

Relational algebra is a procedural language which describe the step by step procedure for evaluating of a query

Relational algebra expression ~~mechan~~ represent the query evaluation

- ① Selection (σ) :- Selecting "Rows"
- ② Projection (π) :- Selecting "Column"

① find ~~the~~ all ^{the} employees whose salary is above 5,000

Employee (Eno, Ename, Salary)

σ Employee
Salary > 5000

Employee
R
it represent
table with all
rows &
columns

② find all the employees name

π Employee
Ename

③ find name of the employees whose salary is above 5K

π (σ Employee)
Ename
Salary > 5000

Q1. Consider the following SQL query

Select l from R where c ;

where l and c represents the list of columns and condition respectively on the relation R

write the equivalent relational Algebra for this

$$\pi_l(\sigma_c R)$$

Q2. Consider the following two statements

S1: $\pi_{\text{Ename}}(\sigma_{\text{Salary} > 5000} \text{Employee})$

S2: Select Ename
from Employee
where Salary > 5000;

Let the no. of tuples in the result of S_1 and S_2 are n_1 and n_2 respectively. Then which of the following relationship holds good.

(a) $n_1 = n_2$

(b) $n_1 \leq n_2$

(c) $n_2 \leq n_1$

(d) $n_1 \neq n_2$

Employee	
Ename	Salary
A	6000
A	9000

(o/p)

Ename
A
A

 n_2

(o/p)

Ename
A

 n_1

$n_1 \leq n_2$

2

Relational algebra assumes the elimination of duplicate on Impletis

Cartesian Product (X)

R	
A	B
1	2
3	4

S	
C	D
5	6
7	6

R x S			
A	B	C	D
1	2	5	6
1	2	7	6
3	4	5	6
3	4	7	6

$R \times S$ returns a relation instance whose schema contains all the fields of R followed by all the fields of S and the Result of $R \times S$ contains all the possible of tuples of R

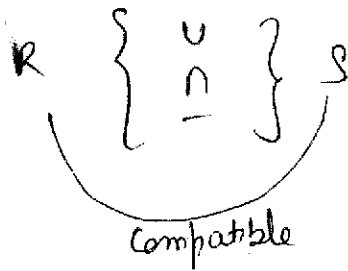
Q. Borrower (cust-name, loan-no)
 Depositor (cust-name, acc-no)
 Loan (loan-no, Branch, Amount)

find name of the customer who have a loan in the branch XYZ

Π
 (cust-name)

(Borrower x Loan)
 $\sigma_{\text{Borrower.loan-no} = \text{Loan.loan-no} \wedge \text{Loan.Branch} = \text{'XYZ'}}$

Set Manipulation operator



Relational algebra supports the use of set operation Union (\cup), Intersection (\cap) and difference ($-$) between the two compatible relations

Q1 Find names of all customers who have an account or loan or both

(a) $\left(\pi_{\text{Cust-name}} \text{ Depositor} \right) \cup \left(\pi_{\text{Cust-name}} \text{ Borrower} \right)$

(b) $\left(\left(\pi_{\text{Cust-name}} \text{ Depositor} \right) \cap \left(\pi_{\text{Cust-name}} \text{ Borrower} \right) \right) \cup \left(\left(\pi_{\text{Cust-name}} \text{ Depositor} \right) - \left(\pi_{\text{Cust-name}} \text{ Borrower} \right) \right) \cup \left(\left(\pi_{\text{Cust-name}} \text{ Borrower} \right) - \left(\pi_{\text{Cust-name}} \text{ Depositor} \right) \right)$
 \nwarrow both account & loan

(c) $\left(\pi_{\text{Cust-name}} \text{ Depositor} \right) - \left(\pi_{\text{Cust-name}} \text{ Borrower} \right)$
 \nwarrow account but no loan

Consider the following two statement

$S_1: \left(\pi_{\text{cust-name}} \text{ Depositor} \right) \cup \left(\pi_{\text{cust-name}} \text{ Borrower} \right)$

In Relational Algebra
→ Eliminates the duplicates

$S_2: \left(\text{Select cust-name from Depositor} \right)$

→ union
 $\left(\text{Select cust-name from Borrower} \right)$
Eliminates the duplicates

The no. of tuple in the result of S_1 & S_2 are n_1 & n_2 respectively then which of the following relationship holds good.

- (a) $n_1 = n_2$
- (b) $n_1 \leq n_2$
- (c) $n_1 \subseteq n_2$
- (d) $n_1 \neq n_2$

Join

Join is defined as a Cartesian product followed by selection then projection.

Variants of Join

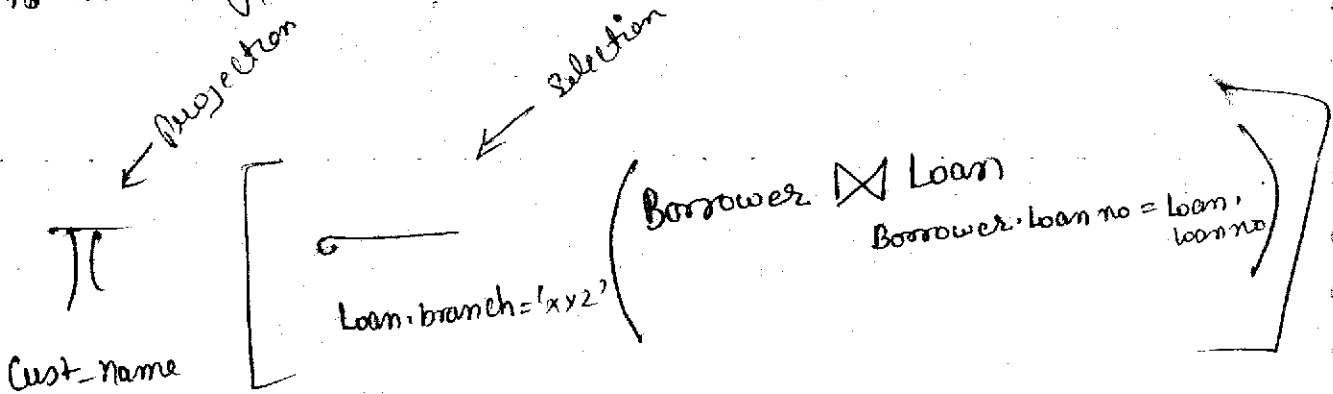
① Conditional Join :- In which the two relations are joined based on some conditions.

R	S
A B	C D

$$R \bowtie_{B>C} S = \overline{B>C} (R \times S)$$

A B C D

Find names of the customers who have a loan in branch XYZ



It is an example of Equijoin (key = key)

② Equi Join

Equi Join in which two Relation are joined only with Equality condition

R		S	
A	B	C	D

can be $\rightarrow \pi_{A,C,D}$

$$R \bowtie_{B=C} S = \pi_{A,B,D} \left(\sigma_{B=C} (R \times S) \right)$$

A	B	C	D
xx	10	10	xx
xx	20	20	xx

duplicate

NOTE

Every Equi Join is also be a Conditional Join but Every Conditional Join need not be an Equi Join

③

Natural Join :- is an Equi Join in which the equality is are specified in all field having the same name

R		S	
A	B	B	D

$$R \bowtie S = \pi_{A,B,D} \left(\sigma_{R.B=S.B} (R \times S) \right)$$

A	B	C	D
xx	10	10	xy
xx	20	20	xy

π
last name

$\sigma_{Branch = 'xy2'} (Borrower \bowtie loan)$

no need to specify condition

Let R and S be two relation with m and n numbers of tuples ~~at~~ each then find the maximum and minimum no of tuples in $R \bowtie S$

Ans

max	mn
min	0

R		S	
A	B	B	D

m n $\rightarrow m \times n$

(a)

1 3 $\rightarrow 0$

(b)

The natural join returns the result only Cartesian product when there are no ^{columns} ~~tuples~~ in Common.

when

Rename (ρ)_{rho}

Rename operation is used to represent the relational algebra with the symbolic name

$\pi_{\text{Cust_name}} [\text{Branch} = 'x \times 2' (Borrower \bowtie Loan)]$

← can't represent

① $\rho[A, (Borrower \bowtie Loan)]$

② $\rho[B, (\text{Branch} = 'x \times 2' A)]$

③ $\pi_{\text{Cust_name}} B$

⑧

Consider the following table of data

T_1		
P	Q	R
11	a	6
16	b	9
26	a	7

T_2		
A	B	C
11	b	7
26	c	4
11	b	6

following

find the number of tuples in the result of each of the relational algebra expression.

① $T_1 \bowtie_{P=A} T_2 = 3$

② $T_1 \bowtie_{Q=B} T_2 = 2$

③ $T_1 \bowtie_{P=A \wedge R=C} T_2 = 1$

④ $T_1 \bowtie T_2 = 9$

$\pi_{\text{Cust-name}} \left[\sigma_{\text{Branch}='xyz'} \left(\text{Borrower} \bowtie_{100} \text{loan} \right) \right] = 10,000$
 testy is in 10,000 then Project

or
 $\pi_{\text{Cust-name}} \left[\text{Borrower} \bowtie_{100} \left(\sigma_{\text{Branch}='xyz'} \text{loan} \right) \right]$

400 ← testy in 400 8 then Project

It is optimized query of above one

⑨

pg-61 (b) Ans

Q11 (u)

B is FK in C is PK

$$\pi_B(R_1) - \pi_C(R_2)$$

tuples in B not in C
not possible

- (b) x
- (c) not always equal
- (d) x - no value present in FK may not present in PK (not possible)

Q1

(a) $\pi_{\text{Person-name}} \left(\sigma_{\text{City} = \text{'Miami'}} \text{Employee} \right)$

(b) $\pi_{\text{Person-name}} \left(\sigma_{\text{Salary} > 100,000} \text{works} \right)$

(c) $\left[\pi_{\text{Person-name}} \left(\sigma_{\text{City} = \text{'Miami'}} \text{Employee} \right) \right] \cap \left[\pi_{\text{Person-name}} \left(\sigma_{\text{Salary} > 100,000} \text{works} \right) \right]$

Q2

(a) $\pi_{\text{Person-name}} \left[\sigma_{\text{Company-name} = \text{'First Bank Corporation'}} \text{works} \right]$

(b) $\pi_{\text{Person-name, city}} \left[\left[\sigma_{\text{Company-name} = \text{'FBC'}} \text{works} \right] \bowtie \text{Employee} \right]$

(c) $\pi_{\text{Person-name, city, Street}} \left[\left(\sigma_{\text{Comp-name} = \text{'FBC'}} \wedge \text{Salary} > 10000 \right) \text{works} \right] \bowtie \text{Employee}$

7

name	Sex	Marks	Condition fails	name	Sex	Marks
X A	M	600		A	M	600
X B	M	400	X	B	M	400
X C	F	900	X	C	F	900
D	F	700	X	D	F	700
E	F	500	X	E	F	500
F	F	300	X	F	F	300

$$= (C, D, E, F) - (E, F)$$

$$= (C, D)$$

name of all females
 name of all females
 whose marks are less than
 males marks

- (a) C
- (b) C, D, E
- (c) C, D, E
- (d) C, D

Q Consider the following table of data

ID	name	age
12	A	60
15	S	24
99	R	11

ID	name	age
15	S	24
25	H	40
98	R	20
99	R	11

Id	phone
10	2200
99	2100

How many tuples does the following relational algebra expression returns, assume that the schema $A \cup B$ is same as

$$(A \cup B) \bowtie C$$

$$A.Id > 40 \vee C.Id < 15$$

ID	name	age
12	A	60
15	S	24
99	R	11
25	H	40
98	R	20

$A \cup B$

A ID	C ID
12	10 ✓
12	99 X
15	10 ✓
15	99 X
99	10 ✓
99	99 X
25	10 ✓
25	99 X
98	10 ✓
98	99 ✓

11

Division operation (\div)

Q find name of the faculty who teaches all the course.

Faculty		Teaches		Course	
Fid	Fname	Fid	Cid	Cid	Cname
1	A	1	98	98	DB
2	B	1	99	99	CD
3	C	2	98	100	DS
		3	99		
		1	100		

$$\left[\pi_{\text{Fname, Cid}} (\text{Faculty} \bowtie \text{Teaches}) \right] \div \left[\pi_{\text{Cid}} (\text{Course}) \right]$$

Fname	Cid
A	98
A	99
B	98
C	99
A	100

Cid
98
99
100

o/p \rightarrow

Fname
A

Definition

Consider the two relation instances A & B in which A has exactly two fields X and Y and B has just one field Y with the same domain as in A with defined the division operation $A \div B$ as the set of all X values such that for every Y value in B there is a tuple X, Y in A

A	
X	Y
X ₁	Y ₁
X ₂	Y ₁₀₀
X ₃	Y ₁
X ₁	Y ₂
X ₂	Y ₂
X ₁	Y ₃

(a)

B ₁
Y
Y ₁
Y ₂
Y ₃

$A \div B$
X
X ₁

b

$$\begin{array}{r} B_2 \\ \hline Y \\ \hline Y_1 \\ \hline Y_2 \end{array}$$

$$\begin{array}{r} A \div B_2 \\ \hline X \\ \hline X_1 \\ \hline X_2 \end{array}$$

c

$$\begin{array}{r} B_3 \\ \hline Y \\ \hline Y_1 \end{array}$$

$$\begin{array}{r} A \div B_3 \\ \hline X \\ \hline X_1 \\ \hline X_2 \\ \hline X_3 \end{array}$$

use division

find the name of publisher who publishes all the Category of books

Book (ISBN, title, Author, Price, ~~Category~~ ID, Category)
 Publisher (Pid, Pname, City)

$$\left[\pi_{Pname, Category} (Publisher \bowtie Book) \right] \div \left[\pi_{Category} Book \right]$$

a) $(A, B) \div B = A$

b) $(A, B, C) \div C = (A, B)$

c) $(A, B, C) \div (B, C) = A$

d) $(A, B) \div C = \phi$

e) $(A, B) \div (B, C) = \phi$

f) $(A, B) \div (A, B, C) = \phi$

} not possible

Q Let R_1, R_2 are two relation with n_1 & n_2 rows respectively where $n_2 > n_1$, then find the min & max no. of rows in each of the following relational algebra expressions.

Expression

Min

Max

① $\sigma_{age > 15} R_1$

0

n_1

If all column contains same name then element duplicate

② $\pi_{name} R_2$

1

n_2

If n_1, n_2 contains all n_1

③ $R_1 \cup R_2$

n_2

$n_1 + n_2$

④ $R_1 \cap R_2$

0

n_1

⑤ $R_1 - R_2$

0

n_1

⑥ $R_1 \bowtie R_2$

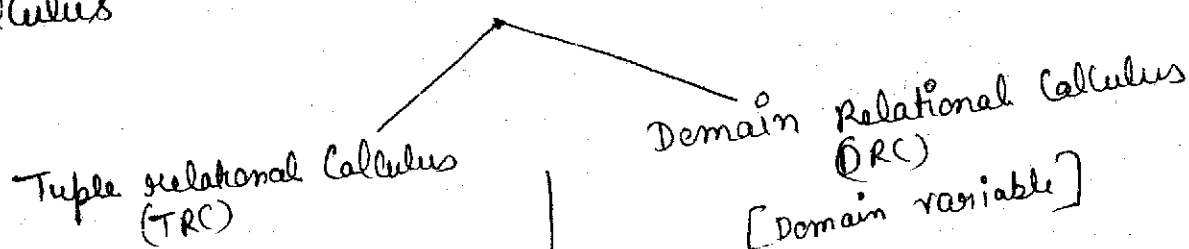
0

$n_1 \times n_2$

Relational Calculus

RC which describes the desired answer without explicit express it about how answer is to be computed this ~~norm~~ norm procedural style of query is called declarative.

There are two variants of R.C and are based on predicate Calculus



(a) → [Tuple variable]

Tuple variable :- It is a variable that takes on the tuples of a relation schema as values.

Domain variable :- It is a variable that ranges over the values in the domain of some attributes

(b) → form of TRC query

$\{ \overline{T} / P(T) \}$

↑ ↑
 Tuple variable formula which

form of DRC query

$\{ \overline{\langle x_1, x_2, \dots, x_n \rangle} / P(\langle x_1, x_2, \dots, x_n \rangle) \}$

↑ ↑
 domain variable formula which described

Borrower (Cust-name, loan-no)

Loan (loan-no, Branch, Amount)
 L B A

representing all the columns of loan

Q1 find all the loans of an amount above 5000

$$TRC: \{ T / T \in \text{Loan} (T.\text{amount} > 5000) \}$$

$$DRC: \{ \langle L, B, A \rangle / \langle L, B, A \rangle \in \text{Loan} (A > 5000) \}$$

Q2 find the loan no of loans of an amount above 5000

here some col

$$TRC: \{ T / \exists L \in \text{Loan} (L.\text{amount} > 5000 \wedge T.\text{Loan.no} = L.\text{Loan.no}) \}$$

required in o/p

$$DRC: \{ \langle L \rangle / \exists B, A (\langle L, B, A \rangle \in \text{Loan} (A > 5000)) \}$$

it is assignment
(any of col loan no. to the loan no of T)

Q3 find name of the customer and his amount of loan who have a loan in Branch XYZ

selection

$$TRC: \{ T / \exists B \in \text{Borrower} (\overset{(1)}{T.\text{Cust.name} = B.\text{Cust.name}} \wedge \exists L \in \text{Loan} (L.\text{Branch} = 'XYZ' \wedge L.\text{Loan.no} = B.\text{Loan.no} \wedge \overset{(2)}{T.\text{amount} = L.\text{amount}})) \}$$

assignment

$$DRC: \{ \langle C, A \rangle / \exists L (\langle C, L \rangle \in \text{Borrower}) \wedge \exists B (\langle L, B, A \rangle \in \text{Loan} (B = 'XYZ')) \}$$

Q Interpret the following "TRC" expressions

$$\{T/\exists S \in \text{Student} (T.\text{sname} = S.\text{sname}) \wedge \exists C \in \text{course} (C.\text{sno} = S.\text{sno} \wedge C.\text{courseName} = 'CS')\}$$

Ans finding the names of all student studying in the course CS

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Q3

(a) $\pi_A(R)$

$$\{T/\exists R \in R (T.A = R.A)\}$$

1	2
2	4

(b) $\{T/\exists R \in R (T.B = 17)\}$

1	2
1	1
2	2
2	1

(c) $\{T/\exists R \in R (T.A = R.A \wedge T.B = R.B \wedge T.C = R.C) \wedge \exists S \in S (T.D = S.D \wedge T.E = S.E \wedge T.F = S.F)\}$

(d) $\{T/\exists R \in R (T.A = R.A) \wedge \exists S \in S (T.F = S.F \wedge S.C = S.D)\}$

here conversion of \rightarrow Algebra Exp to TRC

(17)

$$\pi_A(R) = \{T/\exists R \in R\}$$

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04

here algebra exp to DRC

(a) $\{ \langle A \rangle / \exists B, C (\langle A, B, C \rangle \in R_1) \}$

here A is required

(b) $\{ \langle A, B, C \rangle / \langle A, B, C \rangle \in R_1 (B=17) \}$

here nothing require
so take all
with condition

(c) $\{ \langle A, B, C \rangle / \langle A, B, C \rangle \in R_1 \vee \langle A, B, C \rangle \in R_2 \}$

we don't
use \vee symbol
in this so
take (both)

(d) $\{ \langle A, B, C \rangle / \langle A, B, C \rangle \in R_1 \wedge \langle A, B, C \rangle \in R_2 \}$

A
 \wedge

(e) $\{ \langle A, B, C \rangle / \langle A, B, C \rangle \in R_1 \wedge \langle A, B, C \rangle \notin R_2 \}$

(f) $\{ \langle A, B, C \rangle / \exists P (\langle A, B, P \rangle \in R_1) \wedge \exists Q (\langle Q, B, C \rangle \in R_2) \}$

05
(a) $\{ \langle a \rangle / \exists b (\langle a, b \rangle \in R \wedge b=7) \}$

