Use of Neural Networks (Autoencoding) for Stock Picking for the implementation of the Markovitz Portfolio Theory

Sai Kumar M 2014B3A70951H
Arvind A S 2014B3A70381H

Abstract

One of the major obstacles faced in financial forecasts is the dimensionality problem. Therefore, if we can pick stocks with smaller reconstruction errors, then any financial technique tried on them should work better. The Markovitz Theorem gets affected by the same problem, we hence are trying to find out if better results can be obtained if we use stocks with smaller reconstruction error.

Modern Portfolio Theory

Markovitz portfolio theory or the modern portfolio theory hypothesizes that risk-averse investors can maximise return for a given level of risk or conversely minimise risk for a given level of return.

It suggests that it is possible to construct an efficient 'frontier' of optimal portfolios using historic data of a collection of stocks by varying risk and return at each point. This efficient portfolio would give the composition of stocks (weights) that maximise expected return for specified risk measured by standard deviation. The reduction in risk occurs through diversification of stocks as each stock's price is differently correlated with the price of the other. This reduces the effect of adverse price changes of one stock on the value of the portfolio to be minimal.

Since the efficient portfolio frontier can be approximated to a second-degree polynomial, we solve for weights using a convex optimiser. The input for the convex optimiser has to be in the form of a quadratic equation with constraints in the form of matrices. An example is shown below.

Suppose we have to create a portfolio of two stocks, and suppose our surface function is $min_{x,y} \frac{1}{2} x^2 + 3x + 4y$

Subject to

x, y >= 0 implying stocks cannot be shorted (no negative weights)

x + y = 1 implying all available cash should be invested.

These parameters can be converted into matrix representation as follows

$$\frac{1}{2} \{x, y\} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \{x, y\} + \{3, 4\} \{x, y\}$$

And the conditions can be converted into

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

The weights would be defined by the matrix $[x,y]^T$. P is the first matrix in the surface function containing the term of the highest order, Q is the second matrix, G is the left side of the inequality in the constraints equation and H is the right.

By inputting these constraints into the convex solver in python, we get the resultant weights required in the form of a matrix. Expanding the matrix unknown matrix to n x 1 dimension, we can create the function for a portfolio containing 'n' stocks with n + 1 constraints.

As $\min_X X^T PX + QX$

$GX \leq H$

Autoencoding

Autoencoding is a technique that is used to solve issues related to high dimensionality. In particular, it is a non-linear of finding the number of dimensions (attributes) that can be reduced for any analysis without major loss in accuracy. This can be seen by calculating the reconstruction error.

In Autoencoding, the information is encoded into a hidden layer with a reduced set of dimensions and then decoded back into the original number of dimensions. This encoding and decoding will lead to a loss in accuracy. The difference is reconstruction error. There are several ways to calculate the reconstruction error including absolute difference, square of errors etc.

Therefore, we can infer that information with smaller reconstruction errors can be characterised with a smaller number of dimensions than those with larger reconstruction errors.

Methodology:

First, we obtain the data in wide format and check for inconsistencies in the data. Once the inconsistencies have been handled, companies are chosen that satisfy the survivorship bias of 240 months starting from January 1980.

Autoencoding is performed over all such available companies and 30 companies with least reconstruction error are chosen to construct a portfolio called a reconstructed portfolio. Another 30 companies satisfying survivorship bias and not exclusive to the reconstructed portfolio are chosen randomly by a random generator function to construct a random portfolio for comparison.

Next, Markovitz portfolio weights for target returns ranging from 1-2% per month are calculated for both portfolios with constraints that all money is invested and no shorting of stock is allowed. The companies in both these portfolios are held for the next 120 months with rebalancing at the end of every month for each target return.

Results

For all target rates (from 1.0% to 1.5% differing by 0.1%), the portfolio of stocks chosen according to autoencoding did better than the portfolio of stocks chosen randomly. The performance ability was measured by finding the sum of square of differences between the target rate and the actual rate of return.

