Question 7: Given 4 cards with 13 possible face values, calculate the probability of 1 pair, 2 pair, 3 of a kind and 4 of a kind. Simulate the results and compare to calculations.

Given: A simulation proposition of 4 cards with 13 possible face values.

Probability calculation approach: Assuming that we are using a standard deck of 52 cards, we select 4 cards at random. The formula for the total possible selection of cards from our pulls is as such: $\binom{52}{4} = \frac{52!}{4!(52-4)!} = \frac{52*51*50*49}{4*3*2*1} = 270,725$. If we're just using the 13 possible values, calculations would differ. For the sake of simplicity, let's use a real deck.

Probability of One Pair: For the probability of one pair (and for probabilities following), we will follow the following pattern: Given 13 values, we have $\binom{13}{1}$ ways to choose the value of the pair. We need one pair given 4 cards, so the binomial will be $\binom{4}{1}$. Pair this with the binomial for the other two pairs from our random pull, and the formula will look like $\binom{13}{1} * \binom{4}{2} * \binom{12}{2}$, which when computed equals 82,368. Divide this with the total selection, and our probability yields **0.3042**.

Probability of Two Pairs: Two pairs is much like the previous, except now we need the binomial for the second pair (4 cards, 2 pairs). The formula is as follows: $\binom{13}{2} * \binom{4}{2} * \binom{4}{2}$ which extends to 78 x 6 x 6 which equals 2,808. Divide by the total and your probability should be **0.0104**.

Probability of Three of a kind: Three of a kind differs in that we need 3 of the same value and 1 different value. The formula is as such: $\binom{13}{1} * \binom{4}{3} * 12 * 4$, which equates to 2,496. Once you divide this by the total, your probability yields **0.0092**.

Probability of Four of a kind: Like the prior, except now we want ALL of the same value. The formula is as such: $\binom{13}{1} * \binom{4}{4}$ which equals 13. Dividing this by the total gets you $\mathbf{0.048}$.

Therefore: Our probabilities are as such: P(One Pair) = 0.3042, P(Two Pair) = 0.0104, P(Three of a kind) = 0.0092, P(Four of a kind) = 0.048.