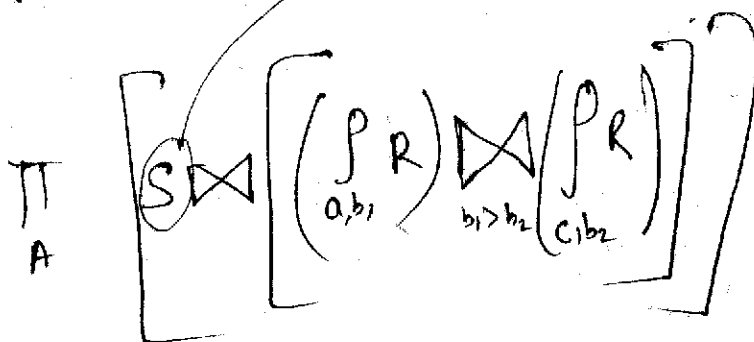
Table
projection
selection

$\pi_A (\sigma_{B=7} R)$

(b) $R \bowtie S$

$\sigma_{\begin{pmatrix} p_s \\ a_c \end{pmatrix}}$

(c)



Transaction

Transaction is a collection of operations that form a single logical unit of work

eg →

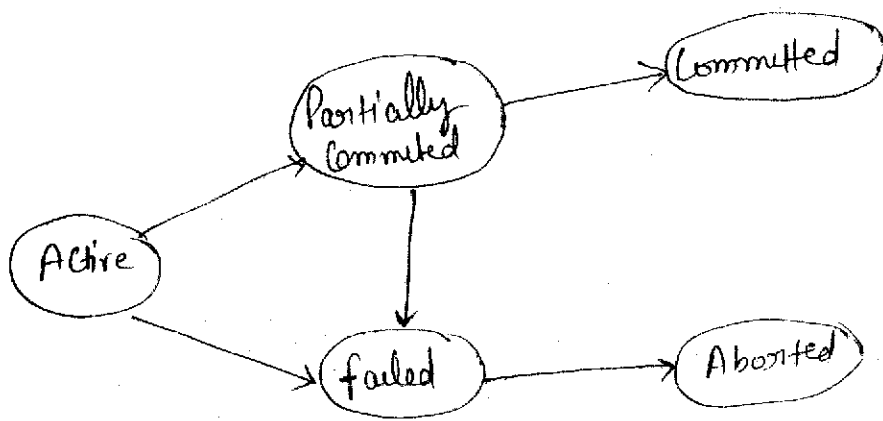
$A \xrightarrow{100} B$	
Read(A)	
$A = A - 100$	
Write(A)	
Read(B)	↓ fail
$B = B + 100$	
Write(B)	
<u>Commit</u>	

ACID

Transaction Properties

- ① Atomicity :- All/None
Transaction manager (always monitor each transaction)
- ② Consistency :- Correctness → user/Application Programmer
- ③ Isolation :- Each T's is unaware of other T's → Concurrency Control mgr
- ④ Durability :- All operations done by a T's must become permanent → Recovery mgr

F

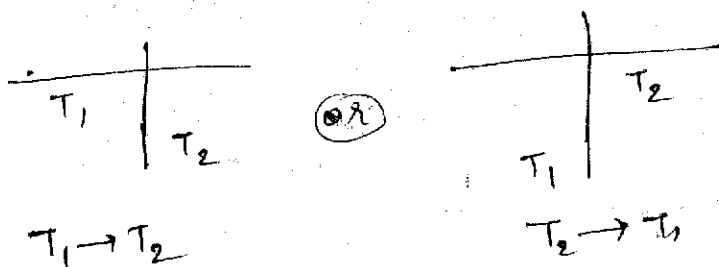


Transaction States

Schedule :- represents the order in which the operations of a transactions are executed.

① **Serial Schedule** :- It consists of operations belonging to various transaction where the instructions belonging to one single transaction appears together in that schedule

eg:- If there are two Transaction then two possibility of serial schedules

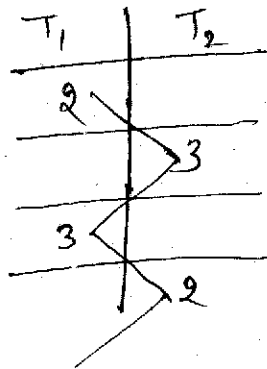


If there are n -transactions then the possible no. of serial schedules are $n!$

eg:- 3 transactions, ≤ 3 possible serial schedules

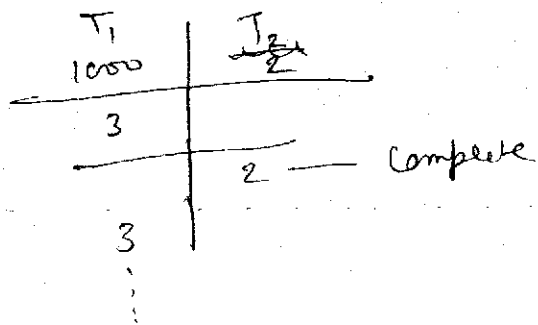
Every serial schedule leads to consistent state in the database system

- ② Concurrent schedule :- If two transactions are executing concurrently the OS may make it one _____ one transaction little while than perform a context switch execute the second transaction for a little while then switch back to the first transaction and so on



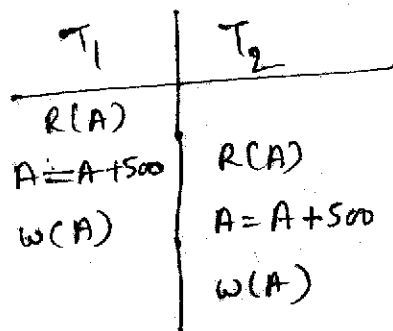
why concurrency

- ① To minimize the longer waiting time



T_1 1000 T_2 2
 $T_1 \rightarrow T_2$
 T_2 has to wait

- ② In computer system it is possible that when one transaction is executing IO operation then other can execute CPU operation and vice versa.



Effective Utilization of Resources

Q1
find the possible no. of concurrent schedules over two transactions each with two operations of each.

T_1
 $R_1(A)$
 $w_1(A)$

T_2
 $R_2(A)$
 $w_2(A)$

$$\frac{(2+2)!}{2! \times 2!} = \frac{4!}{2 \times 2} = \frac{24}{4} = 6$$

$T_1 \rightarrow T_2$:
 $R_1(A) w_1(A) R_2(A) w_2(A)$
 $R_1(A) R_2(A) w_1(A) w_2(A)$
 $R_1(A) R_2(A) w_2(A) w_1(A)$

$T_2 \rightarrow T_1$:
 $R_2(A) w_2(A) R_1(A) w_1(A)$
 $R_2(A) R_1(A) w_2(A) w_1(A)$
 $R_2(A) R_1(A) w_1(A) w_2(A)$

6 Concurrent schedules possible

Q2
find the no. of concurrent schedules over three transactions with two, three, four operations respectively

T_1 T_2 T_3
2 3 4

$$\frac{(2+3+4)!}{2! \times 3! \times 4!} = \frac{9!}{2 \times 6 \times 24} = 1260$$

generalization

$$\frac{(n_1 + n_2 + n_3 + \dots + n_n)!}{n_1! \times n_2! \times n_3! \times \dots \times n_n!}$$

~~The number of~~

If there are n -transaction with n_1, n_2, \dots, n_n operation then total no. of concurrent schedule is

$$\frac{(n_1 + n_2 + n_3 + \dots + n_n)!}{(n_1! \times n_2! \times \dots \times n_n!)}$$

no. of non-serial schedule is

$$\frac{n_1 + n_2 + \dots + n_n}{(n_1! \times n_2! \times \dots \times n_n!)} - n!$$

Serial Schedule = $n!$

Any concurrent schedule that when executed must be equivalent to some serial schedule to ensure the consistency of the database.

Problems due to Concurrent execution of transaction.

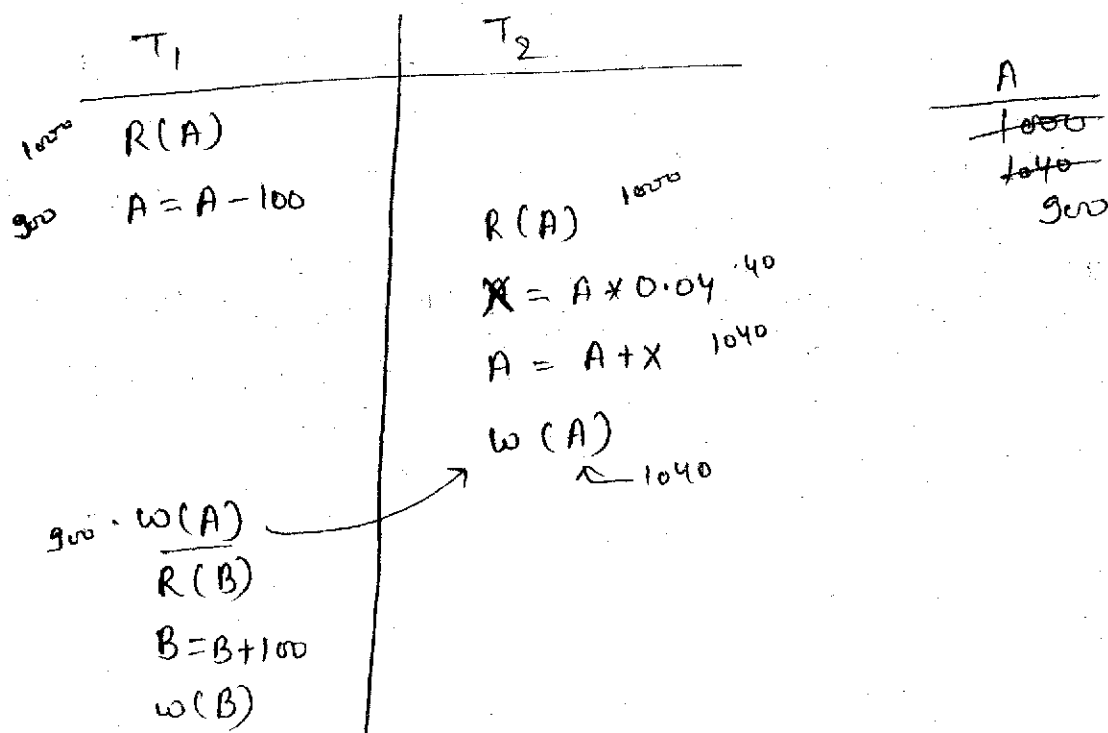
① ~~Lost~~ Lost-update Problem (write write conflict)

Suppose that the operations of T_1 & T_2 are interleaved in such a way that T_2 reads the value of A before T_1 updates its value in the database.

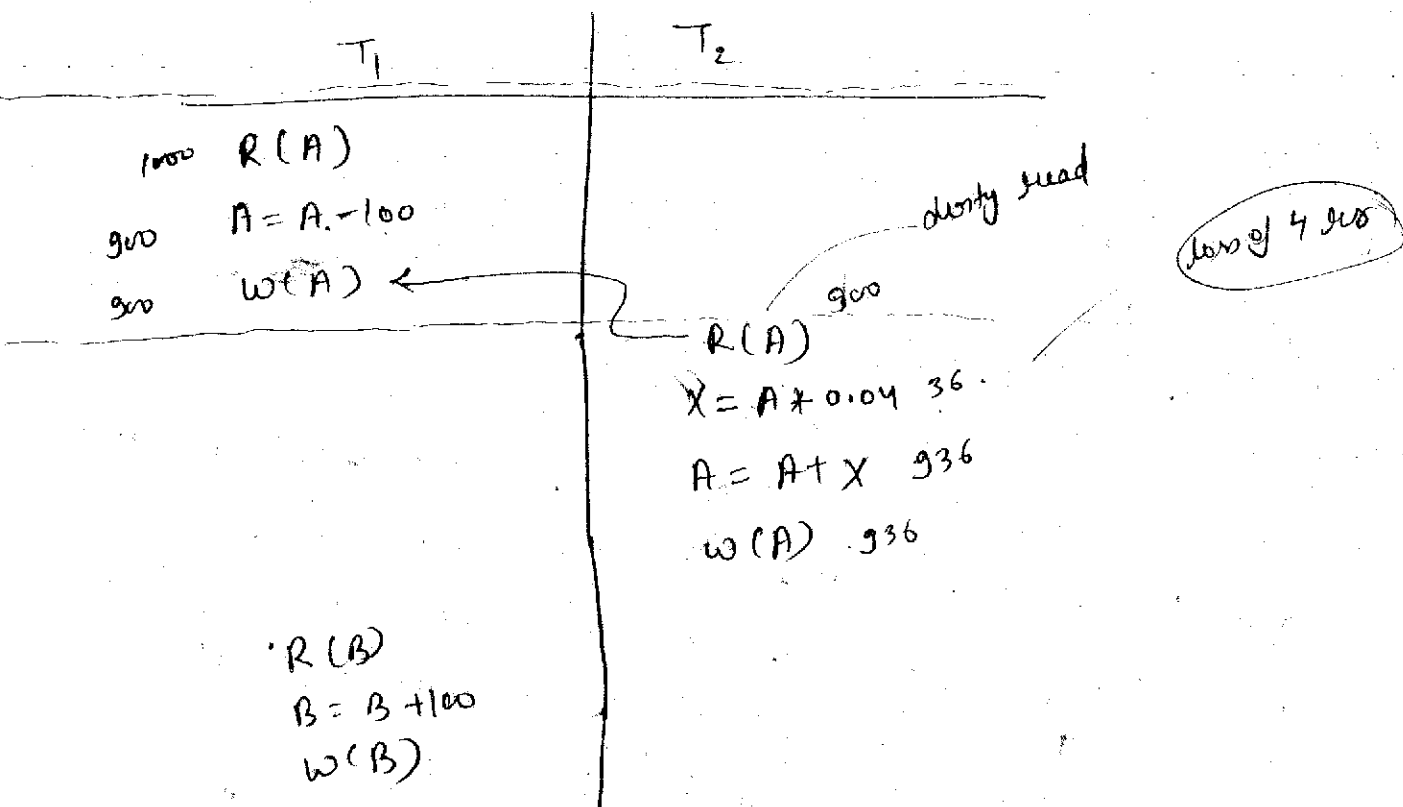
Now when T_2 updates the value of A in the database after it if T_1 updates its value in database the value of A updated by transaction T_2 is overwritten by transaction T_1 and hence is lost this is known as lost update problem or ww conflict.

(or)

If update of one transaction is overwritten by the update of another transaction is called lost update problem



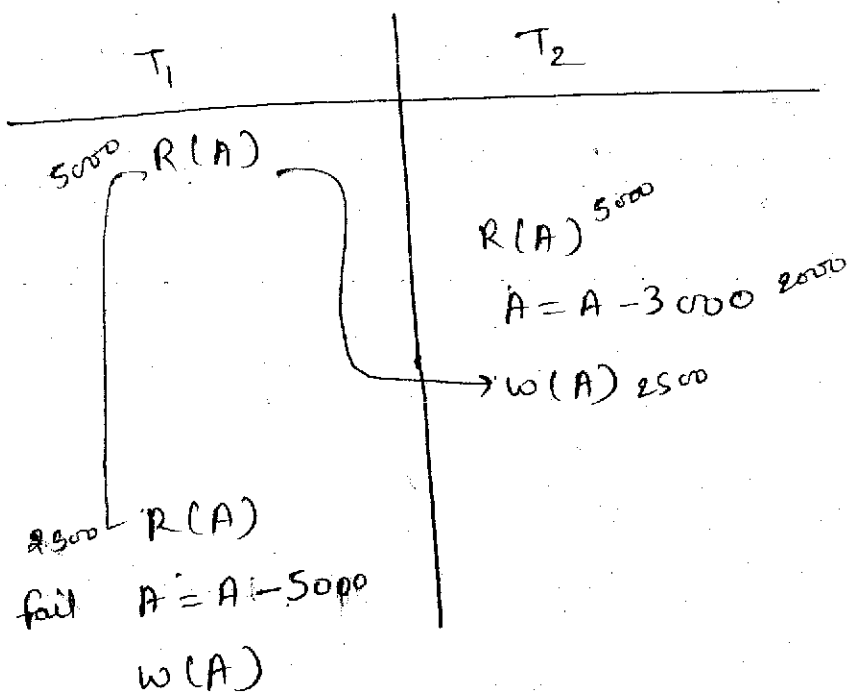
② Dirty Read problem (WR conflict)



Reading an uncommitted data is called dirty read

③ UnRepeatable Read Problem (R-w conflict)

when a transaction tries to read the value of a data item twice & another transaction updates the same data item in between the two read operation of the first transaction as a result the first transaction reads varied values of same data item during its execution. this is known as unrepeatable read.



Consider the following schedule.

S1: $R_2(x)$ $R_2(y)$ $R_1(x)$ $R_1(y)$ $\underline{w_1(x)}$ $R_2(x)$

unrepeatable read

as if this write operation commit or no dirty read

S2: $R_2(x)$ $R_2(y)$ $\underline{w_2(x)}$ $R_1(x)$ $R_2(y)$ dirty read

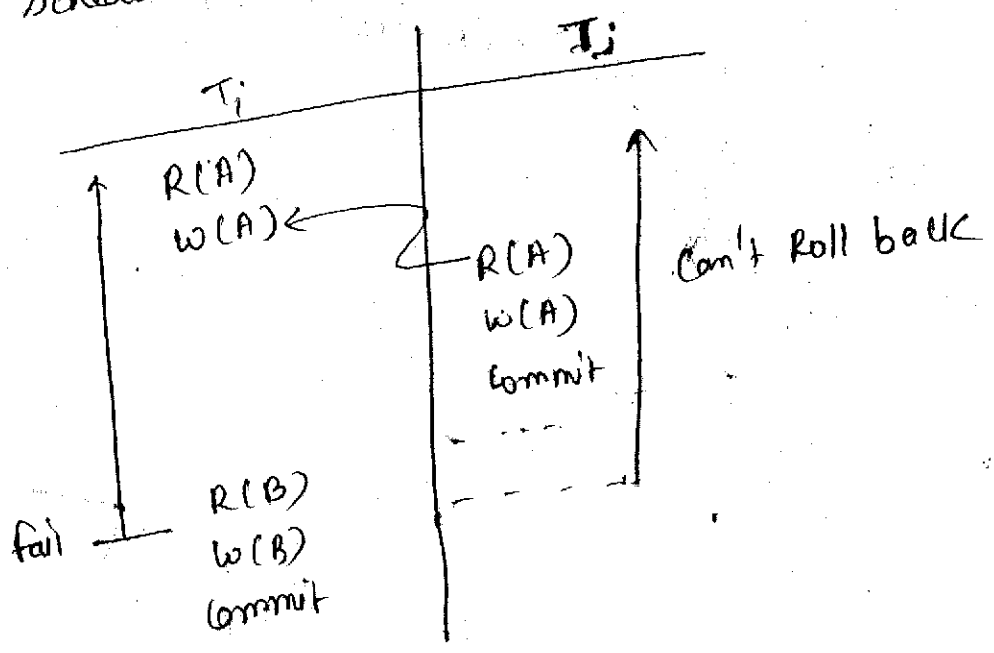
S3: $R_2(x)$ $R_2(y)$ $R_1(x)$ $R_1(y)$ $\underline{w_1(x) \cdot w_2(x)}$ lost update

which of the above schedules results in unrepeatable read prob

Schedules based on recoverability

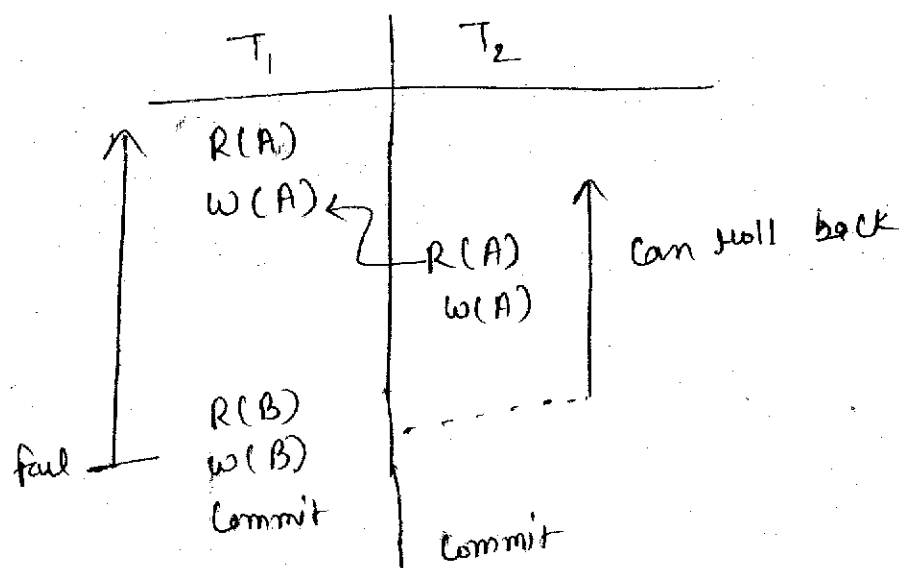
① Non-Recoverable Schedule

Let T_i and T_j be two transaction in which T_j reads the data that was return by T_i . if T_i fails after T_j commits roll back of T_j is not possible such situation is the example of non-recoverable schedule.



