9.5.11

EE25BTECH11049 - Sai Krishna Bakki

Question:

Two pipes running together can fill a tank in 100/9 minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Solution:

Given:

Let the time taken by the faster pipe to fill the tank be **'x'** minutes and the time taken by the slower pipe to fill the tank be **'x+5'** minutes.

The amount of the tank each pipe fills in one minute is its work rate.

- Work rate of the first pipe $= \frac{1}{x}$
- Work rate of the second pipe $= \frac{1}{x+5}$

When working together, they fill the tank in $\frac{100}{9}$ minutes. Therefore, their combined work rate is the reciprocal, $\frac{9}{100}$ of the tank per minute.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100} \tag{1}$$

$$\frac{2x+5}{x^2+5x} = \frac{9}{100} \tag{2}$$

$$\implies y = 9x^2 - 155x - 500 = 0 \tag{3}$$

which can be expressed as the conic

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-155}{2} \\ \frac{-1}{2} \end{pmatrix}, f = -500 \tag{5}$$

To find the roots of (3), we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{6}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

The parameter κ for the points of intersection is found using the formula:

$$\kappa = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{8}$$

where $g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f$.

using (8). The values of κ are given by

$$\kappa_i = \frac{1}{9}\left(\frac{155}{2} \pm \sqrt{\left(\frac{-155}{2}\right)^2 + 4500}\right) \tag{9}$$

$$\implies \kappa_1 = 20, \kappa_2 = \frac{-25}{9} \tag{10}$$

Hence the points of intersection are

$$\mathbf{h} + \kappa\mathbf{m} = \begin{pmatrix} 20 \\ 0 \end{pmatrix}\begin{pmatrix} \frac{-25}{9} \\ 0 \end{pmatrix} \tag{11}$$
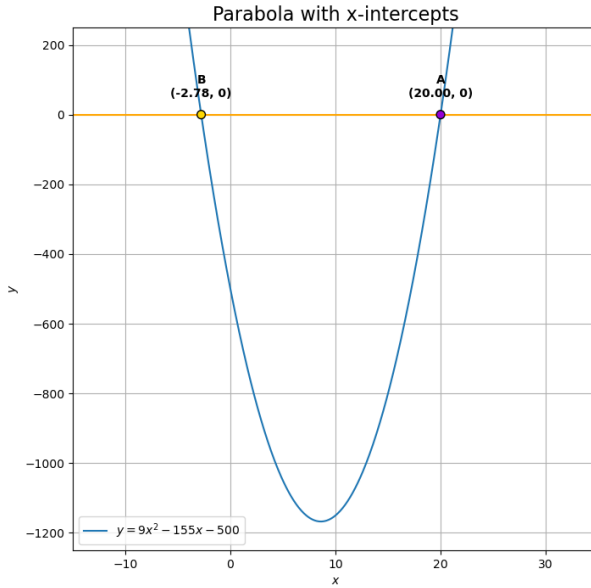
Hence the solutions of (3) are x=20 and x=$\frac{-25}{9}$.

Fig. 1