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Question

Let $\mathbf{R}$ be an $n \times n$ nonsingular matrix. Let $\mathbf{P}$ and $\mathbf{Q}$ be two $n \times n$ matrices such that $\mathbf{Q} = \mathbf{R}^{-1}\mathbf{P}\mathbf{R}$. If $\mathbf{x}$ is an eigenvector of $\mathbf{P}$ corresponding to a nonzero eigenvalue λ of $\mathbf{P}$, then

- $\mathbf{R}\mathbf{x}$ is an eigenvector of $\mathbf{Q}$ corresponding to eigenvalue λ of $\mathbf{Q}$
- $\mathbf{R}\mathbf{x}$ is an eigenvector of $\mathbf{Q}$ corresponding to eigenvalue $\frac{1}{\lambda}$ of $\mathbf{Q}$
- $\mathbf{R}^{-1}\mathbf{x}$ is an eigenvector of $\mathbf{Q}$ corresponding to eigenvalue λ of $\mathbf{Q}$
- $\mathbf{R}^{-1}\mathbf{x}$ is an eigenvector of $\mathbf{Q}$ corresponding to eigenvalue $\frac{1}{\lambda}$ of $\mathbf{Q}$

Theoretical Solution

Start with the eigenvector equation for P:

$$\mathbf{P}\mathbf{x} = \lambda\mathbf{x} \tag{1}$$

Pre-multiply both sides by $\mathbf{R}^{-1}$:

$$\mathbf{R}^{-1}(\mathbf{P}\mathbf{x}) = \mathbf{R}^{-1}(\lambda\mathbf{x}) \tag{2}$$

Now, we need to relate this to $\mathbf{Q}$. We know that $\mathbf{Q} = \mathbf{R}^{-1}\mathbf{P}\mathbf{R}$. Let's introduce the identity matrix $\mathbf{I} = \mathbf{R}\mathbf{R}^{-1}$ into our equation.

$$\mathbf{R}^{-1}\mathbf{P}(\mathbf{R}\mathbf{R}^{-1})\mathbf{x} = \lambda(\mathbf{R}^{-1}\mathbf{x}) \tag{3}$$

$$(\mathbf{R}^{-1}\mathbf{P}\mathbf{R})(\mathbf{R}^{-1}\mathbf{x}) = \lambda(\mathbf{R}^{-1}\mathbf{x}) \tag{4}$$

Substitute $\mathbf{Q} = \mathbf{R}^{-1}\mathbf{P}\mathbf{R}$ into the equation:

$$\mathbf{Q}(\mathbf{R}^{-1}\mathbf{x}) = \lambda(\mathbf{R}^{-1}\mathbf{x}) \tag{5}$$

$\therefore$ $\mathbf{R}^{-1}\mathbf{x}$ is an eigenvector of $\mathbf{Q}$ corresponding to the eigenvalue λ of $\mathbf{Q}$.