10.6.4

EE25BTECH11049 - Sai Krishna Bakki

Question:
Draw a circle of radius 5cm. From a point 8cm away from its centre, construct a pair of tangents to the circle.
Solution:
Let's take center as origin **O** and a point 8cm aways from its center as $\mathbf{h} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$.
The equation of a circle is given by

$$g(\mathbf{x}) = \|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.1}$$

for

$$\textbf{center} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{since } \mathbf{c} = -\mathbf{u} \tag{0.2}$$

we get

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.3}$$

we also know for any circle

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.4}$$

radius(r) = 5cm, we know that $r^2 = \|u\|^2 - f$ which gives us

$$f = -25 \tag{0.5}$$

By using below equation, we can determine the direction vectors of the tangent lines from an external point

$$\mathbf{m}^T\left[(\mathbf{Vh} + \mathbf{u})(\mathbf{Vh} + \mathbf{u})^T - \mathbf{V}g(\mathbf{h})\right]\mathbf{m} = 0 \tag{0.6}$$

$$g(h) = 39 \tag{0.7}$$

where $\mathbf{m} = \begin{pmatrix} m_x \\ m_y \end{pmatrix}$ is the direction vectors of a tangent line.
Substituting values in (**??**), we get

$$\begin{pmatrix} m_x & m_y \end{pmatrix}\begin{pmatrix} 25 & 0 \\ 0 & -39 \end{pmatrix}\begin{pmatrix} m_x \\ m_y \end{pmatrix} = 0 \tag{0.8}$$

$$25m_x^2 - 39m_y^2 = 0 \tag{0.9}$$

$$\tag{0.10}$$

The slopes of the tangent line is given by $k = \frac{d_x}{d_x}$. we solve for the slopes:

$$k^2 = \frac{25}{39} \implies k = \pm\frac{5}{\sqrt{39}} \tag{0.11}$$

Now, normal vectors of tangent lines are

$$\mathbf{n_1} = \begin{pmatrix} 5 \\ \sqrt{39} \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 5 \\ -\sqrt{39} \end{pmatrix} \tag{0.12}$$

Equations of tangent lines which passes through a point 8cm away from the center are

$$\mathbf{n_1^T x} = c, \mathbf{n_2^T x} = c \tag{0.13}$$

substituting **h** in line equation to get c,we get

$$c = 40 \tag{0.14}$$

$$\begin{pmatrix} 5 & \sqrt{39} \end{pmatrix} \mathbf{x} = 40 \tag{0.15}$$

$$\begin{pmatrix} 5 & -\sqrt{39} \end{pmatrix} \mathbf{x} = 40 \tag{0.16}$$

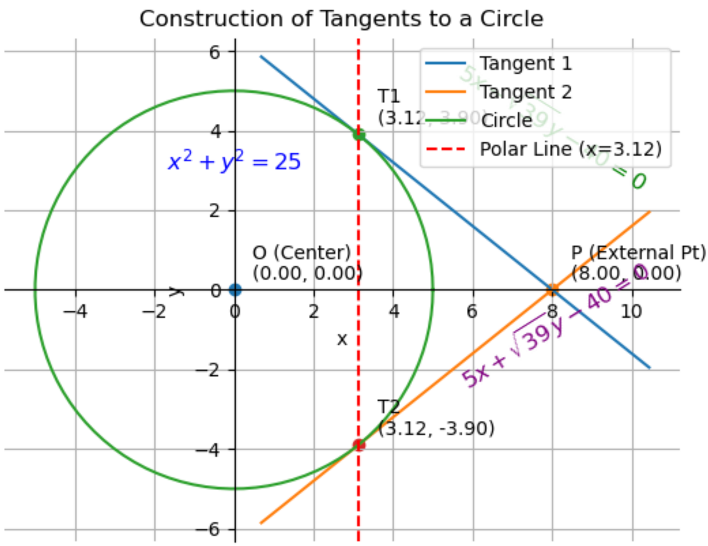
Now, solve for points of contact, for that we use the following formulae

$$\mathbf{q_i} = \left(\pm r \left(\frac{\mathbf{n_i}}{||\mathbf{n_i}||} \right) - \mathbf{u} \right) \tag{0.17}$$

we get

$$\mathbf{q_1} = \begin{pmatrix} \frac{25}{8} \\ \frac{5\sqrt{39}}{8} \end{pmatrix}, \mathbf{q_2} = \begin{pmatrix} \frac{25}{8} \\ -\frac{5\sqrt{39}}{8} \end{pmatrix} \tag{0.18}$$

Fig. 0.1


Construction of Tangents to a Circle
Tangent 1
Tangent 2
Circle
Polar Line (x=3.12)
T1
(3.12, 3.90)
T2
(3.12, -3.90)
$x^2 + y^2 = 25$
O (Center)
(0.00, 0.00)
P (External Pt)
(8.00, 0.00)
$5x - \sqrt{39}y - 40 = 0$
$5x + \sqrt{39}y - 40 = 0$
x
y