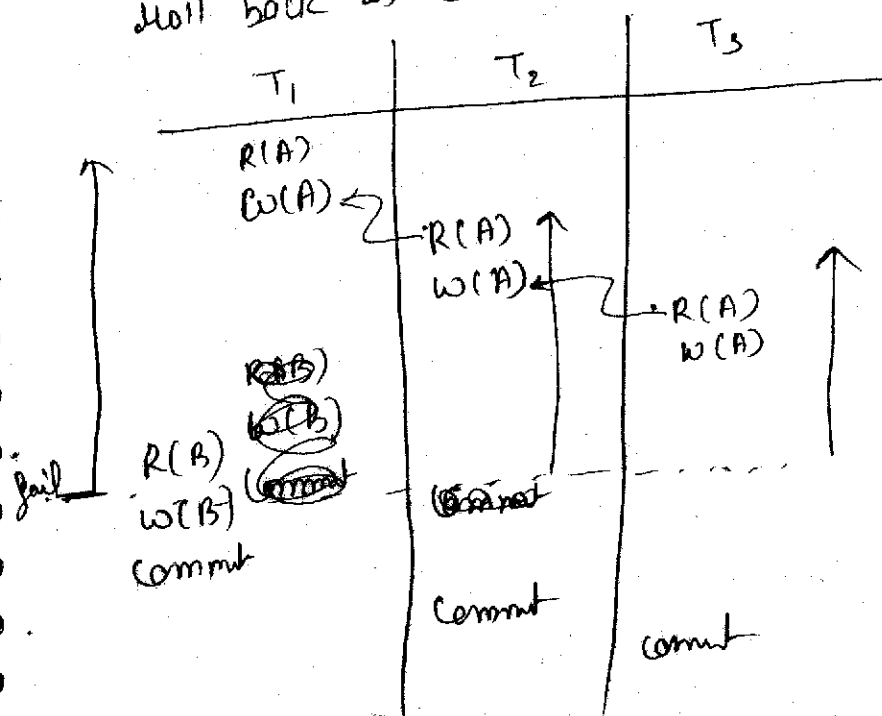
② Recoverable schedule

A recoverable schedule is one where ^{for} each pair of transaction T_i and T_j such that T_i reads a value that was returned by T_j . The commit operation of T_i should appear after the commit operation of T_j .



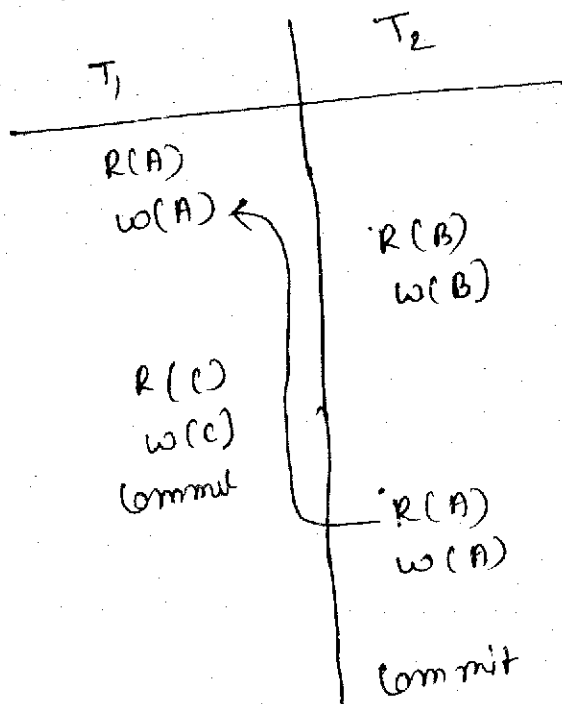
③ Cascading Rollbacks (Cascading Aborts)

A single transaction failure leads to multiple transaction to roll back is called a schedule with cascading rollbacks or CA.



④ Cascade Schedule

A schedule is said to be cascade where for each pair of transaction T_i and T_j such that T_j reads a data item that was previously returned by T_i and the commit operation of T_i appears before the read operation of T_j .

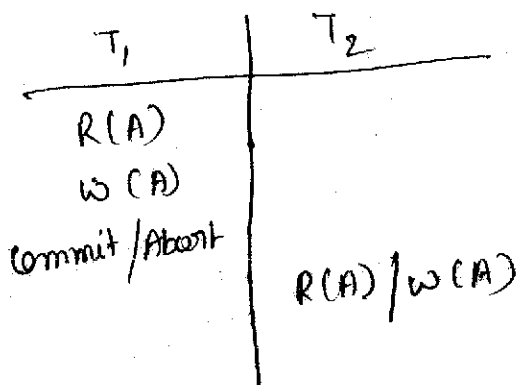


NOTE

Every cascade schedule is also recoverable.

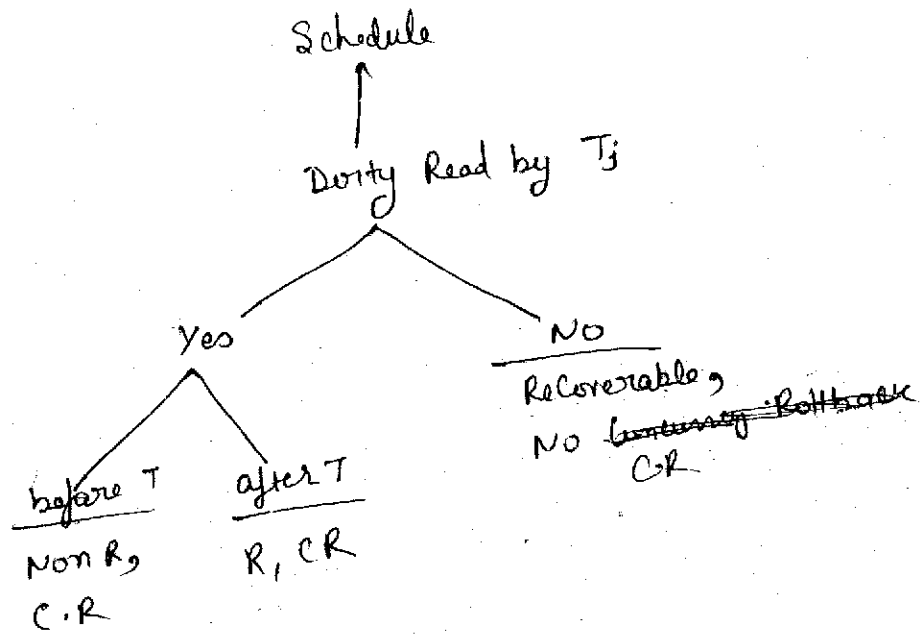
⑤ Strict Schedule

A schedule is said to be strict if a value returned by a transaction T is not read or overwritten by other transaction until either T aborts or commits.

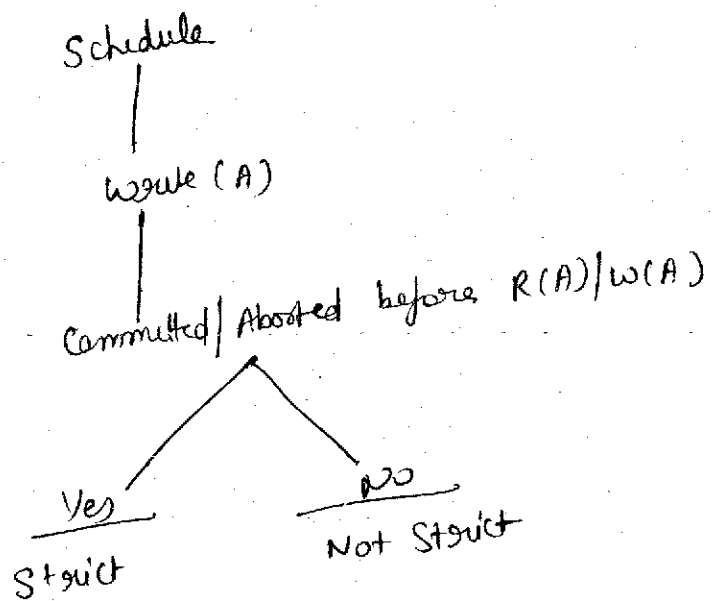


Every strict schedule is both recoverable and cascadeless

T_1	T_2
$w(A)$	$R(A)$
Comit	Comit



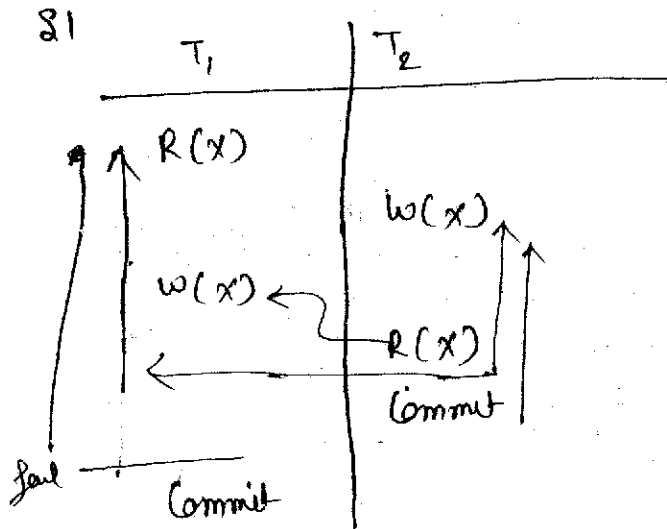
T
$w(A)$



2

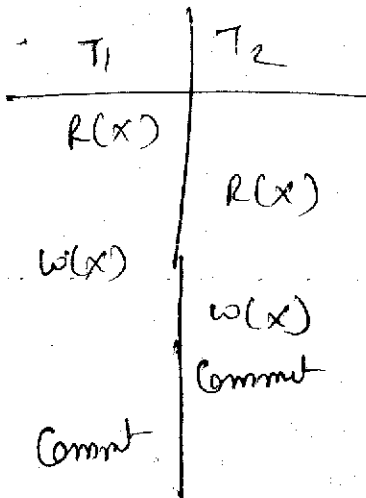
for each of the following schedule test for Recoverability.

①



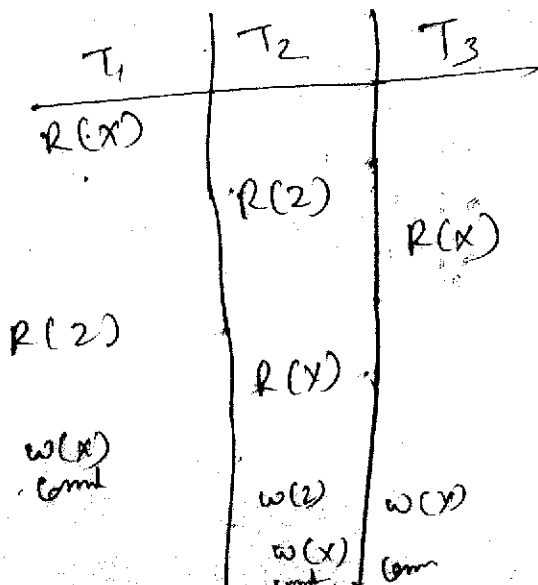
before
non R
CR
not strict

②



R_1
no CR
not strict

③



have no dirty read
so
Recoverable,
No CR
Strict

30

Serializable Schedule

Any Concurrent schedule that when executed equals to some serial schedule is said to be serializable schedule.

Conflict Serializable Schedule

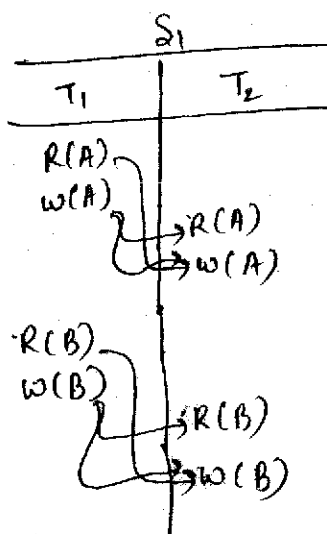
Conflict operation

- ① $w_1(A) \quad w_2(A)$
- ② $w_1(A) \quad R_2(A)$
- ③ $R_1(A) \quad w_2(A)$

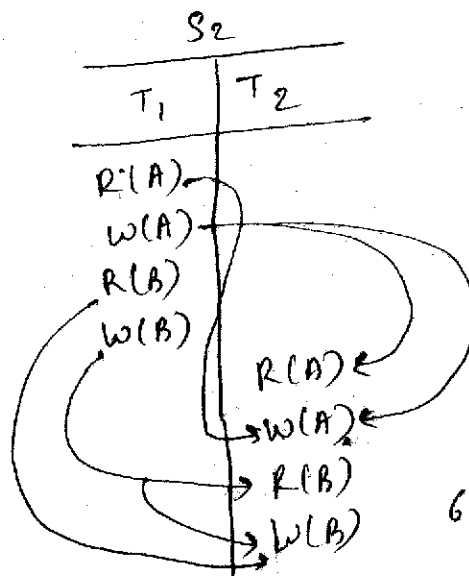
Conflict Equivalent Schedules

The two schedules are said to be Conflict Equivalent if all the conflicting operations in both S_1 and S_2 must be executing in the same order.

$$S_1 \subseteq S_2$$



6 Conflicts



6 Conflicts

$$S_1 \subseteq S_2$$

51

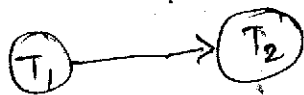
If a Concurrent schedule Conflict equal to a serial schedule is said to be Conflict serializable schedule.

S_1 is Conflict serializable ($T_1 \rightarrow T_2$)

Test for Conflict Serial schedule

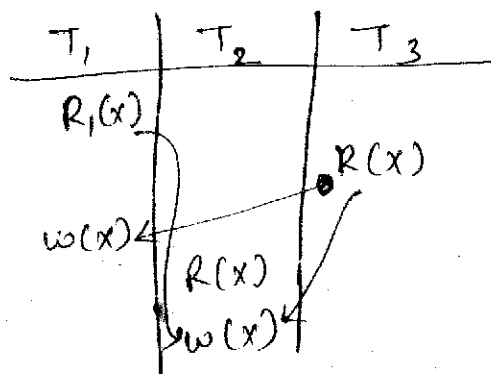
- ① Construct a precedence graph for the schedule in which
Each vertex corresponds to one transaction and each
edge corresponds to a
(let there is a conflict from T_i to T_j then draw
a directed edge from T_i to T_j)
- ② If the graph contains no cycles then schedule is said
to be Conflict serializable otherwise no conflict
serializable
- ③ The serializability order is determined based on the
directed edges

eg → Precedence graph for S_1 (prev Page)

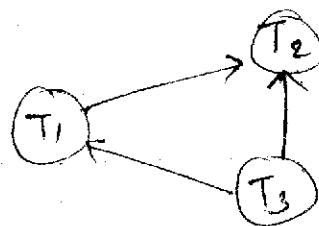


S_1 is C.S ($T_1 \rightarrow T_2$)

①



no cycle so
C.S



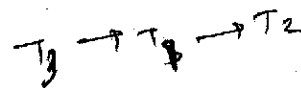
remove whose outdegree is 0

T_2

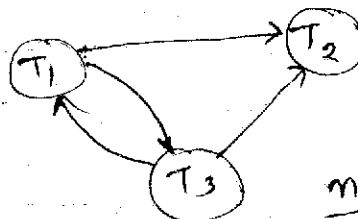
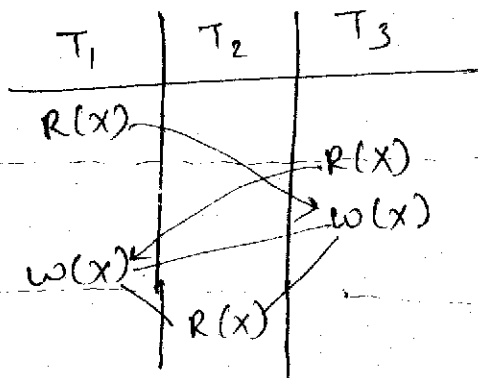
then T_3

then T_1

so

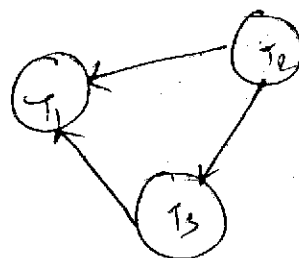
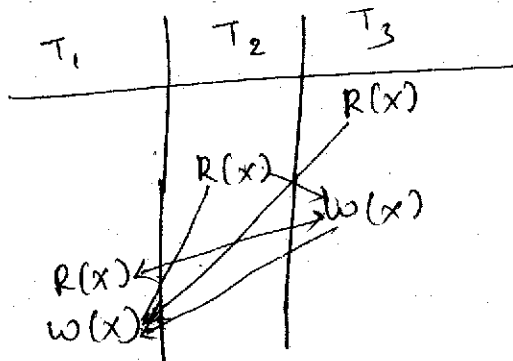


