OPTIMIZATIONS USING LINEAR PROGRAMMING

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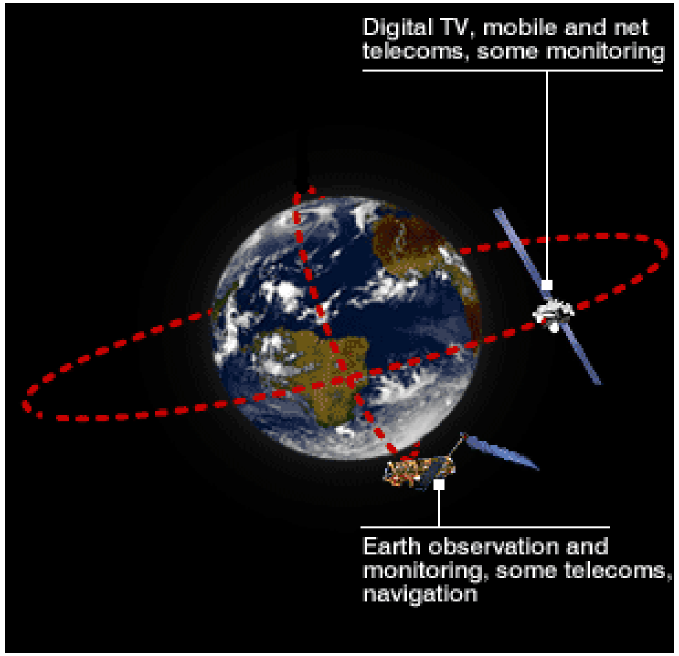


Find an schedule for the satellite that maximizes
the number of photographs taken, subject to the on-board recording capacity

Maximize Returns
Subject to
the investment risk is minimized

Optimization lies at the heart of machine learning:

- **Minimize the loss function subject to overfitting of the model and under fitting of the model.**
- **Find the parameters that minimize loss function**

Similar wide range of optimization problems can be seen in:

- **Supply Chain Industry**
- **Transportation Industry(Uber)**
- **Gambling**

Types of Optimization:

I. Convex Optimization

II. Non Convex Optimization

Examples of convex optimization in ML(Global minimum is guaranteed)

- Linear Regression/ Ridge regression, Lasso, SVM

Examples of non-convex optimization in ML:

- neural networks
- Gaussians mixtures

Support Vector Machines:

- Learning the SVM can be formulated as an optimization:

$$\max_{\mathbf{w}} \frac{2}{||\mathbf{w}||} \text{ subject to } \mathbf{w}^\top \mathbf{x}_i + b \begin{array}{ll} \geq 1 & \text{if } y_i = +1 \\ \leq -1 & \text{if } y_i = -1 \end{array} \text{ for } i = 1 \ldots N$$

- Or equivalently

$$\min_{\mathbf{w}} ||\mathbf{w}||^2 \text{ subject to } y_i \left(\mathbf{w}^\top \mathbf{x}_i + b \right) \geq 1 \text{ for } i = 1 \ldots N$$

Addressing optimization problems:

- linear, quadratic and convex programming methods can be used to solve convex problems.
- linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints.

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{subject to} & A\mathbf{x} \le \mathbf{b} \\ \text{and} & \mathbf{x} \ge \mathbf{0} \end{array}$$

- Linear Constraints

Fantasy Football using Linear Programming:

