5.13.74

EE25BTECH11049 - Sai Krishna Bakki

Question:
The trace of a square matrix is defined to be the sum of its diagonal entries. If **A** is a 2 x 2 matrix, such that the trace of **A** is 3 and the trace of $\mathbf{A}^3$ is -18, then the value of the determinant of **A** is
Solution:
Given:

$$\text{tr}(\mathbf{A}) = 3 \tag{0.1}$$

$$\text{tr}(\mathbf{A^3}) = -18 \tag{0.2}$$

$$\text{tr}(\mathbf{I}) = 2 \tag{0.3}$$

Let the eigenvalues of 2x2 matrix A be λ_1 and λ_2, we know that trace is the sum of eigenvalues.

$$\lambda_1 + \lambda_2 = 3 \tag{0.4}$$

we are given that $\text{tr}(\mathbf{A^3}) = -18$. Since the eigenvalues of $\mathbf{A}^3$ are λ_1^3 and λ_2^3, the trace of $\mathbf{A}^3$ is their sum.

$$\lambda_1^3 + \lambda_2^3 = -18 \tag{0.5}$$

We can use the algebraic identity for the sum of cubes to connect our two equations (0.4) and (0.5).

$$\lambda_1^3 + \lambda_2^3 = (\lambda_1 + \lambda_2)((\lambda_1 + \lambda_2)^2 - 3\lambda_1\lambda_2) \tag{0.6}$$

Substituting the equations (0.4) and (0.5) in above equation, we get

$$\lambda_1\lambda_2 = 5 \tag{0.7}$$

But determinant of **A** is $\lambda_1\lambda_2$.
Therefore, the value of the determinant of **A** is 5.