12.686

EE25BTECH11049 - Sai Krishna Bakki

Question:

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are vectors of length 4.

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}$$

It is known that $\mathbf{B}$ is not a scalar multiple of $\mathbf{A}$. Also, $\mathbf{C}$ is linearly independent of $\mathbf{A}$ and $\mathbf{B}$. Further, $\mathbf{D} = 3\mathbf{A} + 2\mathbf{B} + \mathbf{C}$. The rank of the matrix $\begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{pmatrix}$ is

Solution:

The rank of the matrix is defined as the number of linearly independent columns or rows it contains. Let's analyze the linear independence of the columns of the given matrix, which are the vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$.

Given:

1) $\mathbf{A}, \mathbf{B}$ are linearly independent
2) $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are linearly independent
3) $\mathbf{D} = 3\mathbf{A} + 2\mathbf{B} + \mathbf{C}$ where $\mathbf{D}$ is linearly dependent on $\mathbf{A}, \mathbf{B}, \mathbf{C}$

$$\mathbf{D} = \begin{pmatrix} 3a_1 + 2b_1 + c_1 \\ 3a_2 + 2b_2 + c_2 \\ 3a_3 + 2b_3 + c_3 \\ 3a_4 + 2b_4 + c_4 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} \tag{1}$$

$$\mathbf{M} = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{pmatrix} \tag{2}$$

$$\mathbf{M} = \begin{pmatrix} a_1 & b_1 & c_1 & 3a_1 + 2b_1 + c_1 \\ a_2 & b_2 & c_2 & 3a_2 + 2b_2 + c_2 \\ a_3 & b_3 & c_3 & 3a_3 + 2b_3 + c_3 \\ a_4 & b_4 & c_4 & 3a_4 + 2b_4 + c_4 \end{pmatrix} \tag{3}$$

There is one linearly dependent column so there are three linearly independent columns.

$\therefore$ The rank of $\mathbf{M}$ is 3.