Furthermore, in terms of risk (measured by standard deviation), the portfolio of stocks chosen due to reconstruction error had lower standard deviation and hence lower risk than the portfolio of rando stocks.

Graphs

Given below are the graphs for the portfolio returns over a 120-month period (rolling).

Random Portfolio – RED

Portfolio (based on reconstruction error) - BLUE

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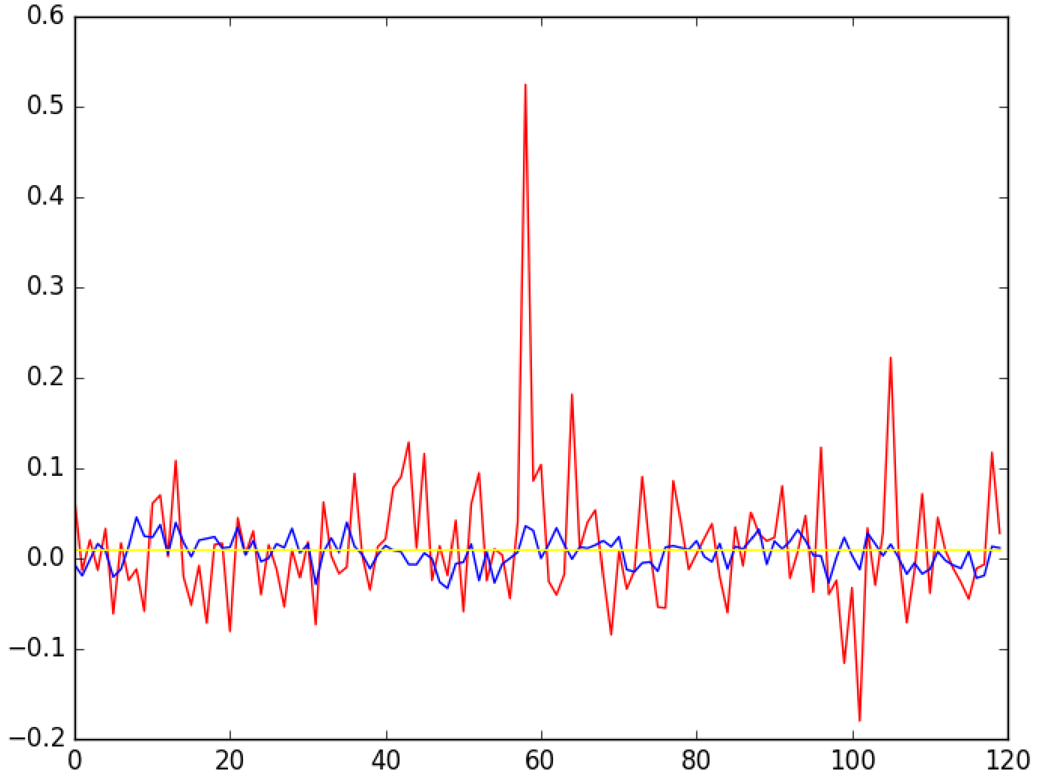