②



not C.S

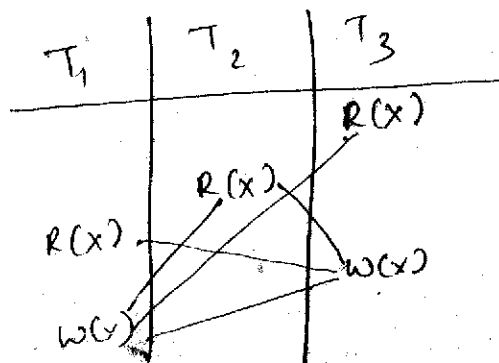
③



C.S

$T_2 - T_3 - T_1$

④

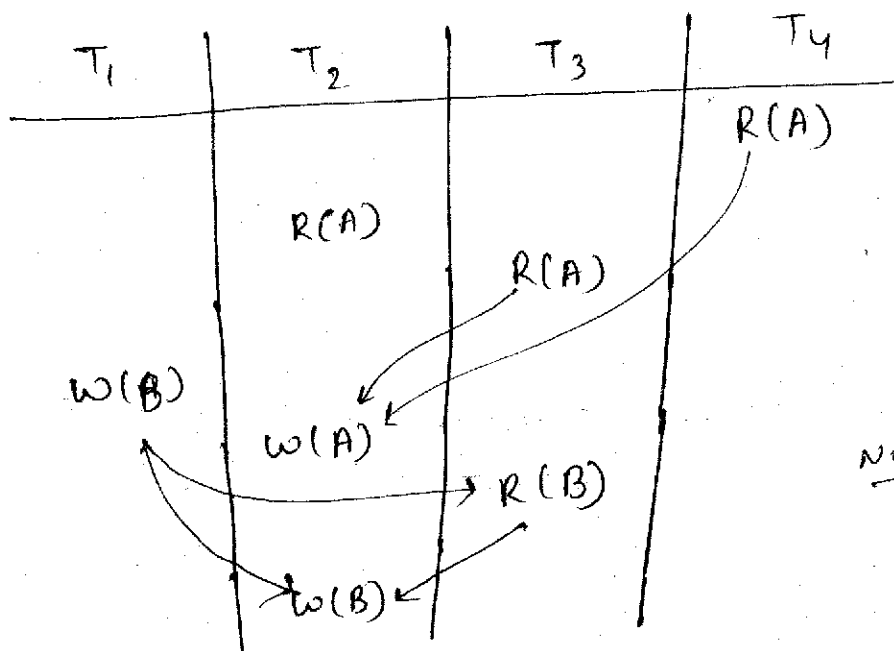


not C.S

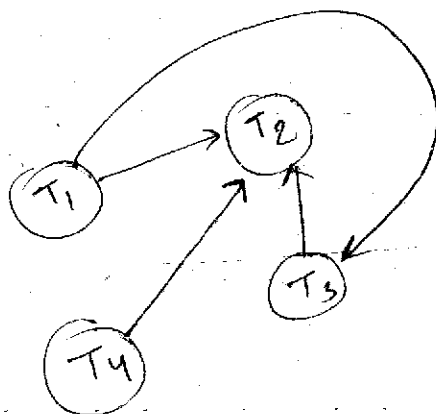
133

Consider the following schedule

one Concurrent Schedule
may be Conflict
Equivalent to more
than one schedule

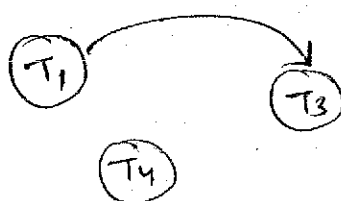


no cycle so
C.S



Remove T2

X - X - X - T2



Two possibility

① remove T4 or T3



X - X - T4 - T2

remove T3

X - T3 - T4 - T2

rem T2

T1 - T3 - T4 - T2

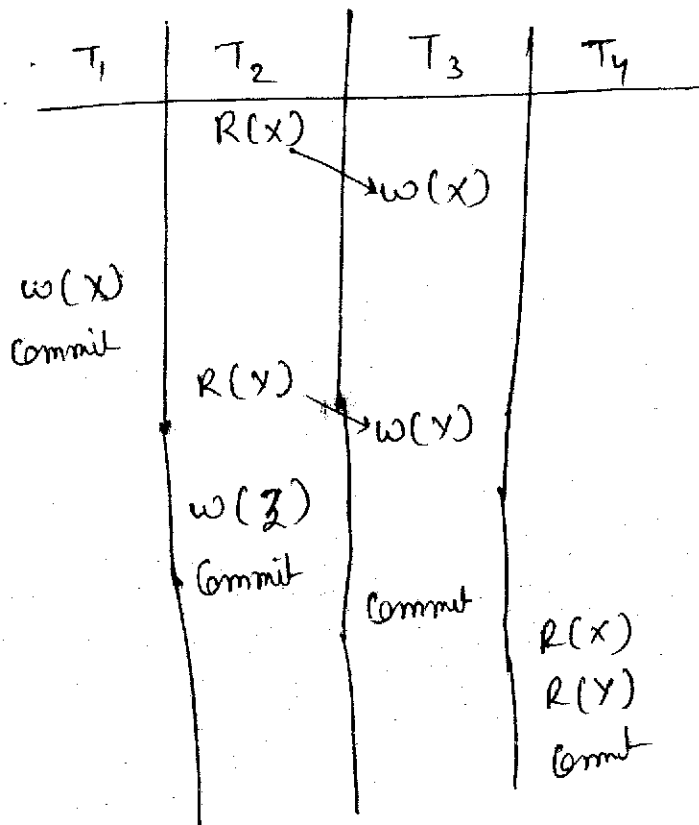


X - X - T3 - T2

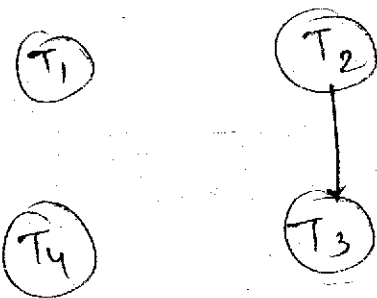
Possible

- ① T1 - T4 - T3 - T2
- ② T4 - T1 - T3 - T2
- ③ T1 - T3 - T4 - T2

39



no cycle
C.S



Case 1

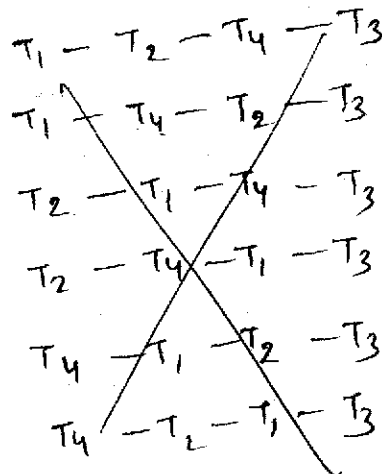
removed T3

T_1

T_2

T_4

$C3=6$



$T_4 - T_2 - T_1 - T_3$

$T_1 - T_2 - T_4 - T_3$

$T_1 - T_4 - T_2 - T_3$

$T_2 - T_1 - T_4 - T_3$

$T_2 - T_4 - T_1 - T_3$

$T_4 - T_1 - T_2 - T_3$

35

Case 2

~~more~~

12

$T_2 - T_3 - T_4 - T_1$

$T_2 - T_4 - T_3 - T_1$

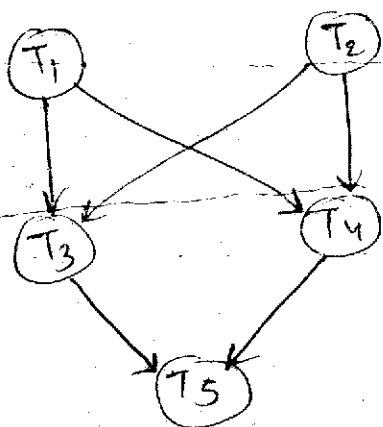
$T_4 - T_2 - T_3 - T_1$

$T_1 - T_2 - T_3 - T_4$

$T_2 - T_3 - T_1 - T_4$

$T_2 - T_1 - T_3 - T_4$

Q. Consider the following graph find the no. of conflict serial schedules for the precedence graph



$T_2 - T_1 - T_4 - T_3 - T_5$

$T_1 - T_2 - T_4 - T_3 - T_5$

$T_1 - T_2 - T_3 - T_4 - T_5$

$T_2 - T_1 - T_3 - T_4 - T_5$

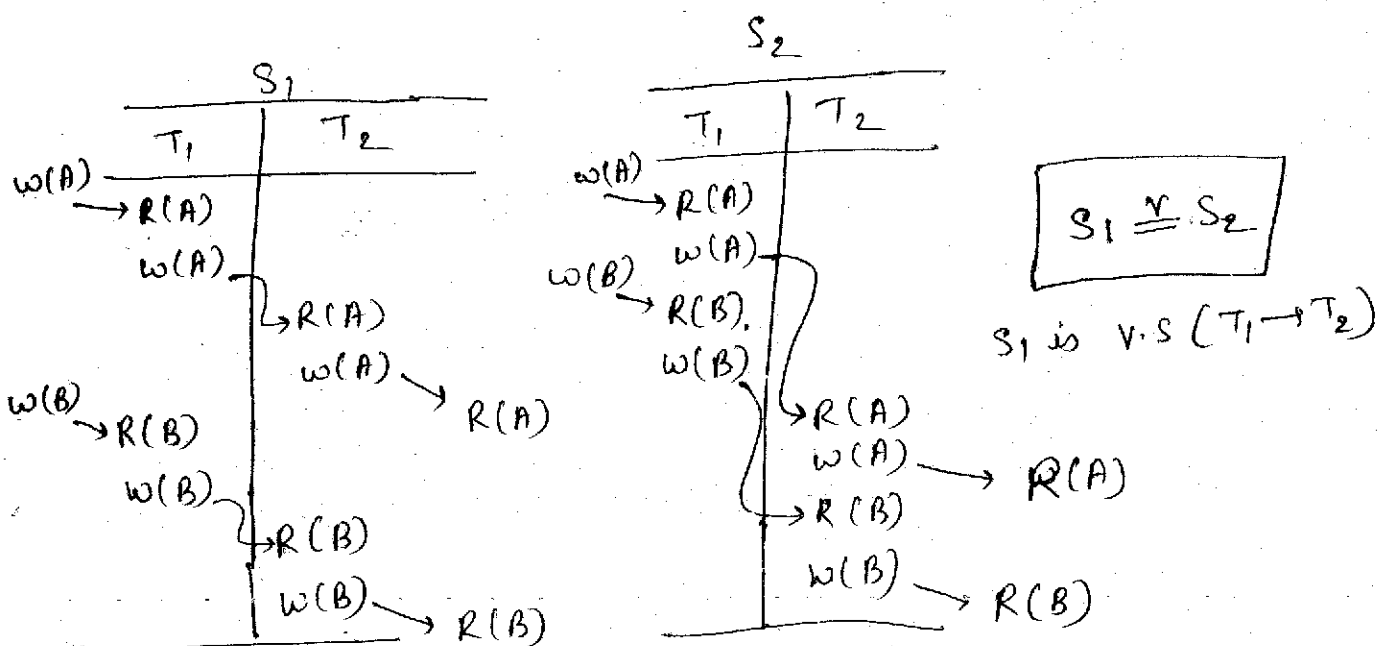
4

3

② View Serializability

View Equivalent Schedules

The Two Schedules S_1 and S_2 are said to be View Equivalent iff all the write-read sequences must be executed in the same order for each data item in both the schedules



$T_1 \rightarrow T_2$

S_1 is CS ($T_1 \rightarrow T_2$)

S_1 is V.S ($S_1 \rightarrow S_2$)

A Concurrent schedule is view equivalent to some serial schedule is said to be view serializable schedule

eg $\rightarrow S_1$ is view serializable ($T_1 \rightarrow T_2$)

Q2 Test is the following schedule is view serializable

S3	
T1	T2
w(A)	
	w(A)
R(A)	

S4 (T1 → T2)	
T1	T2
w(A)	
	w(A)
R(A)	

S5	
T1	T2
	w(A)
R(A)	

S3 ≠ S4

S3 ≠ S5



S3 is not VS
S3 is not CS

so S3 is not VS

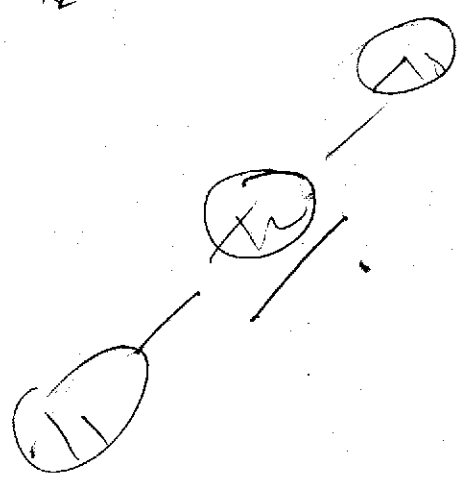
S6		
T1	T2	T3
w(A)		
	w(A)	
R(A)		
		w(A)

T1	T2	T3
w(A)		
	w(A)	
R(A)		
		w(A)

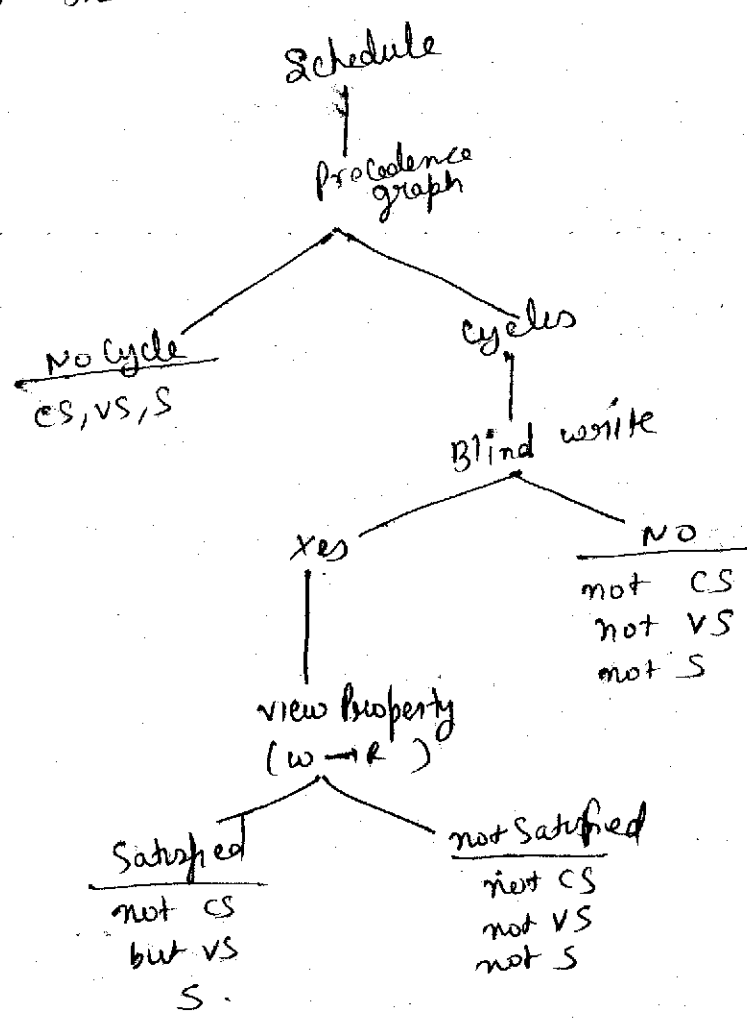
T1 execute by T2
T2 execute by T3

S6 is v.s (T1 → T2 → T3)

S6 is not CS



- ① A transaction that performing a write op^r without reading it of some data is called blind write op^r
- ② Every conflict serializable schedule is also view serializable but every view serializable need not be conflict serializable
- ③ A schedule that is not CS but VS then there must be at least one blind write operation
- ④ A schedule that is CS or VS or both is said to be serializable schedule
- ⑤ A schedule that is not CS and there is no blind write op^r then the schedule must not be VS



pg-24
Q-3

(a)

(b)

(c)

~~RR~~

~~RR~~

Relaxable
C.R
Not Strict

dirty Read
from $T_3 \rightarrow T_2$

~~RR~~

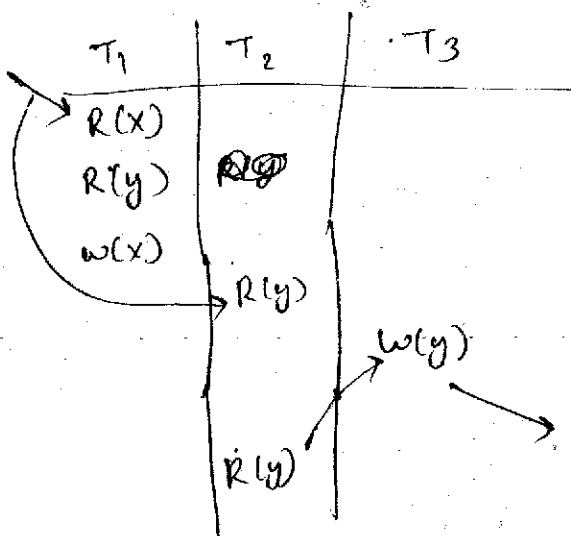
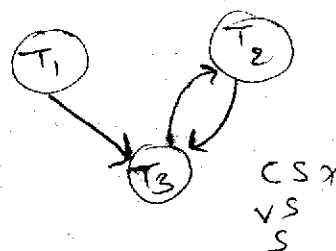
(d)

2-3
3-

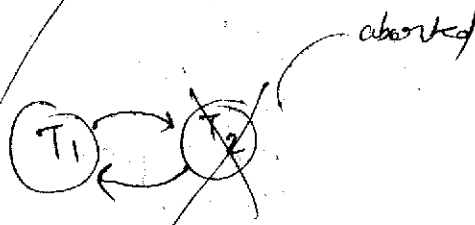
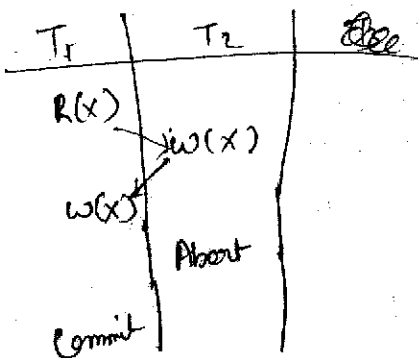
Cycle →

dirty read 3-2

Relax
C.R
Not Strict



If we change
One abort by $T_1 \rightarrow w(x)$
then Strict



only T_1 left

T_1

no $T_1 \rightarrow T_2$ possible in future
C.S no VS, S modified
R, no R

no Strict

(40)

3

T_1	T_2
$R(x)$	$w(x)$
$w(x)$	$w(x)$
$commit$	$commit$

cycle
 so not CS
 NOT V.S
 No
R, No C.R
 Not Strict

9

T_1	T_2
$w(x)$	$R(x)$

T_1 abort
 so CS
 VS
 S
~~not~~ dirty read
 R, CR

Not Strict

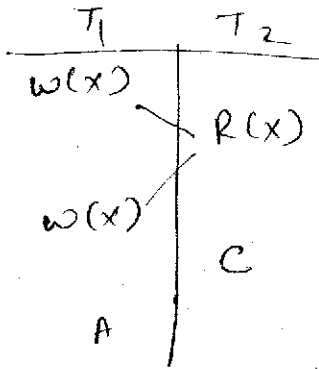
11

T_1	T_2
$w(x)$	$R(x)$
$w(x)$	C
C	

NOT CS
 NOT VS
 NOT S
dirty Read
 NOT R
 C.R is there
 Not Strict

41

(i)



2 transaction, if one abort then we can say it is CS

T_1 abort so CS
VS
S

dirty read
NOT Recoverable
C.R required
NOT Strict

(J)

CS
VS
S
Strict
So R
No C.R

(K)

CS
VS
S

no dirty read
strict
So R
No C.R
every strict is

(L)

NOT CS
NOT VS
NOT S

dirty read is there (from 1-3)
but not committed by
So R
C.R required
NOT Strict

Concurrency Control

The data item can be shared by the data items in the mutually exclusive manner i.e only one transaction can access the data item

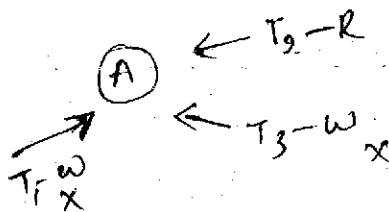
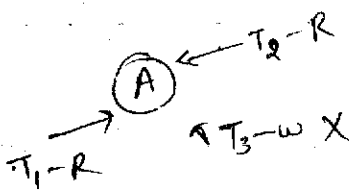
Lock based Protocols

2-lock modes

- ① Shared lock (Lock-S) :- Only Read data
- ② Exclusive lock (Lock-X) :- Both Read and write data

B	S	X
S	✓	X
X	X	X

← Compatible matrix of lock modes

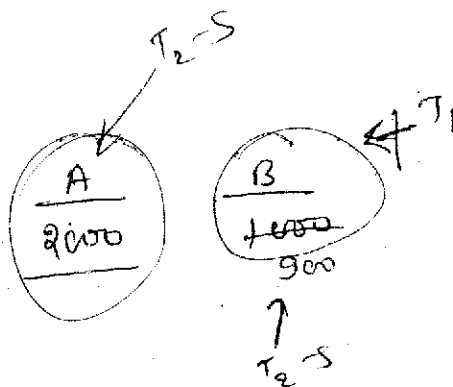


eg → $T_1: B \xrightarrow{100} A$
 Lock-X(B)
 R(B)
 $B = B - 100$
 W(B)
 unlock-R(B)
 Lock-X(A)
 R(A)
 $A = A + 100$
 W(A)
 unlock(A)

