12.62

EE25BTECH11049 - Sai Krishna Bakki

Question:
The eigenvalues of the matrix

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are
Solution:
Given

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

To find eigenvalues of the matrix $\mathbf{A}$

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \tag{2}$$

$$(\mathbf{A} - \lambda\mathbf{I})\,\mathbf{x} = 0 \tag{3}$$

$$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0 \tag{4}$$

$$\begin{vmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \tag{5}$$

$$(2-\lambda)\left((2-\lambda)(1-\lambda) - 0\right) - 3(3)(1-\lambda) = 0 \tag{6}$$

$$(1-\lambda)\left((2-\lambda)^2 - 9\right) = 0 \tag{7}$$

$$\lambda = 1, -1, 5 \tag{8}$$

$\therefore$ The eigenvalues of the matrix are 1,-1 and 5.