Numerical Algorithms - Second implementation

Simão José Varela Moreno

2 | IMPLEMENTATION. LEAST SQUARES

The aim of the implementation is to show the behaviour of the three kinds of Q factorizations for a matrix A given theoretically.

First, we will factorize Hilbert matrix in several dimensions and we compare the error in the orthogonality of Q by means of $\|Q^t Q - I\|_F$ with respect to the condition number of matrix H_n for different values of n, when Q is computed with the three considered procedures. We complete the following table:

		$\|Q^t Q - I\|_F$		
n	$\mu(H_n)$	Householder	Gram-Schmidt	Modified Gram-Schmidt
2	19,2815	$0{,}0256 \cdot 10^{-14}$	0	0
10 → ⋮	$1{,}6025 \cdot 10^{13}$	$0{,}1138 \cdot 10^{-14}$	6,2869	0.002
15	$2{,}4960 \cdot 10^{17}$	$0{,}1912 \cdot 10^{-14}$	9,7815	1,7090

We will present a figure with the three methods in log scale with the errors. We plot condition number against error.

Second, we will compare the operative cost, with random matrixes factorizations with size $n \times n$. We fill the table

n	Reflectores	Gram-Schmidt	Gram-Schmidt modificado
16	0,0009	0,0001	0,0001
32	0,0007	0,0004	0,0003
64 → ⋮	0,0019	0,0003	0,0004
512	1,1328	0,1350	0,4260

And a second figure in log scale. We plot the dimension of the matrix against the spent CPU-time.

FUNCTIONS ON MATLAB

- rand, create a random matrix.
- ncond, compute the condition number, each norm.
- norm, compute the norm of a vector ('fro', means Fröbenius norm).
- hilb, create the Hilbert matrix.
- loglog, equivalent to plot with log scale both axes.
- subplot, divides the figure window in several graphics.