5.5.1

EE25BTECH11049 - Sai Krishna Bakki

Question:

If $\mathbf{A} = \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and use it to solve the following system of equations

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

Solution:

$$\begin{pmatrix} 5 & -1 & 4 & | & 1 & 0 & 0 \\ 2 & 3 & 5 & | & 0 & 1 & 0 \\ 5 & -2 & 6 & | & 0 & 0 & 1 \end{pmatrix} \xleftarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 5 & -1 & 4 & | & 1 & 0 & 0 \\ 2 & 3 & 5 & | & 0 & 1 & 0 \\ 0 & -1 & 2 & | & -1 & 0 & 1 \end{pmatrix} \tag{1}$$

$$\xleftarrow[R_2 \leftarrow R_2 + 3R_3]{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 5 & 0 & 2 & | & 2 & 0 & -1 \\ 2 & 0 & 11 & | & -3 & 1 & 3 \\ 0 & -1 & 2 & | & -1 & 0 & 1 \end{pmatrix} \tag{2}$$

$$\xleftarrow{R_3 \leftarrow -R_3} \begin{pmatrix} 5 & 0 & 2 & | & 2 & 0 & -1 \\ 2 & 0 & 11 & | & -3 & 1 & 3 \\ 0 & 1 & -2 & | & 1 & 0 & -1 \end{pmatrix} \tag{3}$$

$$\xleftarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 5 & 0 & 2 & | & 2 & 0 & -1 \\ 0 & 1 & -2 & | & 1 & 0 & -1 \\ 2 & 0 & 11 & | & -3 & 1 & 3 \end{pmatrix} \tag{4}$$

$$\xleftarrow{R_1 \leftarrow \frac{1}{5}R_1} \begin{pmatrix} 1 & 0 & 2/5 & | & 2/5 & 0 & -1/5 \\ 0 & 1 & -2 & | & 1 & 0 & -1 \\ 2 & 0 & 11 & | & -3 & 1 & 3 \end{pmatrix} \tag{5}$$

$$\xleftarrow{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 0 & 2/5 & | & 2/5 & 0 & -1/5 \\ 0 & 1 & -2 & | & 1 & 0 & -1 \\ 0 & 0 & 51/5 & | & -19/5 & 1 & 17/5 \end{pmatrix} \tag{6}$$

$$\xleftarrow{R_3 \leftarrow \frac{5}{51}R_3} \begin{pmatrix} 1 & 0 & 2/5 & | & 2/5 & 0 & -1/5 \\ 0 & 1 & -2 & | & 1 & 0 & -1 \\ 0 & 0 & 1 & | & -19/51 & 5/51 & 17/51 \end{pmatrix} \tag{7}$$

$$\xrightarrow[R_2 \leftarrow R_2 + 2R_3]{R_1 \leftarrow R_1 - \frac{2}{5}R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left| \begin{matrix} 28/51 & -2/51 & -17/51 \\ 13/51 & 10/51 & -17/51 \\ -19/51 & 5/51 & 17/51 \end{matrix} \right. \tag{8}$$

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} 28/51 & -2/51 & -17/51 \\ 13/51 & 10/51 & -17/51 \\ -19/51 & 5/51 & 17/51 \end{pmatrix} \tag{9}$$

Now, Finding system of equations

$$\mathbf{AX} = \mathbf{C} \tag{10}$$

where $\mathbf{C} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{C} \tag{11}$$

$$\mathbf{X} = \begin{pmatrix} 28/51 & -2/51 & -17/51 \\ 13/51 & 10/51 & -17/51 \\ -19/51 & 5/51 & 17/51 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \tag{12}$$

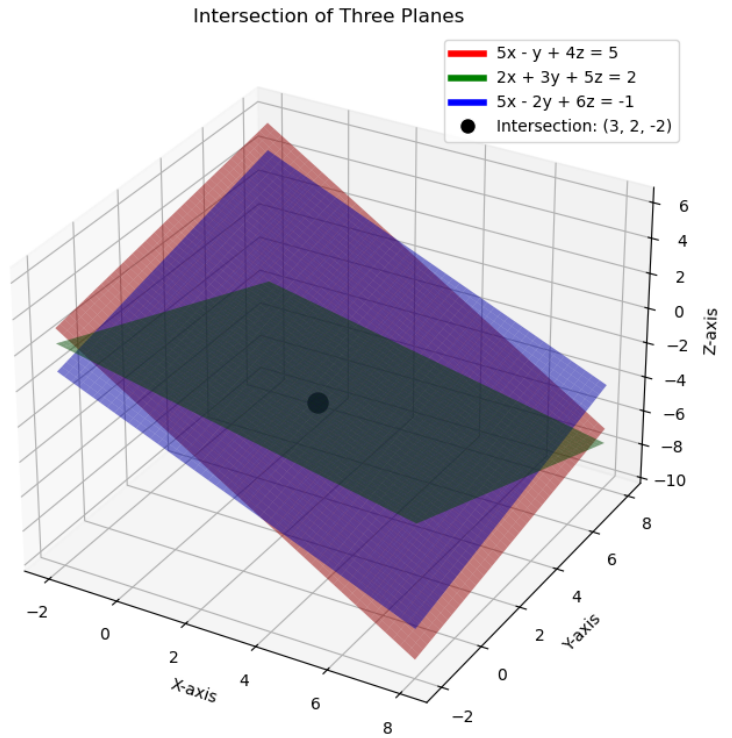$$\therefore \mathbf{X} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \tag{13}$$

Fig. 1