Numerical Algorithms - Seventh lab session

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7.1) Compute the eigenvalues and eigenvectors of the following matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

<u>Resolution</u>: $A x = \lambda x$ → eigenvalues (λ) → eigenvectors (x)

$(A - \lambda I)x = 0 \Leftrightarrow \boxed{\det(A - \lambda I) = 0}$ Equation (Polynomial 3rd degree) solve: 3 roots → λ

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda I = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}; \quad A - \lambda I = \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & -\lambda \end{pmatrix}.$$

* $\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & 0 & 0 & 1-\lambda & 0 \\ 0 & -\lambda & 1 & 0 & -\lambda \\ 0 & 0 & -\lambda & 0 & 0 \end{pmatrix}$

$0 \quad 0 \quad 0 \quad (1-\lambda)(-\lambda)(-\lambda)$

$$(1-\lambda)(-\lambda)(-\lambda)$$
$$= (1-\lambda)(-\lambda)^2$$
$$= \lambda^2(1-\lambda)$$
$$= \lambda(\lambda - \lambda^2)$$
$$\Rightarrow \lambda = 0 \lor \lambda - \lambda^2 = 1$$

* $\operatorname{Ker}(A - \lambda I) = \{x \in \mathbb{R}^3 \mid A - \lambda I_x = 0\}$

$$\operatorname{Ker}\begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & -\lambda \end{pmatrix} = \begin{cases} 1 - \lambda = 0 \\ -\lambda + 1 = 0 \\ -\lambda = 0 \end{cases}$$

For $\lambda = 0$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} x = 0 \\ y \in \mathbb{R} \\ z = 0 \end{cases}$ $\operatorname{Ker}(A - \lambda I) = \{(0, y, 0), y \in \mathbb{R}\}$ $= \langle (0,1,0)^t \rangle$

$(0,1,0)$ is eigenvector for $\lambda = 0$

For $\lambda = 1$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} x \in \mathbb{R} \\ y = z = 0 \end{cases}$

$\operatorname{Ker}(A - \lambda I) = \{(x, 0, 0), x \in \mathbb{R}\}$ $= \langle (1,0,0)^t \rangle$