and g(y) is piece-wise continuous function).

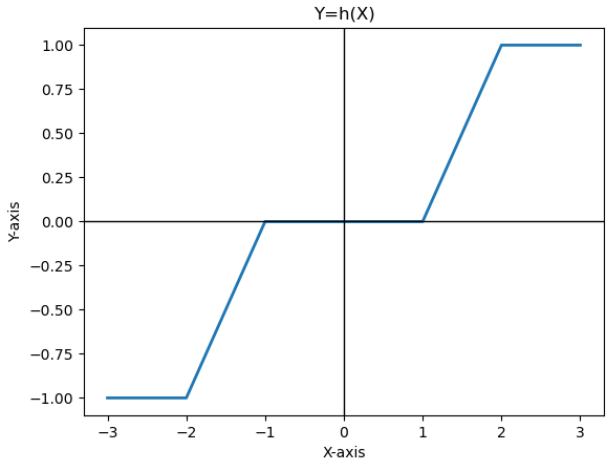

Fig. 0

1) $a\delta(y-1)+b\delta(y+1)+g(y)$

2) $a\delta(y+1)+b\delta(y)+c\delta(y-1)+g(y)$

3) $a\delta(y+2)+b\delta(y)+c\delta(y-2)+g(y)$

4) $a\delta(y+2)+b\delta(y-2)+g(y)$

Solution:

$$Y = h(X) \tag{1}$$

$$Y = \begin{cases} -1 & x < -2 \\ 0 & -1 < x < 1 \\ 1 & x > 2 \\ h(x) & otherwise \end{cases} \tag{2}$$

Probability distribution function of Y is

$$f_Y(y) = \sum p_Y(y_i)\delta(y-y_i) \tag{3}$$

Let,

$$p_Y(y_i) = \begin{cases} a & y_i = -1 \\ b & y_i = 0 \\ c & y_i = 1 \end{cases} \tag{4}$$

$g(y)$ be the component of pdf of Y for x ∈ (-2,-1) ∪ (1,2)

$$f_Y(y) = p_Y(-1)\delta(y+1) + p_Y(0)\delta(y) + p_Y(1)\delta(y-1) + g(y) \tag{5}$$

$$= a\delta(y+1) + b\delta(y) + c\delta(y-1) + g(y) \tag{6}$$