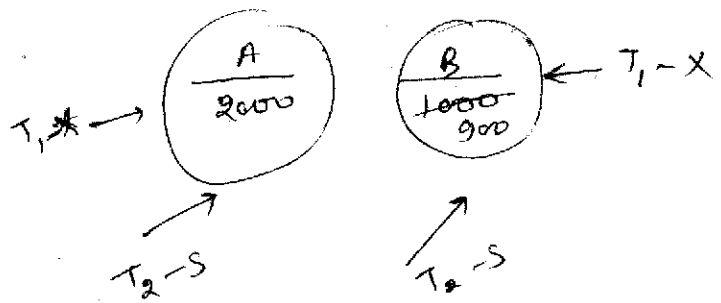
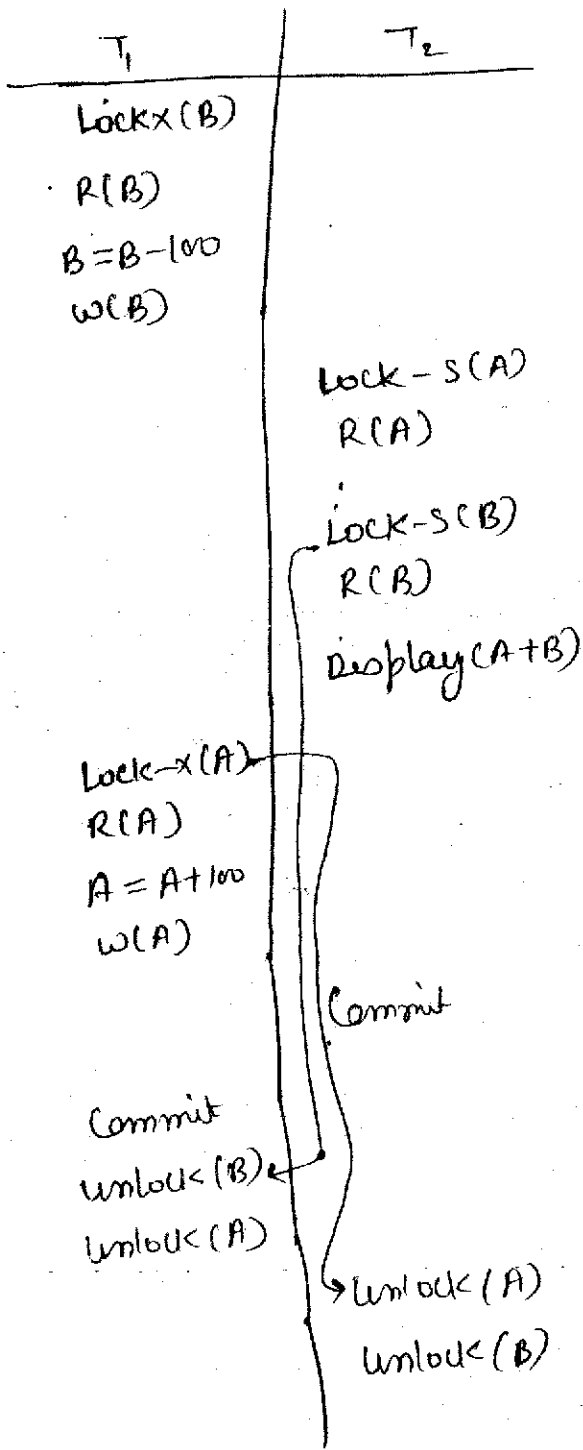
$T_2: \text{Display}(A+B)$
 Lock-S(A)
 R(A)
 unlock(A)
 Lock-S(B)
 R(B)
 unlock(B)
 display(A+B)

eg →

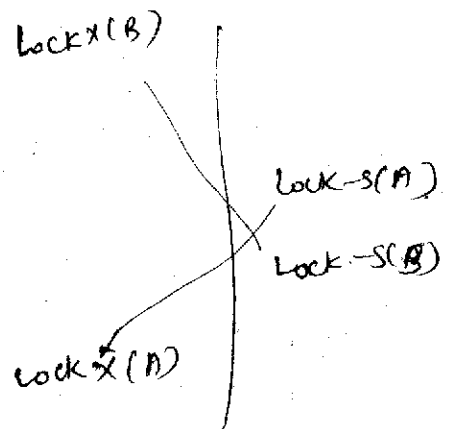
T_1	T_2
<p>1000 Lock-X(B)</p> <p>900 R(B)</p> <p>900 B = B - 100</p> <p>900 W(B)</p> <p>900 Unlock(B)</p>	<p>Lock-S(A)</p> <p>R(A) 2000</p> <p>Unlock(A)</p> <p>Lock-S(B)</p> <p>→ R(B) 900</p> <p>Unlock(B)</p> <p>Display (A+B) 2900</p> <p>Commit</p>
<p>Lock-X(A)</p> <p>R(A)</p> <p>A = A + 100</p> <p>W(A)</p> <p>Unlock(A)</p> <p>Commit</p>	



inconsistent result
due to early unlock of B

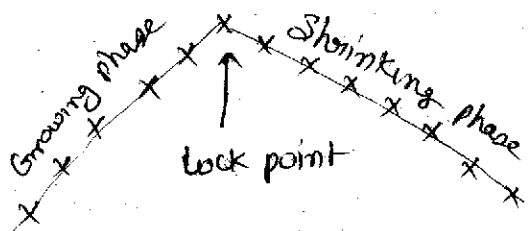


"Deadlock"



2-phase locking protocol (2PL)

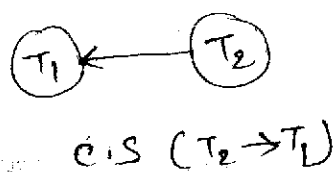
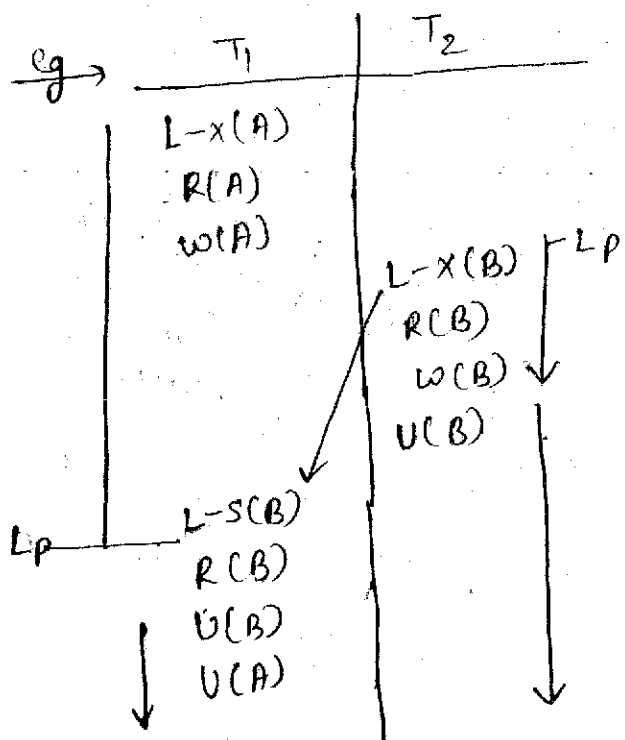
- ① Growing phase; - T's can acquire the locks but can't release
- ② Shrinking phase; - T's can release the lock but can't acquire



Lock Point

The point is a transaction that performed its final lock

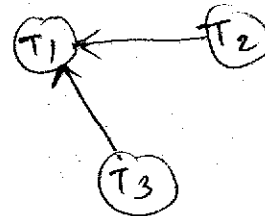
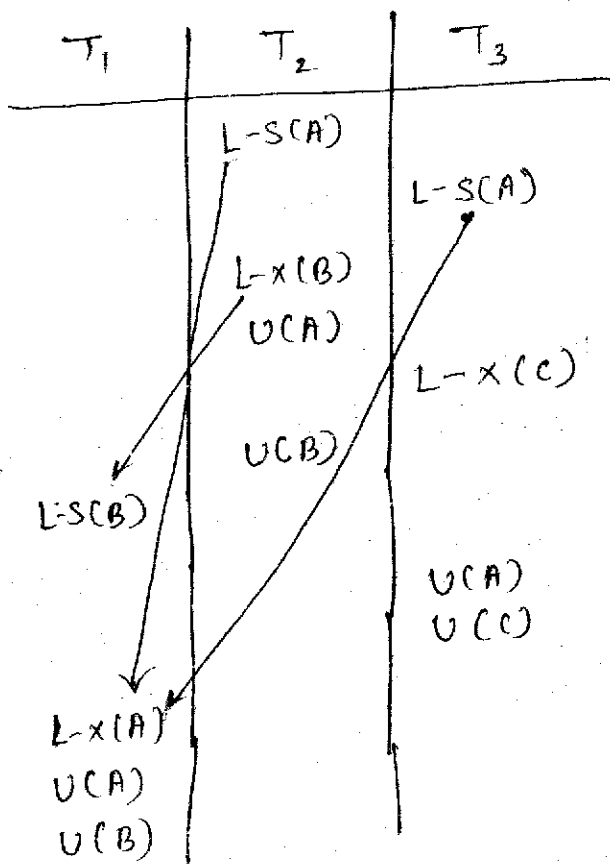
Any schedule that is possible under 2PL must ensures conflict serializability



Dead lock can occur under 2PL

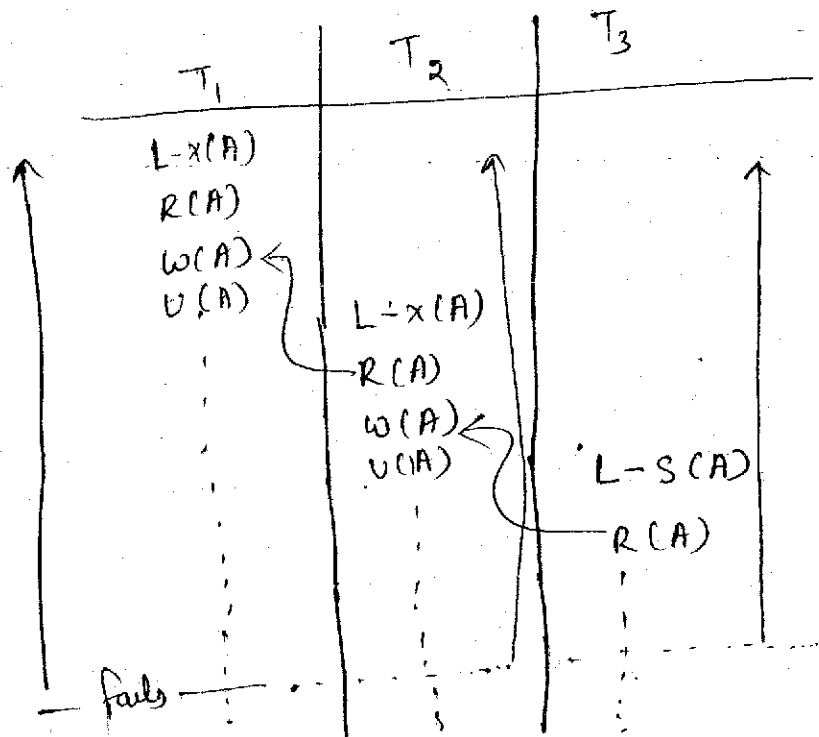
see Prev Page eg

Find the Conflict serializable order for the following schedule.



C.S

$\left\{ \begin{array}{l} T_2 - T_3 - T_1 \\ T_3 - T_2 - T_1 \end{array} \right\}$



here Roll back is bec.
of unlock by commit
if we unlock Resources
after commit then
Roll back no occurs

Cascading Roll back may occur under 2PL protocols

CR can be avoided by a modification of 2PL called Strict 2-PL or Rigorous 2PL

Strict 2PL

It requires that in addition to locking being 2 phase all the exclusive mode locks taken by a transaction must be held until it commits

Rigorous 2PL

It requires that all locks taken by the transaction must be held until it commits

Consider the following schedules

S ₁	S ₂	S ₃	S ₄
\uparrow L-x(A) R(A) W(A) \rightarrow U(A) \downarrow L-S(B) R(B) U(B) Commit	L-x(A) R(A) W(A) L-S(B) R(B) U(A) U(B) Commit	\downarrow L-S(A) R(A) \downarrow L-x(B) R(B) U(A) W(B) Commit U(B)	\downarrow L-S(A) R(A) \downarrow L-x(B) R(B) W(B) Commit U(A) U(B)

which of the above schedules ~~results~~ is according to Strict 2PL

not 2PL Basic 2PL (2PL) Strict 2PL Strict 2PL Rigorous 2PL

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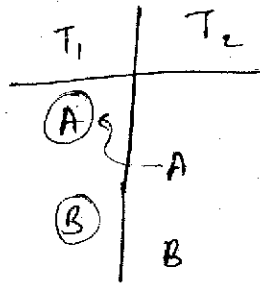
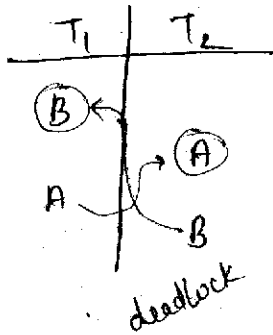
Every Rigorous 2PL is Strict 2PL but Every Strict 2PL need not be Rigorous 2PL

Most database system uses either strict or rigorous 2PL

Graph based Protocol

It is a deadlock free protocol

eg →



Deadlock not possible as order matters

- It requires that we need to have prior knowledge about the ordering about the data items that is how each transaction is going to access the data items to acquire such prior knowledge we

If $d_i \rightarrow d_j$ is an ordering then any transaction that requires both d_i and d_j must access d_i before accessing d_j

- The ordering about the data items is represented using a directed graph called the database graph

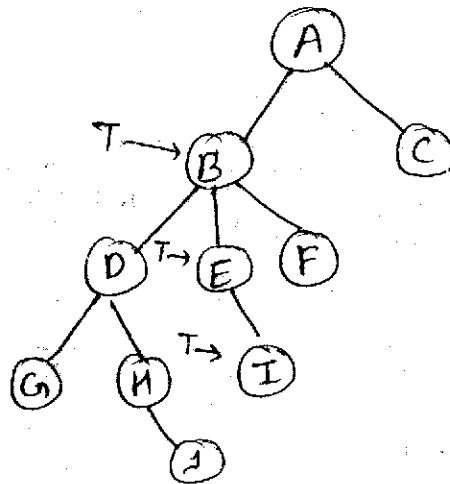
One of the simple protocol called two phase protocol which is restricted to employee only ~~exp~~ exclusive mode of locks and must also observed the following rules

a) first locks by transaction must be any data item

$$D = \{d_1, d_2, \dots, d_n\}$$

$$d_i \rightarrow d_j$$

Directed graph



b) Subsequently a transaction T can lock a data item only if the parent of the data item is currently locked by T

c) Unlocking may be done ~~at~~ any time

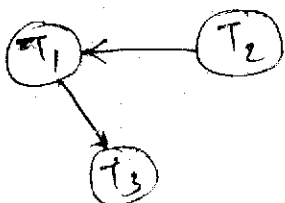
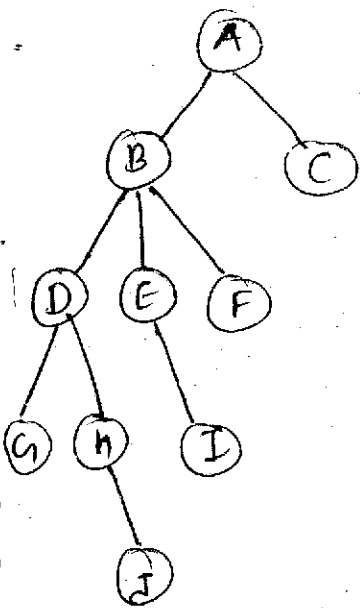
d) A data item that has been locked and unlocked by the same transaction T that can not be relocked by the same transaction.

i.e. Each data item can be locked by a transaction at most once.

NOTE

Any schedule that is possible under two phase protocol must ensure the Conflict Serializability.

Find the serializability order of the following schedule based on the database graph given below.

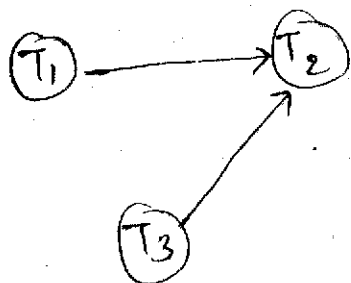
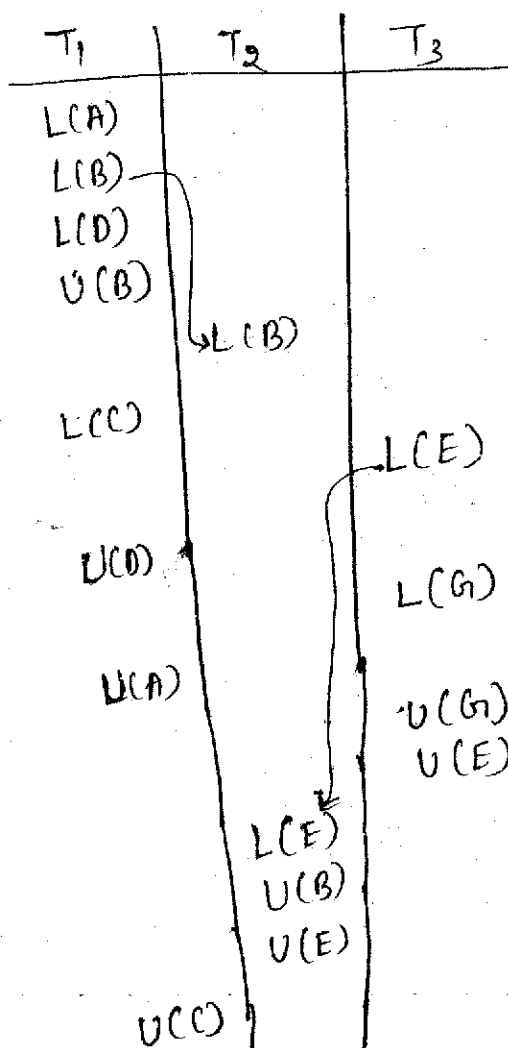
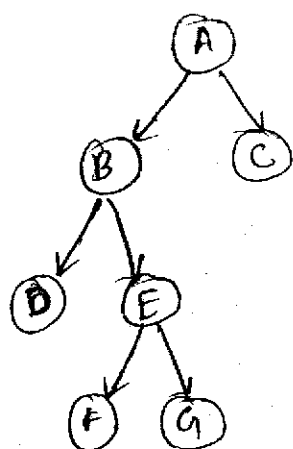


Since No cycle so
C.S

$(T_2 \rightarrow T_1 \rightarrow T_3)$

T_1	T_2	T_3
$L-x(B)$ $L-x(E)$ $L-x(D)$ $U(B)$ $U(E)$ $L-x(G)$ $U(D)$ $U(G)$	$L-x(D)$ $L-x(H)$ $U(D)$ $U(H)$	$L-x(B)$ $L-x(E)$ $U(E)$ $U(B)$

Find the serializability order of the following schedule based on the database graph given below.



No cycle so

C.S

$$\left(\begin{array}{l} T_1 \rightarrow T_3 \rightarrow T_2 \\ T_3 \rightarrow T_1 \rightarrow T_2 \end{array} \right)$$

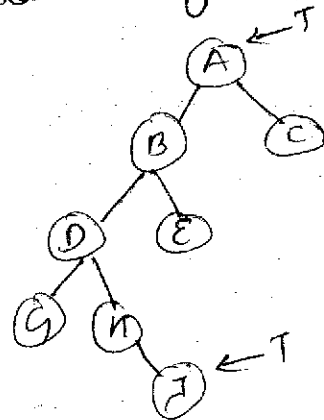
Advantage

- free from deadlock
- Ensure Conflict Serializability

Drawback

- need to have Prior Knowledge of ~~data~~ ordering of data items
- Increased locking overhead

eg → If a Trans need data items A and T then it has to lock B, D, H unnecessarily



Time Stamp based protocols

- ① It determines ordering about the transactions based on the time stamp of a transaction

Time Stamp: — The time when the transaction enters into the system for execution is known as Time Stamp.

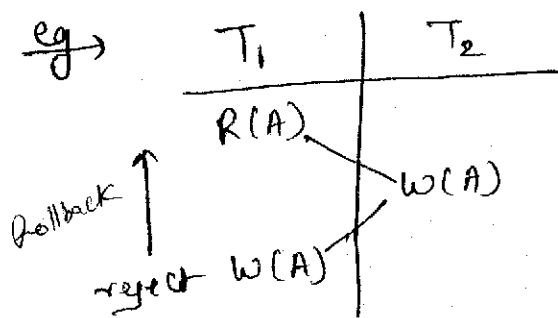
- ② Each transaction is given a fixed unique Time Stamp denoted by $Ts(T_i)$
- ③ If T_j enters after T_i then the relation betⁿ their time stamp is $Ts(T_i) < Ts(T_j)$
- ④ If $Ts(T_i) < Ts(T_j)$ then the system must ensure that the produced schedule is equivalent to a serial schedule $T_i \rightarrow T_j$ (53)

5) If any Transaction that is not executing in the order of their time stamp such operation is rejected and the transaction is rolled back

6) The Aborted Trans will restart in system with fresh time stamp

① Time Stamp ordering protocol

It requires to process all the conflict in Read & write operation in the order of their Time Stamps.



