5 | Direct Methods for Solving Linear Systems

5.1 Solve by means of Gaussian elimination the following systems you should indicate the decomposition $A = LU$

$$\begin{cases} 2x_1 + 2x_2 + 2x_3 + 2x_4 = 12 \\ 4x_1 + 6x_2 + 6x_3 + 6x_4 = 34 \\ 6x_1 + 14x_2 + 16x_3 + 16x_4 = 82 \\ 2x_1 + 2x_2 + 12x_3 + 14x_4 = 42 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 12 \\ 4 & 6 & 6 & 6 & 34 \\ 6 & 14 & 16 & 16 & 82 \\ 2 & 2 & 12 & 14 & 42 \end{array}\right] \begin{array}{l} \\ L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ L_4 \leftarrow L_4 - L_1 \end{array} \rightarrow \left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 12 \\ 0 & 2 & 2 & 2 & 10 \\ 0 & 8 & 10 & 10 & 46 \\ 0 & 0 & 10 & 12 & 30 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow L_3 - 4L_2]{} \left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 12 \\ 0 & 2 & 2 & 2 & 10 \\ 0 & 0 & 2 & 2 & 6 \\ 0 & 0 & 10 & 12 & 30 \end{array}\right] \xrightarrow{L_4 \leftarrow L_4 - 5L_3} \left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 12 \\ 0 & 2 & 2 & 2 & 10 \\ 0 & 0 & 2 & 2 & 6 \\ 0 & 0 & 0 & 2 & 0 \end{array}\right]$$

$$\begin{cases} 2x_1 + 2x_2 + 2x_3 + 2x_4 = 12 \\ 2x_2 + 2x_3 + 2x_4 = 10 \\ 2x_3 + 2x_4 = 6 \\ 2x_4 = 0 \end{cases} \quad \begin{cases} \text{---} \\ x_2 = 10 - 2(3) = 2 \\ x_3 = 3 \\ x_4 = 0 \end{cases} \quad \begin{cases} x_1 = \frac{12 - 2(4) - 2(3)}{2} = 1 \\ x_2 = 2 \\ x_3 = 3 \\ x_4 = 0 \end{cases}$$

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 6 & 6 & 6 \\ 6 & 14 & 16 & 16 \\ 2 & 2 & 12 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 1 & 0 & 5 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$A \qquad L \qquad U$