

12.582

EE25BTECH11049 - Sai Krishna Bakki

Question:

The position vector OP of point $\mathbf{P} = (20,10)$ is rotated anti-clockwise in the X-Y plane by an angle $\theta = 30^\circ$ such that point \mathbf{P} occupies position \mathbf{Q} . The coordinates (x,y) of \mathbf{Q} is

Solution:

Given

$$\mathbf{P} = \begin{pmatrix} 20 \\ 10 \end{pmatrix}, \theta = 30^\circ \quad (1)$$

we use

$$\mathbf{x}_n = \mathbf{R}\mathbf{x}_0 \quad (2)$$

where \mathbf{R} is Rotation matrix

$$\mathbf{Q} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \mathbf{P} \quad (3)$$

$$\mathbf{Q} = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} \begin{pmatrix} 20 \\ 10 \end{pmatrix} \quad (4)$$

$$\mathbf{Q} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 20 \\ 10 \end{pmatrix} \quad (5)$$

$$\mathbf{Q} = \begin{pmatrix} 10\sqrt{3} - 5 \\ 10 + 5\sqrt{3} \end{pmatrix} \quad (6)$$

Using approximation, the coordinates of \mathbf{Q} is

$$\mathbf{Q} = \begin{pmatrix} 12.32 \\ 18.66 \end{pmatrix} \quad (7)$$

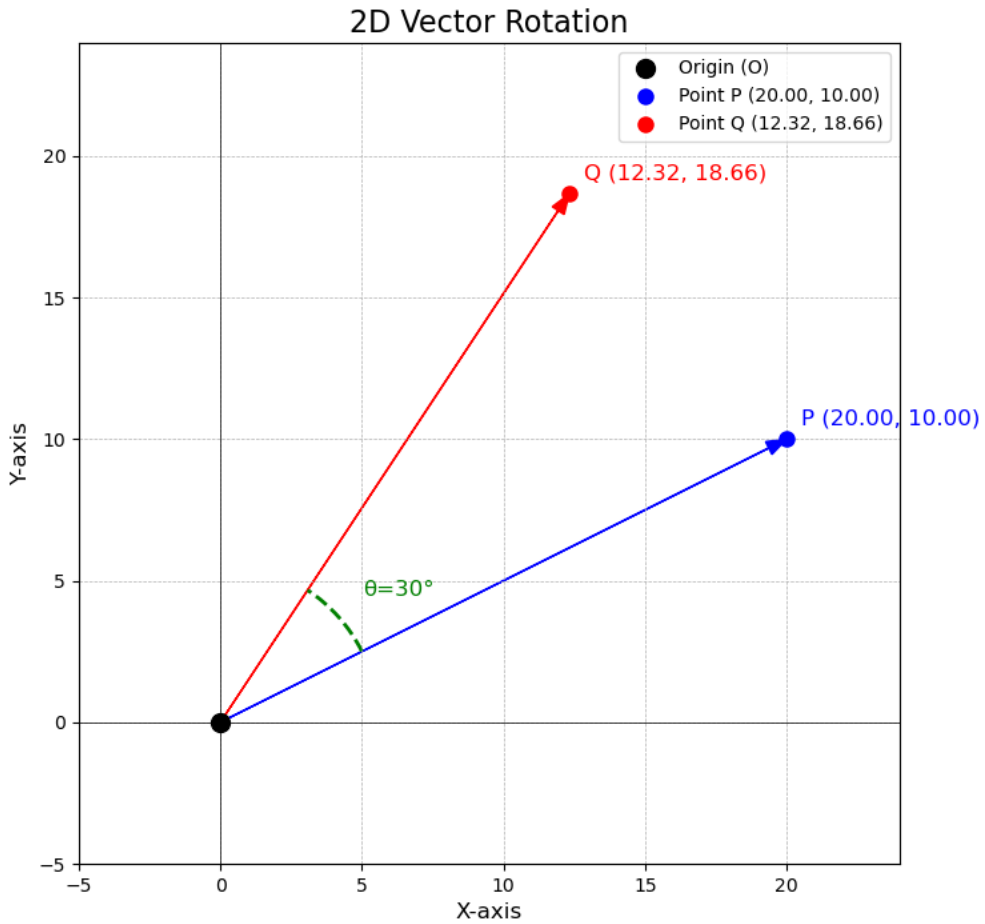


Fig. 1