Req:

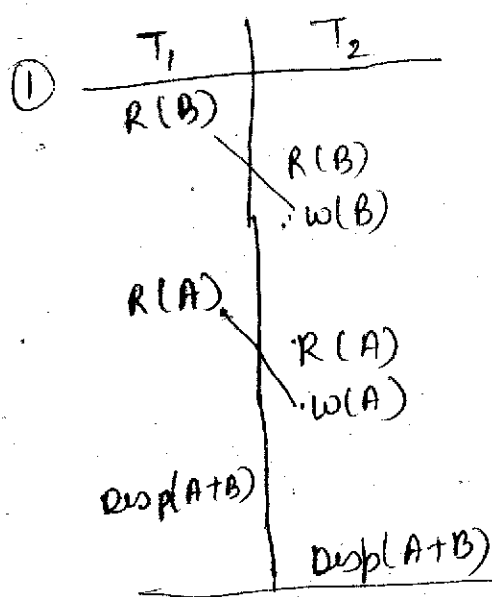
① $T_s(T_1) < T_s(T_2)$

② $T_1 \rightarrow T_2$

③ The schedule is not possible under T.O.P

④ The schedule is possible under Thomas write Rule (T.W.R)

Test the following schedule possible under Time Stamp O.P



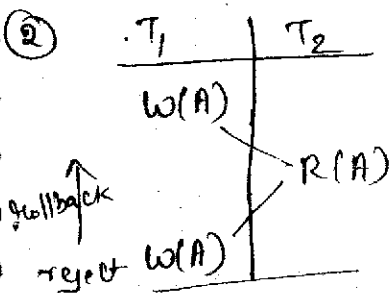
① $T_S(T_1) < T_S(T_2)$

② $T_1 \rightarrow T_2$ (order of time stamp)

③ both the conflicts are from T_1 to T_2 ($T_1 \rightarrow T_2$)

So The Schedule is possible under T.O.P

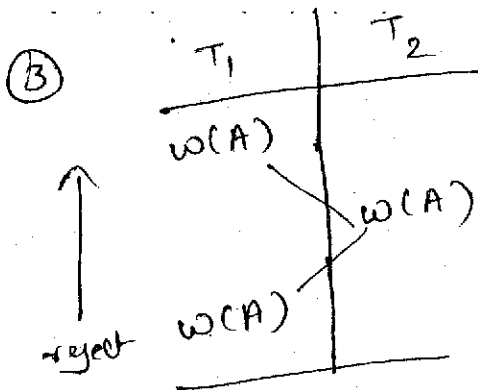
④ Also possible under TWR



① $T_S(T_1) < T_S(T_2)$

② $T_1 \rightarrow T_2$

③ not possible under T.O.P



① $T_S(T_1) < T_S(T_2)$

② $T_1 \rightarrow T_2$

③ Not possible under T.O.P

④ not possible

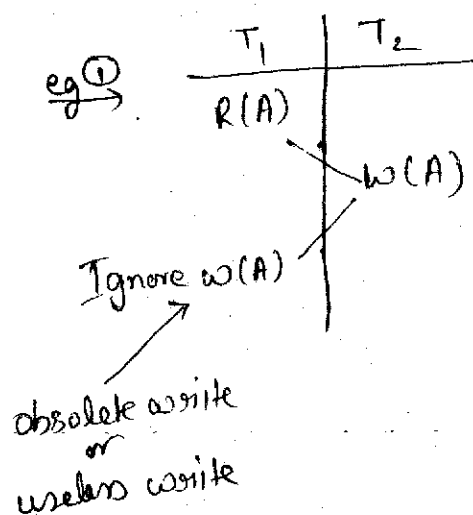
Advantage

- Ensures the Conflict Serializability

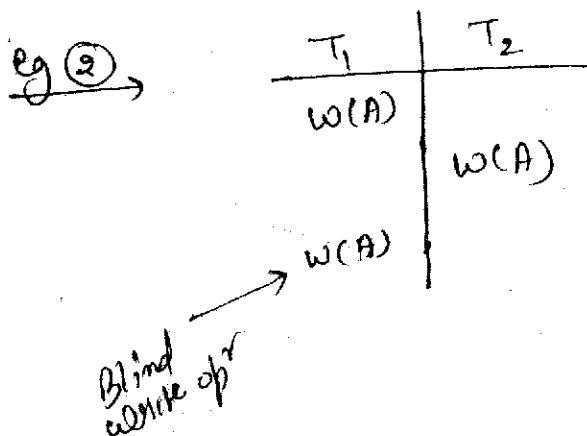
Disadvantage

- Starvation may occur due to continuously aborting (rollback) & restarting the Transaction

② Thomas write Rule (T.W.R)



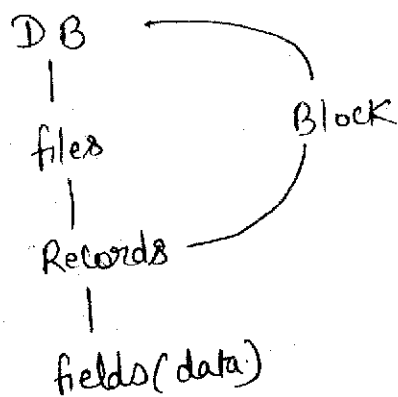
This protocol is a modification of time Stamp ordering protocol. It requires to ignore the obsolete write operations also called useless write.



Not possible under TOP
and also
not possible under T.W.R

$T_1 \rightarrow T_2$	Rollbacks	Allowed
T.O.P	① $R_2(A) \ w_1(A)$ ② $w_2(A) \ R_1(A)$ ③ $w_2(A) \ w_1(A)$	① $R_2(A) \ R_1(A)$
T.w.R	① $R_2(A) \ w_1(A)$ ② $w_2(A) \ R_1(A)$ ③ $w_2(A) \ \underline{w_1(A)}$ ↑ if not obsolete	① $R_2(A) \ R_1(A)$ ② $w_2(A) \ \underline{w_1(A)}$ ↑ if obsolete

file Organization

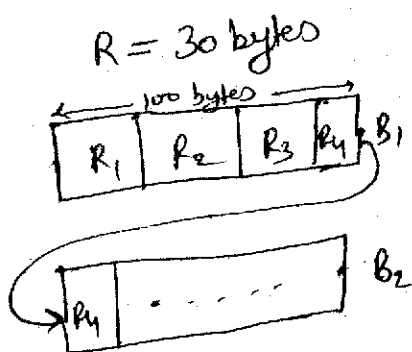


Blocking factor:- It is the avg no. of file records stored in a disc block.

for storing records into blocks we have two strategy.

① Spanned Strategy

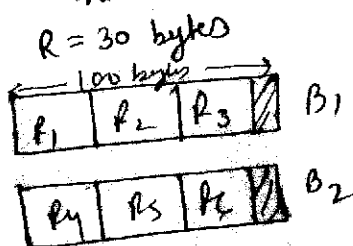
In which partial part of a record can be stored in to a block



Adv:- No wastage of memory
Disadv:- More no. of Block access
Suitable:- Variable length records

② Unspanned Strategy

In which no records can be stored in more than one block



Adv:- Less no. of Block access
Disadv:- wastage of memory
Suitable:- fixed length records.

Organization of records in a file

- ① unordered file organization :- Any record can be placed any where in the file where there is a place for the records usually records are inserted at the end of file.
It uses sequential

Advantage

- Insertion is Easy.

Disadvantage

- Searching is expensive.

- ② Ordered file Organization :- Records ~~can be~~ are ordered based on some search Key value.
It uses Binary search

Advantage

- Searching is efficient

Disadvantage

- Insertion is expensive

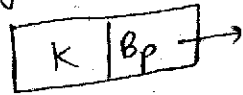
R no
1
2
3
5 ← 4
8
7
.
.
1
100

Insertion,
modification,
updaton
requires
reorganization
of ~~data~~ records

Index

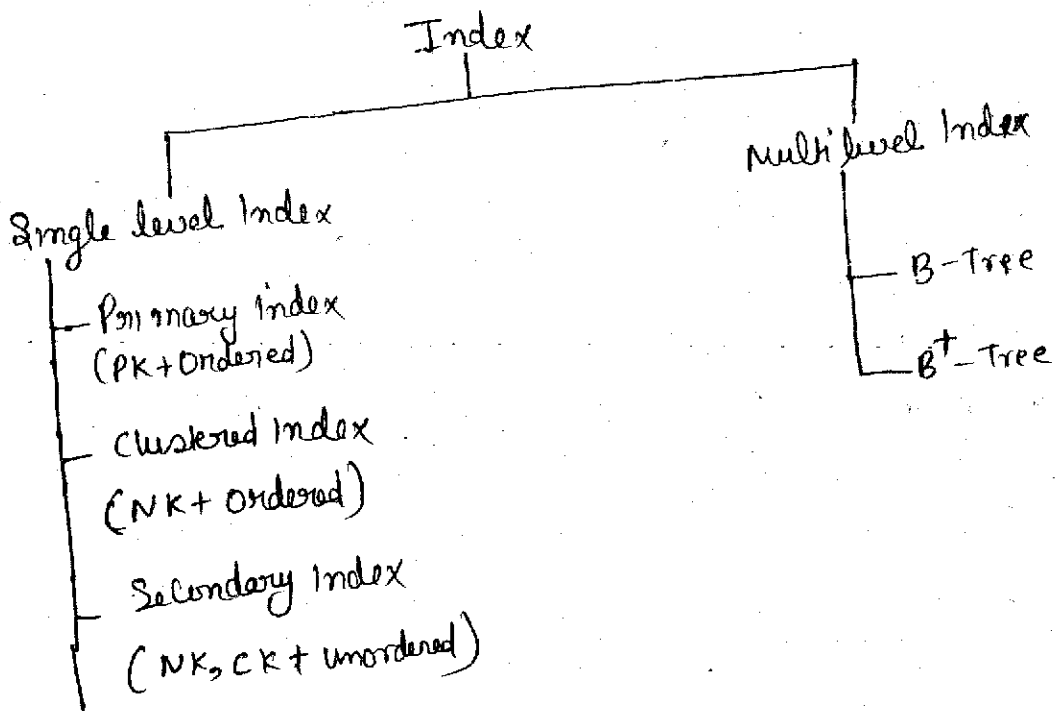
Index is used to speedup the retrieval of records in response to certain search condition

- ① Index is a collection of records with two fields Key and block pointer



- ② Index ~~can~~ ^{may} be created on any field of the file (Primary Key, Candidate key or non key)

- ③ Index is an ordered file on which we performed binary search



Other classification of Index

- ① Sparse index

- ② Dense index

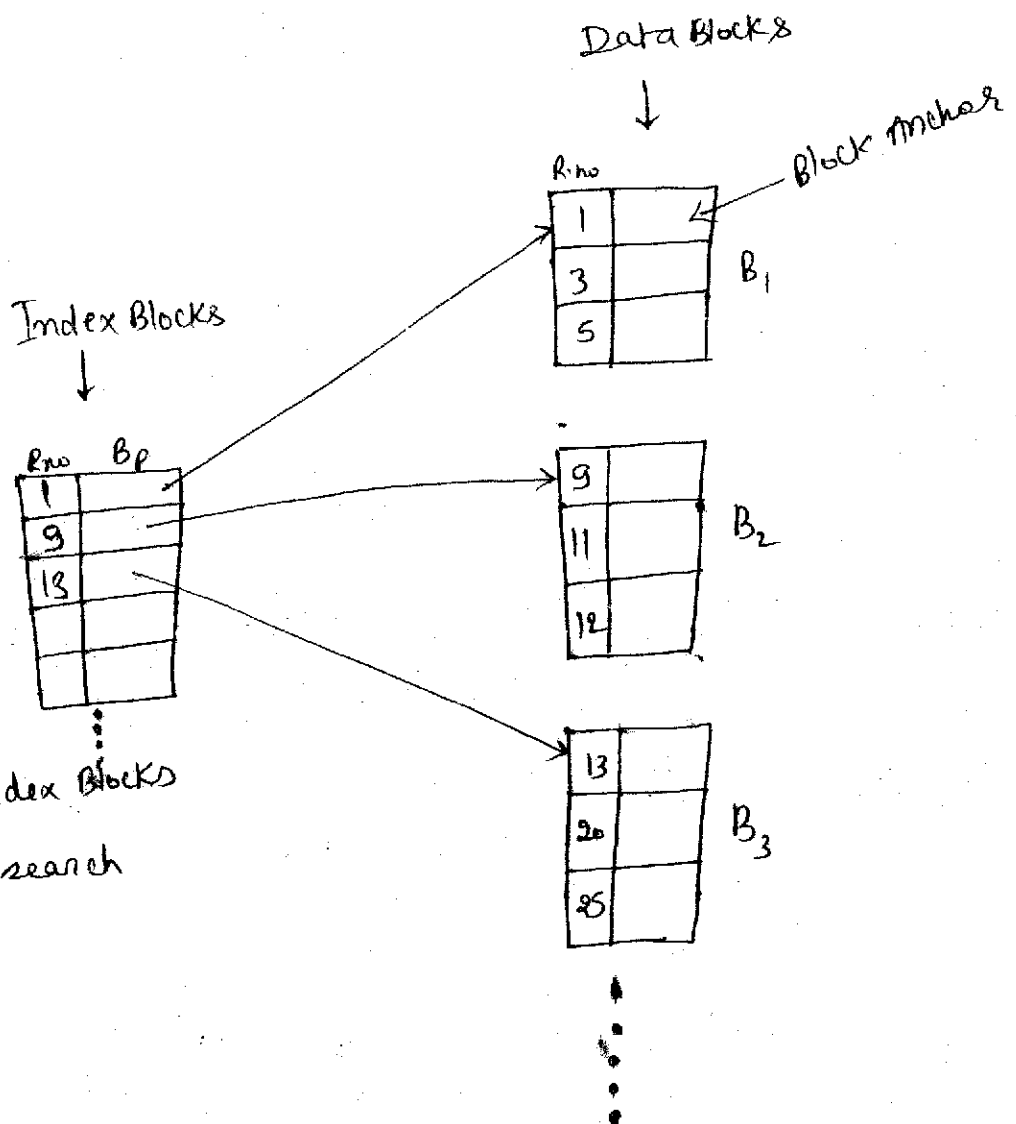
① Sparse Index :- It has an Index ^{entry} only for some search key values

② Dense Index :- It has an Index entry for every search key values.

Primary Index

PI is an ordered file whose records are of fixed length with two fields the first field of the same data type as the ordering key called the primary key of the data file and the second field is pointer to disk block.

- ① An Index entry is created for first record of each block called "block Anchor"
- ② The no. of Index records are equals to no. of blocks in the data file (data blocks)
- ③ The type of Primary Index is called sparse Index



- * $B_i \rightarrow$ Index Blocks
- * Binary search

* The avg no of Block access

$$= \log_2 B_i + 1$$

- * B-Block
- * Binary search
- * Any no. of Block access

Suppose that we have an ordered file with 30,000 records stored on a disc with block size 1024 bytes. find record size of fixed size and as are unspare record with records 100 bytes

Now suppose that if we create Primary Index on the ordering key field of the file of size 9 bytes and a Block pointer is 6 bytes.

① Then find the avg no. of block accesses without & with Primary Index.

Given, $r = 30,000$
 $R = 100 \text{ bytes}$ — fixed, unspared

$B = 1024 \text{ bytes}$

$K = 9 \text{ byte}$, $B_p = 6 \text{ bytes}$

no. of unspared (given)

$$\text{Blocking factor} = \left\lfloor \frac{1024}{100} \right\rfloor = 10 \text{ records/Block}$$

$$\text{No. of datablocks} = \left\lceil \frac{30,000}{10} \right\rceil = 3000$$

$$\begin{aligned} \text{The avg no. of Block access} &= \lceil \log_2 3000 \rceil \\ &= 12 \text{ block access} \end{aligned}$$

$$\begin{aligned}\text{Size of Index record} &= K + B_p \\ &= 9 + 6 = 15 \text{ Bytes}\end{aligned}$$

$$\text{No of index records} = 3000$$

$$\text{Blocking factor} = \left\lfloor \frac{1024}{15} \right\rfloor = 68 \text{ Index records / Blocks}$$

$$\text{No. of index Blocks} = \left\lceil \frac{3000}{68} \right\rceil = 45$$

$$\begin{aligned}\text{The avg no. of Block access} &= \lceil \log_2 45 \rceil + 1 \\ &= 6 + 1 \\ &= 7 \text{ block access}\end{aligned}$$

Clustered Index

If records of a file are ordered on a non key field which does not have distinct value for each record that field is called clustering field.

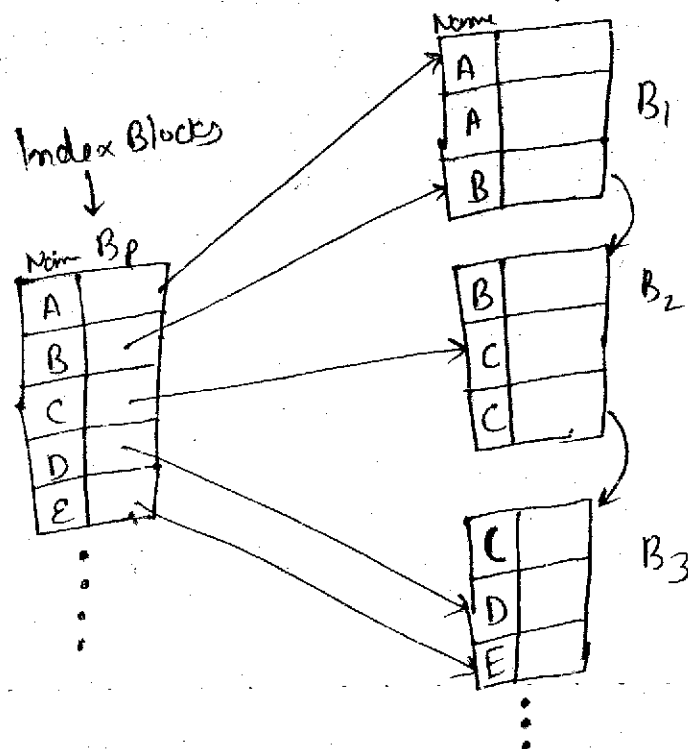
we can create different type of Index called Clustering Index to speed up the retrieval of records that have the same value for the clustering field.

Clustering Index is an ordered file with two field. The first field is of the same ^{type} as the clustering field of the data type & the second field is a block pointer.

An Index entry is created for each distinct value of a non key and

Each block pointer points to the first block in the data file that has the record with that clustering field

The type of clustered index is called sparse index



* B_i - Index Blocks

* Binary Search

* B - Blocks

* Binary Search

The avg no. of Block access

$$\geq \log_2 B_i + 1$$

* avg no. of Block access

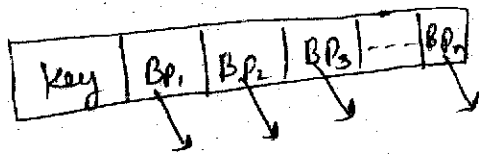
$$\geq \log_2 B$$

Secondary Index

Secondary Index Provide 2e

secondary Index may be ~~called~~^{on} Candidate Key or a non key or unordered file

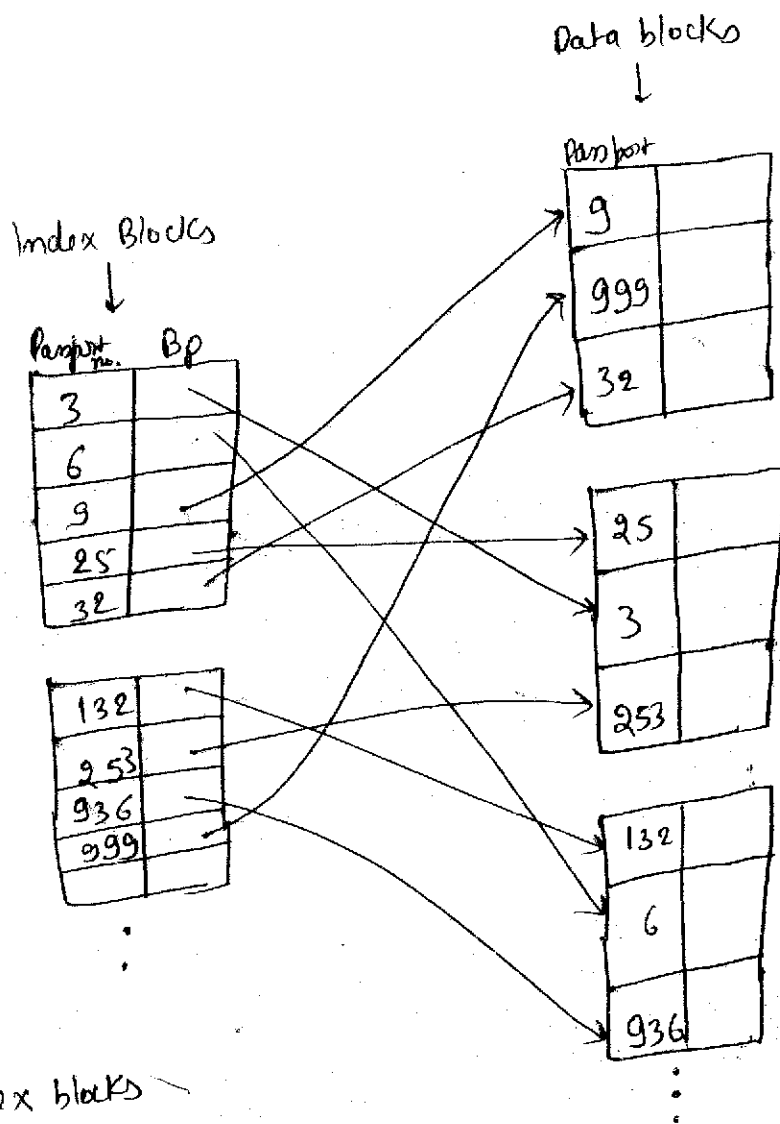
① Secondary index (NK + unordered)



② Secondary index (CK + unordered)

The no. of Index records are equals to the no. of records in a data file i.e an index entry is created for each record in a data file

The type of This Index is called dense index



* B_i - Index blocks

* Binary Search

* The avg no of Block access

$$= \log_2 B_i + 1$$

* B - Blocks

* Linear Search

* Avg no of block access $= \frac{B}{2}$

Q2 If a secondary Index is created on the primary key field of the file of Q1 then find the avg no. of block accesses using with or without Index.

