Basic Statistics Level-2

Saikrishna Budi

Linkedin : http://linkedin.com/in/sai-krishna-budi-3b0341209

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Sol:

Since the work begins after the 10 minutes the remaining time left will be 50 minutes.

Probability that the service manager cannot meet his commitment=P(x>50) =1-P(x<=50)

Where "X" is the time taken to complete the work. By converting 50 to z-score

Standard nominal variable Z=(X-μ)/σ= (50-45)/8

=0.73237

=73.237%

Probability that the service manager will not meet his commitment =100-73.237

=26.76

=0.2676

Option **B**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:

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A.) More employees at the processing center are older than 44 than between 38 and 44.

For 44

Probability of employees >44=P(X>44) =1-P(X<=44)

Z=(X-μ)/σ= (44-38)/6 =6/6

Z=1

P(Z>=1) = 84.13 %

Probability of employees greater than 44=100-84.13=15.86

For 38

Probability of employees >38=P(X>38)=1-P(X<=38)

Z=(X-μ)/σ=(38-38)/6 =0/6

Z=0

P(Z=0) = 50 %

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-50

=84.13-50=34.13

Therefore the statement that "More employees at the processing center are older than 44 than between 38 and 44" is **TRUE.**

B.) A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:

Z=(X-μ)/σ

= (30-38)/6

= -1.33333

P(Z<-1.3333) =0.09176

=9.176%

So, the number of employees under 30 with probability 0.9176=400*0.9176=36.86

=36

The statement about "A training program for employees under the age of 30 at the center would be expected to attract about 36 employees" is **TRUE.**

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3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Sol:

- The difference between 2 X1 and X1 + X2 is X1 - X2.
- The distribution of X1 - X2 is also normal, with a mean of μ - μ = 0 and a variance of σ2 + σ2 = 2σ2. Therefore, the standard deviation of X1 - X2 is sqrt (2) * σ.
- The parameters of the distribution of X1 - X2 are 0 for the mean and sqrt (2) * σ for the standard deviation.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Sol:

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. By finding the Standard Normal Variable Z (Z Value), we can calculate the X values

$Z=(X- \mu) / \sigma$

For Probability 0.005 the Z Value is -2.57 (from Z Table).

$Z * \sigma + \mu = X$

$Z(-0.005)*20+100 = -(-2.57)*20+100$

$= 151.4$

$Z(+0.005)*20+100 = (-2.57)*20+100$

$= 48.6$

So, the correct option is **D**

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:

A.) Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

- Mean profit is RS 540 million
- Standard deviation is RS 225 million
- State norms interval (0.95,540,225)
- Range is RS 99.0081034, 980.991896

B.) Specify the 5^{th} percentile of profit (in Rupees) for the company

formula $X=X=\mu+Z\sigma$;

Where in form Z table,5 percentile =-1.645

$X=540(-1.645) *225$

$X=169.875$

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C.) Which of the two divisions has a larger probability of making a loss in a given year?

Probability of division making a loss p(X<0)

By using Stats.norm.cdf (0,5,3)

We got the result is 0.0477903