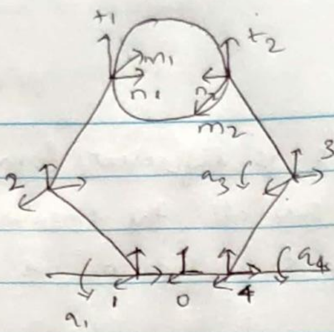
(1)

$$R_1 = \begin{pmatrix} \hat{n}_1 & \hat{m}_1 & \hat{t}_1 \end{pmatrix}$$

$$R_2 = \begin{pmatrix} \hat{n}_2 & \hat{m}_2 & \hat{t}_2 \end{pmatrix}$$



for finding jacobian.

impact of $q_1 q_2 q_3 q_4$

@ c_1 c_2.

$$\underset{c_1,\,hand}{\overset{N}{\xi}} = \underbrace{J_h}_{\text{hand jacobian}} \dot{q} \; \xrightarrow{\quad} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix}$$

$$J_h = \begin{pmatrix} \hat{z}_1 \times (O_0^{c_1} - O_0^1) & \hat{z}_2 \times (O_0^{c_1} - O_0^2) & \hat{z}_3 \times (O_0^{c_1} - O_0^3) & \hat{z}_4 \times (O_0^{c_1} - O_0^4) \\ \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 \\ \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 \\ z_1 \times (O_0^{c_2} - O_0^1) & z_2 \times (O_0^{c_2} - O_0^2) & \hat{z}_3 \times (O_0^{c_2} - O_0^3) & z_4 \times (O_0^{c_2} - O_0^4) \end{pmatrix}$$

where $O_0^{c_1} = \begin{pmatrix} -0.015 \\ 0.05 \\ 0 \end{pmatrix}$ $\quad O_0^{c_2} = \begin{pmatrix} 0.015 \\ 0.05 \\ 0 \end{pmatrix}$

$$\hat{z}_1 = \hat{z}_2 = \hat{z}_3 = \hat{z}_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$O_0^1 = \begin{pmatrix} -0.015 \\ 0 \\ 0 \end{pmatrix} \quad O_0^2 = \begin{pmatrix} -0.0583 \\ 0.025 \\ 0 \end{pmatrix} \quad O_0^3 = \begin{pmatrix} 0.0583 \\ 0.025 \\ 0 \end{pmatrix}$$

$$O_0^4 = \begin{pmatrix} 0.015 \\ 0 \\ 0 \end{pmatrix}$$

Substituting above values

$$J_h = \begin{pmatrix} -0.05 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$
J_h = \begin{bmatrix}
-0.05 & -0.025 & -0.025 & -0.05 \\
0 & 0.043 & -0.043 & -0.03 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
-0.05 & -0.025 & -0.025 & -0.05 \\
0.03 & 0.073 & -0.0433 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

using G^T from previous. HW. $\Rightarrow J_{hand\text{-}object} = (G^T)^+ J_h$

$$
J_{handobject} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
-0.015 & -0.015 & -0.015 & -0.015 \\
0.05 & 0.025 & 0.025 & 0.05 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-0.0007 & -0.0029 & 0.0029 & 0.0029
\end{bmatrix}
$$

Now. $\xi_o^N = J_{hand\text{-}object}\, \dot{q}$

$$\dot{q} = (J_{handobject})^+ \xi_o^N$$

from previous $\xi_o^N = \begin{bmatrix} 0 \\ -0.0075 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\dot{q} = \begin{bmatrix} -0.25 \\ 0.5 \\ -0.25 \\ 0.5 \end{bmatrix}$$

(2)

3D $\longrightarrow$ 2D Simplification of Grasp matrix formulation

$$G^T_{ci} = H R^{ci}_N P_i$$

$$\xi^N_{ci} = \begin{bmatrix} v^N_{ci} \\ w^N_o \end{bmatrix} = \begin{bmatrix} v_{ci,x} \\ v_{ci,y} \\ w^N_{o\,z} \end{bmatrix} = \{ \, \} \quad c_i, o$$

$$v^N_o = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \qquad c_i = c_x\hat{i} + c_y\hat{j} + c_z\hat{k}$$

$$w^N_o = w_x\hat{i} + w_y\hat{j} + w_z\hat{k} \qquad O^N = o_x\hat{i} + o_y\hat{j} + o_z\hat{k}$$

assuming 2D Planar.

$$v^N_o = v_x\hat{i} + v_y\hat{j} \qquad c_i = c_x\hat{i} + c_y\hat{j}$$

$$w^N_o = w_z\hat{k} \qquad O^N = o_x\hat{i} + o_y\hat{j}$$

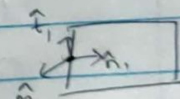
$$v^N_{ci} = v^N_o + w^N_o \times (c^N_i - O^N)$$

$$v^N_{ci} = v_{x,o}\hat{i} + v_{y,o}\hat{j} + w_z(c_x - o_x)\hat{j} + w_z(c_y - o_y)(-\hat{i})$$

in matrix form.

$$\xi^N_{ci} = \begin{bmatrix} v^N_{ci\,x} \\ v^N_{ci\,y} \\ w^N_{z\,ci} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -(c_y - o_y) \\ 0 & 1 & c_x - o_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{x,o} \\ v_{y,o} \\ w_{z,o} \end{bmatrix}$$

