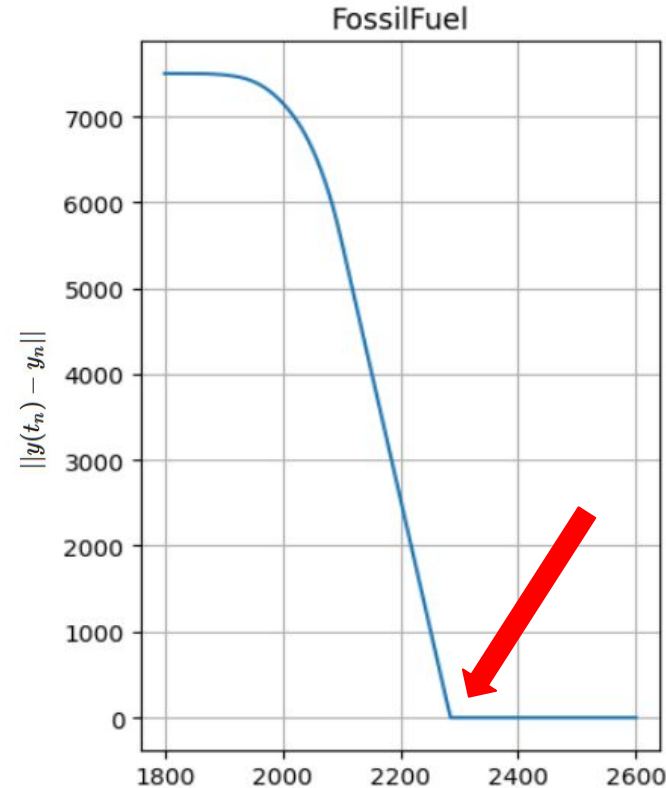


Study of the differential system



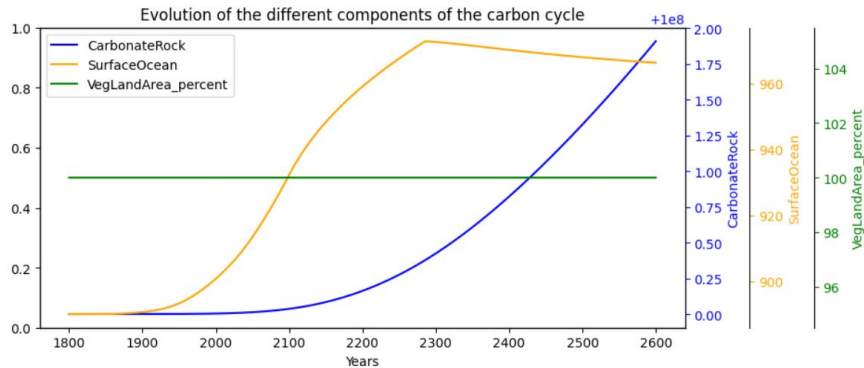
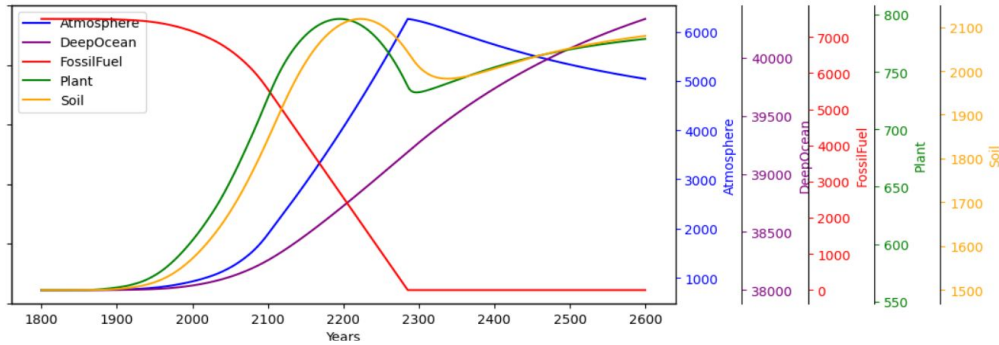
$$\text{Photosynthesis} \propto V L A p \frac{\left(\frac{280A}{A_i} - 40\right)}{\frac{280A}{A_i} + 80} \left(\frac{280A}{A_i} + 280\right) \left(\frac{280A}{A_i} + 280\right)$$