- **Build an ideal Fantasy Football team out of 465 players.**

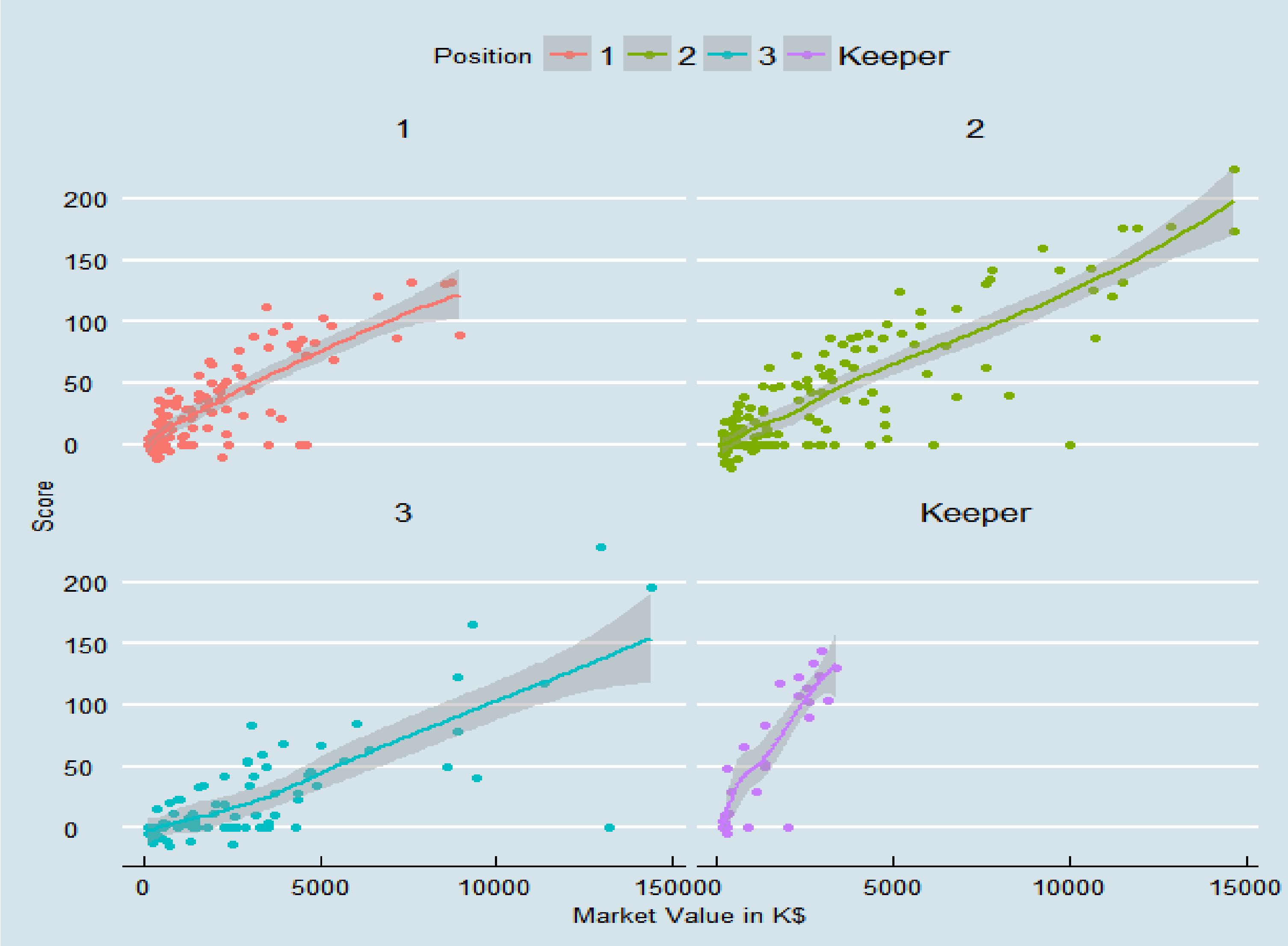
Objective:

- **Maximize the score of the team**
- **Subject to:**
  - **Budget<=20 M$**
  - **1 keeper,**
  - **5 defenders,**
  - **5 midfielders,**
  - **3 scorers.**

Data set: comunio.de, German fantasy football site

	Name	Position	Score	MarketValue	buy
1	Arnautovic	3	68	3980000	0
2	Arnold	2	18	2830000	0
3	Arslan	2	0	380000	0
4	Aubameyang	3	0	13200000	0
5	Audel	2	0	200000	0
6	Avdic	3	0	160000	0
7	Avevor	1	0	170000	0
8	Ayhan	1	0	480000	0
9	B. RÃ¶cker	1	0	220000	0
10	Badelj	2	49	2260000	0
11	Badstuber	1	56	1550000	0
12	Baier	2	72	2260000	1
13	Bajner	3	0	320000	0

ggplot of market value vs score of players:



Solving optimization using lpsolve package in R

```
f.obj <- football_data$Score  ###objective max team score

f.con <- t(football_data$MarketValue)  ### constraints max MV <= Budget
player <- rep(1, nrow(football_data))  ## constraints max number of players!

f.con <- rbind(f.con, player)

## constrain that per postion can only be a certain number of players be set up. (e.g. just one keeper)
## define matrix   - as a one hot (dummy coding what position the player holds)
A <- as.data.frame(model.matrix(MarketValue ~ Position -1, football_data) )

f.con <- rbind(f.con, t(as.matrix(A)))

f.dir <- c("<=", "<=", "=", "<=", "<=" ,"<=")
f.rhs <- c(20000000, 13, 1, 5, 5, 3)  ## right hand side .. not more than Budget, Players, and players per

### solve the problem
solved<- lp("max", f.obj, f.con, f.dir, f.rhs, all.bin=TRUE)  ## just binary variables!
```

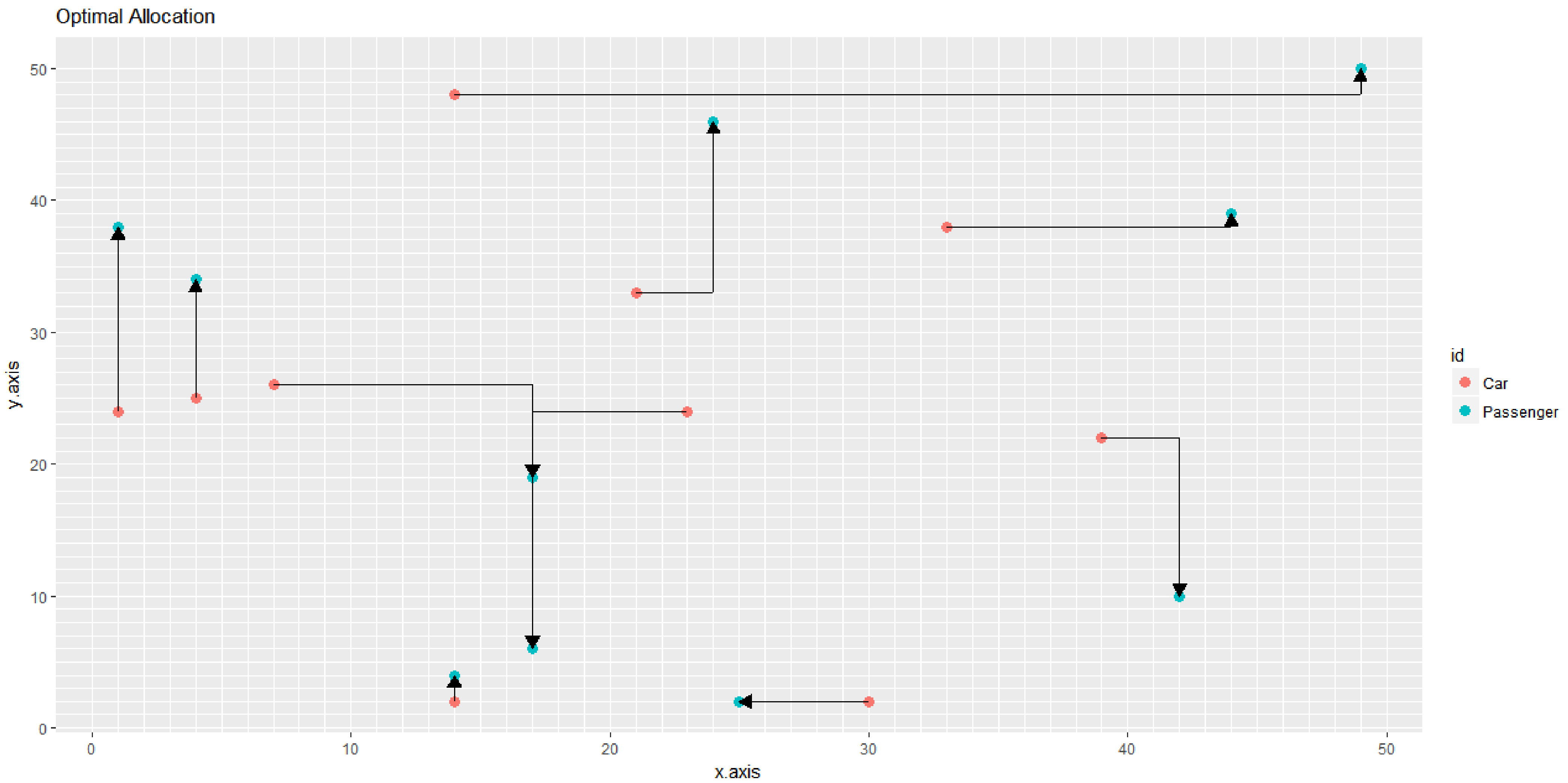
Final team:

- ❖ Team Market value: 19800000
- ❖ Team total score: 784
- ❖ Number of players bought: 13
- ❖ Players:

Badstuber, Baier, Balitsch, Ede,Gomez, Hasebe, klavan, krmas, piszczek,R. SchAfer, Soto, Svensson, Werner

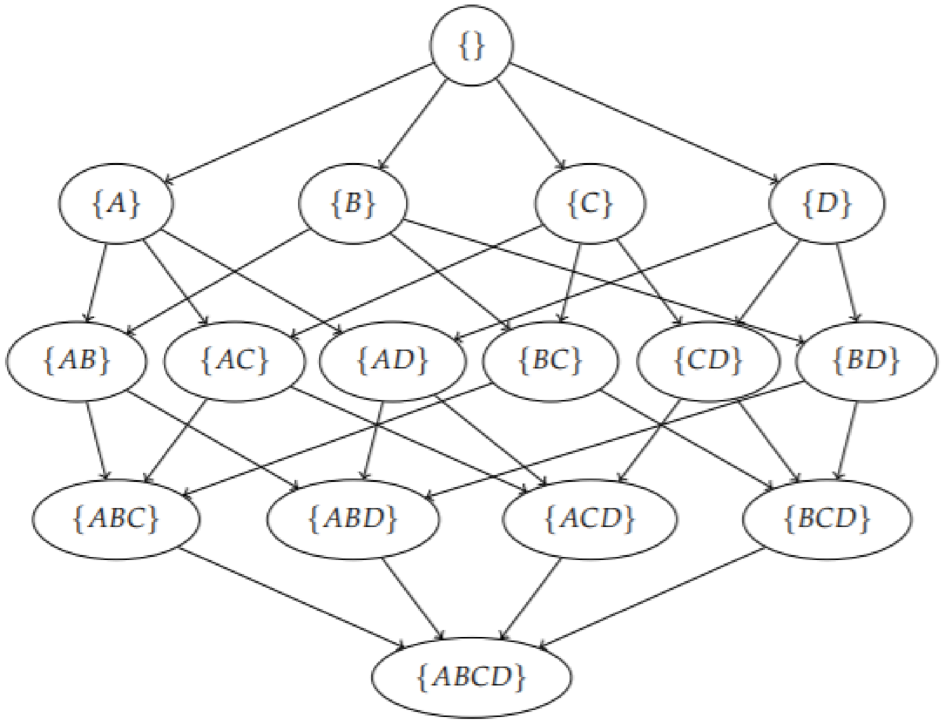
```
> sum(df[df$buy == 1,]$MarketValue)  ## what is the Budget
[1] 19800000
> sum(df[df$buy == 1,]$Score) ## what is the Score
[1] 784
> sum(df[df$buy == 1,]$buy)   ## number of players bought
[1] 13
> paste(df[df$buy == 1,]$Name, collapse=", ")
[1] "Badstuber, Baier, Balitsch, Ede, Gomez, Hasebe, Klavan, Krmas, Piszczek, R. SchÃ¤fer, Soto, Svensson, Werner
"
```

Uber Network Optimization:



Transcript profiling data→ sparse linear genetic network

3. Bayesian Network Structure Learning



linear programming:
estimate the structure of network from: transcript profiling data

maximize Score(G)
s.t. G ∈ DAGS(V)

V:-nodes corresponding to the variables X1, . . . , Xn
DAGS(V) is the space of DAGs over V.

A scoring function is a function whose outcome is a measure related to the probability of the substructure (sv → v) to be in the network, according to the dataset D.

THANK YOU!!!