Given

$$n = 30,000$$

$$R = 100 \text{ bytes, fixed, unspanned}$$

$$B = 1024$$

$$K = 9 \text{ bytes, } B_p = 6 \text{ bytes}$$

$$\text{Blocking factor} = \left\lfloor \frac{1024}{100} \right\rfloor = 10 \text{ records/block}$$

$$\text{No of data blocks} = \left\lceil \frac{30000}{10} \right\rceil = 3000$$

$$\text{The avg no. of Block access} = \frac{3000}{2} = 1500$$

$$\begin{aligned} \text{Size of Index record} &= K + B_p \\ &= 9 + 6 = 15 \text{ bytes} \end{aligned}$$

$$\text{no of index records} = 30,000$$

$$\text{Blocking factor} = \left\lfloor \frac{1024}{15} \right\rfloor = 68 \text{ index records/blocks}$$

$$\text{No. of index blocks} = \left\lceil \frac{30000}{68} \right\rceil = 442$$

$$\text{The avg no. of Block access} = \lceil \log_2 442 \rceil + 1$$

$$= 9 + 1$$

$$= 10 \text{ Block access}$$

(68)

Multilevel Index

- ① An index over an index is called multiple level index
- ② On any of the first level index we create a primary index called second level index on which we create a primary index called third level index and so on until all the index records fit in one disc block
- ③ Number of index ~~block~~ records in the n^{th} level are equals to no. of blocks in the $(n-1)$ level where $n \geq 2$
- ④ The no. of blocks to access is equals to $(l+1)$ where l is the no. of levels i.e.
At each level we access one block and one block in the data file.

Q3 Suppose if we create a ~~multi~~ level index on the key field of the file of question 2. Then find the no. of block accesses.

$$N = 30,000$$

$$R = 100 \text{ bytes}$$

$$B =$$

$$K =$$

$$\begin{aligned} \text{Size of index record} &= K + Bp \\ &= 9 + 6 = 15 \end{aligned}$$

1st No of index records = 30000

$$\text{Blocking factor} = \left\lfloor \frac{1024}{15} \right\rfloor = 68 \text{ index records/blocks}$$

$$\text{No. of index blocks} = \left\lceil \frac{30000}{68} \right\rceil = 442$$

2nd no. of index records = 442

$$\text{no. of index blocks} = \left\lceil \frac{442}{68} \right\rceil = 7$$

3rd no. of index records = 7

$$\text{no. of index blocks} = \left\lceil \frac{7}{68} \right\rceil = 1$$

$$\text{The no. of block access} = 3 + 1$$

$$= 3 + 1$$

$$= 4 \text{ block access}$$

$$\frac{19-82}{0=7}$$

$$\text{Size of index record} = k + Bp$$

$$= 6 + 10 = 16$$

$$\text{No of index records} = 16,384$$

$$\text{Blocky factor} = \left\lfloor \frac{1024}{16} \right\rfloor = 64$$

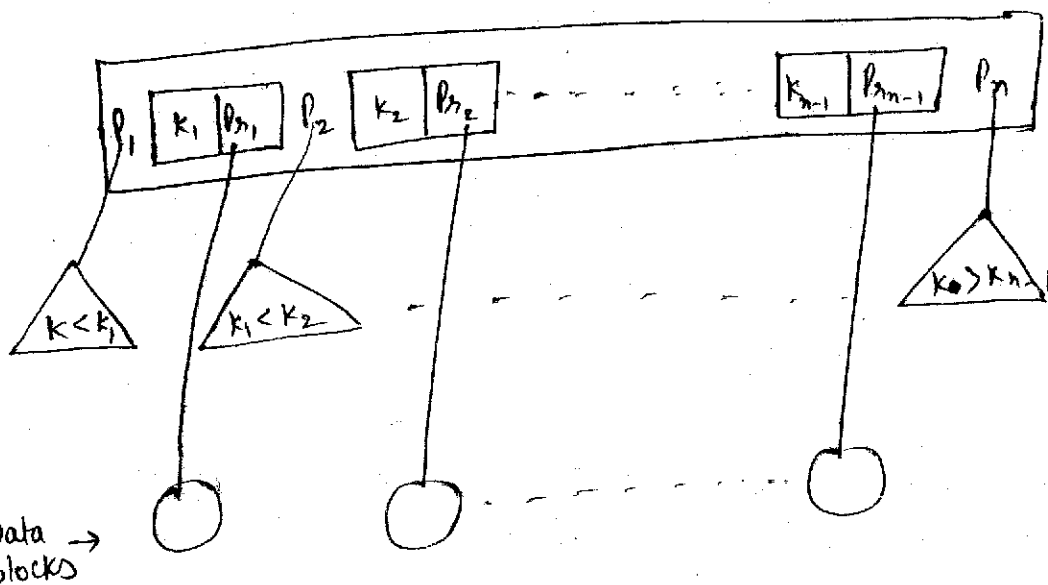
$$\text{No. of index blocks} = \frac{16}{64} = 256$$

Implementation multilevel Index Using B-Tree

B-Tree

- ① B-Tree is a height balanced search tree
- ② The node structure of a B-Tree corresponds to one block in the index file

Structure of a B-Tree node



$p_1, p_2, \dots, p_n \leftarrow$ Block pointers / Index pointer / true pointer

$k_1 < k_2 < \dots < k_{n-1} \leftarrow$ keys

$p_{k1}, p_{k2}, \dots, p_{kn-1} \leftarrow$ Data pointers / Related pointers

- ① B-Tree provides direct access i.e. The key to search is found at some level of next levels of index and access the data blocks
it bypasses all the

Order of B-Tree (let n)

Order of B-Tree is the no. of tree pointers in a node

$n \leftarrow$ tree pointers

$(n-1) \leftarrow$ Index records

$$n * P + (n-1) * (K + P_r) \leq B$$

Q4

Calculate the order of a B-Tree stored on a disc with block size 512 bytes, Key size 9 bytes and record pointers of size 7 bytes and block pointer of size 6 bytes.

$$n * 6 + (n-1) * (9 + 7) \leq 512$$

$$6n + 16n - 16 \leq 512$$

$$22n \leq 528$$

$$n \leq 24$$

Q5

Calculate the approximate no. of entries that can be stored in a B-Tree of level-3 of Q4 [Assume that each node is 69% full]

order of 69% full

$$= 24 * 0.69$$

$$= 16$$

Root	1 node	16 Pointers	15 Keys
Level 1	16 nodes	256	240
Level 2	256	4096	3840
Level 3	4096	65536	61440

$$\text{Total index records} = 15 + 240 + 3840 + 61440$$

$$= 65,535$$

(78)

Q6 find the no. of levels required to store 20,000 index records in a B-Tree of order 12.

Root	1 node	12 pointer	11 keys
Level 1	12	144	132
Level 2	144	1728	1584
Level 3	1728	20736	19008

3 level required

or

$$\log_{12} 20000 \approx 4$$

$$\log_{\text{nodes}} \text{no. of records} = \text{no. of levels (including root)}$$

4

B-Tree Insertion

Order : n

Max : n pointers

$(n-1) \leftarrow \text{Key}$

Min : $\lceil \frac{n}{2} \rceil \leftarrow \text{Pointers}$

$\lceil \frac{n}{2} \rceil - 1 \leftarrow \text{Keys}$

except
for root

$$\left(\begin{array}{c} \lceil \frac{n}{2} \rceil - 1 \\ \lceil \frac{n}{2} \rceil \end{array} \right)$$

eg

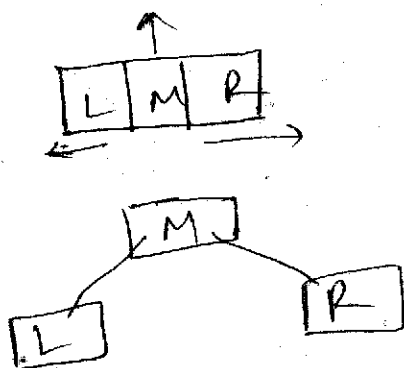
order : 5

Max : 5 pointers
4 Keys

Min : 3 pointers
2 Keys

- ① A new element ~~or~~ Key is always inserted in the leaf node
- ② If the node is full we split the node into two nodes moving the middle element ^{to} ~~in~~ to ^{one} level above and insert it at its proper place

eg

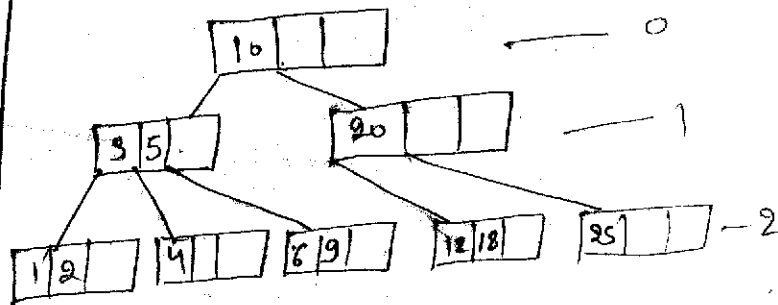
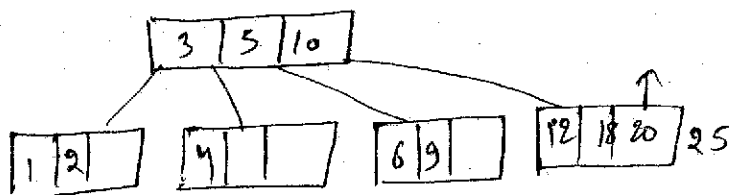
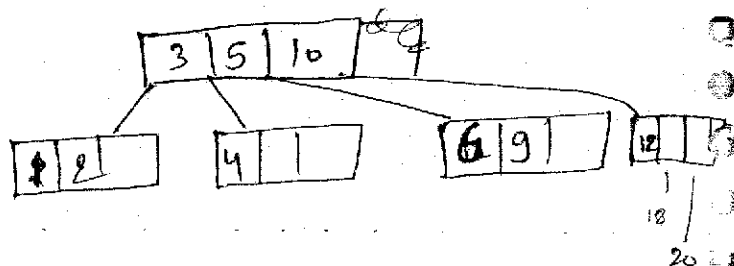
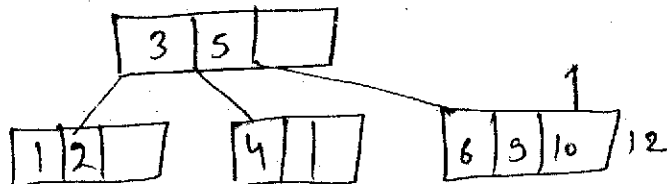
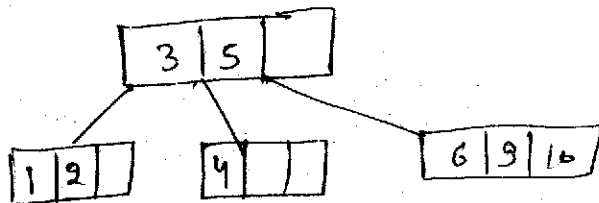
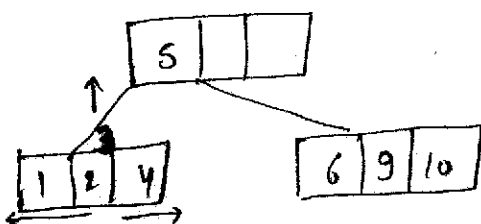
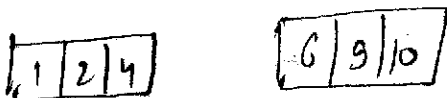
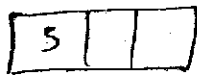
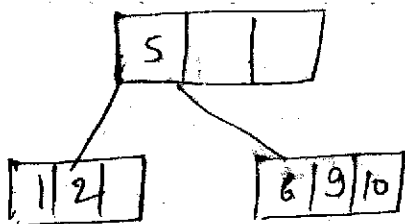
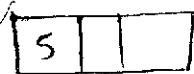
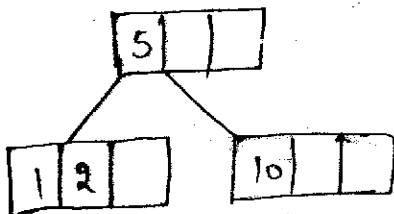
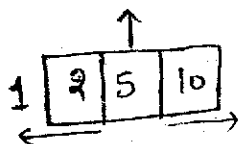


Q7 Insert the following keys in the B-Tree of order 4

Keys: 2, 5, 10, 1, 6, 9, 4, 3, 12, 18, 20, 25

Max: 4 pointers, 3 Keys

Min: 2 pointers, 1 Key



76

no. of splits = 5

no. of Root splits = 2

no. of leaf splits = 4

total no. of nodes = 8

No of nodes with full factor 100% = 0

Root = 10

no of levels = 2 levels

Consider a B-Tree of order 3

20, 15, 10, 5, 8, 30, 1, 40

Max: 3 pointer, 2 Keys

No. of Splits = 4

No. of Root splits = 2

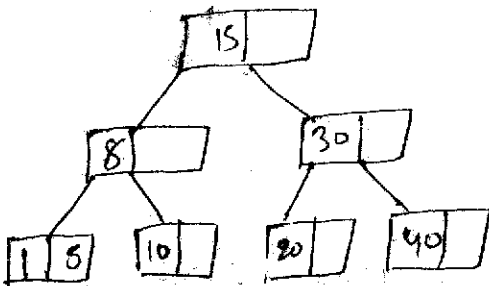
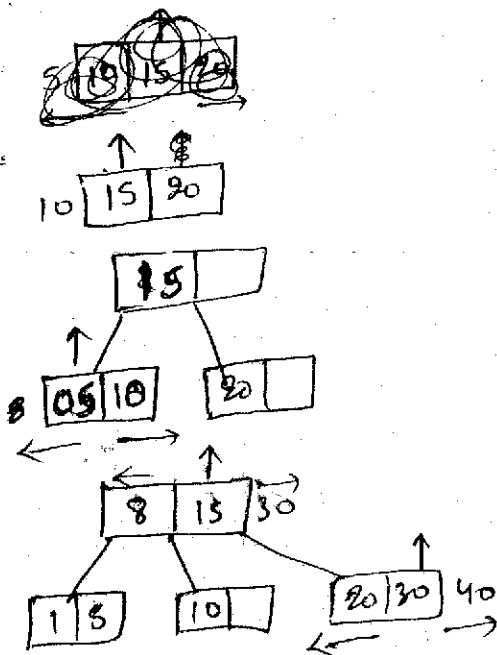
No. of leaf splits = 3

total no. of nodes = 7

No. of nodes with full factor 100% = 1

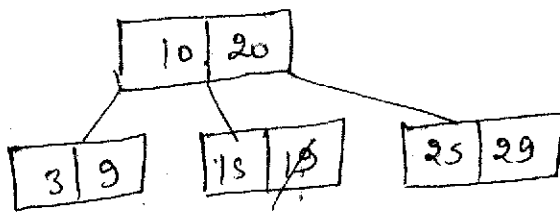
Root = 15

no. of levels = 2

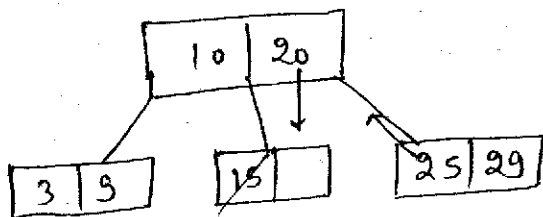


B-Tree Deletion

Consider the following B-Tree



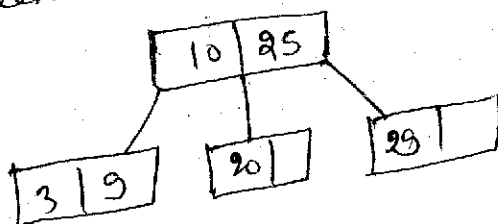
Case 1 Delete 19



If deleting the key results in more than ^{or equals to} min no. of keys, no pointer modifications required i.e.

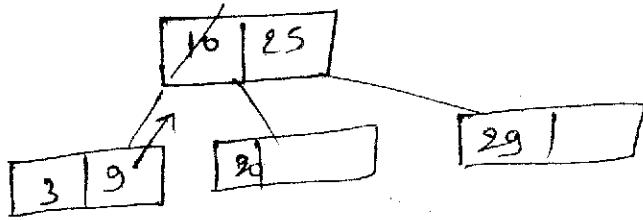
If the node contains more than min keys then delete it

Case 2 Delete 15

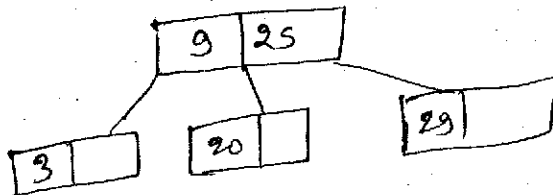


If deleting the key results in less than the min no. of keys then borrow the keys sibling

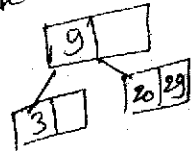
Case ③ delete 10 (deleting internal node)



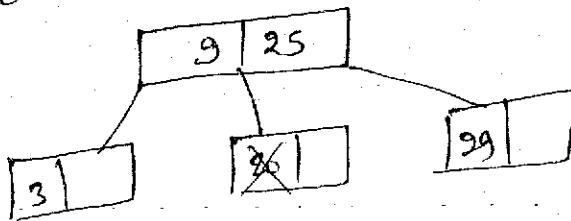
If key to be deleted i.e not a leaf node then find the smallest key greater than the key to delete or largest key less than the key to delete & replace.



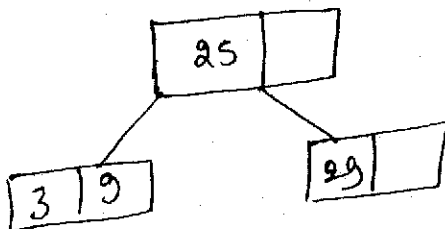
If we delete 25 then



Case ④ delete 20

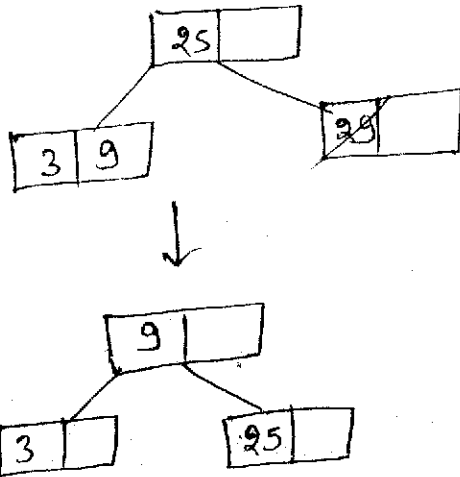


If sibling do not have spare (extra) keys then merge the node parent either with its left siblings or right siblings



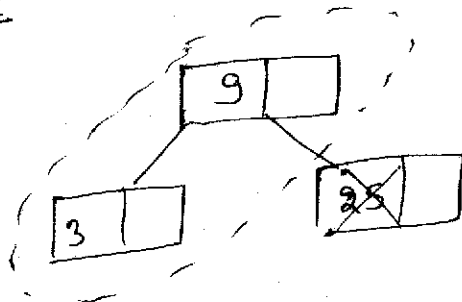
Case 5

delete 29

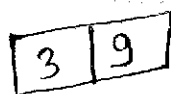


Case 6

delete 25



The Case in which the parent also do not have sufficient keys then merge the node parent and sibling to make one node.



