Numerical Optimization
Assignment 2

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December 2022

1 Simplex Method

- Solve a constrained linear program using the simplex method :
  1. Formulate the objective function.
  2. Set up the constraints and cast the entire problem as a linear program
  3. Convert the linear program to standard form. Ensure that all requirements are met for the LP to be in standard form..

1. Our objective function can be defined as:

$$f(x) = c^T x = 35x_1 + 40x_2 + 20x_3 + 30x_4$$

2. and constraints :

   a :

$$100x_1 + 120x_2 + 70x_3 + 80x_4 \leq 100000$$

   b :

$$7x_1 + 10x_2 + 8x_3 + 8x_4 \leq 8000$$

   c :

$$x_1 + x_2 + x_3 + x_4 \leq 1000$$

   d all variables are restricted to:

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

3. Casting the problem to a Linear Program :

$$\begin{aligned}
&\max c^T x = 35x_1 + 40x_2 + 20x_3 + 30x_4 \\
&100x_1 + 120x_2 + 70x_3 + 80x_4 \leq 100000 \\
&7x_1 + 10x_2 + 8x_3 + 8x_4 \leq 8000 \\
&x_1 + x_2 + x_3 + x_4 \leq 1000 \\
&x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0
\end{aligned}$$

4. Standard Form :

$$\begin{aligned}
&\min c^T x = -35x_1 - 40x_2 - 20x_3 - 30x_4 + 0s_1 + 0s_2 + 0s_3 \\
&100x_1 + 120x_2 + 70x_3 + 80x_4 + s_1 = 100000 \\
&7x_1 + 10x_2 + 8x_3 + 8x_4 + s_2 = 8000 \\
&x_1 + x_2 + x_3 + x_4 + s_3 = 1000 \\
&x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0 \quad s_1 \geq 0 \quad s_2 \geq 0 \quad s_3 \geq 0
\end{aligned}$$

$$A = \begin{bmatrix} 100 & 120 & 70 & 80 & 1 & 0 & 0 \\ 7 & 10 & 8 & 8 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 100000 \\ 8000 \\ 1000 \end{bmatrix}$$

5. Basic feasible solution :
   Equality constraint will have the form Ax = b:

$$\begin{bmatrix} 100 & 120 & 70 & 80 & 1 & 0 & 0 \\ 7 & 10 & 8 & 8 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 100000 \\ 8000 \\ 1000 \end{bmatrix}$$

We choose B as the basic columns then the solution is given as :

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\{x_1, x_2, x_3, x_4\} = 0$$

$$\{s_1, s_2, s_3\} = B^{-1}b$$

$$B^{-1}b = \begin{bmatrix} 100000 \\ 8000 \\ 1000 \end{bmatrix}$$

$$x = \{x_1, x_2, x_3, x_4, s_1, s_2, s_3\} = \{0, 0, 0, 0, 100000, 8000, 1000\}$$

The solution is a non-negative basic solution thus it is a basic feasible solution.

6. We chose pivoting rule (a)(index at which $\bar{c} < 0$ is the most negative) from the slides for entering index, since it reduces the cost at the fastest rate and it is easy to implement.

7. Basic solution: $x = [0, 0, 0, 0, 100000, 8000, 1000]$. Feasibility check is in the program.

8. Simplex method steps:

It.	Basic indices	Non-basic indices	Basic solution	Cost
0	$[4, 5, 6]$	$[0, 1, 2, 3]$	$[0, 0, 0, 0, 100000, 8000, 1000]$	0
1	$[1, 4, 6]$	$[0, 2, 3, 5]$	$[0, 800, 0, 0, 4000, 0, 200]$	-32000.0
2	$[0, 1, 6]$	$[2, 3, 4, 5]$	$[250, 625, 0, 0, 0, 0, 125]$	-33750.0
Final	$[0, 1, 3]$	$[2, 4, 5, 6]$	$[500, 250, 0, 250, 0, 0, 0]$	-35000.0

The final optimal profit he can achieve is 35000CU.

9. The result for all our slack variables is 0.

10. The LP solver from scipy calculates a best objective cost of -35000 and $x = [1000, 0, 0, 0, 0, 1000, 0]$ as solution. As expected it gets the same profit of 35000CU, but terminates on a point and has one slack variable ($x_7 = 1000$) that is not 0. The implication of the slack variable is that less workforce is required with the solution of