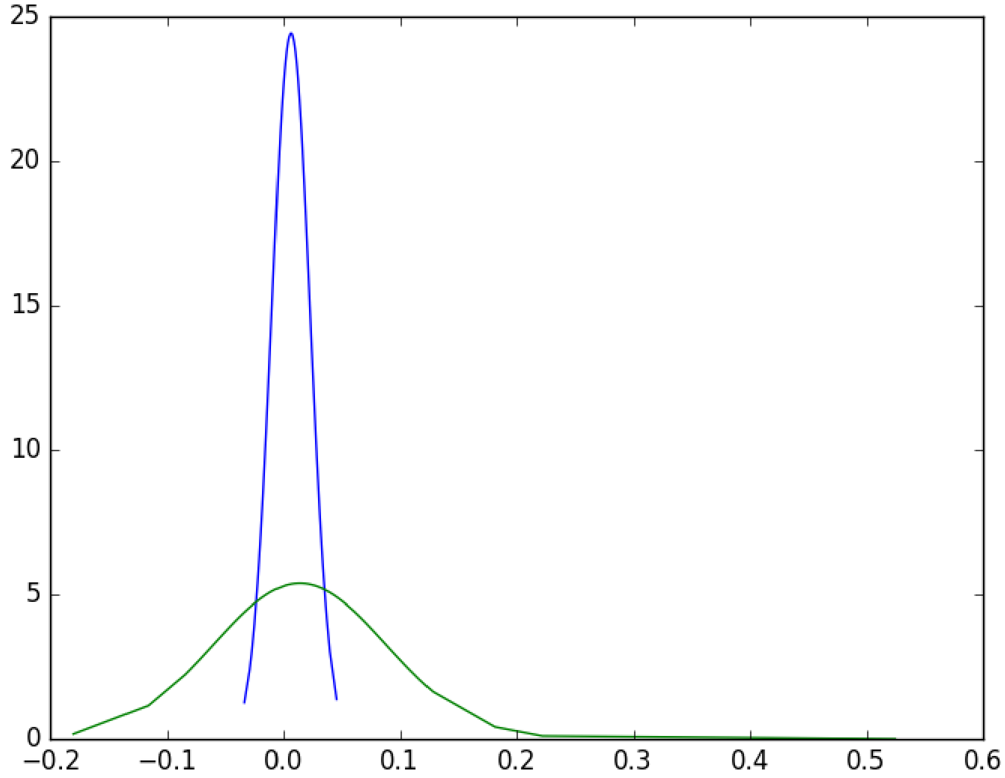
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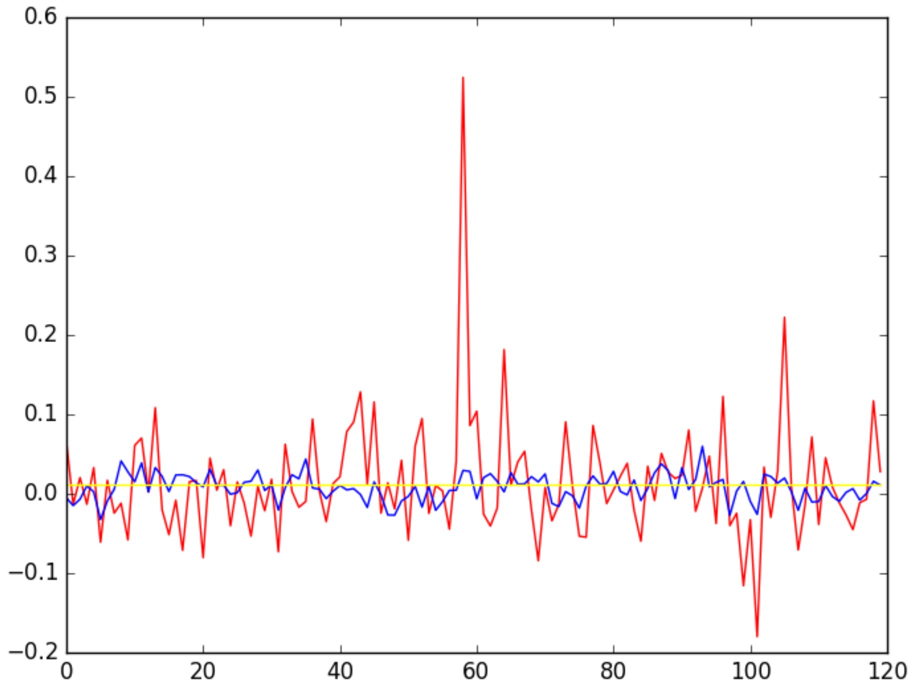
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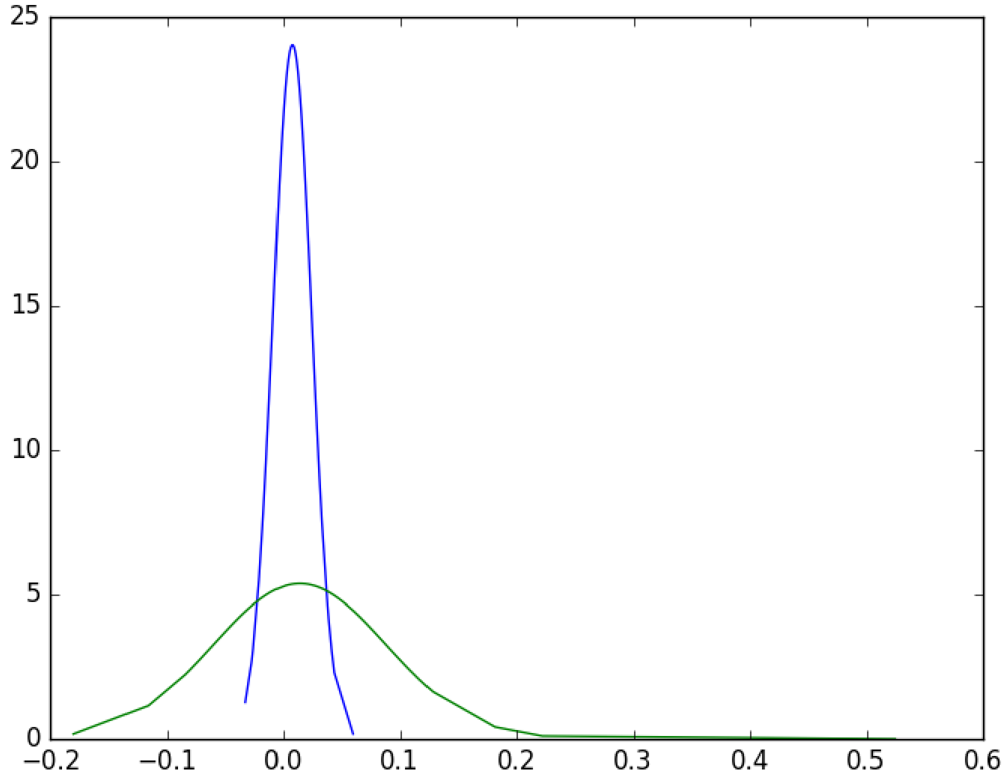
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