

$$\begin{aligned}
f_t &= \sigma(W_f x_t + U_f h_{t-1} + b_f) \\
i_t &= \sigma(W_i x_t + U_i h_{t-1} + b_i) \\
o_t &= \sigma(W_o x_t + U_o h_{t-1} + b_o) \\
c'_t &= \tanh(W_c x_t + U_c h_{t-1} + b_c) \\
c_t &= f_t \circ c_{t-1} + i_t \circ c'_t \\
h_t &= o_t \circ \tanh(c_t)
\end{aligned} \tag{1}$$

Variables:

$M$ : input vector dimension

$N$ : output vector dimension

( $H_i$ : hidden layer  $i$  vector dimension)

$x_t$  [ $M \times 1$ ]: input vector

$h_t$  [ $N \times 1$ ]: output vector

$h_0 = 0$

$c_t$  [ $N \times 1$ ]: cell state vector

$c_0 = 0$

$W$  [ $N \times M$ ],  $U$  [ $N \times N$ ] and  $b$  [ $N \times 1$ ]: parameter matrices and vector (W is for weight, U is for update?, and b is for bias?)

$f_t$ ,  $i_t$  and  $o_t$ : gate vectors

$f_t$  [ $N \times 1$ ]: Forget gate vector. Weight of remembering old information.

$i_t$  [ $N \times 1$ ]: Input gate vector. Weight of acquiring new information.

$o_t$  [ $N \times 1$ ]: Output gate vector. Output candidate.

$$\begin{aligned}
\delta h_t + &= target_t - h_t \\
\delta o_t &= \delta h_t \circ \tanh(c_t) \\
\delta c_t + &= \delta h_t \circ o_t \circ \tanh'(c_t) \\
\delta i_t &= \delta c_t \circ c'_t \\
\delta f_t &= \delta c_t \circ c_{t-1} \\
\delta c'_t &= \delta c_t \circ i_t \\
\delta c_{t-1} &= \delta c_t \circ f_t \\
\delta \hat{i}_t &= \delta c_t \circ c'_t \\
\delta \hat{f}_t &= \delta c_t \circ c_{t-1} \\
\delta \hat{c}'_t &= \delta c'_t \circ (1 - c_t^2) \\
\delta \hat{i}_t &= \delta i_t \circ i_t \circ (1 - i_t) \\
\delta \hat{f}_t &= \delta f_t \circ f_t \circ (1 - f_t) \\
\delta \hat{o}_t &= \delta o_t \circ o_t \circ (1 - o_t) \\
\delta W_{i,f,o,c} &= \delta \hat{i}, \hat{f}, \hat{o}, \hat{c}'_t x_t^T \\
\delta U_{i,f,o,c} &= \delta \hat{i}, \hat{f}, \hat{o}, \hat{c}'_t h_{t-1}^T \\
\delta b_{i,f,o,c} &= \delta \hat{i}, \hat{f}, \hat{o}, \hat{c}'_t \\
\delta h_{t-1} &= \sum U_{i,f,o,c}^T \delta \hat{i}, \hat{f}, \hat{o}, \hat{c}'_t
\end{aligned} \tag{2}$$

$$\begin{aligned}
\delta h_T &= 0 \\
\delta c_T &= 0
\end{aligned}$$