Computational Methods for Statistics

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1 Z-Test

1. Suppose μ radar guns are set up along a stretch of road to catch people driving over the speed limit of μ_0 kilometers per hour. Each radar gun is known to have a normal measurement error N(0, σ^2). For a passing car, let $\tilde{X_n}$ be the average of the n readings. The police's default assumptions that car is not speeding.

   (a) Describe the above story in the context of statistical hypothesis testing. Are the most natural null and alternative hypotheses simple or compound?

   The idea is using statistical hypothesis to determine whether a car is speeding or not.

   $$\begin{aligned} \mu &\implies \textit{Speed Limit} \\ \mu_0 &\implies \textit{Speed of the driver} \end{aligned}$$

   Radar guns will have a mean of 0 and a standard derivation of σ^2 which translates to:

   $$\begin{aligned} \mu_0 &\leq \mu \ \textit{Driver not Speeding} \\ \mu_0 &\geq \mu \ \textit{Driver is Speeding} \end{aligned}$$

   (b) The police would like to set a threshold on $\tilde{X_n}$for issuing tickets so that no more than 3% of the tickets are given in error.

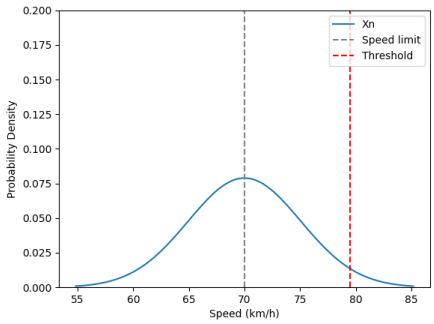
      i. Use the statistical hypothesis testing described in 1a to determine what threshold the police should set when using n = 3 radars. Use $\mu_0 = 70$ and draw σ from N (7, 1) by setting the seed of the random number generator to your student registration ID.
      ID number = 11933108.

      To calculate the Threshold i made a function in python using "norm.ppf" and set the values for it as: $\mu_0 = 70$, n = 3 $\sigma = 8.75834$, $\alpha = 0.003$. This translates to this formula where z is calculated automatically.x

      $$c = \mu + z_{1-\alpha}\frac{\sigma}{\sqrt{n}}$$

      The Threshold to be set from the Police will equal 79.51052

ii. Plot a graph illustrating your reasoning in part (i).

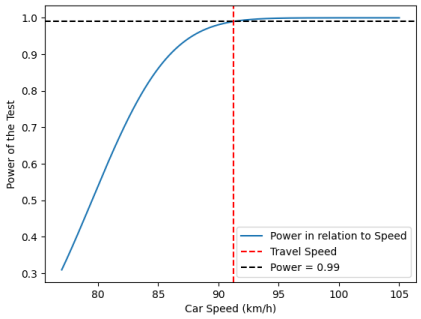


(c) What is the power of this test with the alternative hypothesis that the car is travelling at 10% above the permitted speed, that is at $1.1\mu_0$ kilometers per hour? How many radars are needed to achieve a power of 0.95 with $\alpha = 0.03$?

The Power of this test if a car is travelling at $1.1 * \mu_0$, so at 77 km/h will be 0.28097. While to achieve a power of 0.95, 20 Radars are going to be needed.

(d) Plot the power of the test with n = 3 radars as a function of the car travelling at speeds from the range $[1.1\mu_0 , 1.5\mu_0]$ kilometers per hour. At what travelling speed the test achieves the power of 0.99?

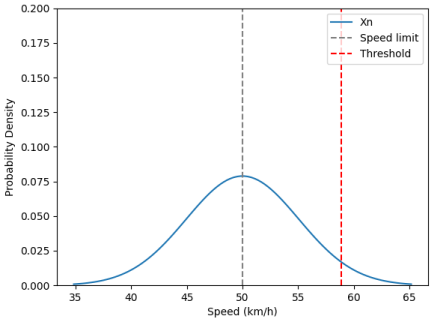
To plot the Graph i calculated the power for every in-between value form 77 to 105 km/h. When the power hits 0.99 i saved the speed which we get this value. In our case this Speed will be 91.266.

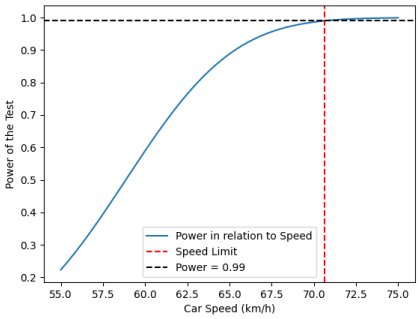


(e) Repeat all the steps for $\mu_1 = 50$ and $\mu_2 = 100$ kilometers per hour with $\alpha = 0.04$ (that is no more than 4% of the tickets are given in error).

i. TEST 2 : $\mu_1 = 50$, $\alpha = 0.04$

a) Threshold for this test will be 58.852 km/h





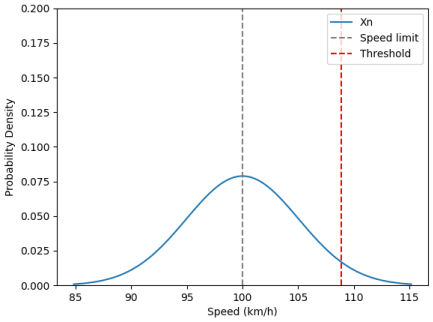
b) Power for this test at 10% above speed limit : 0.2823

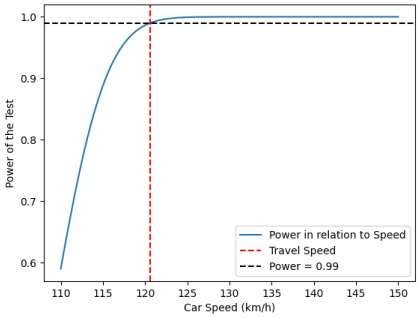
c) To reach a power of at least 0.95 at least 36 Radars would be needed.

d) Graph indicating the power in speed interval from 55 to 75 km/h. Travel speed when the power reaches 0.99 value: 70.62 km/h.

ii. TEST 3 : $\mu_1 = 50$, $\alpha = 0.04$

a) Threshold for this test will be 108.853 km/h





b) Power for this test at 10% above speed limit : 0.5898

c) To reach a power of at least 0.95 at least 9 Radars would be needed.

d) Graph indicating the power in speed interval from 110 to 150 km/h. Travel speed when the power reaches 0.99 value: 120.61 km/h.