1. The system isn't linear
2. The function f isn't Lipschitz (cf Photosynthesis...)
3. The function isn't C^1 :

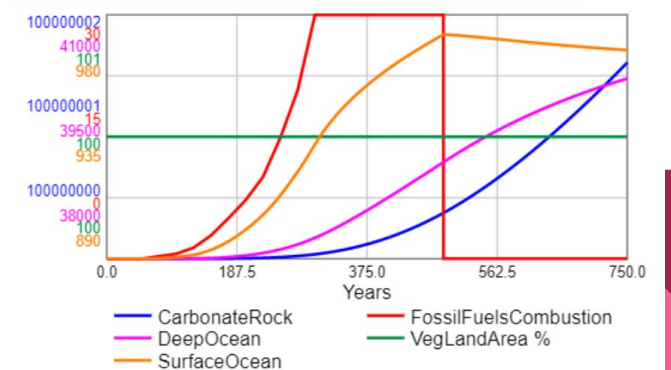
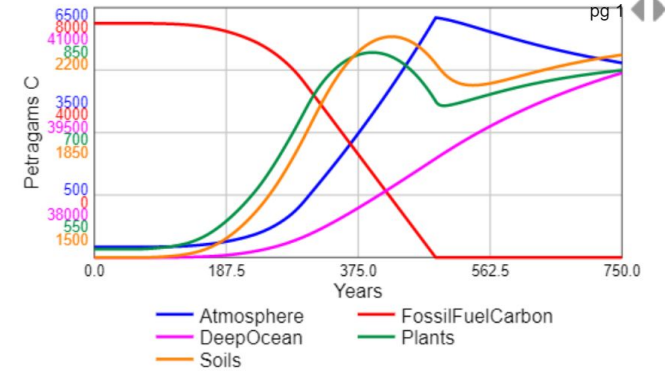


Our Methods

Simulation :



Website :



Runge-Kutta

q=1 (Euler) :

$$\begin{array}{c|c} 0 & 0 \\ \hline & 1 \end{array}$$

q=2 (Midpoint) :

$$\begin{array}{c|cc} 0 & 0 & 0 \\ \alpha & \alpha & 0 \\ \hline & 1 - \beta & \beta \end{array}$$

$\alpha = \frac{1}{2}$ et $\beta = 1$

q=4 (Classic R-K) :

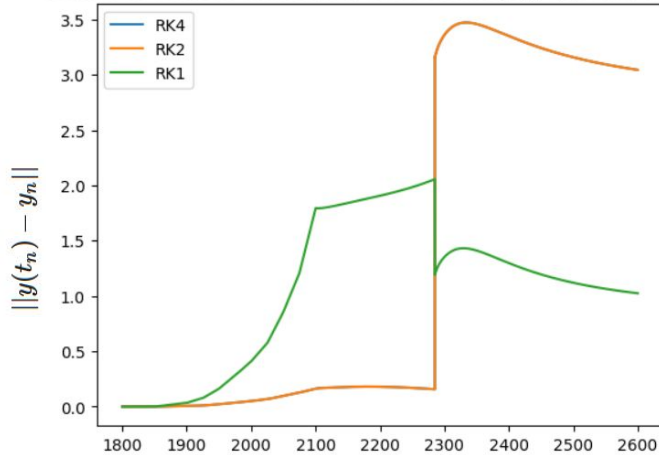
$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$$

