2.10.77

EE25BTECH11049 - Sai Krishna Bakki

Question:
The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are

1) Collinear
2) Vertices of a parallelogram
3) Vertices of a rectangle
4) None of these

Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} -a \\ -b \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{D} = \begin{pmatrix} a^2 \\ ab \end{pmatrix} \tag{4.1}$$

Condition for the points to be vertices of a parallelogram is

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{4.2}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{C} - \mathbf{D} = \begin{pmatrix} a - a^2 \\ b - ab \end{pmatrix} \tag{4.3}$$

But

$$\mathbf{B} - \mathbf{A} \neq \mathbf{C} - \mathbf{D} \tag{4.4}$$

If $\mathbf{B} - \mathbf{A} \neq \mathbf{C} - \mathbf{D}$ then the points cannot be vertices of a rectangle too because every rectangle is a specific type of parallelogram.

Condition for the points to be collinear is

$$\text{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{D} \end{pmatrix} = 1 \tag{4.5}$$

$$\text{rank}\begin{pmatrix} a & a - a^2 \\ b & b - ab \end{pmatrix} \tag{4.6}$$

$$\begin{pmatrix} a & a - a^2 \\ b & b - ab \end{pmatrix} \xleftrightarrow{R_2 \to \frac{-b}{a} R_1 + R_2} \begin{pmatrix} a & a - a^2 \\ 0 & 0 \end{pmatrix} \tag{4.7}$$

The number of non zero rows in the row reduced matrix (also known as *echelon form*) is defined as the rank.
For the above matrix, Rank is one.
Therefore, we can conclude that four points are collinear.

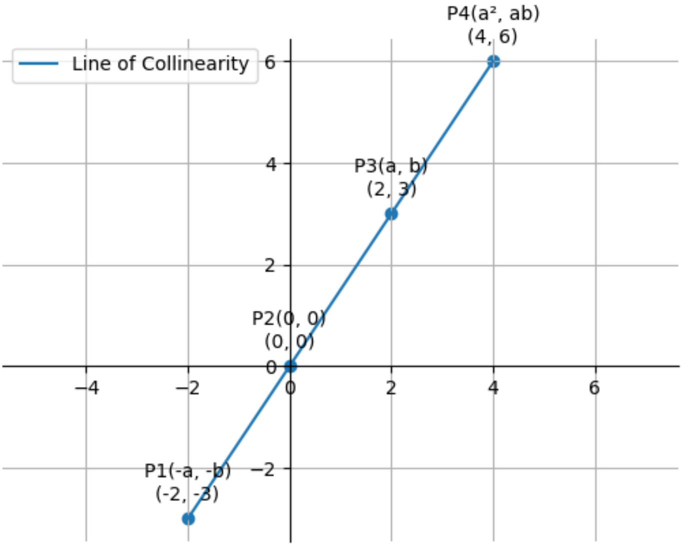
P4(a², ab)
(4, 6)
Line of Collinearity
P3(a, b)
(2, 3)
P2(0, 0)
(0, 0)
P1(-a, -b)
(-2, -3)
6
4
2
0
−2
−4
−2
0
2
4
6

Fig. 4.1