Implementation of Multilevel Index using B⁺ tree

B⁺ Tree (Ordered access)

Two modifications of B-Tree node

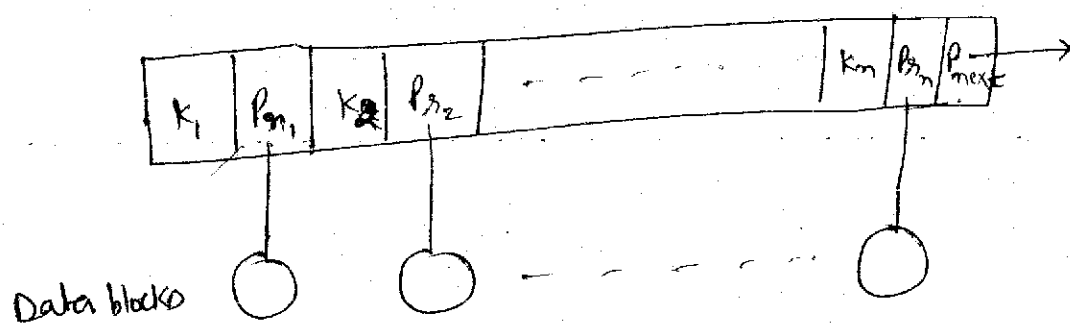
① Leaf node → Remove tree pointers, Contains only Key, Record pointers.

② Internal nodes → Remove record pointers, Contains only Key, tree pointers

NOTE

The structure of leaf and internal node is a B⁺ tree is different

① Structure of Leaf node



$K_1 < K_2 < \dots < K_n$ ← Key

P_1, P_2, \dots, P_{nn} ← Record pointer / Data pointer

P_{next} ← Block pointer / tree pointer / index pointer

Order of leaf node (let n)

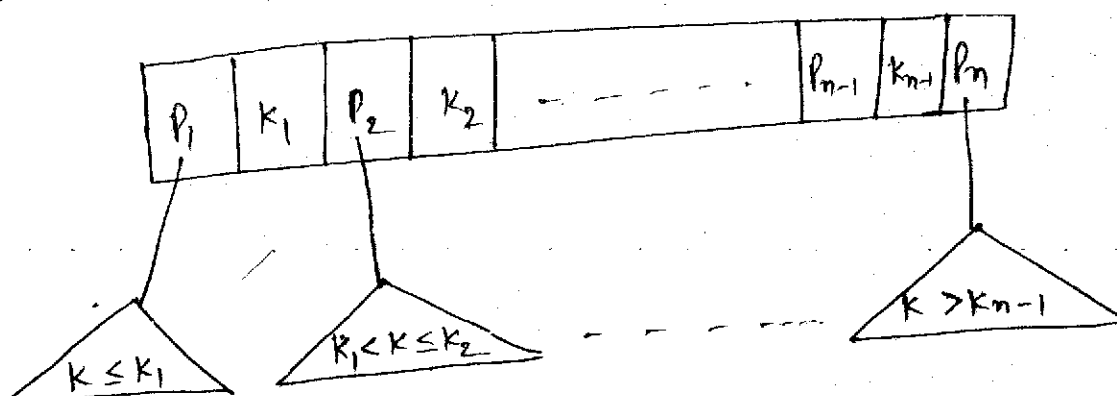
Order of leaf node is the no. of index records in a node

$n \leftarrow$ Keys, Record pointer

$1 \leftarrow$ Block pointer (p_{next})

$$n * (K + P_r) + p_{next} \leq B$$

② Structure of Internal node



$p_1, p_2, \dots, p_n \leftarrow$ True pointer / Block pointer / Index pointer

$K_1 < K_2 < \dots < K_{n-1} \leftarrow$ Keys

Order of

Order of Internal node (Let n)

Order of internal node is the no. of tree pointers in it

$n \leftarrow$ tree pointers

$(n-1) \leftarrow$ keys

$$n * p + (n-1) * k \leq B$$

NOTE

Order of leaf and internal node of a B^+ Tree is different

$$B = 512, K = 9, P_s = 7, P = 6$$

Q8

Calculate the order of B+ tree. Suppose that the search key is field is 9 bytes. The block size is 512 bytes. A record pointer is 7 bytes. and the block pointer is 6 bytes.

order of internal node →

$$9 \times (n-1) + 6 \times n \leq 512$$

$$16n - 9 \leq 512$$

$$16n \leq 521$$

$$n \leq 34$$

order of leaf node

$$n(9+7) + 6 \leq 512$$

$$16n \leq 506$$

$$n \leq 31$$

Q9

Calculate the approximate no. of entries that can be stored in a B+ tree of level-3 of Q8. [Assume that each node is 69% full]

order of leaf node

$$= 31 \times 0.69$$

$$= 21$$

order of internal nodes

$$= 34 \times 0.69$$

$$= 23$$

			no. need this level
Root	1 node	23 pointers	↓
Level 1	23 nodes	529	—
Level 2	529 "	12,167	—
Leaf Level	12,167	12,167 × 21 = 2,55,507	—

Compare it with B-tree

more index ~~are~~ stored here
approx [4 times]

and is less level

84

Q1

Q1

$K = 8$ bytes

$B = 512$

$P = 4$

here as key int node →

$$m \times 4 + 8n - 8 \leq 512$$

$$12n \leq 520$$

$$n \leq 43$$

Q2

$P = 6, K = 14, B = 512$

$$n \times 6 + (n-1)14 \leq 512$$

$$20n \leq 526$$

$$n \leq 26$$

Q4

$B = 1024$

$P_1 = 7$

$K = 9$

$P = 6$

$$n(K+P) + P_{next} \leq 1024$$

$$n(9+7) + 6 \leq 1024$$

$$16n \leq 1018$$

$$n \leq 63$$

Q5

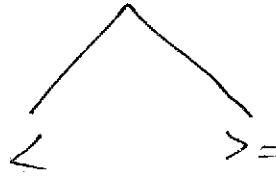
Q5

B⁺ Tree Insertion.

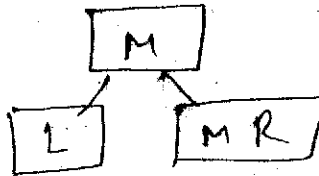
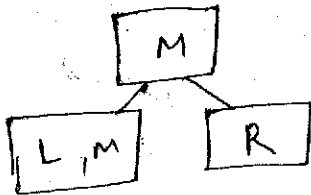
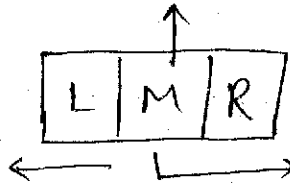
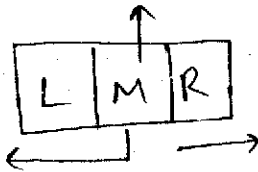
Property :-



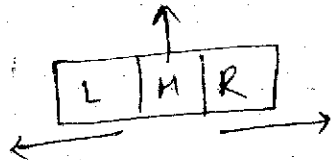
(3)



Leaf Split



Internal Split



PC

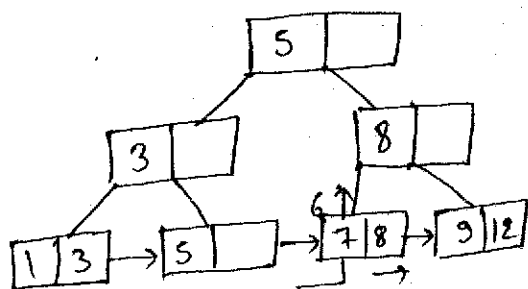
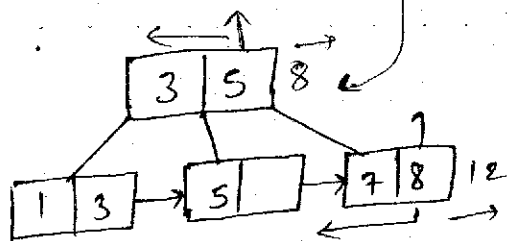
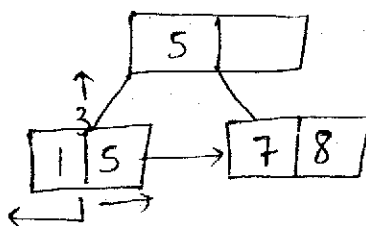
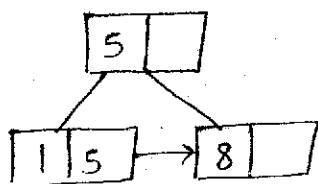
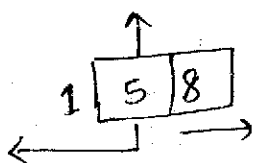
Q10

Insert the following keys in a B⁺ tree of ~~order 3~~
Internal node order 3 and leaf node order 2

order of Internal 3 → 3 pointers, 2 Keys

order of Leaf 2 → 2 Keys

Keys: 8, 5, 1, ~~7~~, 3, 12, 9, 6



① No of splits = 5

② No of Leaf node splits = 4

③ Root node splits = 2

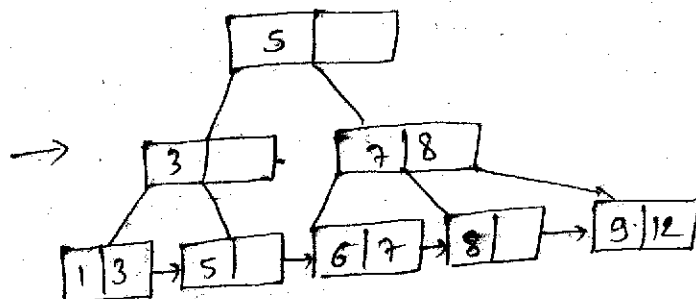
④ Internal node = 1

⑤ no of nodes = 8

⑥ no of nodes of 100% fill factor = 4

⑦ Root node = 5

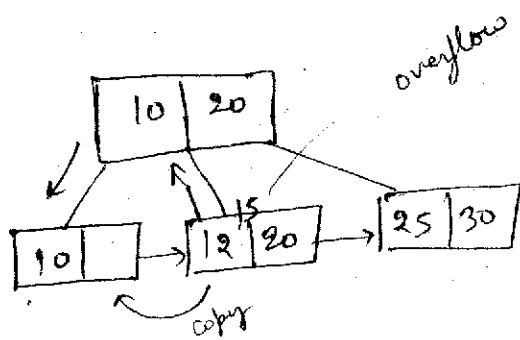
⑧ Levels = 2



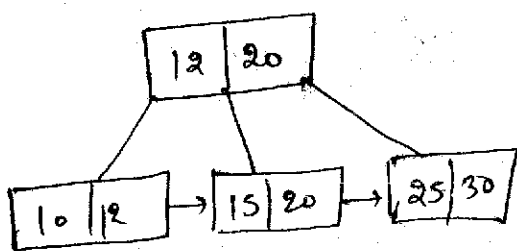
87

B⁺ tree Rotation

B⁺ tree can incorporate rotation to reduce the no. of ^{node} ~~page~~ splits. a rotation occurs when a leaf node is full but of its sibling is not full rather than splitting the leaf node we move a records to its siblings adjusting the pointers as necessary. typically the left sibling is checked first (if exist) and then the right sibling

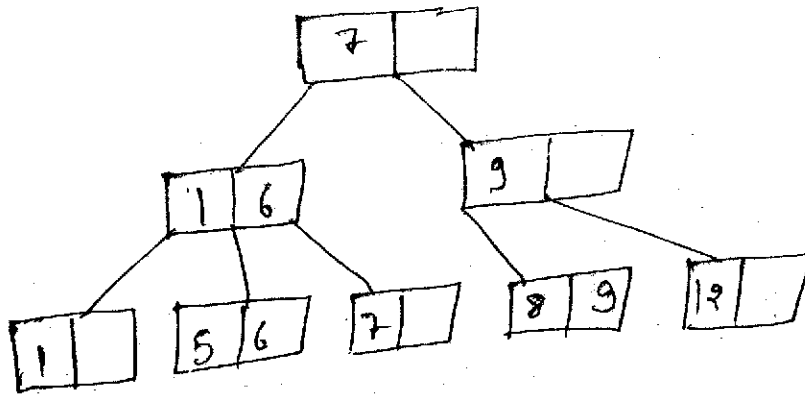


Insert 15

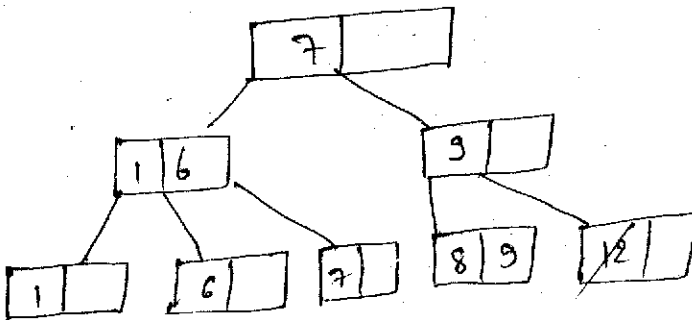


← This is Special case (avoid this)
If asked in exam (do it by rotation)
then only do this case
otherwise do normally

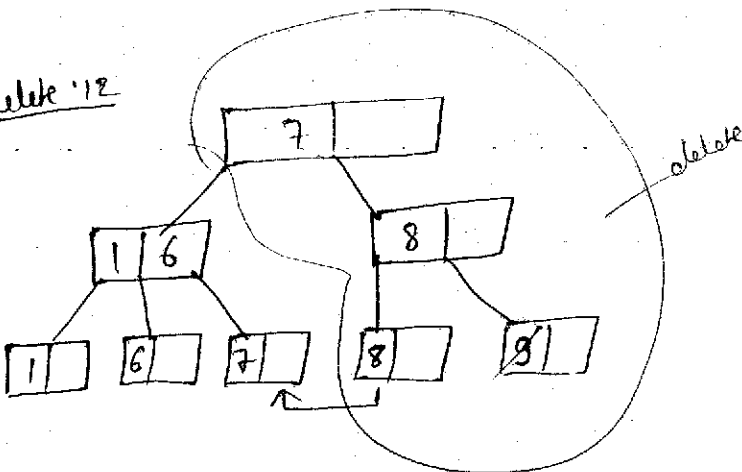
B+ tree Deletion



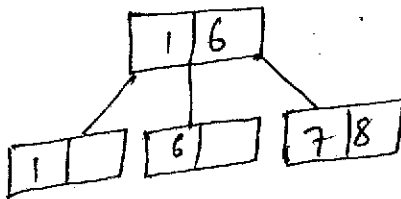
delete 5, 12, 9



delete 12



delete 9

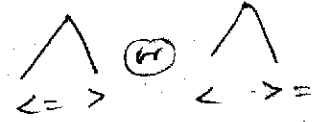


Same prop of B-Tree

① ② ③ } same

+

① Property



② Height balance

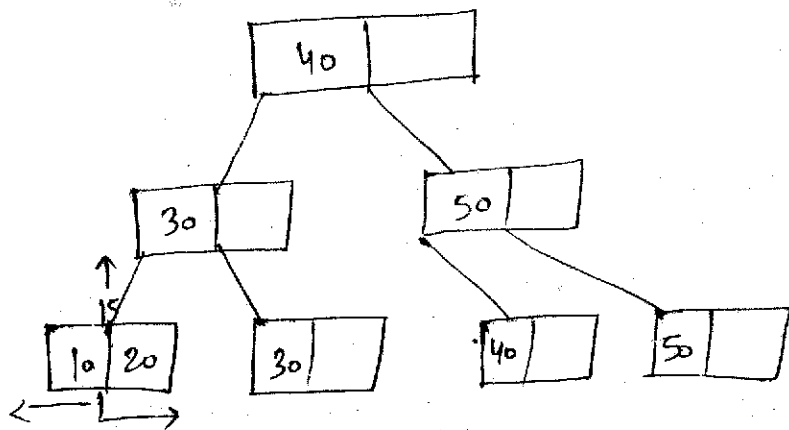
③ Min order

If we merge 8 to 7 then
if we delete right sub tree of 7
then link of 7 node becomes
1 (violate prop) so
delete root 7 node

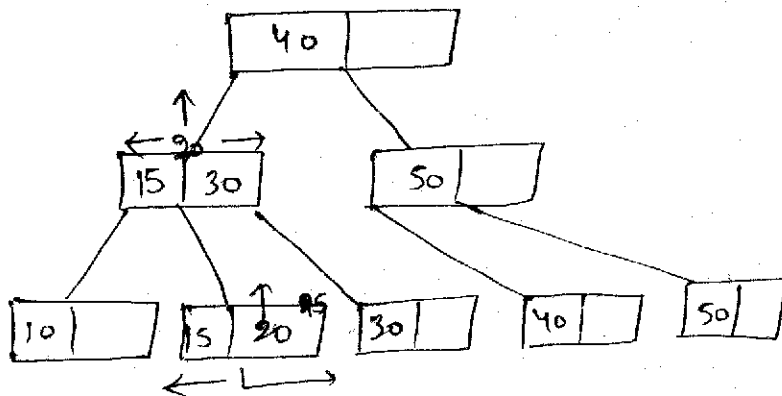
8

Pg-82

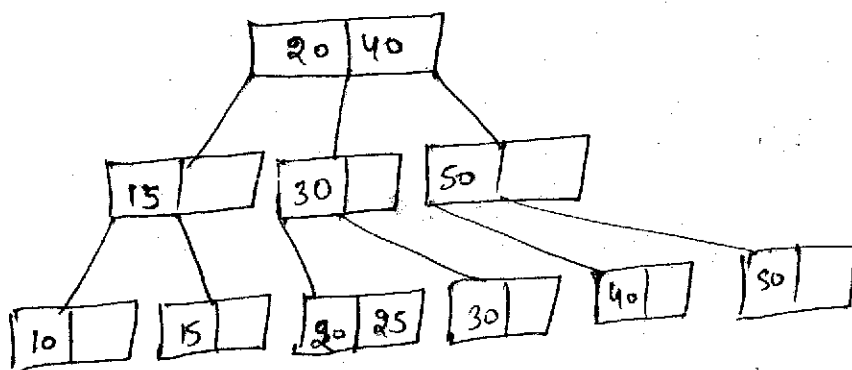
05, 6



Insert 15



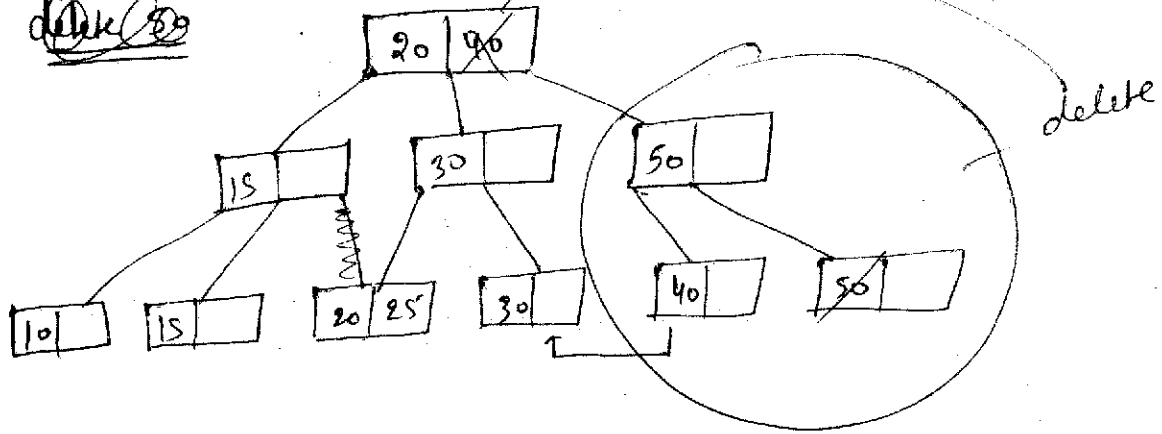
Insert 25



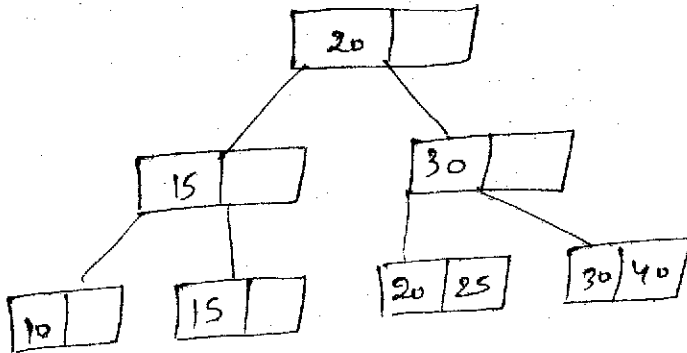
20

6

delete 30



delete 50



(10)