$$T = 800 \text{ years}$$

$$n \approx 15 \times T$$

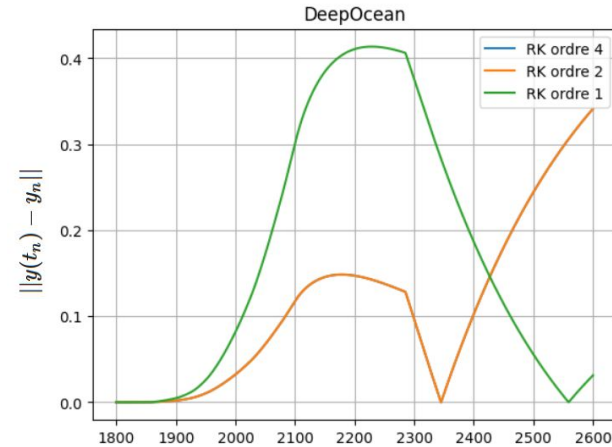
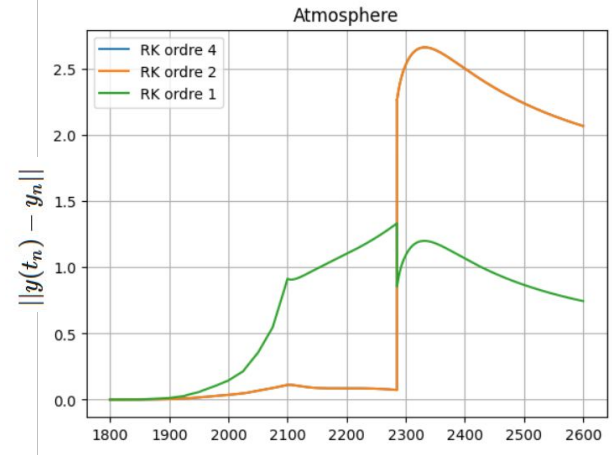
$$t_0 = 1800 \text{ years}$$

Difference between real solution and methods: $|z(t) - \text{approximation}(t)|$



$z(t)$? We take RK 4 with $n \gg 1$.

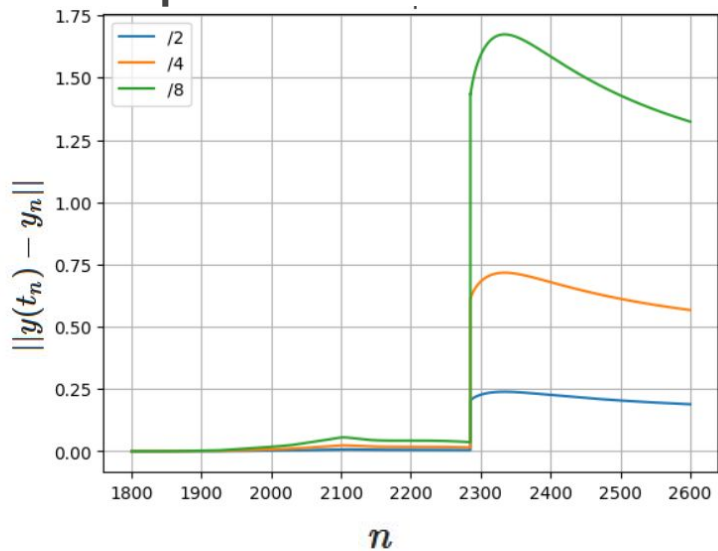
$$n = 150 \times T$$





Error

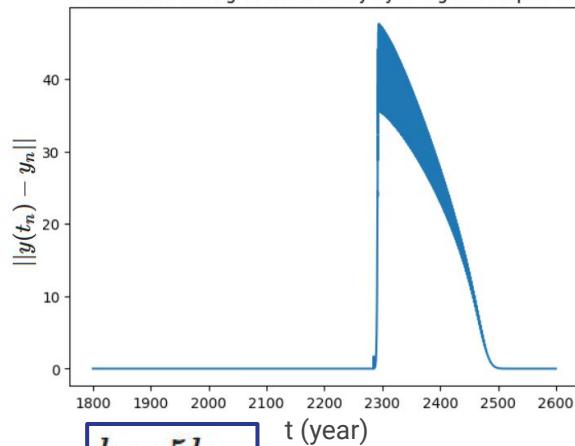
Error fluctuation upon changing the step size



$z(t)$? We take RK 4 with $n \gg 1$.