Now $\xi^{ci}_{ci} = R^{ci}_N P_{i,2D} \, \xi^N_{ci}$



$$R_1 = [\hat{n}_1, \hat{t}_1, \hat{m}_1]$$

$$G^T = H R^{ci}_N P_{i,2D}$$

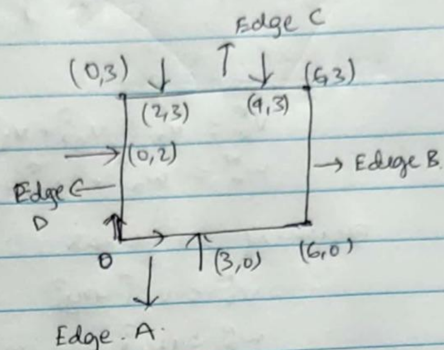
where $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $R^{ci}_N = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_{i,2D} = \begin{bmatrix} 1 & 0 & (-(c_y - o_y)) \\ 0 & 1 & c_x - o_x \\ 0 & 0 & 1 \end{bmatrix}$$

θ - amount of rotation

code is self explanatory with appropiate functionnames and variable names. and added comments whenever needed

from the code output after the 5th vector is introduced. the quality metrics are as follows



~~[illegible]~~ ~~[illegible]~~

$\max(Q_{MSV}) = 2.34$

$\text{argmax}(Q_{MSV}) = (6,0)$

$\max(Q_{volume}) = 27.018$

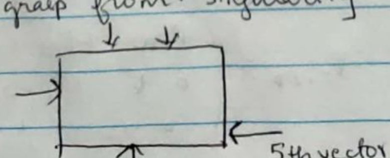
$\text{argmax}(Q_{volume}) = (6,0)$

$\max(Q_{isotropy}) =$ ~~[illegible]~~ 0.47

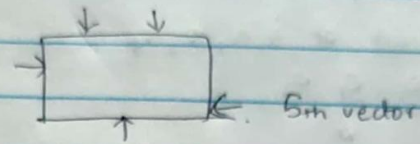
$\text{argmax}(Q_{isotropy}) =$ ~~[illegible]~~ $(3.8, 0)$

These metrics are trying to analyse grasp stabity based on the Grasp matrix singular values.

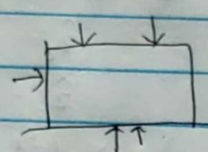
① as the minimum singular value is high. the farther away is the grasp from singularity



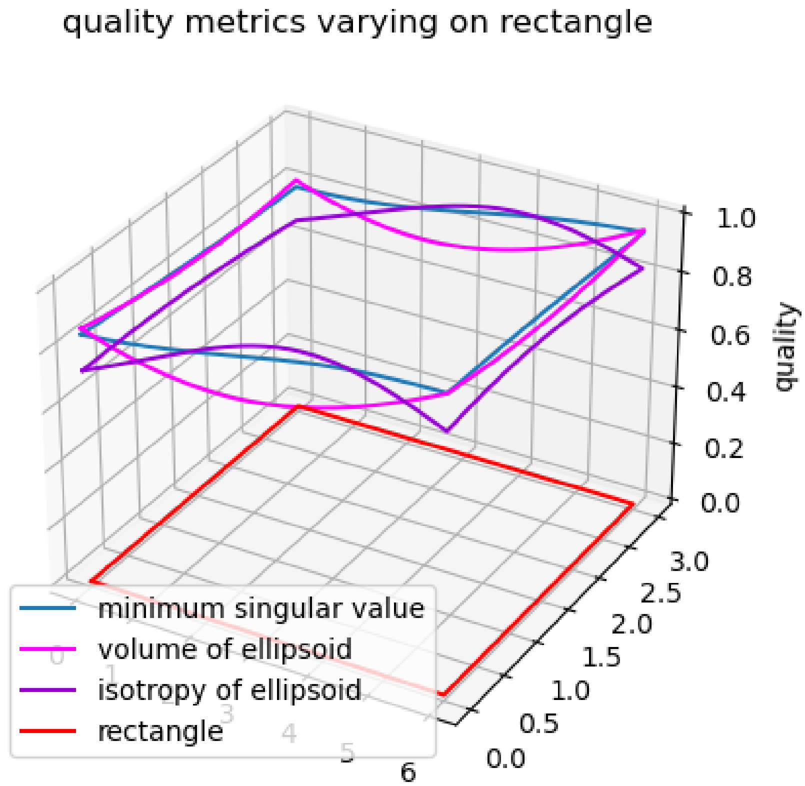
② this is different from Q_{volume} which is trying to make sure all the singular values are maximized but results in same location as previous one.



③ finally, isotropy is measuring $\frac{\sigma_{min}}{\sigma_{max}}$, which when maximized is directly making the Grasp stable across different configurations



However this doesn't prevent the configuration from being unstable and can clearly be highly susceptible to wrenches.


quality metrics varying on rectangle
quality
1.0
0.8
0.6
0.4
0.2
0.0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
3
4
5
6
minimum singular value
volume of ellipsoid
isotropy of ellipsoid
rectangle