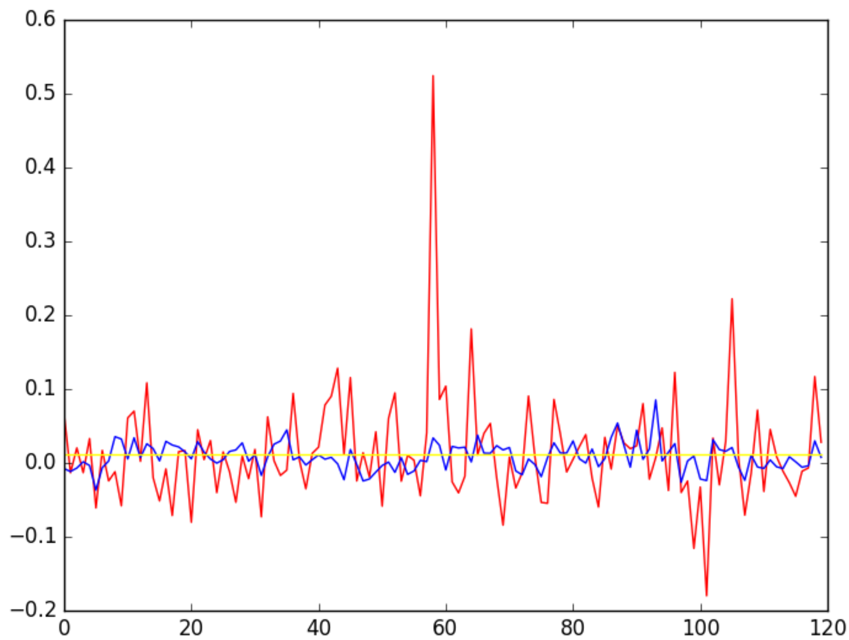
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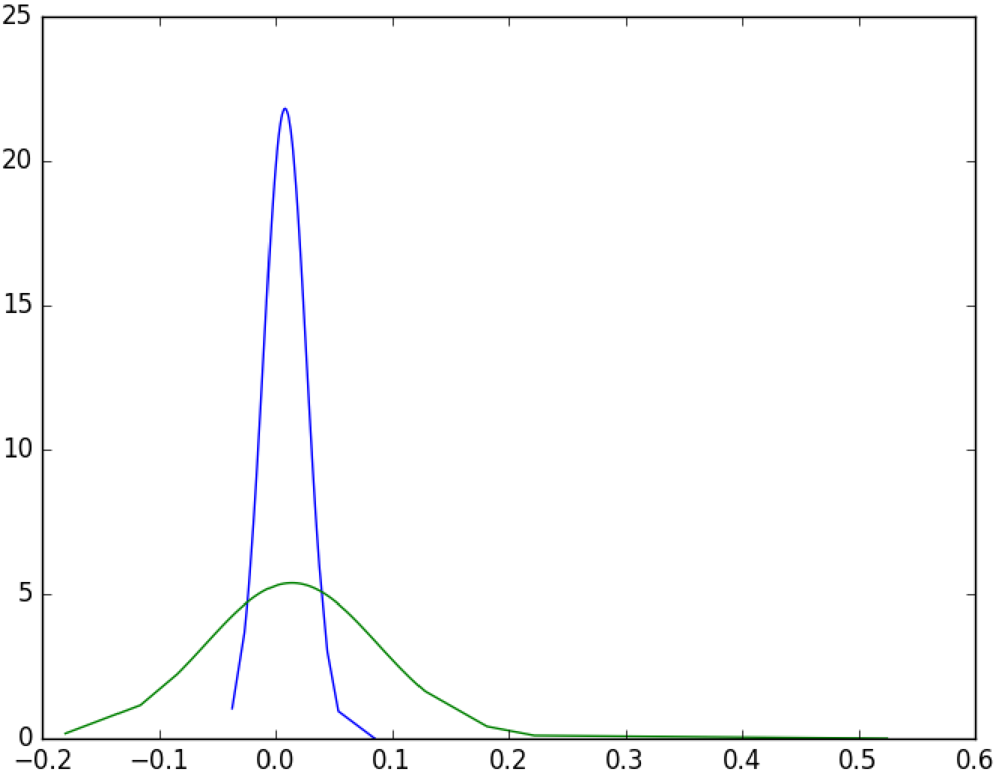
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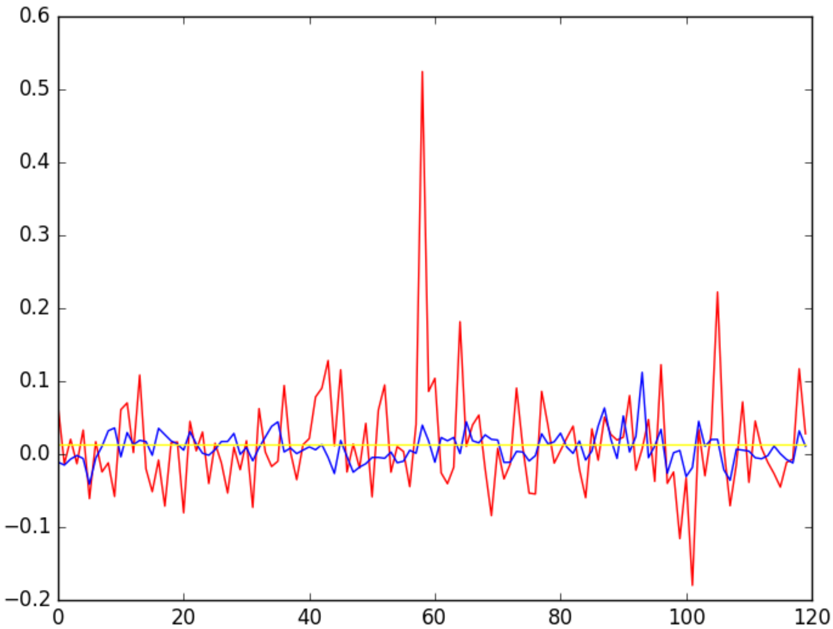
