4.5 Find the accuracy of the following quadrature rule

$$\int_0^1 f(x)\,dx \approx \frac{1}{2} f(0) + \frac{1}{2} f(1) - \frac{1}{12} f''(0)$$

$$I = \int_0^1 f(x)\,dx$$

* $I = 1 - 0 = 1$ $\quad f(x) = x^0 = 1$ $\quad I_{pm}(f) = \frac{1}{2}(1) + \frac{1}{2}(1) - \frac{1}{12}(0) = \boxed{1}$ ✓

* $I = \frac{1^2 - 0^2}{2} = \frac{1}{2}$ $\quad f(x) = x^1 = x$ $\quad I_{pm}(f) = \frac{1}{2}(0) + \frac{1}{2} - \frac{1}{12}(0) = \boxed{\frac{1}{2}}$ ✓

* $I = \frac{1^3 - 0^3}{3} = \frac{1}{3}$ $\quad f(x) = x^2$ $\quad I_{pm}(f) = \frac{1}{2}(0) + \frac{1}{2} - \frac{1}{12}(2) = \frac{1}{2} - \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$ ✓

* $I = \frac{1^4 - 0^4}{4} = \frac{1}{4}$ $\quad f(x) = x^3$ $\quad I_{pm}(f) = \frac{1}{2}(0) + \frac{1}{2} - \frac{1}{12}(0) = \boxed{\frac{1}{2}} \neq \frac{1}{4}$ ✗

∴ The accuracy is 3.