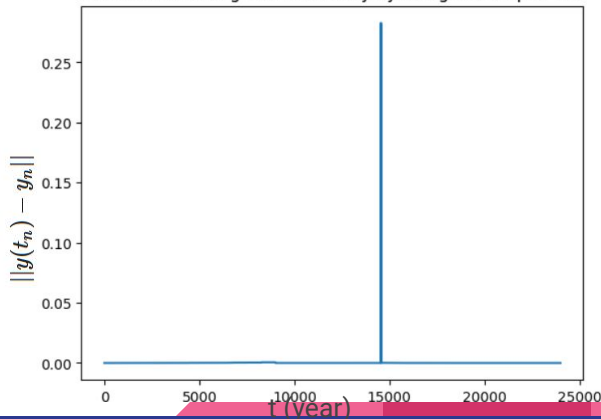
$$h = 25h_{\infty}$$

Norm of the error generated locally by doing one step of RK



$$h = 5h_{\infty}$$

Norm of the error generated locally by doing one step of RK4



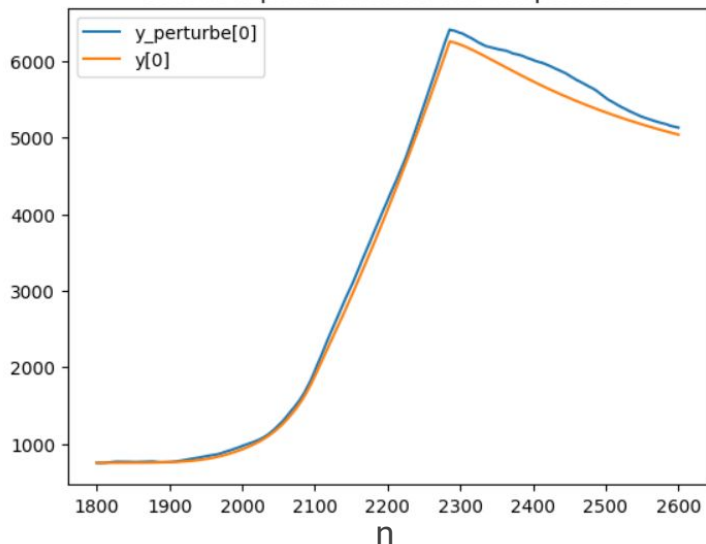
Stability

$$\max_{0 \leq n \leq N} \|\tilde{y}_n - y_n\| \leq e^{\Lambda T} \left(\|\tilde{y}_0 - y_0\| + \sum_{n=0}^N \|e_n\| \right).$$

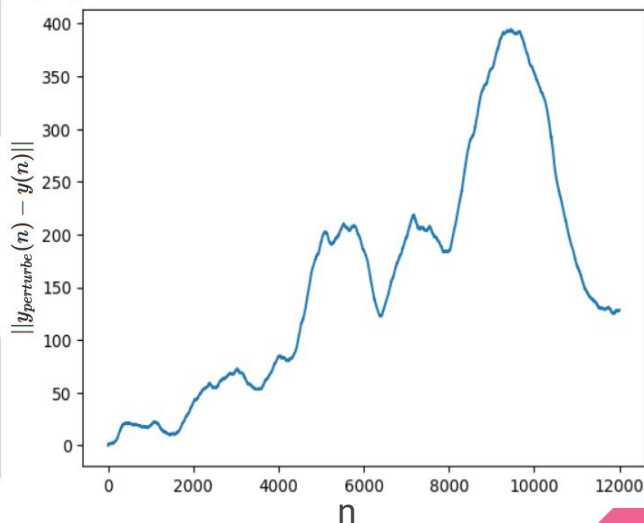
$N_{test} = 25$ and $\epsilon_n \sim N(0, I_8)$

Max : 0.20549020694871434
Mean : 0.11468413407887101
Var : 0.04145937700262542

Effet de la perturbation sur une composantes



$\|y_{perturbe}(n) - y(n)\|$



(max |y_perturbe - y|)/somme epsilon