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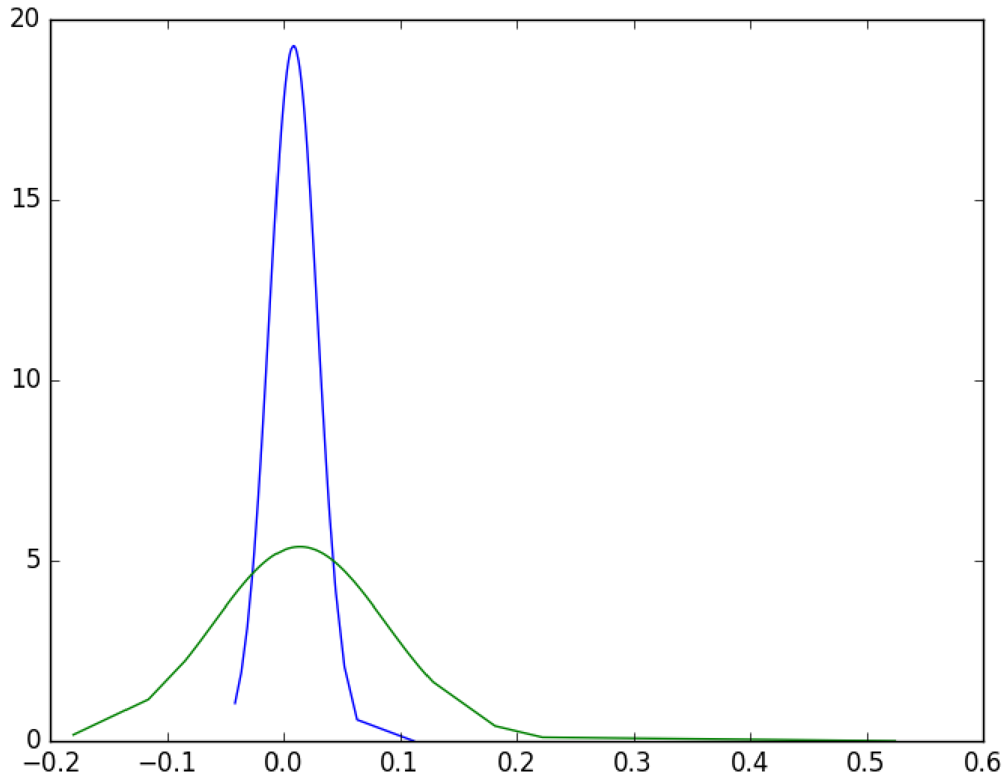
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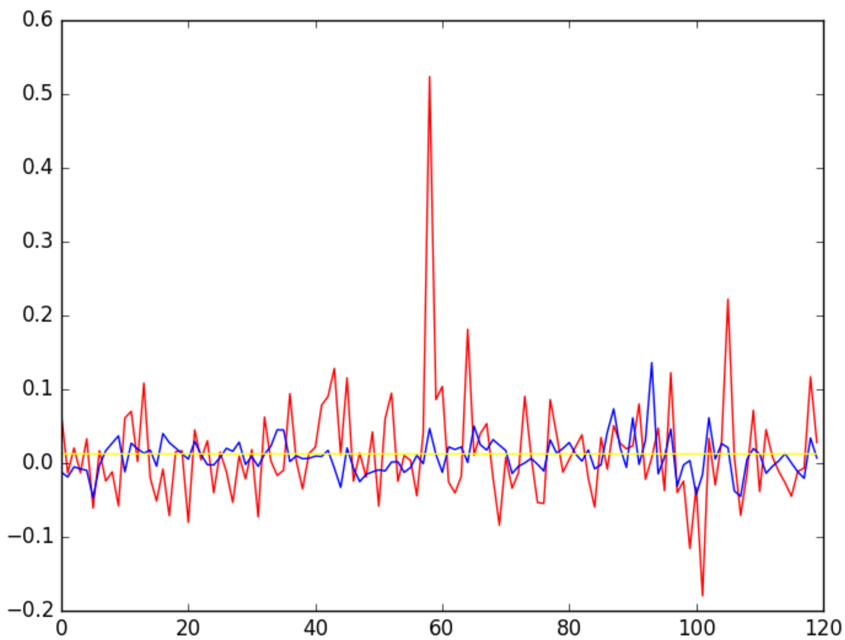
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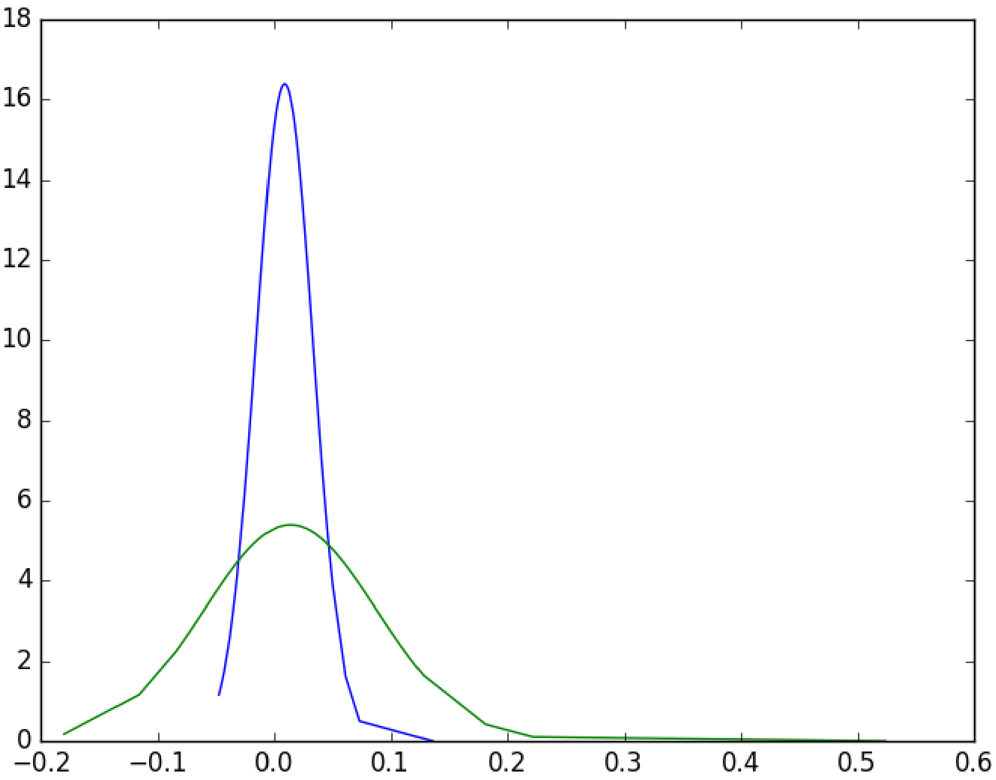

Further Research

Usually stocks of corporation with larger market caps have more price stability then stocks with smaller companies. Hence, the number of dimensions required for these (larger cap stocks) are lower than others. Therefore, an analysis of larger cap vs small cap based on autoencoding could be done.

The current analysis assumes a static set of stocks over the 120-month rolling period. However, it is possible to get better returns if a dynamic portfolio is created instead where portfolio composition is changed every month based on the past data.